Mean Shift Algorithm

Meanshift is falling under the category of a clustering algorithm in contrast of Unsupervised learning that assigns the data points to the clusters iteratively by shifting points towards the mode (mode is the highest density of data points in the region, in the context of the Meanshift). As such, it is also known as the **Mode-seeking algorithm**. Mean-shift algorithm has applications in the field of image processing and computer vision.

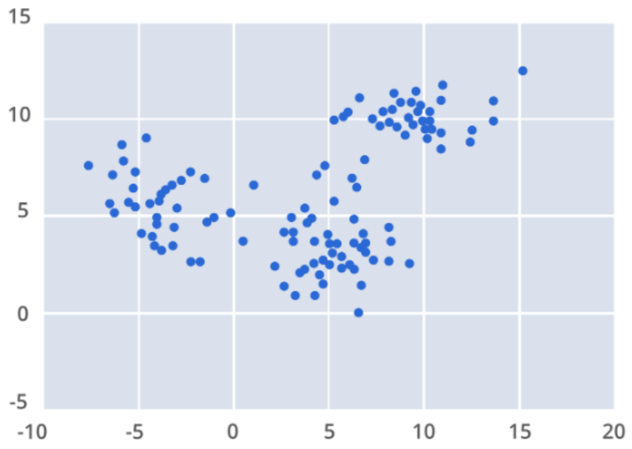
Given a set of data points, the algorithm iteratively assigns each data point towards the closest cluster centroid and direction to the closest cluster centroid is determined by where most of the points nearby are at. So each iteration each data point will move closer to where the most points are at, which is or will lead to the cluster center. When the algorithm stops, each point is assigned to a cluster.

Unlike the popular K-Means cluster algorithm, mean-shift does not require specifying the number of clusters in advance. The number of clusters is determined by the algorithm with respect to the data.

Note: The downside to Mean Shift is that it is computationally expensive O(n^2).

Kernel Density Estimation –

The first step when applying mean shift clustering algorithms is representing your data in a mathematical manner this means representing your data as points such as the set below.

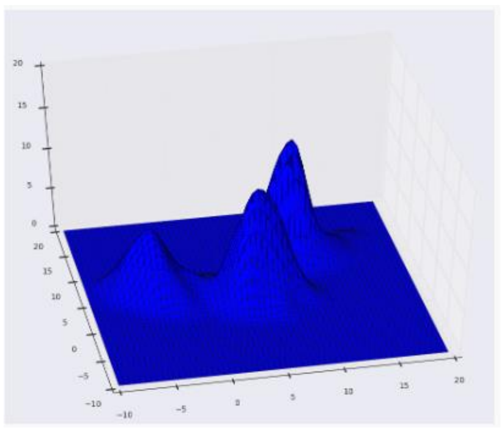


Mean-shift builds upon the concept of kernel density estimation is sort KDE. Imagine that the above data was sampled from a probability distribution. KDE is a method to estimate the underlying distribution also called the probability density function for a set of data.

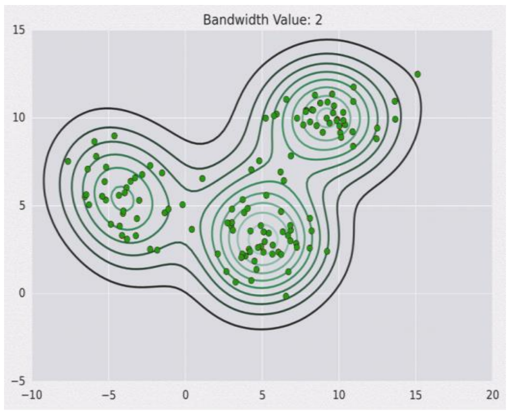
It works by placing a kernel on each point in the data set. A kernel is a fancy mathematical word for a weighting function generally used in convolution. There are many different types of kernels, but the most popular one is the Gaussian kernel. Adding up all of the individual kernels generates a probability surface example density function. Depending on the kernel bandwidth parameter used, the resultant density function will vary.

Below is the KDE surface for our points above using a Gaussian kernel with a kernel bandwidth of 2.

Surface plot:



Contour plot:



Below is the Python implementation :

```
import numpy as np
import pandas as pd
from sklearn.cluster import MeanShift
from sklearn.datasets.samples_generator import make_blobs
from matplotlib import pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

clusters = [[2, 2, 2], [7, 7, 7], [5, 13, 13]]

X, _ = make_blobs(n_samples = 150, centers = clusters,
                                   cluster_std = 0.60)

ms = MeanShift()
ms.fit(X)
cluster_centers = ms.cluster_centers_

# Finally We plot the data points
# and centroids in a 3D graph.
fig = plt.figure()
```

