14/8/2022

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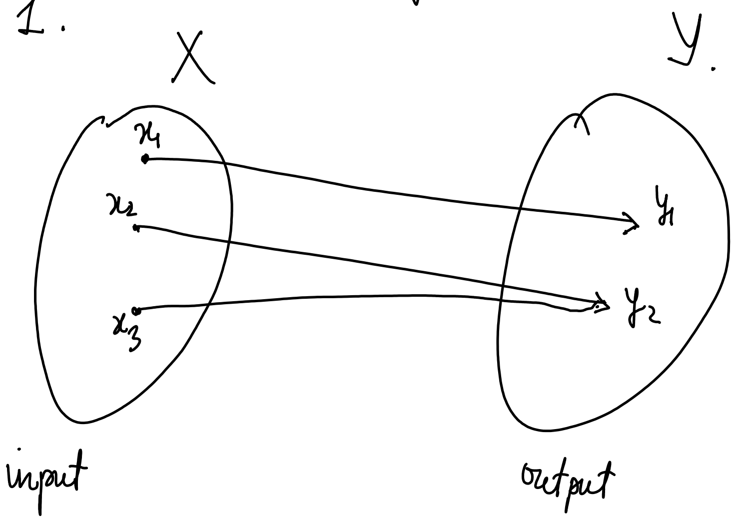
$y = 2.$

Chương 1.



$$A^2 - B^2 = (A-B)(A+B).$$

Ví dụ 2. Tính các giới hạn sau

a) $\lim\limits_{x\to 2}\frac{x-2}{x^2-4}$ b) $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-1}{x}$ c) $\lim\limits_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt[5]{1+x}}{x}$

d) $\lim\limits_{x\to 4}\frac{2-\sqrt{x}}{3-\sqrt{2x+1}}$ e) $\lim\limits_{x\to +\infty}\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x+1}}$ f) $\lim\limits_{x\to +\infty}\left[\sqrt{x+\sqrt{x}}-\sqrt{x}\right]$

g) $\lim\limits_{x\to 1}\left(\frac{3}{1-\sqrt{x}}-\frac{2}{1-\sqrt[3]{x}}\right)$

a) $$\lim_{x\to 2}\frac{x-2}{x^2-4} = \lim_{x\to 2}\frac{x-2}{(x-2)(x+2)} = \lim_{x\to 2}\frac{1}{x+2} = \frac{1}{4}$$

b) $$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x} = \lim_{x\to 0}\frac{(\sqrt{1+x}-1)(\sqrt{1+x}+1)}{x(\sqrt{1+x}+1)} = \lim_{x\to 0}\frac{\cancel{1}+\cancel{x}-\cancel{1}}{\cancel{x}(\sqrt{1+x}+1)}$$

$$= \lim_{x\to 0}\frac{1}{\sqrt{1+x}+1} = \frac{1}{2}$$

$$A^3 - B^3 = (A-B)(A^2+AB+B^2)$$

c) $$\lim_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt[5]{1+x}}{x} = \lim_{x\to 0}\frac{\sqrt[3]{1+x}-1+1-\sqrt[5]{1+x}}{x}$$

$$= \underbrace{\lim_{x\to 0}\frac{\sqrt[3]{1+x}-1}{x}}_{A} + \underbrace{\lim_{x\to 0}\frac{1-\sqrt[5]{1+x}}{x}}_{B}$$

