Problem Chosen	2024	Team Control Number
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Mastering the Match: Unveiling the Secrets of Momentum

Abstract

In the 2023 Wimbledon Men's Singles final, Spanish rising star Carlos Alcaraz defeated Grand Slam champion Novak Djokovic, sparking in-depth discussions about the momentum of the match and its impact on predicting the winner. We constructed a comprehensive tennis match momentum evaluation and fluctuation prediction model. It analyzed player conditions through multi-level indicators, assessed players' performance, predicted match momentum, and evaluated its impact on match outcomes.

For Problem 1, a **Kappa Test** confirmed a moderate correlation between service and scoring, and a **multi-dimensional indicator evaluation system** was established, including psychological factors, technical and tactical elements, physical fitness, serving, and other factors (weather, venue, personal factors, etc.). Due to the unique nature of data on mentality and skills, we conducted **sliding window analysis** and **global attention analysis** on qualitative variables related to mentality, and numerically processed symbolic variables related to skills. Subsequently, we employed a **Random Forest model** to determine the weights of the aforementioned multi-dimensional indicators and their internal variables for both players. This facilitated the construction of a predictive model for player performance, successfully forecasting and visually representing the performance of two players in **1701 five-set matches**.

For Problem 2, we first predicted the momentum data for each scoring point using a Random Forest, achieving **an accuracy of 71.36%**. Then, we applied **Permutation Tests** and statistical significance analysis, demonstrating that momentum significantly influences match outcomes. The analysis substantiated this influence at a 0.05 level of statistical significance, confirming that **momentum indeed plays a pivotal role** in the game.

For Problem 3, we used four models (**lightGBM, SVM, ARIMA, and XGBoost**) to predict the momentum fluctuations of the players. We selected the model, **lightGBM**, with **the smallest RMSE and MAE**, indicating the highest accuracy, to determine the predicted momentum values. Furthermore, we identified **the most crucial factor** related to match fluctuations: **control over the serve**. Based on this, some suggestions are provided to Carlos Alcaraz for a new match against a different player.

For Problem 4, by testing the model in **the first 1301 matches** of the 2023 Wimbledon Men's Singles and **the 2503 (quarterfinals) matches** of the 2023 Wimbledon Women's Singles, we evaluated the accuracy and versatility of the model. The study found that the model exhibits **good versatility** for Men's and Women's Singles matches under Wimbledon rules. However, it still needs further optimization for other specific contexts.

Finally, drawing from the model evaluation and **sensitivity analysis** results, we compiled a memorandum. This document offers coaches and athletes scientifically grounded, practical insights into the nuances of tennis match dynamics.

Keywords: Kappa Test, Random Forest, Permutation Test, LightGBM

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1 Introduction

1.1 Problem Background

In the 2023 Wimbledon Gentlemen's final, Carlos Alcaraz demonstrated exceptional athletic performance, particularly in agility and strategic play, defeating the seasoned Novak Djokovic. The scene during the match is as shown in Figure 1.

Figure 1: 2023 Wimbledon Gentlemen's final Scene

In this intense and competitive encounter, both competitors exhibited significant fluctuations in performance across different sets. Djokovic excelled in the first set, but subsequently, Alcaraz, with his remarkable counterattack abilities, successfully reversed the course of the match. The outcome of the match, disrupting Djokovic's winning streak, not only made a statement in the tennis world but also served as a prime example of the complex interplay of momentum[1] in competitive sports.

1.2 Restatement of the Problem

In light of the background information and constraints outlined in the problem statement, our focus shifts to addressing the following problems.

- Develop predictive models to capture tennis scoring sequences and match progression, identify the player with a competitive advantage at critical moments, and quantify their performance. The model must incorporate serving advantage and provide data-driven visualization.
- Employ the developed model to evaluate whether fluctuations in player performance and winning streaks during a match are random. By contrasting model predictions with actual match data, the study will explore the effectiveness and relevance of the 'momentum' concept in tennis matches.
- Develop predictive metrics and models to identify and forecast transition points in match progression, specifically moments of momentum shift. Utilize historical match data to determine key factors affecting match fluctuations and devise strategies for players to face different opponents.
- Validate the predictive accuracy of the model by quantifying its performance across multiple competitions; when the model underperforms, identify and incorporate missing key factors for improvement; and assess the model's generalizability and adaptability across different match environments and other sports.

- Reflecting on the outcomes achieved, draft a concise memorandum of 1-2 pages for coaches, offering advisory notes on the significance of 'momentum' and preparatory tactics to ready play.

1.3 Our work

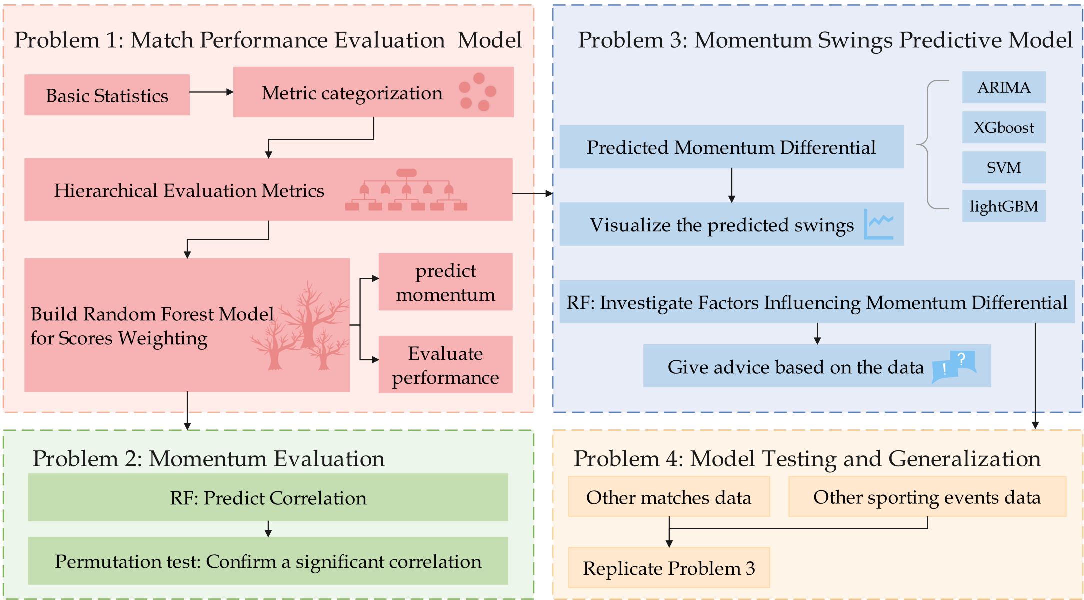


Figure 2: Attention Span for Psychological Aspects

2 Assumptions and Notations

2.1 Assumptions

To simplify our problems, we make the following basic assumptions, each of which is adequately justified.

- **Assumption 1:** The match process exhibits a certain level of randomness and unpredictability.
  Justification: Due to the influence of various complex factors, tennis matches possess inherent randomness and unpredictability. These factors encompass players' technical abilities, physical fitness levels, psychological states, as well as external environmental conditions such as court characteristics, weather changes, and audience emotions. To simulate this uncertainty, random disturbance terms are introduced into the model to more authentically reflect the occasional nature of match processes.

- **Assumption 2:** Every event in the match has a bidirectional impact.
  Justification: Each event in a match (such as break points or unforced errors) affects both sides of the competition. This implies that the occurrence of any event not only impacts the player triggering it but also has a direct or indirect effect on the opponent.

- **Assumption 3:** The preprocessed data is reliable.
  Justification: This assumption is crucial for ensuring the accuracy and reliability of the model solutions. We rely on the quality and completeness of the data to construct and validate our mathematical model, enabling effective analysis and prediction.

2.2 Notations

Table 1: Notions and Symbol Description

Symbol	Description
F_{psy}	Psychological Factors
F_{phy}	Physical Fitness
F_{tac}	Technical and Tactical Elements
F_{ser}	Serving
F_{oth}	Other Factors
θ_i	Weight of indicators
M_{pi}	The momentum of the p_ith player, i=1,2
M_{pre_i}	Predicted momentum values for player p_i, i=1,2
$RMSE$	Root Mean Square Error
MAE	Mean Absolute Error
$MAPE$	Mean Absolute Percentage Error
$MSLE$	Root Mean Squared Logarithmic Error
R^2	R-square Error

3 Task 1:Match Performance Evaluation Model

3.1 The Impact of Server on Point Victor in Tennis

In conducting the correlation study of serve, serve_no, and point_victor (first serve/second serve), the Simple Kappa is used for correlation coefficients instead of Pearson or Spearman. This is because Simple Kappa can be applied to pairwise purely categorical unordered data.

