

# Formal Methods

Program Verification  
and  
Model Checking

# Program Verification and Model Checking

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# Formal Methods

Program Verification  
and  
Model Checking

# Intent

Introduce the basic ideas of formal methods.

Show the direct application.

Draw your attention and interest.

Hopefully inspire some of you.

# Content

Appeal to your intuition.

Leverage multiple powerful techniques.

- programs

- propositional logic and predicate logic

- temporal logic

- models

# Interaction

informal.

Lively and spontaneous.

Ask questions in the flow.

Have interesting conversations.

“There is no such thing as a dumb question!”

Let's Start!

# The Specification

“Develop a procedure in the C programming language to find the larger of the given two integers.”



## The Specification

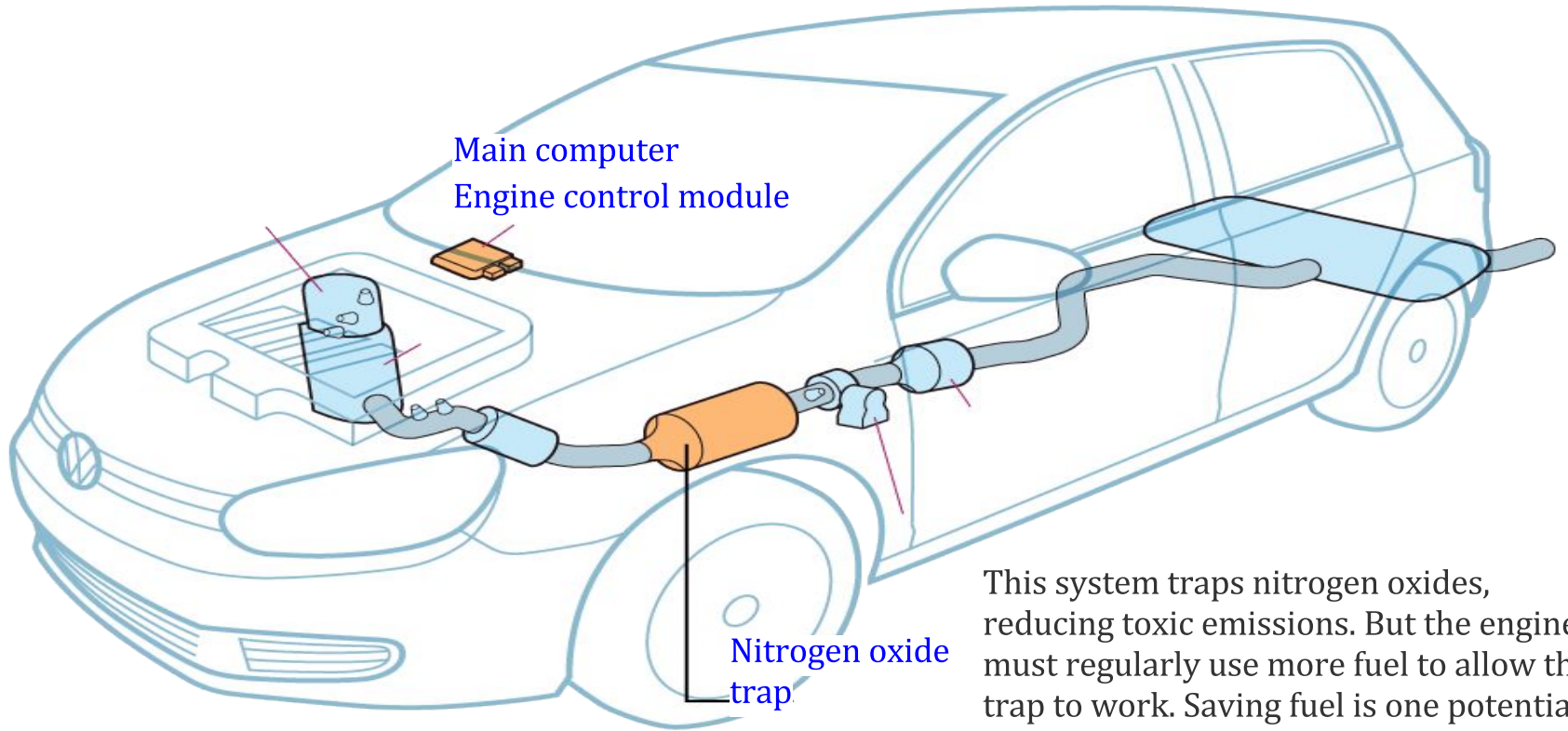
“Develop a procedure in the C programming language to implement the linear searching algorithm. Assume the inputs consist of an array of integers and a key to search.”