$y = \sqrt[3]{1+x}$

$y^3 = 1+x.$

$x = y^3 - 1.$

Ta có $A = \lim\limits_{x\to 0} \dfrac{(\sqrt[3]{1+x}-1)(\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}+1)}{x\cdot\left(\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}+1\right)} = \lim\limits_{x\to 0} \dfrac{\cancel{1}+\cancel{x}-\cancel{1}}{\cancel{x}\left(\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}+1\right)}$

$$= \lim_{x\to 0} \frac{1}{\sqrt[3]{(1+x)^2}+\sqrt[3]{1+x}+1} = \frac{1}{3}.$$

$$\left(A^5 - B^5 = (A-B)\underline{(A^4 + A^3B + A^2B^2 + AB^3 + B^4)}\right)$$

t.tự $B = \dfrac{-1}{5}$

$$\Rightarrow \lim_{x\to 0} \ldots = A + B = \frac{1}{3} + \frac{-1}{5} = \frac{2}{15}$$

d) $\lim\limits_{x\to 4} \dfrac{2-\sqrt{x}}{3-\sqrt{2x+1}}$ e) $\lim\limits_{x\to +\infty} \dfrac{\sqrt{x+\sqrt{x}}}{\sqrt{x+1}}$ f) $\lim\limits_{x\to +\infty} \left[\sqrt{x+\sqrt{x}} - \sqrt{x}\right]$

d) $\lim\limits_{x\to 4} \dfrac{2-\sqrt{x}}{3-\sqrt{2x+1}} = \lim\limits_{x\to 4} \dfrac{(2-\sqrt{x})(2+\sqrt{x})(3+\sqrt{2x+1})}{(3-\sqrt{2x+1})(2+\sqrt{x})(3+\sqrt{2x+1})}$

$$= \lim_{x\to 4} \frac{(4-x)(3+\sqrt{2x+1})}{(9-2x-1)(2+\sqrt{x})}$$

$$= \lim_{x\to 4} \frac{(4-x)(3+\sqrt{2x+1})}{2(4-x)(2+\sqrt{x})} = \lim_{x\to 4} \frac{3+\sqrt{2x+1}}{2(2+\sqrt{x})} = \frac{6}{8} = \frac{3}{4}$$

e) $\lim\limits_{x\to +\infty} \dfrac{\sqrt{x+\sqrt{x}}}{\sqrt{x+1}} = \lim\limits_{x\to +\infty} \dfrac{\sqrt{1+\frac{1}{\sqrt{x}}}}{\sqrt{1+\frac{1}{x}}} = \dfrac{1}{1} = 1.$

f) $\lim\limits_{x\to +\infty} \left(\sqrt{x+\sqrt{x}} - \sqrt{x}\right) = \lim\limits_{x\to +\infty} \dfrac{(\sqrt{x+\sqrt{x}}-\sqrt{x})(\sqrt{x+\sqrt{x}}+\sqrt{x})}{\sqrt{x+\sqrt{x}}+\sqrt{x}}$

$$= \lim_{x\to +\infty} \frac{\cancel{x}+\sqrt{x}-\cancel{x}}{\sqrt{x+\sqrt{x}}+\sqrt{x}}$$

$$= \lim_{x\to +\infty} \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}+1} = \frac{1}{2}$$

g) $\lim\limits_{x\to 1} \left(\dfrac{3}{1-\sqrt{x}} - \dfrac{2}{1-\sqrt[3]{x}}\right)$ Đặt $\sqrt[6]{x} = y \to x = y^6 \to \begin{cases} \sqrt{x} = y^3 \\ \sqrt[3]{x} = y^2 \end{cases}$

$= \lim\limits_{y\to 1} \left(\dfrac{3}{1-y^3} - \dfrac{2}{1-y^2}\right)$ khi $x \to 1$ thì $y \to 1$.

$$= \lim_{y\to 1}\left(\frac{3}{(1-y)(1+y+y^2)} - \frac{2}{(1-y)(1+y)}\right)$$

$$= \lim_{y\to 1}\frac{3(1+y)-2(1+y+y^2)}{(1-y)(1+y)(1+y+y^2)}$$

$$= \lim_{y\to 1}\frac{3+3y-2-2y-2y^2}{(1-y)(1+y)(1+y+y^2)}$$

$$= \lim_{y\to 1}\frac{-2y^2+y+1}{(1-y)(1+y)(1+y+y^2)} = \lim_{y\to 1}\frac{(y-1)(-2y-1)}{(1-y)(1+y)(1+y+y^2)}$$

$$= \lim_{y\to 1}\frac{2y+1}{(1+y)(1+y+y^2)} = \frac{3}{6} = \frac{1}{2}.$$

2.

