CS 601 Spring 2020: Problem Set 1.

Problem 1. Give regular expressions to generate each of the following languages.

a) $\{w \in \{a,b\}^* : \ w \text{ does not end in } ba\}$
b) $\{w \in \{0,1\}^* : w = \alpha \circ \beta \text{ and } \alpha \text{ has an even number of } 1\text{'s and } \beta \text{ has an even number of } 0\text{'s}\}$

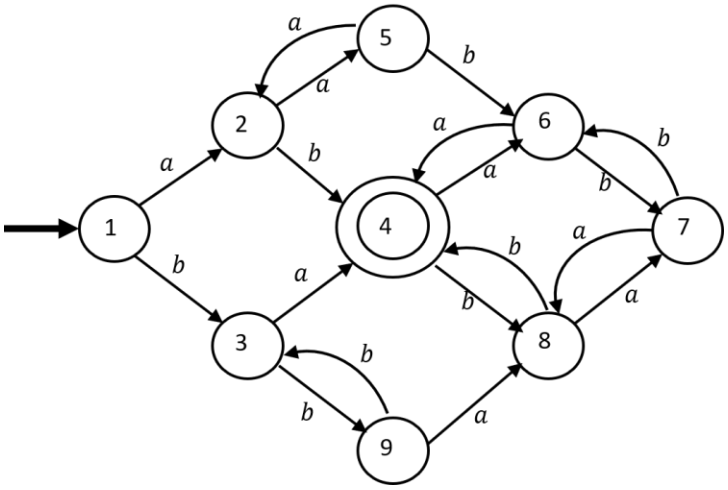
Problem 2. Let D_k denote the set of binary strings that represent numbers divisible by k. For example, input strings $0, 00, 000, 10, 010, 0010, 0100010$ are all divisible by 2 (the least significant bit is the rightmost, or last symbol in the sequence), and therefore are all in the language D_2.

1. Construct a deterministic FSA to recognize the language D_3.
2. Prove that D_k is regular, for every $k \geq 1$.

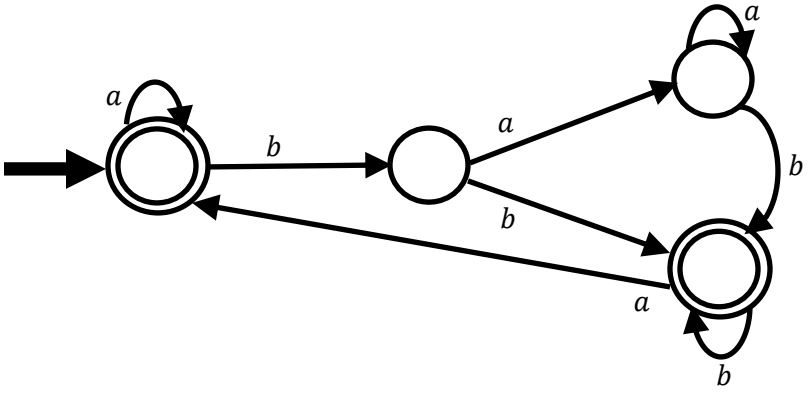
Problem 3. For any string σ, over alphabet Σ, we define the string $SHIFT(\sigma)$ as follows: if $\sigma = aw, a \in \Sigma, w \in \Sigma^*$ then $SHIFT(\sigma) = wa$. For example, $SHIFT(0111) = 1110$, $and \ \ SHIFT(10110) = 01101$.

Prove that if L is regular, then so is $SHIFT(L) = \{SHIFT(\sigma) : \sigma \in L\}$.

Problem 4. Apply the DFA minimization algorithm to the DFA shown below. Show the matrix of distinguishable pairs of states after each iteration of the loop.



Problem 5. Give a regular expression to describe the language of the following DFA. Use the DFA to regular expression conversion theorem and show all steps of your conversion.



Problem 6. Use the pumping lemma to prove that the language $\left\{0^{2^i} : i \geq 0\right\}$ is not regular. The language consists of strings of 0's whose lengths are powers of 2. Be sure to include all details of your proof.