HW1

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1 Problem 1

We're required to prove $(p \wedge (p \rightarrow q) \rightarrow q)$ using resolution.

We will prove this by showing that the resolution of $(p \wedge (p \rightarrow q)) \wedge (\neg q)$[this is the negation of the expression] is unsatisfiable.

We will now convert the expression to CNF.

- replace $p \rightarrow q$ with $\neg p \vee q$
- the expression is now $p \wedge (\neg p \vee q) \wedge \neg q$

we can now apply resolution to the expression. we resolve the last two terms tp get $p \wedge \neg p$ which is the empty clause thus the negation is unsatisfiable and our original expression is valid

2 Problem 2

- The CNF is $\{\{q, \neg r\}, \{\neg p, r\}, \{\neg q, r, p\}\{p, q, \neg q\}, \{\neg r, q\}\}$
  - resolving the first two terms we get: $\{\{q, \neg p\}, \{\neg q, r, p\}, \{p, q, \neg q\}, \{\neg r, q\}\}$
  - resolving the third term we get: $\{\{q, \neg p\}, \{\neg q, r, p\}, \{\neg r, q\}\}$
  - resolving first and second term we get: $\{\{\neg p, r, p\}, \{\neg r, q\}\}$
  - resolving the first term: $\{\{\neg r, q\}\}$
    This cannot be resolved further.
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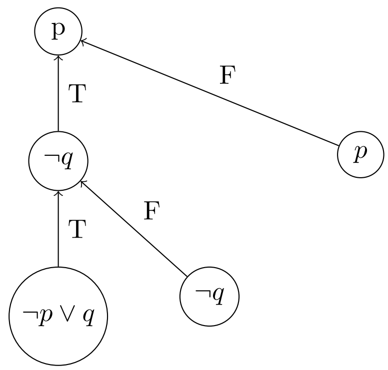
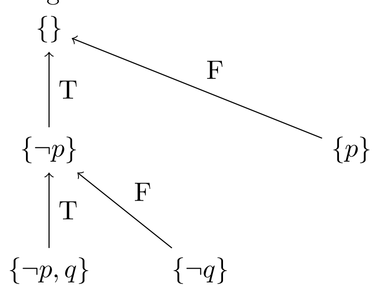
3 Problem 3

the CNF in question 1 was $\{\{p\}, \{\neg p, q\}, \{\neg q\}\}$[note this is the negation of the original expression]

We can represent this as a tree as follows following the order $p > \neg q > \neg p \vee q$:



using this tree we can derive the resolution proof tree as follows:



As always, We start from the bottom children and keep resolving the terms until we reach the root. We can see that the root is empty and thus the expression is unsatisfiable.

thus our original expression is valid.