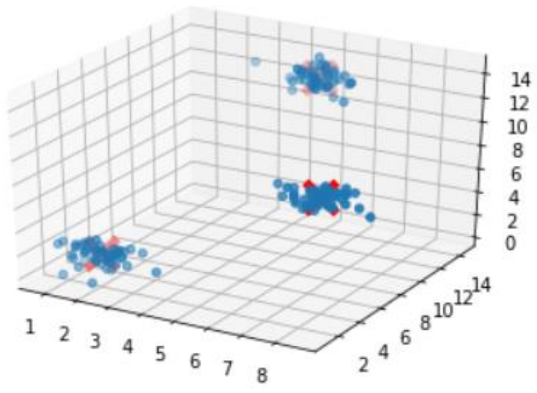
```
ax = fig.add_subplot(111, projection ='3d')

ax.scatter(X[:, 0], X[:, 1], X[:, 2], marker ='o')

ax.scatter(cluster_centers[:, 0], cluster_centers[:, 1],
           cluster_centers[:, 2], marker ='x', color ='red',
           s = 300, linewidth = 5, zorder = 10)

plt.show()
```

Output:



To illustrate, suppose we are given a data set {ui} of points in d-dimensional space, sampled from some larger population, and that we have chosen a kernel K having bandwidth parameter h. Together, these data and kernel function returns the following kernel density estimator for the full population's density function.

$$f_K(\mathbf{u}) = \frac{1}{nh^d} \sum_{i=1}^{n} K\left(\frac{\mathbf{u} - \mathbf{u}_i}{h}\right)$$

The kernel function here is required to satisfy the following two conditions:

1. $\int K(\mathbf{u})d\mathbf{u} = 1$
2. $K(\mathbf{u}) = K(|\mathbf{u}|)$ for all values of $\mathbf{u}$

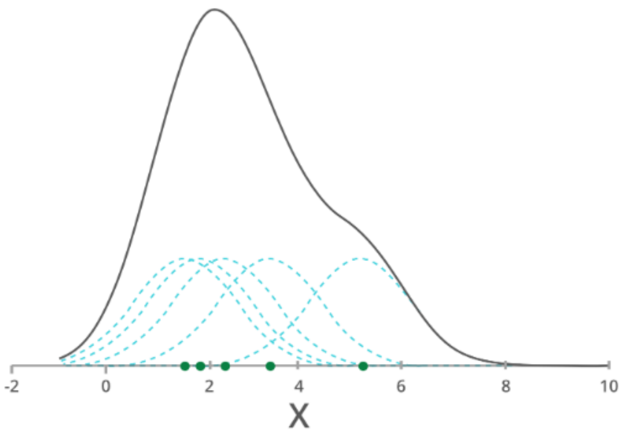
-> The first requirement is needed to ensure that our estimate is normalized.
-> The second is associated with the symmetry of our space.

Two popular kernel functions that satisfy these conditions are given by-

1. Flat/Uniform $K(\mathbf{u}) = \frac{1}{2}\begin{cases} 1 & -1 \leq |\mathbf{u}| \leq 1 \\ 0 & else \end{cases}$

2. Gaussian = $K(\mathbf{u}) = \frac{1}{(2\pi)^{d/2}} e^{-\frac{1}{2}|\mathbf{u}|^2}$

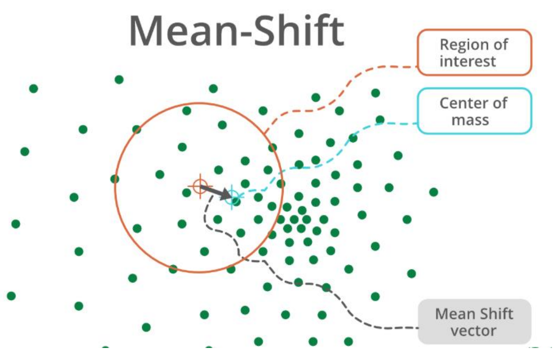
Below we plot an example in one dimension using the Gaussian kernel to estimate the density of some population along the x-axis. We can see that each sample point adds a small Gaussian to our estimate, centered about it and equations above may look a bit intimidating, but the graphic here should clarify that the concept is pretty straightforward.



Example of kernel density estimation using a gussain kernel for each data point: Adding up small Gauggians about each example returns our net estimate for the total density, the black curve.

Iterative Mode Search –

1. Initialize random seed and window W.
2. Calculate the center of gravity (mean) of W.
3. Shift the search window to the mean.
4. Repeat Step 2 until convergence.



General algorithm outline –

```
for p in copied_points:
    while not at_kde_peak:
        p = shift(p, original_points)
```

Shift function looks like this –

```
def shift(p, original_points):
    shift_x = float(0)
    shift_y = float(0)
    scale_factor = float(0)

    for p_temp in original_points:
        # numerator
        dist = euclidean_dist(p, p_temp)
        weight = kernel(dist, kernel_bandwidth)
        shift_x += p_temp[0] * weight
        shift_y += p_temp[1] * weight
        # denominator
        scale_factor += weight

    shift_x = shift_x / scale_factor
    shift_y = shift_y / scale_factor
    return [shift_x, shift_y]
```

Pros:

- Finds variable number of modes
- Robust to outliers
- General, application-independent tool
- Model-free, doesn't assume any prior shape like spherical, elliptical, etc. on data clusters
- Just a single parameter (window size h) where h has a physical meaning (unlike k-means)

Cons:

- Output depends on window size
- Window size (bandwidth) is not trivial
- Computationally (relatively) expensive (approximately 2s/image)
- Doesn't scale well with dimension of feature space.