Table 2: Kappa Consistency Test

	Paired Items	Kappa Value	Standard Error	Z-value
server pairing server no	-0.018^{**}	0.012^{*}	-1.531^{**}	0.126^{*}
server pairing point victor	0.406^{***}	0.012^{*}	32.766^{***}	< 0.001
serve no pairing point victor	-0.017^{***}	0.012^{*}	-1.478^{***}	0.139^{***}

Note: *, **, *** represent the significance levels of 1%, 5%, and 10% respectively

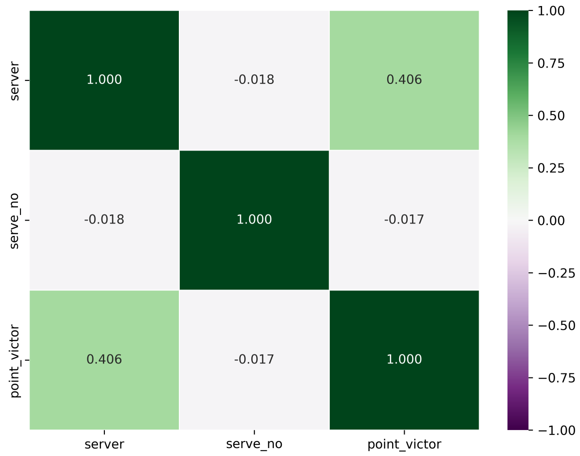


Figure 3: Correlation Matrix Related to Serving in Tennis

In analyzing the relationships between variables, the Kappa coefficient is 0.406, revealing a moderate level of consistency between them. This value is significantly higher, indicating that the relationship between `server` and `point_victor` is statistically meaningful.

In summary, the relationship between `server` and `point_victor` is both significant and of moderate strength. This may have important implications for predicting match outcomes and devising tactical strategies.

3.2 Factors Affecting Tennis Players' Momentum

To accurately assess a tennis player's state at a specific time and somewhat reflect their momentum, we established a multi-level indicator evaluation system. This system includes four aspects: mental score, skill score, physical fitness score, and serving score, along with an additional disturbance item.

Since blending all factors together could result in opaque scoring, we decided to rely on specific data and existing research[2] to individually score these four aspects. However, the disturbance item in the data may have issues of missing or vague information, such as the subjective impact of weather on each player, leading to a strong subjective nature of the disturbance item.

To address this problem, we randomly generated the disturbance item to increase objectivity and tolerance in evaluating player performance under certain external influences. Next, we categorized the given variable data into these four aspects below:

- **Psychological Factors:**
  Mental state enables players to accurately identify opponents' strategies and weaknesses, maintain confidence in their own technical and tactical applications, and harbor a strong desire to win. The impact of psychological factors in matches is reflected in various technical indicators, such as break-point success rates and serve and return point-winning percentages.
- **Technical and Tactical Elements:**
  Contemporary tennis players exhibit a more comprehensive and refined skill set, showcasing heightened levels of power, spin, and precision in their shots. This progress not only enhances the force of the ball but also increases the spin effect and unpredictability

of the landing spot, thereby creating more advantages during matches.

- **Physical Fitness:**
  With the extended match durations, high-intensity exertion, intermittent movement, and frequent striking actions inherent in tennis events, there is a significant need for athletes to maintain a high level of physical fitness. The technical indicators related to physical fitness include the total distance covered during a match, the number of shots per rally, and the speed of serve.
- **Serving:**
  In actual competitions, such as the 2023 Wimbledon Men's Singles final under current study, an analysis of the performance of the two players reveals their outstanding service quality and scoring rates. This can be largely attributed to their formidable serving abilities. Not only do they deliver serves with impressive speed, but they also demonstrate high precision, enabling them to create scoring opportunities at critical moments and control the pace of the match.
- **Other Factors:**
  Tennis match outcomes hinge on factors like environmental conditions and court surfaces, impacting ball behavior.

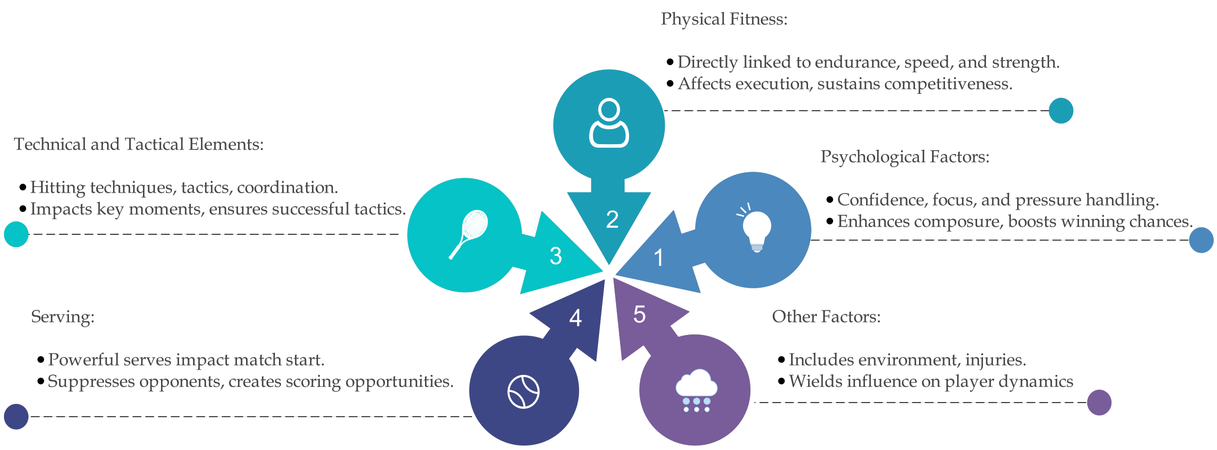


Figure 4: Five Factors Influencing Momentum

3.3 Weighting Relevant Factors with Random Forest

Firstly, we standardized and normalized all data to eliminate dimensional influences, ensuring comparability across various metrics.

In the preprocessing stage of our machine learning pipeline, we employed techniques such as data normalization and standardization to enhance the performance of our model. Data normalization involves scaling the features of the dataset to a specific range, typically between 0 and 1. This is achieved using the following formula:

$$x_{\text{normalized}} = \frac{x - \min(X)}{\max(X) - \min(X)} \tag{1}$$

where x represents the original data point, $\min(X)$ and $\max(X)$ are the minimum and maximum values in the dataset X, respectively.

Additionally, we applied data standardization to ensure that the features have a mean of 0 and a standard deviation of 1. The formula for data standardization is given by:

$$x_{\text{standardized}} = \frac{x - \bar{X}}{\sigma_X} \tag{2}$$

Here, $\bar{X}$ denotes the mean of the dataset X, and σ_X represents the standard deviation.

These preprocessing techniques enable our model to converge faster during training and improve its overall stability and performance on the given dataset.

Subsequently, we employed a Random Forest model[3] to determine the weight of each variable within each category. Analyzing the impact of each category on the outcome of a game, we obtained the importance of each variable within each aspect. In random forests, it is common to gauge the contribution of features to model performance using Gini importance. The Gini importance in random forests is computed by measuring the extent to which each feature decreases the Gini index at the nodes of the trees.

The contribution of each feature to the Gini index is calculated and then averaged weighted to obtain the Gini importance for the entire model. The Gini importance of a specific feature , denoted as X_j, $Gini\_Importance(X_j)$ can be calculated using the following formula:

$$Gini\_Importance(X_j) = \frac{\sum_{\text{each tree}} Gini(node) \times \text{sample size}}{\text{total sample size}} \tag{3}$$

When analyzing the existing data, we found that data related to skill factors, including variables such as serve depth and direction, are represented by symbols. The actual significance and impact on match results are challenging to accurately characterize based on publicly available data for a professional athlete's level. Additionally, we believe that psychological factors are closely linked to a player's recent performance in several matches, and changes in mentality cannot be independently isolated in a short period. Given the specificity of these two types of data, we quantified and objectively assessed both psychological and skill factors.