Develop and Test the C Program!

# VW Emission Fraud

[https://en.wikipedia.org/wiki/Volkswagen\\_emissions\\_scandal](https://en.wikipedia.org/wiki/Volkswagen_emissions_scandal)

<http://www.vw-emissions-fraud.com/>



This system traps nitrogen oxides, reducing toxic emissions. But the engine must regularly use more fuel to allow the trap to work. Saving fuel is one potential reason that Volkswagen's software could have been altered to make cars pollute more...



Program Synthesis.  
Guarded Commands.  
Nondeterminacy.

“... The first moral of the story is that program testing can be used very effectively to show the presence of bugs but never to show their absence.”

# Program Testing

Testing aims to show *incorrectness*, rather than *correctness*.

When a test fails, it succeeds in revealing an error.

When a considerable number of tests fail to detect bugs, our confidence in the program increases, even if the correctness cannot be established.

# Program Testing

Testing aims to show incorrectness, rather than correctness.

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**Code coverage** requires that certain parts of the code be exercised.

# Program Testing versus Verification

Testing aims to show incorrectness, rather than correctness.

When a test fails, it succeeds in revealing an error.

When a considerable number of tests fail to detect bugs our confidence in the program increases, even if the correctness cannot be established.

This is in contrast with formal methods where a program is proved to be correct without executing it!



# Specifications and Programs

Specification is a rigorous definition of the relation between inputs and outputs.

Mathematically precise definition of *what* is required.

Program is an implementation of a solution that meets the specification.

There will be more than one correct implementation for the same specification.

# Program Verification

Verification rigorously establishes if a program is correct with respect to the specification.

- Employs methods of mathematics and logic.

- Provides counter-examples.

Verification may be used to rigorously establish interesting properties of a program.

# Challenges of Programming Languages\*

Ambiguous and unspecified semantics.

Overly complex and complicated semantics.

Hard to reason. Hard to use confidently.

Offer no or very little assistance for verification methods.

Most are inelegant artefacts of the 60s, 70s, 80s and 90s!

\* those regularly taught in typical academic institutes, and used in most software shops.

# Demands of the 21<sup>st</sup> Century

Dependable systems.

Safety-critical systems.

Systems that are correct by construction.

Verified Software!

# Do We Know Enough?

We do have rich experience building large, complex systems.

The theory and tools are out there.

We need more.

We need to build more!

There is a great need to educate programmers!



# Assigning Meaning to Programs

“The semantic definition of a particular set of command types, then, is a rule for constructing, for any command  $c$  a *verification condition*  $V_c(P; Q)$  on the antecedents and consequents of  $c$ .”



# An Axiomatic Basis for Computer Programming

"This involves elucidation of sets of axioms and rules of inference which can be used in proofs of the properties of programming languages."

# First Order Logic

Predicates:  $\text{odd}(x)$ ,  $<$ ,  $>$ ,  $=$ ,

Functions:  $\text{abs}$ ,  $\text{min}$ , ...

Propositional logic symbols:  $\wedge$ ,  $\vee$ ,  $\neg$ ,  $\Rightarrow$

Quantifiers:  $\forall$ ,  $\exists$

Inference Rules!



# More Examples!

My approach at the School of Information Sciences, Manipal.

