CS 601 Spring 2020: Problem Set 3.

Problem 1. (10 points) Show that every infinite TM-recognizable language has an infinite decidable subset.

Problem 2. (10 points) A ***queue automaton*** is like a push-down automaton except that the stack is replaced by a queue. A ***queue*** is a tape allowing symbols to be written only on the left-hand end and read only at the right-hand end. Each write operation (we'll call it a *push*) adds a symbol to the left-hand end of the queue and each read operation (we'll call it a *pull*) reads and removes a symbol at the right-hand end. As with a PDA, the input is placed on a separate read-only input tape, and the head on the input tape can move only from left to right. The input tape contains a cell with a blank symbol following the input, so that the end of the input can be detected. A queue automaton accepts its input by entering a special accept state at any time. Show that a language can be recognized by a deterministic queue automaton if and only if the language is Turing-recognizable.

Problem 3. (10 points) Prove that the language $A\backslash B = \{w : wx \in A, x \in B\}$, where A is a CFL and B is regular is a CFL.

Problem 4. (10 points) We have seen several examples of *undecidable* languages of the form

$$\mathcal{L} = \{\langle M\rangle : M\ is\ a\ TM\ whose\ language\ L(M)\ \ satisfies\ property\ P\}$$

Examples of P include: "$L(M) = \phi$", "$|L(M)| \geq 2$" .

Notice that in each example two conditions are satisfied:

a) If $L(M_1) = L(M_2)$ then $\langle M_1\rangle\ and\ \langle M_2\rangle$ are both in $\mathcal{L}$ or are both not in $\mathcal{L}, and$
b) There are TMs in $\mathcal{L}$ and TMs that are not in $\mathcal{L}$.

Use the recursion theorem to prove a more general result: If P is any property that satisfies conditions a and b, then $\mathcal{L}$ is undecidable. In other words, every interesting property of TMs that depends on the language of the TM is undecidable.

Hint: To begin, let Assume that $\mathcal{L}$ is decidable and let TM $\mathcal{D}$ decide $\mathcal{L}$. Furthermore, since condition b is satisfied by P, let $\langle M\rangle \in \mathcal{L}$ and $\langle N\rangle \notin \mathcal{L}$.

Problem 5. (10 points) Language A is mapping-reducible to language B, written $A \leq_m B$, if there is a computable function $f: \Sigma^* \to \Sigma^*$ such that $\forall w \in \Sigma^*$: $w \in A\ if\ and\ only\ if\ f(w) \in B$.

a) (5 points) Show that $\leq_m$ is a transitive relation.
b) (5 points) Show that if A is TM-recognizable and $A \leq_m \bar{A}$, then A is decidable.
(Hint: first argue that $A \leq_m \bar{A}$ implies that $\bar{A}\ \leq_m A$.)