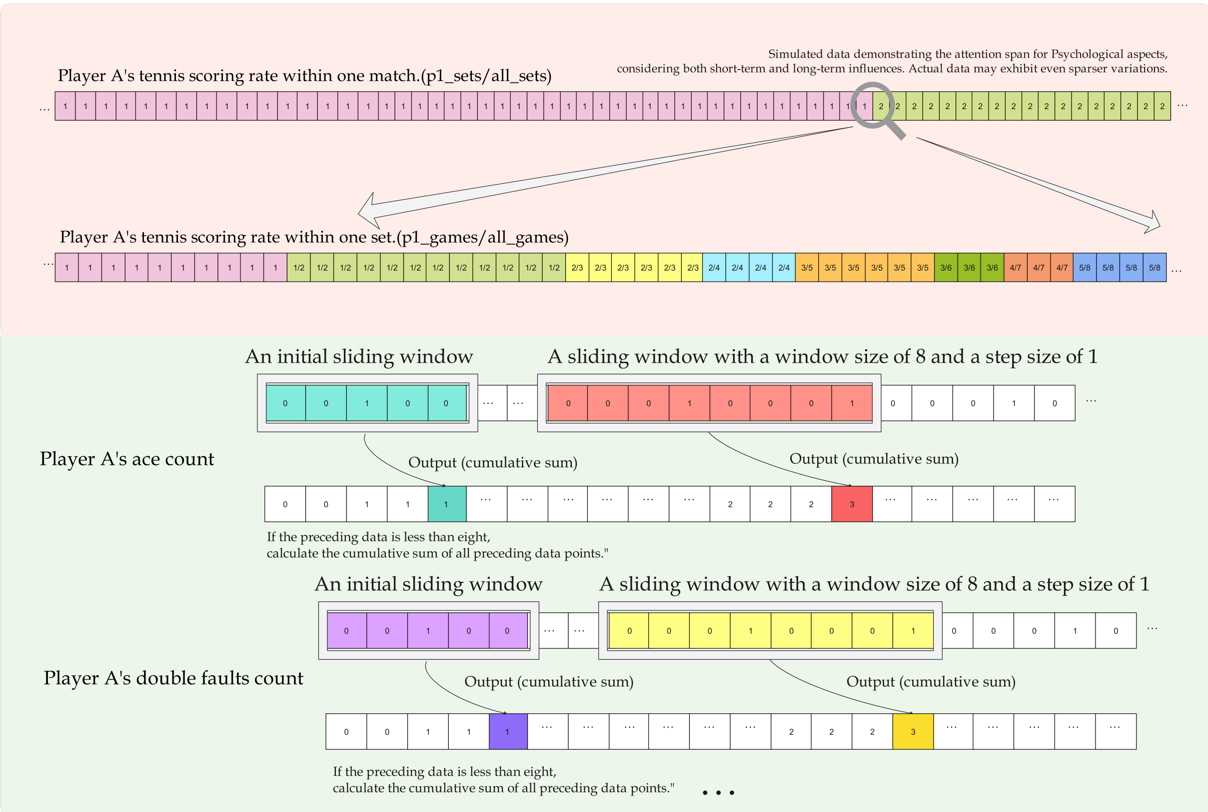
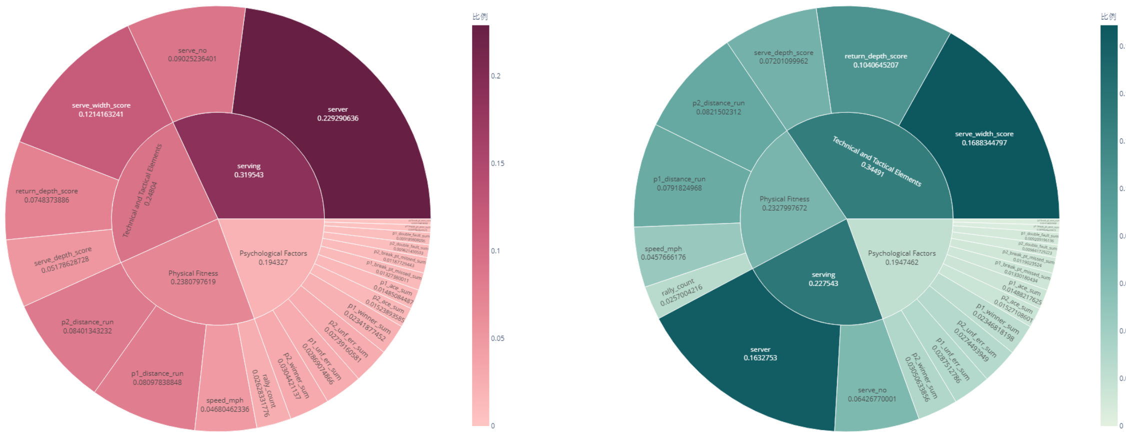


Figure 5: Attention Span for Psychological Aspects

To analyze factors affecting athletes' psychological changes, researchers employed a sliding window analysis method. This approach uses an 8-game window to closely examine how recent events impact mindset, suggesting that current performance has a more significant influence than past isolated events. Psychological theories support that an athlete's confidence is boosted by immediate scoring rates and affected in the long term by individual set percentages, while short-term series scores have a quick but temporary effect on performance.

To assess the impact of technical factors on individual momentum and match outcomes, we employed AI to quantify the technical difficulty and tactical effectiveness of different serving directions, serving depths, and return depths. The final results are presented in terms of scores, replacing the original symbolic variables.

The overall momentum, integrating psychological, physical, skill, and serving factors, is calculated through a linear model, incorporating an error term (err) to reflect interference from other unobserved factors.



(a) Carlos Alcaraz (b) Nicolas Jarry

Figure 6: Weighting Factors for Tennis Players' Momentum

The calculation of psychological factors involves a linear combination, where variables such as ace, double fault, unforced error, and break point situations are assigned positive or negative weights based on their positive or negative impacts on the player.

The weight calculation formulas for four different types of factors have commonalities. Taking the weight calculation formula for psychological factors as an example:

$$M = \sum_{i=1}^{n} w_i \cdot p_i + \text{err} \tag{4}$$

where M represents the overall measure, p_i denotes the individual factors (e.g., p_i for ace, p_2 for double fault, etc.), w_i corresponds to the weights assigned to each factor based on its positive or negative impact, err is the error term introduced to account for random perturbations.

The weights (w_i) are determined by the nature of the factor; positive factors receive positive weights, and negative factors receive negative weights. The error term (err) captures additional random disturbances in the overall measure.

In summary, we can illustrate the weight distribution of the two tennis players for four key factors in the chart below. These factors encompass technical skills, physical condition, mental qualities, and strategic tactics. Each factor is further divided into several measurable indicators. The chart visually presents these weights, facilitating easy comparison and analysis.

3.4 Establishment of the Prediction Formula

To quantify the impact of these dimensions, we initially computed values for the four scoring dimensions at each time point based on predetermined weightings and existing performance indicators. This involved combining the raw data with corresponding weight coefficients, taking into account specific metrics within each dimension.

Subsequently, a random forest model was employed. Utilizing the newly calculated composite scores as input features, we trained the model to predict the players' momentum and ultimate performance outcomes. The training process of the random forest model not only allowed us to identify the relative importance of each scoring dimension to the final results but also provided precise estimates for these weights.

$$M = \sum_{i=1}^{4} w_i \left(\sum_{i=1}^{n} \theta_i \cdot p_i \right) + \text{err} \tag{5}$$

This approach not only enhanced the accuracy of predictions but also deepened our understanding of the determinants of athletic performance, offering valuable insights for athlete training and game strategies.

3.5 Visualization Display

Figure 7 illustrates the dynamics of point-by-point performance between two players, Player1 and Player2, during a match (match__id=2023wimbledon1701) at the 2023 Wimbledon Tennis Championships. The horizontal axis denotes the match time, while the vertical axis represents the momentum values, with positive values indicating offensive momentum and negative values indicating defensive momentum. The varying color shades from light to dark depict the progression of the match from the first to the fifth set.

A macroscopic analysis reveals three distinct phases in the performance of Player1 and Player2 throughout the match:

Initial Phase: In the early stages of the match (first and second sets), the curves of the two players intertwine closely, indicating a comparable momentum between them without any discernible advantage. This phase likely reflects mutual probing and adaptation of strategies.

Intermediate Phase: Moving into the third and fourth sets, the curves start to diverge, demonstrating Player1 gradually gaining ascendancy. Particularly in the third set, Player1's momentum rises, while Player2's momentum declines, suggesting that Player1 begins to dictate the pace of the match during this phase.

Late Phase: In the final set, Player1's momentum significantly strengthens, while Player2's momentum further weakens. This suggests that Player1 exhibits superior psychological and physical resilience during crucial moments of the match, ultimately establishing a decisive advantage.

Taken together, the visualization highlights Player1's ability to progressively establish and sustain momentum, particularly in the mid to late stages of the match. Player1's success may be attributed to various factors, including more effective match strategies, stronger mental resilience, better physical condition, or errors made by the opponent. The cumulative effect of these factors contributes to the gradual tilting of the match momentum in favor of Player1.