C programming language

Data Structures

[More Dafny Samples.](#)

# From Programs to Processes



# Temporal Logic

"In mathematics, logic is static... When one designs a dynamic computer system that has to react to ever changing conditions, ... one cannot design the system based on a static view. It is necessary to characterize and describe dynamic behaviors that connect entities, events, and reactions at different time points. Temporal Logic deals therefore with a dynamic view of the world that evolves over time."

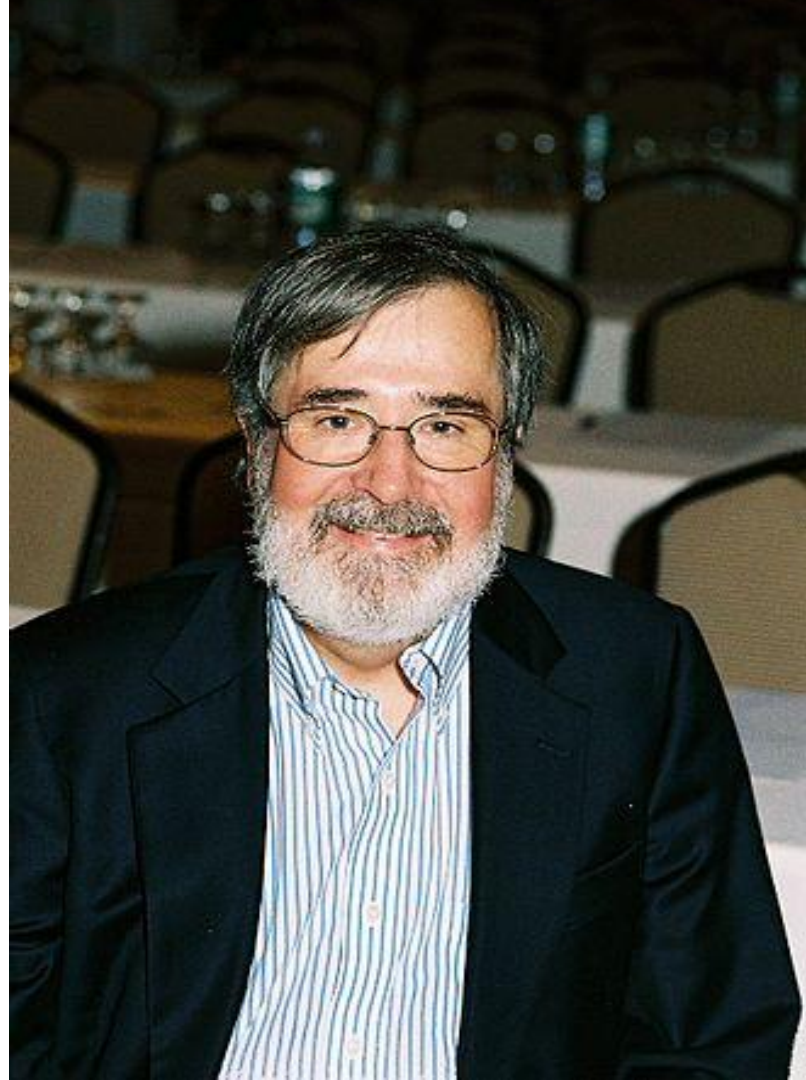


# Specifying Systems

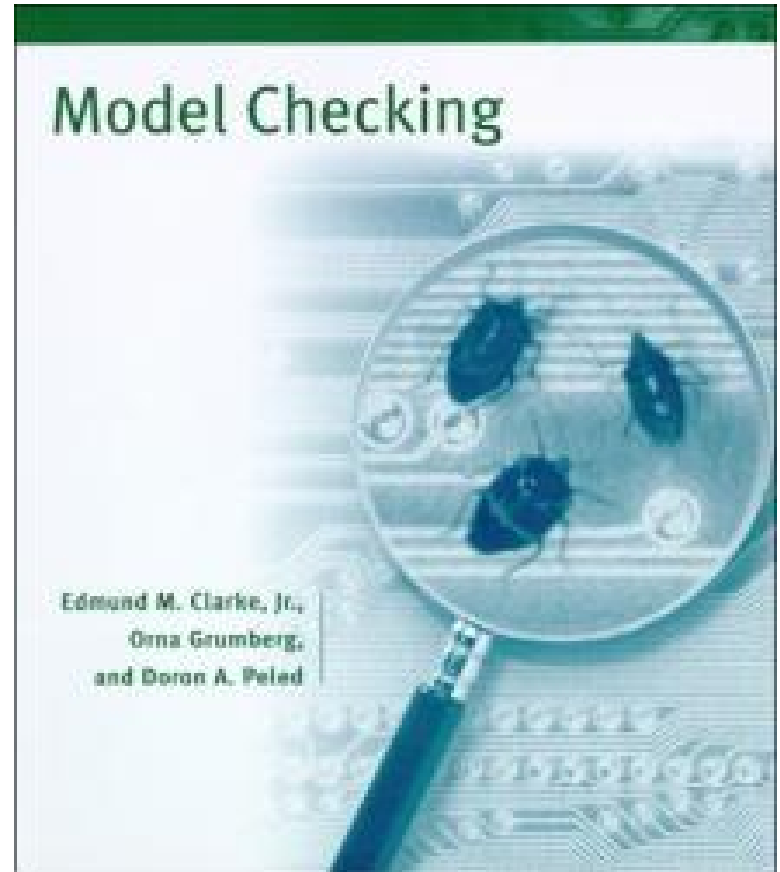
The TLA+ Language and Tools  
for Hardware and Software Engineers



Leslie Lamport



Edmund M. Clarke



Joseph Sifakis



Allen Emerson



# Reactive Programs

A reactive program's role is to maintain an ongoing interaction with its environment rather than to compute a final value and terminate.

Concurrency is a fundamental element in reactive programs.

By definition a reactive program runs concurrently with its environment.

# Examples of Reactive Programs

Web Servers and Web Browsers.

Operating Systems.

Embedded real-time systems.

Computer Games.



# Properties of Reactive Programs

Safety properties

“nothing bad ever happens”

Liveness properties

“something good will eventually happen”

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## Liveness properties

progress, fairness, no livelocks, ...

These properties are defined only over infinite execution sequences of reactive programs.

# Verifying Reactive Programs

Model checking is a successful technology.

We will use a logic based model checking tool.

- Temporal logic for specification

- Variants of state machine (automaton) for modeling

