CS 601 Spring 2020: Problem Set 2.

Problem 1. (10 points) For each of the following languages, either give a CFG generating it, or a high-level description of a PDA that recognizes it:

a) The complement of $\{a^n b^n \mid n \geq 0\}$

b) $\{x_1 \# x_2 \# \cdots x_k \mid k \geq 1, each\ x_i \in \{a,b\}^*\ and\ for\ some\ i,j\ x_i = x_j^R\}$

Problem 2. (10 points) Let C be a context-free language, and R be a regular language. Show that the language $C \cap R$ is context free. Start with a PDA $(Q, \Sigma, \Gamma, \delta, q_{start}, F)$ for C and a DFA $(Q', \Sigma', \delta', q'_{start}, F')$ for R, then describe a PDA for $C \cap R$. Your description may be informal and high-level (i.e. you don't need to define detailed transitions), but must be precise.

Problem 3. (10 points) Use the result of Problem 2 to prove that the language $L = \{w \in \{a,b,c\}^*:\ w \text{ has equal numbers of } a's, b's, \text{and } c's\}$ is not context free.
You may assume that the language $\{a^n b^n c^n : n \geq 0\}$ is not context free.
(Hint: design a regular expression R such that $L \cap R$ is not context free.)

Problem 4. (10 points) **A** 2-stack PDA is a PDA with two stacks; its input tape is 1-way read only. In one step push and pop operations can be performed on both stacks.

a) Describe, at a high-level, a 2-stack PDA to recognize the language $\{a^n b^n c^n : n \geq 0\}$.

b) Can a 2-stack PDA recognize the language $\{a^n b^n c^n d^n : n \geq 0\}$.

c) Can a 2-stack PDA recognize every Turing-recognizable language? Explain your reasoning.

Problem 5. (10 points) Define the languages

$$L_{add} = \{a^i b^{i+j} c^j : i,j \geq 0\}$$
$$L_{mult} = \{a^i b^{ij} c^j : i,j \geq 0\}$$

For each language, what is the smallest class it belongs to (regular, context-free, or TM-decidable)? Justify your answer – for example, if you claim context-free, then give a CFG/PDA for it and also prove that it is not regular.