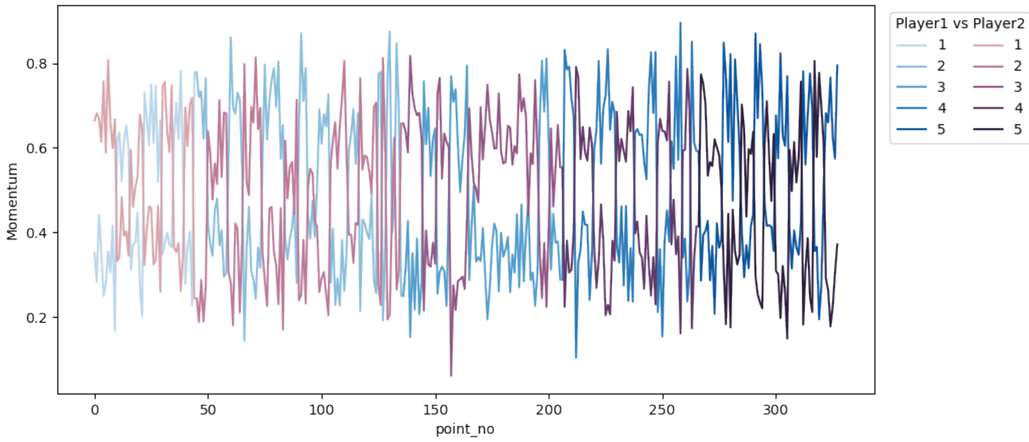


Figure 7: Analysis of Momentum Changes Throughout the Entire Game

To gain a deeper understanding of the impact of players' momentum on the outcome of the match, we will conduct a brief analysis of short-term momentum changes in each set of the game. We will meticulously review the score variations, the contest for crucial points, and the strategic adjustments made by both athletes in each set. This will allow us to unveil key moments and turning points in the match. Through such analysis, we aim to provide a clearer picture of how the confrontation between players evolves with time and set progression, and how they respond to pressure, capitalize on opportunities, and ultimately determine the outcome of the game.

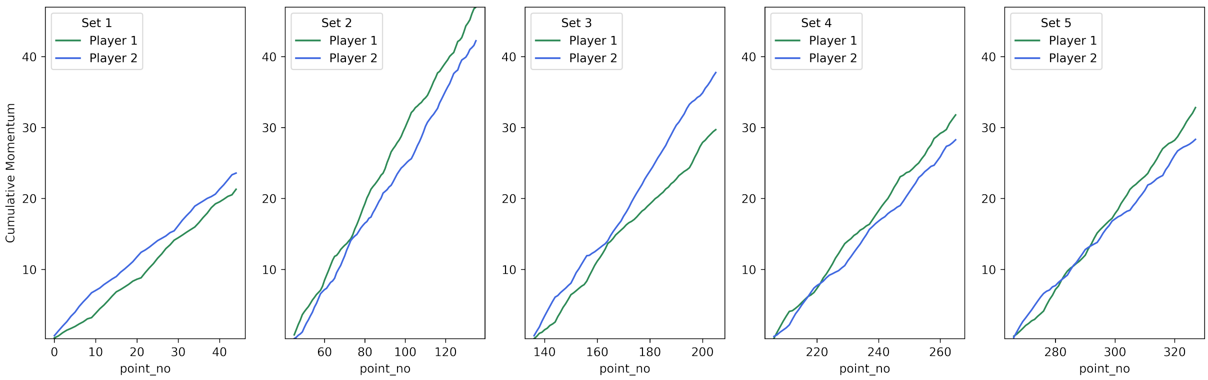


Figure 8: Attention Span for Psychological Aspects

Figure 8 illustrates the cumulative momentum of two players in each set of a match at the Wimbledon Tennis Championships which match_id=2023_wimbledon_1701. The horizontal axis represents the match time, while the vertical axis represents the cumulative momentum values. The blue line represents Player1, and the green line represents Player2. The upward trajectory of the lines indicates that, as the match progresses, the momentum of each player gradually accumulates.

Set1:

In the early stages of the match, Player2 demonstrated a strong offensive momentum, which is clearly reflected in the distribution of scoring points. The data indicates relatively sparse scoring points with a small point_no value, suggesting Player2's robust offensive performance, rapidly widening the score gap, and limiting the number of scored points. This offensive momentum ultimately manifested in the score, establishing a significant advantage for Player2 in the first set.

Set2:

The second set exhibited data with high density and volatility, reflecting the tense atmosphere of the match. Momentum played a pivotal role in this phase, with frequent alternations in energy levels between the two players, highlighting the intense competition. Towards the end of the match, the pronounced fluctuations in momentum underscored the complexity of the game, and the marginal advantage indicated that Player1 successfully capitalized on momentum in crucial moments to secure victory.

Set3:

Overall, there was a noticeable inconsistency between momentum and match scores, potentially related to the specific circumstances of the game. During this phase, Player2's performance seemed suboptimal, likely influenced by personal factors such as a lack of concentration, which exhibited a lower correlation with the metrics in our model. While Player1 was unable to control the situation in the third set, the abnormal performance of Player2 allowed Player1 to secure victory by exploiting the opponent's weaknesses.

Set4:

From the fourth game onwards, Player1 took control of the situation, and this change in momentum is directly reflected in the energy data. In the initial phase, Player1's momentum exceeded 0.8, indicating a high probability of Player1 actively dominating the match. In the latter part of the game, the frequent changes in momentum suggested a reversal in the match dynamics, and the relatively high level of momentum maintained by Player1 possibly reflected a stable mindset throughout the match.

Set5:

At the beginning of the match, Player2 took the lead, but subsequent changes in momentum were clearly evident in the gradual reduction of energy levels. Player1 gradually gained the advantage and sustained momentum throughout the match. Ultimately, Player1 secured victory by successfully leveraging the advantage provided by momentum. The continuous demonstration of momentum proved crucial throughout the match, especially during pivotal moments.

4 Task 2: Momentum Evaluation

4.1 Implementation of the Permutation Test

Permutation test is a statistical method based on the random permutation of observed values. It estimates the probability distribution of a certain statistic under the null hypothesis by randomly permuting the sample numbers[4]. This allows for hypothesis testing.

In our study, we employed the permutation test to assess the impact of 'momentum' on match outcomes. By randomly shuffling momentum data and recalculating the model predictions, we constructed a reference distribution. Through multiple iterations of this permutation process, we ensured the stability and reliability of the statistic, enabling a more accurate investigation into whether momentum has a significant effect on match results.

The following code is the implementation of a permutation test.

Algorithm 1: Permutation Test

Data: Real results of the test set y_test
Predicted results of the model on the test set y_pred
Result: List of accuray values generated during iterations acc_list
Probability p

$n < -1000$
for $i < -1$ *to* n **do**
    y_perm <- random_permutation(y_test)
    acc_list[i] <- accuracy_score(y_perm, y_pred)
    $p < -\frac{1}{n}\sum_{i=0}^{n} I(acc\_list >= actual\_acc)$
end
return p

4.2 Permutation Test: Assessing the Impact of Momentum

In order to assess the role of "momentum" more precisely in the competition, we employed a Random Forest machine learning model for prediction. Specifically, we used the momentum of both competing parties as input features for the model to predict the match outcomes (win or loss). To examine the actual impact of momentum, we conducted permutation testing: wherein the momentum data was randomly shuffled and re-input into the model for prediction. This process was repeated multiple times to construct the predictive distribution under the null hypothesis.

To verify whether 'momentum' plays a role in tennis matches, we established the following hypotheses:

- **Null Hypothesis (H0):** Momentum does not play a role in matches, and players' performances are random.
- **Alternative Hypothesis (H1):** Momentum plays a role in matches, and players' performances are influenced by the previous moments of the game.

Below is a description of the formula for calculating the p-value in permutation testing:

Let T be the observed test statistic, and T_i be the test statistic calculated in the i-th permutation, where n is the total number of possible permutations. Then the formula for calculating the p-value is:

$$p = \frac{1}{n}\sum_{i=1}^{n} I(T_i \geq T) \tag{6}$$

I is an indicator function, which takes the value of 1 when $T_i \geq T$, and 0 otherwise. This formula is used to determine whether to reject the null hypothesis based on the probability value known as the p-value.

We calculated the prediction accuracy of the model after each permutation and compared it with the predictions on the original non-permuted data. The range of these statistical measures will be utilized to assess the impact of "momentum" on predictive performance.

Accuracy is commonly calculated using the following formula:

$$\text{Accuracy} = \frac{\text{TP} + \text{TN}}{\text{TP} + \text{TN} + \text{FP} + \text{FN}} \tag{7}$$

This formula measures the ratio of correct predictions to the total number of predictions, expressed as a percentage. It serves as a fundamental metric for evaluating the overall performance of a machine learning model.

In our study, we conducted model predictions and compared them with the predictive distribution obtained through permutation testing to calculate the p-value. Specifically, we focused on the actual accuracy denoted as "Actual Accuracy=0.7136," indicating a model accuracy of 71.36% on the test dataset. Accuracy serves as a crucial metric for assessing model performance, representing the proportion of correct predictions made by the model. This accuracy percentage is a vital indicator for evaluating the model's performance and reliability, providing insights into the effectiveness of our predictions. Figure 9 visually illustrates the p-value calculations, further supporting our assessment of the model's performance.