Chú ý

*) Một số giới hạn thường gặp:

1/. $\lim_{x\to 0}\frac{sinx}{x} = 1$

2/. $\lim_{x\to +\infty}\left(1+\frac{1}{x}\right)^x = \lim_{x\to -\infty}\left(1+\frac{1}{x}\right)^x = e$

3/. $\lim_{u\to 0}(1+u)^{\frac{1}{u}} = e$

Ví dụ 3. Tính các giới hạn sau (*áp dụng* $lim_{x\to 0}\frac{sinx}{x} = 1$)

$\sin^2 x = \frac{1-\cos 2x}{2}$

a) $\lim_{x\to 0}\frac{tanx}{x}$ b) $\lim_{x\to 0}\frac{1-cosx}{x^2}$ c) $\lim_{x\to 0}\frac{sinax}{sinbx}$

a) $$\lim_{x\to 0}\frac{\tan x}{x} = \lim_{x\to 0}\frac{\sin x}{\cos x}\cdot\frac{1}{x} = \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{1}{\cos x}$$

$$= \lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{1}{\cos x} = 1.1 = 1.$$

b) $$\lim_{x\to 0}\frac{1-\cos x}{x^2} = \lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2}$$

$$= \lim_{x\to 0}\left(\frac{\sin\frac{x}{x}}{\frac{x}{2}}\right)^2\cdot\frac{1}{2}$$

$\sin^2\frac{x}{2} = \frac{1-\cos x}{2}$

$\frac{\sin^2\frac{x}{2}}{\frac{x^2}{2^2}} = \sin^2\frac{x}{2}\cdot\frac{2^2}{x^2}$

$$= \frac{1}{2}.$$

c) $$\lim_{x\to 0}\frac{\sin ax}{\sin bx} = \lim_{x\to 0}\frac{\sin ax}{ax}\cdot\left(\frac{bx}{\sin bx}\right)\cdot\frac{a}{b}$$

$$= \underbrace{\lim_{x\to 0}\frac{\sin ax}{ax}}_{1}\cdot\frac{1}{\underbrace{\lim_{x\to 0}\frac{\sin bx}{bx}}_{1}}\cdot\frac{a}{b} = \frac{a}{b}.$$

Ví dụ 4. Tính các giới hạn sau (*áp dụng 2/. và 3/.*)

a) $\lim\limits_{x\to+\infty}\left(\frac{x^2-1}{x^2+1}\right)^{x^2}$ b) $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}$

c) $\lim\limits_{x\to 0}(1-2x)^{\frac{1}{x}}$ d) $\lim\limits_{t\to+\infty}\left(\frac{t}{t+1}\right)^{t}$

$$\frac{x^2-1}{x^2+1} = \frac{x^2+1-2}{x^2+1} = 1-\frac{2}{x^2+1}$$

a) $$\lim_{x\to+\infty}\left(1+\frac{-2}{x^2+1}\right)^{x^2} = \lim_{x\to+\infty}\left[\left(1+\frac{1}{\frac{x^2+1}{-2}}\right)^{\cancel{\frac{x^2+1}{-2}}}\right]^{\frac{x^2\cdot\cancel{(-2)}}{\cancel{x^2+1}}}$$

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$$= e^{\lim\limits_{x\to+\infty}\frac{-2x^2}{x^2+1}}$$

$$= e^{\lim\limits_{x\to+\infty}\frac{-2}{1+\frac{1}{x^2}}} = e^{-2} = \frac{1}{e^2}.$$

$$(a^m)^n = a^{m.n}$$

$$\underline{\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x = e.}$$