HW4

Dev Goyal

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Problem 1

Task 1:

$$\begin{aligned}\varphi(r,s) = (\forall r', s'.\ (r*(90*s+20*(1-s))+(1-r)*(30*s+60*(1-s)) \\ \geq r'*(90*s+20*(1-s))+(1-r')*(30*s+60*(1-s))) \\ \wedge(s*(r*10+70*(1-r))+(1-s)*(r*80+(1-r)*40 \\ \geq s'*(r*10+70*(1-r))+(1-s')*(r*80+(1-r)*40))) \\ \wedge(r\geq 0)\wedge(s\geq 0)\wedge(r\leq 1)\wedge(s\leq 1)\wedge(r'\geq 0)\wedge(s'\geq 0)\wedge(r'\leq 1)\wedge(s'\leq 1))\end{aligned} \tag{1}$$

Task 2:

the link for the Z3 code is: https://github.com/devg24/CS474/tree/main/HW4
The code outputs sat and gives the equilibrium as r = 0.3 and s = 0.5.

Task 3:

Let r,s be mixed strategies for each player and let F,B denote two outcomes. This is to say that player 1 chooses f with probability r and b with probability 1-r and player 2 chooses f with probability s and b with probability 1-s. let p_{ff} denote the payoff for player 1 when both players choose f and p_{fb} denote the payoff for player 1 when player 1 chooses f and player 2 chooses b. Let p_{bf} denote the payoff for player 1 when player 1 chooses b and player 2 chooses f and p_{bb} denote the payoff for player 1 when both players choose b. Let q_{ff} denote the payoff for player 2 when both players choose f and q_{fb} denote the payoff for player 2 when player 1 chooses f and player 2 chooses b. Let q_{bf} denote the payoff for player 2 when player 1 chooses b and

player 2 chooses f and q_{bb} denote the payoff for player 2 when both players choose b.:

$$\begin{aligned}\psi \equiv \exists r, s. \forall p_{ff}, p_{fb}, p_{bf}, p_{bb}, q_{ff}, q_{fb}, q_{bf}, q_{bb}, r', s' \\ (r * (p_{ff} * s + p_{fb} * (1-s)) + (1-r) * (p_{bf} * s + p_{bb} * (1-s)) \\ \geq r' * (p_{ff} * s + p_{fb} * (1-s)) + (1-r') * (p_{bf} * s + p_{bb} * (1-s))) \\ \wedge (s * (r * q_{ff} + q_{fb} * (1-r)) + (1-s) * (r * q_{bf} + (1-r) * q_{bb} \\ \geq s' * (r * q_{ff} + q_{fb} * (1-r)) + (1-s') * (r * q_{bf} + (1-r) * q_{bb}))) \\ \wedge (r \geq 0) \wedge (s \geq 0) \wedge (r \leq 1) \wedge (s \leq 1) \wedge (r' \geq 0) \wedge (s' \geq 0) \wedge (r' \leq 1) \wedge (s' \leq 1)\end{aligned} \tag{2}$$

the link for the Z3 code is: https://github.com/devg24/CS474/tree/main/HW4

Problem 2

The set of terms T is:

{a, f(a), g(a), f(g(a)), f(f(a)), f(f(f(a)))}

the set of equalities E is:

{ (f(f(f(a))), f(a)), (f(a),g(a)), (f(g(a)), a)}

the set of inequalities I is:

{(a,f(a))}

We start with the equivalence relation:

{ {a}, {f(a)}, {g(a)}, {f(g(a))}, {f(f(a))}, {f(f(f(a)))} }

since f(a) and g(a) are equal, we can merge the sets:

{ {a}, {f(a), g(a)}, {f(g(a))}, {f(f(a))}, {f(f(f(a)))} }

since f(g(a)) and a are equal, we can merge the sets:

{ {a, f(g(a))}, {f(a), g(a)}, {f(f(a))}, {f(f(f(a)))} }

since f(f(f(a))) and f(a) are equal, we can merge the sets:

{ {a, f(g(a))}, {f(a), g(a), f(f(f(a)))}, {f(f(a))} }

since f(a) and g(a) are in the same equivalence class, we have to merge the equivalence class of f(f(a)) and f(g(a)):

{ {a, f(g(a)), f(f(a))}, {f(a), g(a), f(f(f(a)))} }

since f(f(a)) and g(a) are in T so we have to merge f(f(f(a))) and f(g(a)) which yields the following equivalence relation:

{ {a, f(g(a)), f(f(a)), f(f(f(a))), f(a), g(a)} }

since a and f(a) are in I so this means that the model is unsatisfiable.