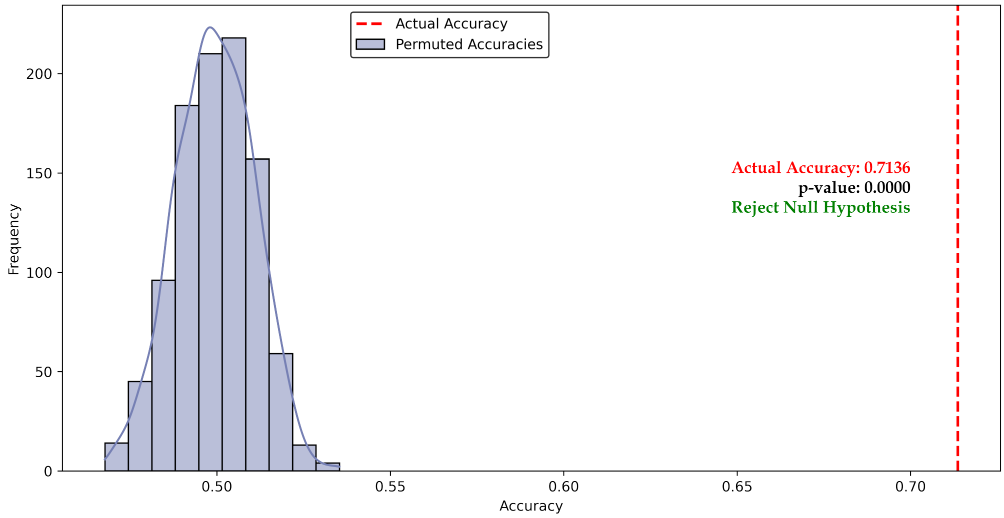


Figure 9: Permutation Test Results: Evaluating the Influence of Momentum

The results indicate that the p-value is below 0.05. Therefore, we reject the null hypothesis and support the alternative hypothesis, suggesting that 'momentum' plays a significant role in matches.

5 Task 3: Momentum Swings Predictive Model

5.1 Player Momentum Fluctuations and Prediction

In competitive sports, particularly in tennis matches, the ability to predict potential fluctuations in the game is crucial for formulating tactics and making real-time decisions. This section aims to construct a data-driven predictive model that can accurately capture the turning points of potential energy between two players during a match.

To achieve this goal, we gathered comprehensive statistical data for players p1 and p2 from 1701 matches and conducted a comparative study using four different modeling approaches: random forest, LightGBM, support vector machine, and linear regression. These models were trained and tested for various match scenarios, undergoing rigorous evaluation through cross-validation.

LightGBM, a variant of the gradient boosting decision tree (GBDT), builds upon decision tree algorithms with a leaf-wise splitting strategy. Unlike the traditional level-wise approach, the leaf-wise method in LightGBM efficiently minimizes loss during tree growth, leading to significantly enhanced classification accuracy compared to other established boosting algorithms. LightGBM introduces two innovative techniques: Exclusive Feature Bundling (EFB) and Gradient-based One-Side Sampling (GOSS).

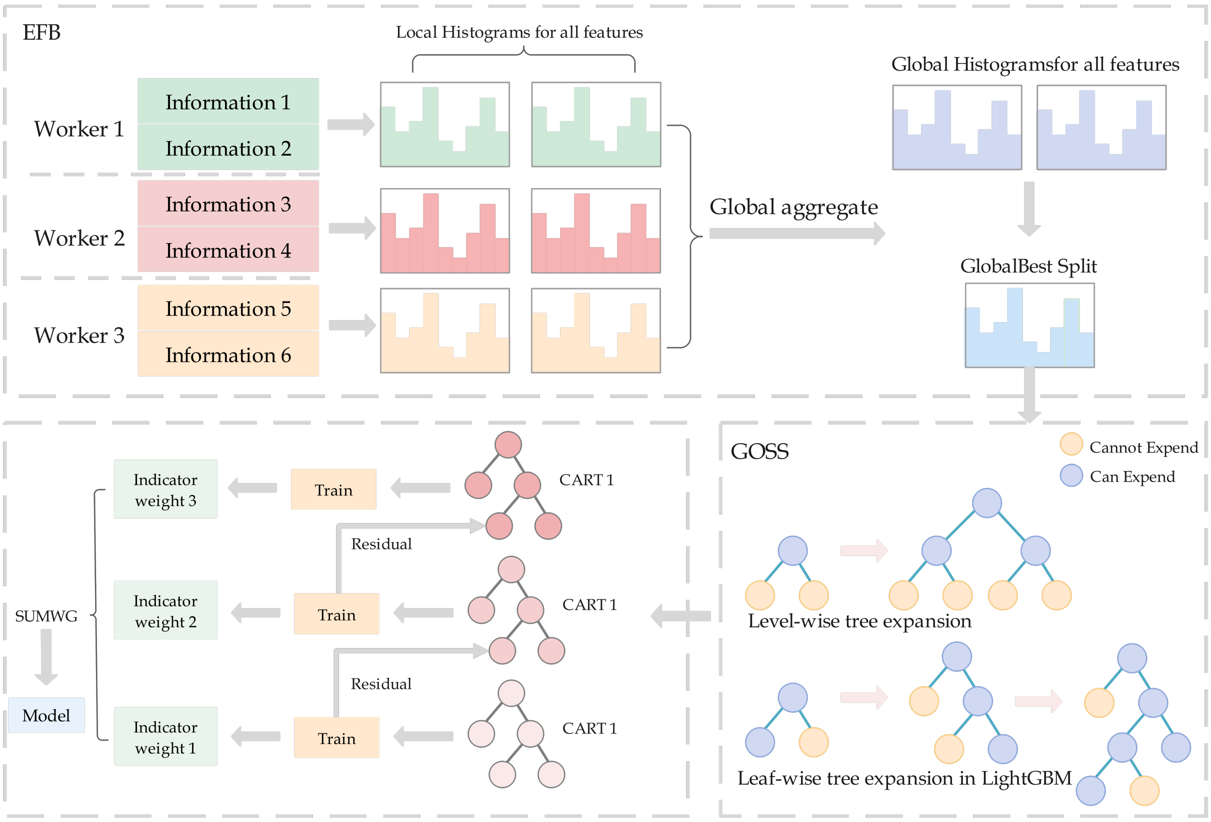


Figure 10: Momentum Swings Predictive Model

The prediction of a Gradient Boosting Decision Tree (GBDT), denoted as $\Gamma(\mathbf{x})$, is obtained by summing the outputs of a set of decision tree models, expressed as $\mathbf{e}_t(\mathbf{x})$, according to Equation (8).

$$\Gamma(\mathbf{x}) = \sum_{t=1}^{T} \mathbf{e}_t(\mathbf{x}) \tag{8}$$

To construct a GBDT model that effectively fits the given loss function $\Phi(y, \Gamma(x)$, the aim is to determine the approximate function $\hat{\Gamma}$ that minimizes the loss.

$$\hat{\Gamma} = \arg_{\hat{\Gamma}} \min \mathrm{E}_{y,S} \Phi(y, \Gamma(x)) \tag{9}$$

LightGBM diverges from the traditional GBDT approach by employing the Gradient-based One-Side Sampling (GOSS) method to split internal nodes instead of relying on information gain. The specific formula for this splitting strategy is provided as follows.

$$\mathrm{V}_j(\mathrm{d}) = \frac{1}{\mathrm{n}} \left[\frac{\left(\sum_{\mathrm{x_i} \in \mathrm{A}_l} g_i + \frac{1-\alpha}{\beta} \sum_{\mathrm{x_i} \in \mathrm{B}_l} g_i \right)^2}{\mathrm{n}_l^{\mathrm{j}}(\mathrm{d})} + \frac{\left(\sum_{\mathrm{x_i} \in \mathrm{A}_r} g_i + \frac{1-\alpha}{\beta} \sum_{\mathrm{x_i} \in \mathrm{B}_r} g_i \right)^2}{\mathrm{n}_{\mathrm{r}}^{\mathrm{j}}(\mathrm{d})} \right] \tag{10}$$

For a detailed explanation, please refer to the citation [5] in the paper.

The results indicate that the LightGBM model outperforms other models in terms of predictive accuracy [6]. This superiority is attributed to its exceptional ability to handle large-scale datasets and lower error metrics (RMSE and MAE).

Table 3: Comparison of Error Metrics for Four Modeling Approaches

model	LightGBM		SVM		XGboost		ARIMA	
Indicators	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
p1	0.082	0.063	0.153	0.116	0.153	0.116	0.191	0.142
p2	0.079	0.061	0.145	0.112	0.145	0.112	0.182	0.131

Subsequently, through additional visual analysis, we compared the potential energy values predicted by the LightGBM model with the actual potential energy values from Task I, providing a clear demonstration of the model's predictive performance and the specific dynamics of potential energy fluctuations during different time intervals, as illustrated in Figure 11.