# Modeling Concurrent Systems

A Kripke structure  $M$  over a set of atomic propositions  $AP$  is a four tuple  $M = (S, I, R, L)$  where

- $S$  is a finite set of states.

- $I \subseteq S$  is a set of initial states.

- $R \subseteq S \times S$  is a transition relation that must be total.

- $L: S \rightarrow 2^{AP}$  is a function that labels each state with a set of atomic propositions true in that state.

# Temporal Logic

A formalism for describing sequences of transitions between states in a reactive system.

A formula may specify that eventually some designated state is reached or that an error state is never reached.

Time is not explicitly mentioned.

# Temporal Logic

## Path quantifiers

A - “for all computation paths”

E - “for some computation path”

## Temporal operators

G “always” or “globally”

F “eventually” or “in the future”

X “next time”

U “until”

# Temporal Logic

EF(Start && !Ready)

“it is possible to get to a state where Start holds but Ready does not hold”

AG(Req  $\Rightarrow$  AF Ack)

“If a request occurs, then it *will be* eventually acknowledged”



# Temporal Logic

AG(EF Restart)

“From any state it is possible to get to the Restart state”

AG(AF DeviceEnabled)

“DeviceEnabled holds infinitely often on every computation path”

# Model Checking

Given a Kripke structure  $M = (S, R, L)$  that represents a finite-state concurrent system and a temporal logic formula  $f$  expressing some desired specification, find the set of all states in  $S$  that satisfy  $f$ :

$$\{s \in S \mid M, s \models f\}$$

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The system satisfies the specification if all initial states are in this set.

Thank You!