HW4

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Problem 1

Task 1:

$$\begin{aligned}\varphi(r,s) = (\forall r', s'.\ (r*(90*s+20*(1-s))+(1-r)*(30*s+60*(1-s)) \\ \geq r'*(90*s+20*(1-s))+(1-r')*(30*s+60*(1-s))) \\ \wedge(s*(r*10+70*(1-r))+(1-s)*(r*80+(1-r)*40 \\ \geq s'*(r*10+70*(1-r))+(1-s')*(r*80+(1-r)*40))) \\ \wedge(r\geq 0)\wedge(s\geq 0)\wedge(r\leq 1)\wedge(s\leq 1)\wedge(r'\geq 0)\wedge(s'\geq 0)\wedge(r'\leq 1)\wedge(s'\leq 1))\end{aligned} \tag{1}$$

Task 2:

The code outputs sat and gives the equilibrium as r = 0.3 and s = 0.5.

Task 3:

Let r,s be mixed strategies for each player and let F,B denote two outcomes. This is to say that player 1 chooses f with probability r and b with probability 1-r and player 2 chooses f with probability s and b with probability 1-s. let p_{ff} denote the payoff for player 1 when both players choose f and p_{fb} denote the payoff for player 1 when player 1 chooses f and player 2 chooses b. Let p_{bf} denote the payoff for player 1 when player 1 chooses b and player 2 chooses f and p_{bb} denote the payoff for player 1 when both players choose b. Let q_{ff} denote the payoff for player 2 when both players choose f and q_{fb} denote the payoff for player 2 when player 1 chooses f and player 2 chooses b. Let q_{bf} denote the payoff for player 2 when player 1 chooses b and

player 2 chooses f and q_{bb} denote the payoff for player 2 when both players choose b.:

$$\begin{aligned}
\psi \equiv \exists r, s. \forall p_{ff}, p_{fb}, p_{bf}, p_{bb}, q_{ff}, q_{fb}, q_{bf}, q_{bb}, r', s' \\
(r * (p_{ff} * s + p_{fb} * (1-s)) + (1-r) * (p_{bf} * s + p_{bb} * (1-s)) \\
\geq r' * (p_{ff} * s + p_{fb} * (1-s)) + (1-r') * (p_{bf} * s + p_{bb} * (1-s))) \\
\wedge (s * (r * q_{ff} + q_{fb} * (1-r)) + (1-s) * (r * q_{bf} + (1-r) * q_{bb} \\
\geq s' * (r * q_{ff} + q_{fb} * (1-r)) + (1-s') * (r * q_{bf} + (1-r) * q_{bb}))) \\
\wedge (r \geq 0) \wedge (s \geq 0) \wedge (r \leq 1) \wedge (s \leq 1) \wedge (r' \geq 0) \wedge (s' \geq 0) \wedge (r' \leq 1) \wedge (s' \leq 1)
\end{aligned} \tag{2}$$