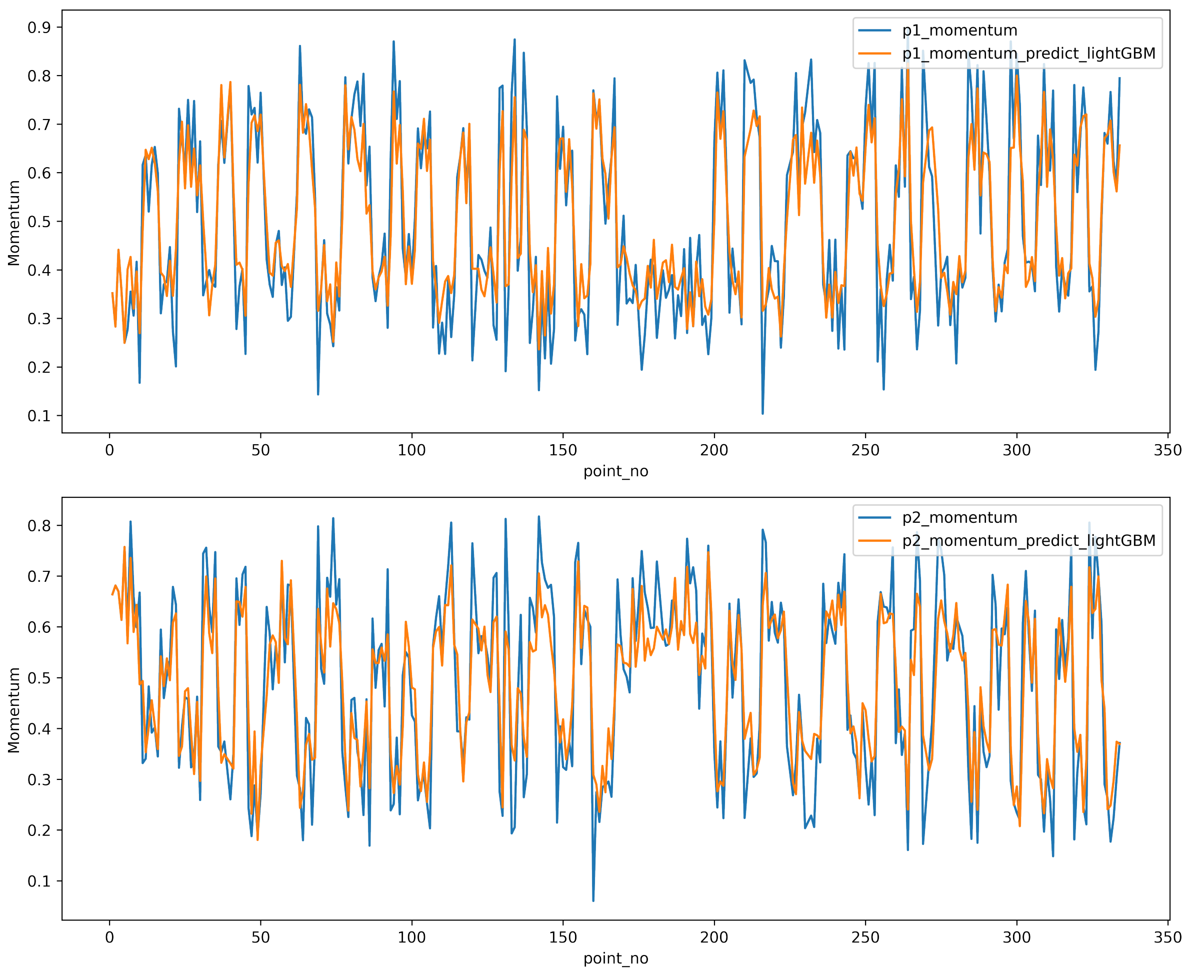


Figure 11: Comparative Analysis of Momentum and Predicted Performance for Two Players

At this point, the introduced variable, denoted as p1_p2_momentum_predict_lightGBM_diff, represents the difference between the predicted potential energy of player 1 (p1) and player 2 (p2), referred to as the "predicted potential energy difference." By analyzing this difference, we gain a better understanding of the model's predictive ability over a specific time period and how predictive performance varies with time. When this predicted potential energy difference is greater than zero, it indicates player 1 gaining momentum, whereas when it is less than zero, it signifies player 2 gaining momentum. This way, we not only demonstrate who holds the advantage over a certain period but also quantify the extent of this advantage. To visually represent the fluctuations in momentum for each player during a specific timeframe, we have highlighted key points on the line chart, as illustrated in Figure 12.

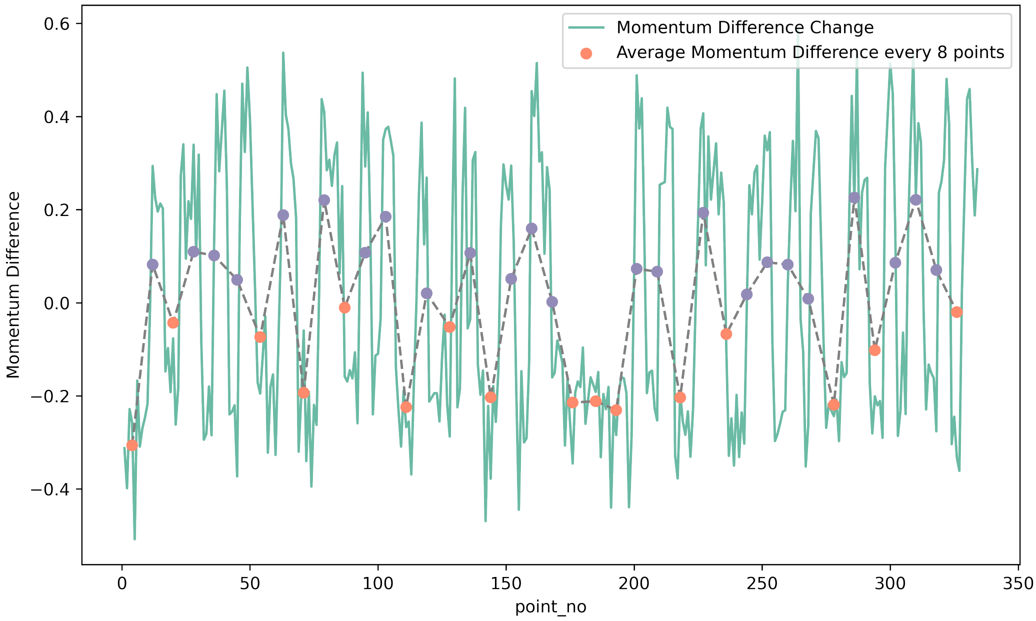
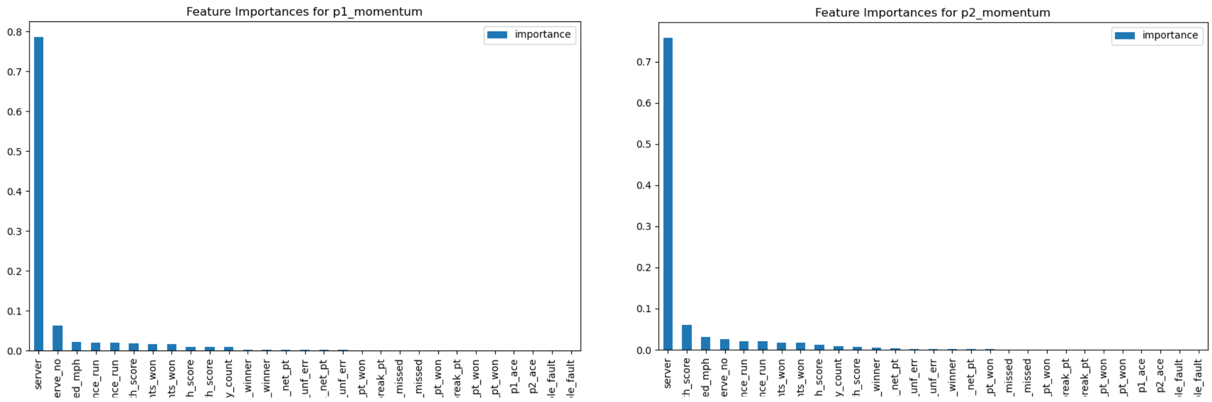


Figure 12: Player Momentum at Each Scoring Point(match_id=1701)

To better illustrate which player gains momentum over a specific period, we have specifically highlighted the average momentum difference at eight key scoring points in the graph. The visualizations align closely with the actual momentum trends of the players as reflected in the match outcomes, indicating that the model possesses good predictive capabilities.

5.2 Match Momentum's Decisive Impact: control over the serve

To gain a deeper understanding of the causes behind momentum fluctuations during matches, we focused on analyzing and identifying statistical indicators most relevant to changes in momentum. Building upon the foundation laid in the first section, we expanded the scope of our dataset, defining the dependent variable as momentum and incorporating all available match statistics as independent variables. Using the feature importance assessment functionality of the random forest model, we successfully identified factors that have the greatest impact on momentum, with serving dominance emerging as a key determinant. The specific weights of each factor are illustrated in Histogram 13.



(a) Carlos Alcaraz (b) Nicolas Jarry

Figure 13: Weighting Factors for Tennis Players' Momentum

Through data analysis, it has been determined that controlling the serve significantly impacts the overall performance of Player 1, accounting for a substantial 78.63%. Similarly, this influence is pronounced for Player 2, with a significant proportion of 75.79%. This proportion far exceeds the impact of any other factor. Therefore, we conclude that mastering the serve plays a decisive role in shaping the competitive momentum for players in tennis matches.

5.3 Strategies to Optimize Carlos Alcaraz's Match Performance

To optimize Carlos Alcaraz's performance in matches, we propose a series of strategies aimed at enhancing his understanding of various aspects of his game and leveraging his strengths. These strategies aim to ensure consistent output, mastery of momentum, and an increased probability of victory, especially when facing new opponents. The following are key recommendations for adjusting momentum and improving overall performance:

- **Exploit Endurance Advantage and Serve Efficiency:**
  Capitalize on Alcaraz's exceptional endurance by focusing on serve speed and accuracy, increasing the difficulty for opponents to return serves, thereby laying a solid foundation for the match.

- **Mental Resilience and Serving:**
  Maintain composure under the pressure of first serve errors, utilizing psychological techniques like positive thinking and deep breathing to regulate emotions and quickly restore confidence for the second serve.

- **Tactical Adjustments and Adaptation:** Flexibly adjust serving strategies based on opponent reactions and match rhythm. Analyze opponents' weaknesses in real-time and exploit them during serves to create more scoring opportunities.

- **Sustain Physical Fitness and Recovery's Impact on Serving:**
  Maintain excellent physical condition to ensure high-level serving performance throughout the match. Adequate rest, nutrition, and recovery training are crucial for sustaining physical fitness and serving performance.

Through these strategies, Carlos Alcaraz can respond more comprehensively and flexibly to various situations in matches, elevate his competitive level, and increase his chances of success.

6 Task 4: Model Testing and Generalization

6.1 2023 Wimbledon Men's Singles First Match Momentum Prediction

To validate the accuracy and universal applicability of the momentum fluctuation prediction model established for Task3, we applied it to another match for verification. We will continue to use the aforementioned LightGBM model to analyze and predict the momentum fluctuations in the first match of the 2023 Wimbledon men's singles final, specifically match_id=2023-wimbledon-1301.

The values of the evaluation metrics using the LightGBM model are shown in Table 4.

Table 4: LightGBM Model Evaluation Metrics

player	RMSE	MAE	MAPE	MSLE	R^2
1	0.082	0.063	16.018%	0.003	0.821
2	0.083	0.062	16.385	0.003	0.810

To more accurately depict the fluctuations in player momentum during specific time intervals, we utilized line graphs to illustrate key data points, as shown in Figure 14. In this graph, purple data points represent Player 1, while orange data points represent Player 2.

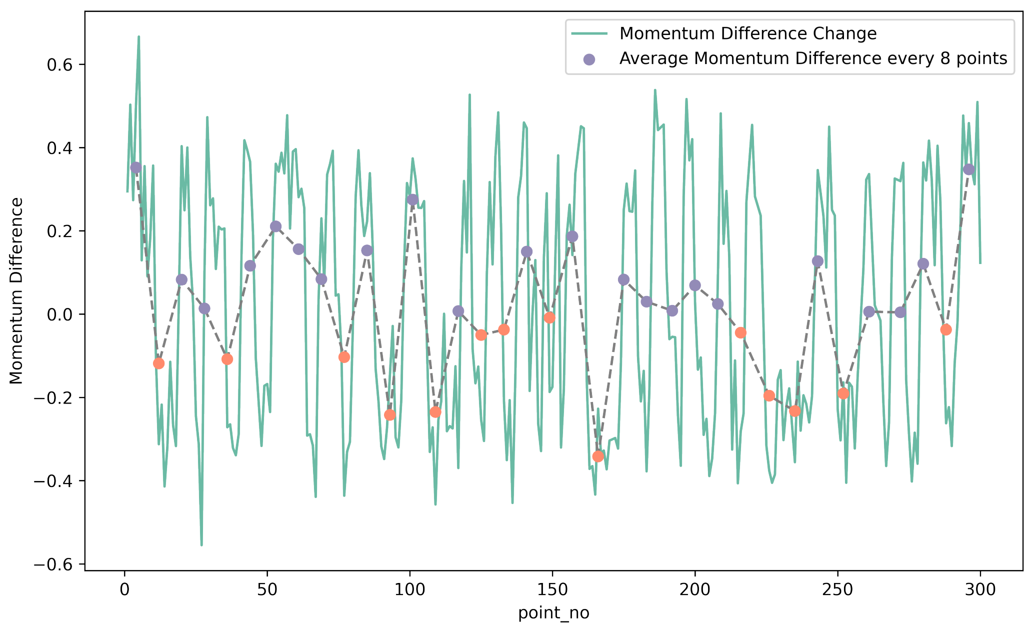


Figure 14: Player Momentum at Each Scoring Point(match_id=1301)

In the initial set, Player 1 secured a 6:3 victory. During this period, the number of purple-marked data points, representing Player 1, in the scoring range of 1-63 significantly outnumbered the orange-marked data points, indicating a higher average momentum for Player 1 and contributing to their success in the first set.

As the match progressed to the second set, both players fought fiercely, resulting in a 6:6 tie. Ultimately, Player 2 clinched victory with an 8:6 score in the tiebreaker. In this set, the quantities of purple and orange data points were comparable, signifying a closely contested battle where both players exhibited similar momentum.

Moving on to the third set, Player 1 once again emerged triumphant with a 6:3 score. Notably, within the scoring range of 156-212, the number of purple data points surpassed that of orange data points, showcasing Player 1's momentum advantage and contributing to their success in the third set.

In the fourth set, Player 1 secured a 6:5 victory. While the quantities of orange and purple data points remained close throughout the set, indicating a balanced momentum between both players, a notable increase in the value of purple data points in the final stages indicated that Player 1's momentum surpassed Player 2's at a crucial moment. This pivotal shift provided Player 1 with a decisive advantage, ultimately leading to their victory in the entire match.

In summary, the player momentum prediction model we developed exhibited exceptional performance in capturing momentum shifts between players during the match. Its robust adaptability to diverse match conditions and player styles enhances its utility in match analysis and tactical decision-making.

If the model underperforms, consider optimizing data quality and feature engineering; adjusting model complexity and hyperparameters; and exploring ensemble methods to enhance predictive accuracy.

6.2 2023 Wimbledon Women's Quarterfinals Momentum Prediction

To validate the generalizability of our momentum fluctuation prediction model to other matches, we collected complete match data for the quarterfinal match between Ons Jabeur and Elena Rybakina in the 2023 Wimbledon Women's Singles from the official Wimbledon Championship website[7]. We employed the same methodology for modeling and analysis.

The values of the evaluation metrics using the LightGBM model are shown in Table 5.

Table 5: LightGBM Model Evaluation Metrics

player	RMSE	MAE	MAPE	MSLE	R^2
1	0.098	0.081	17.382%	0.004	0.632
2	0.089	0.072	15.471%	0.003	0.692

To more precisely represent the changes in momentum experienced by players during specified time periods, we employed line graphs to highlight crucial data points, as depicted in Figure 15. In this chart, the purple points signify Player 1, whereas the orange points denote Player 2.

In the initial set, the match was tightly contested, reaching a 6:6 tie. The subsequent tiebreaker concluded with Player 2 securing a 5:7 victory (1-72). The momentum difference between the two players remained close to zero during this set, indicating an evenly matched struggle.

Moving on to the second set, Player 1 managed to clinch a 6:4 victory (73-134). Although the momentum remained closely contested, Player 1 held a slight lead, ultimately securing the set in their favor.

The third set witnessed a dominant performance from Player 1, winning decisively with a score of 6:1 (135-182). Throughout the majority of this set, Player 1 maintained a stronger momentum than Player 2, securing another victory in the match.

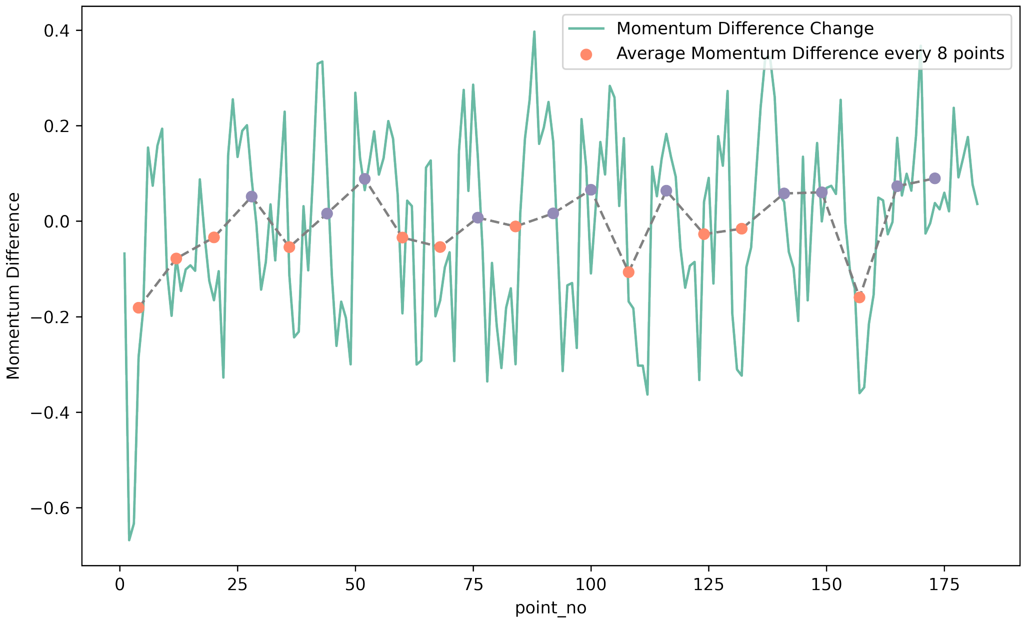


Figure 15: Player Momentum at Each Scoring Point(match_id=2503)

The actual scoring situation mentioned above is largely consistent with the results of the prediction model. This indicates that the model we have established possesses broad applicability and can be applied to other matches. Therefore, such a predictive model provides valuable guidance for athletes to perform better in competitions.

7 Sensitivity Analysis

To detect the influence of psychological factors on momentum, we have chosen to calculate the weight of momentum, denoted as w_i, as the parameter for detection. Specifically, in the predicted momentum of a match, we selected 60 data points at equal intervals. Similarly, for the weight of w_i, we selected 3 data points at equal intervals within the range of 0 to $2w_i$.

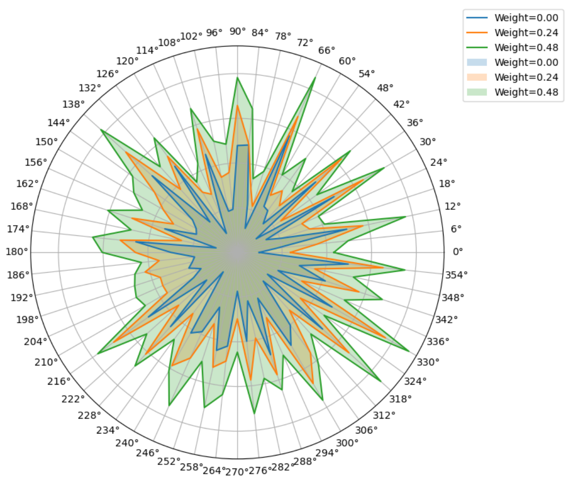


Figure 16: Player Momentum at Each Scoring Point

From the Figure 16, it is observed that the model is relatively sensitive to changes in letter frequency, indicating that it exhibits significant variability in response to input fluctuations.

8 Model Evaluation

8.1 Strengths

- **Comprehensive Evaluation:** A multi-level system objectively assesses mentality, skills, fitness, and serving, offering nuanced insights into player states and match momentum.
- **Effective Data Handling:** Normalization and feature engineering ensure model stability, providing nuanced insights with a focus on set scoring rates.
- **Random Forest Analysis:** Enhances interpretability, offering clear importance levels for each aspect, aiding decision-makers in strategic choices.
- **Permutation Test:** Scientifically evaluates momentum's impact on match results, providing a reliable method for coaching hypotheses.
- **Temporal Momentum:** The model integrates temporal fluctuations, aiding coaches and players in understanding momentum shifts comprehensively.

8.2 Weaknesses

- **Missing Data Impact:** Despite perturbation terms, the model's interpretability is limited due to missing player characteristic data.
- **Limited Generalization:** Improvement needed for the model's adaptation to diverse match types and data sources.

Memorandum

To: coaches

From: Team #2407590

Subject: In-depth Analysis of Momentum and Match Response Recommendations

Date: February 5, 2024

To whom it may concern,

Recently, our research team conducted an in-depth analysis of the phenomenon of "momentum" in tennis matches. This memorandum aims to share our research findings and provide recommendations on how to help players deal with various challenges in matches.

The research indicates that momentum is one of the key qualitative factors influencing the outcome of tennis matches, and its role should not be underestimated. We categorized factors affecting players' performance into psychological, physical, technical, serving, and other uncontrollable factors. Each factor influences a player's momentum at specific moments. Particularly, we found that mastering serving rights can directly impact a player's probability of winning points, making it a crucial element in determining the course of a match. By employing an advanced LightGBM predictive model, we are able to track and predict momentum fluctuations during matches. We have validated the accuracy and applicability of the model across multiple matches.
Based on these analytical results, we offer the following strategic recommendations to coaching teams:

- **Psychological Resilience Training:** Strengthen players' psychological adaptability to help them manage match pressure and expectations, including fostering a positive mindset, enhancing focus, and improving the ability to recover from mistakes.
- **Simulation Training:** Create training environments that closely resemble real match situations, allowing players to adapt to matches in different momentum situations, such as simulating leading, trailing, or critical point scenarios.
- **Data Analysis:** Utilize data analysis tools to assess players' performance under specific momentum conditions, facilitating adjustments to training plans and match strategies.
- **Feedback Mechanism:** Establish a timely feedback system to ensure players quickly receive feedback on technical and tactical performance, along with optimization suggestions under different momentum situations.

We believe that through a deep understanding of momentum and the implementation of corresponding strategies, players' performances in critical moments will be enhanced, thereby strengthening the overall competitiveness of the team. We look forward to further discussing these recommendations with coaching teams and implementing them in future training and matches.

Thank you for your attention and support.

Yours Sincerely,

Team #2407590

References

[1] Moss, B. , & ODonoghue, Peter. (2015). Momentum in us open men's singles tennis. International Journal of Performance Analysis in Sport, 15(3), 884-896.

[2] Ma, S. M. , Liu, C. C. , Tan, Y. , & Ma, S. C. . (2013). Winning matches in grand slam men's singles: an analysis of player performance-related variables from 1991 to 2008. J Sports, 31(11), 1147-1155.

[3] Li, X., Wang, Y., Basu, S., Kumbier, K., & Yu, B. (2019). A Debiased MDI Feature Importance Measure for Random Forests. arXiv.Org, arXiv.org, 2019.

[4] Hemerik, J. , & Goeman, J. . (2014). Exact testing with random permutations.

[5] Dong, S., Khattak, A., Ullah, I., Zhou, J., & Hussain, A. (2022). Predicting and Analyzing Road Traffic Injury Severity Using Boosting-Based Ensemble Learning Models with SHAPley Additive exPlanations.ăInternational journal of environmental research and public health,ă19(5), 2925. https://doi.org/10.3390/ijerph19052925

[6] Ke, G., Meng, Q., Finley, T., Wang, T., Chen, W., Ma, W., Ye, Q., & Liu, T. (2017). LightGBM: A Highly Efficient Gradient Boosting Decision Tree. Neural Information Processing Systems.

[7] https://www.wimbledon.com/index.html

Report on Use of AI

- **Utilizing AI to Resolve Code Errors:**
  While developing the code for our mathematical model, we encountered a series of programming challenges. These issues ranged from syntax errors and runtime exceptions to logical errors, resulting in programs that failed to run correctly or produced results inconsistent with our expectations. Addressing these problems often required delving into the details of the code, manually searching for and fixing issues. This process was not only time-consuming but also prone to oversight. Therefore, we employed AI to assist us in resolving these error issues.

- **Comprehensive Implementation of Technical Factor Quantification Using AI Recommendations:**
  We integrated AI to assist in quantifying technical factors in tennis matches, particularly focusing on strategies related to serving and returning. We quantified variables such as serve_width, serve_depth, and return_depth based on the difficulty of serving, returning, the pressure imposed on opponents, and the impact on serve success rates in high-level matches. This quantification facilitated subsequent modeling.

- **Literature and Data Search:**
  We efficiently employed AI literature search tools to sift through and analyze relevant literature and data. These tools not only rapidly identified the most pertinent research papers and technical articles but also provided abstracts and key information extractions based on our research focus, significantly enhancing the efficiency and quality of literature searches.