

Refracting Rays

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Introduction

This experiment was inspired by the many different applications of lasers. When I observed that a narrow beam of laser remains narrow over long distances (negligible divergence), it made me think that it can be used for many accurate measurements. I was curious to see how it will refract in transparent liquids. The following points come in favor of my experiment.

- 1) A laser beam is monochromatic, collimated and coherent as opposed to a normal beam of light. The red laser I was using has a wavelength of around 650nm. (Muldoon, 2001)
- 2) After conducting a literature review I found out that the refractive index is usually measured using yellow light. However I learned that the difference between refractive indices for yellow and red is only of the order of 0.002 which is 0.15%. (the refractive index of water for yellow light is 1.333 and for red light is 1.331)

Refractive index for water with yellow light (570nm) (Madigan, 2011) = 1.333 (Polyanskiy, Refractive index database, 2012)

Refractive index for water with red light (650nm) (Madigan, 2011) = 1.331 (Polyanskiy, Refractive Index Database, 2012)

Difference = 1.333 - 1.331 = 0.002 over 1.333 = 0.15%

- 3) Since it is collimated, monochromatic and coherent the divergence as well as the dispersion is extremely small. Therefor the beam can be made very narrow and still powerful. This increases accuracy of measurements.
- 4) Since red light gets less scattered compared to the rest of visible light, it can travel longer through the medium. I made use of this property (by using mirrors), so that I could get more displacement. This, in turn, increased accuracy.

This led to a project on measuring the refractive index of different liquids using a red (650nm) laser

My Innovations in this project:

- 1) Used a drafting machine with vernier to set precise angles for the laser beam
- 2) Used mirrors inside the solution to increase the displacement and therefore accuracy
- 3) Derived an equation to calculate the angle of refraction from the displacement
- 4) Found an equation relating the refractive index and the salinity of water

Question

How does the salinity of water affect its refractive index?

Hypothesis

The refractive index of water increases as the salinity of the water increases.

Logical Reasoning:

When light passes through a dense material, it starts to travel slower. This causes the beam of light to refract towards the normal at the point of incidence. When water has more dissolved solids, it has a higher density.

Theory

Note:

In this theory section, many of the citations are from Wikipedia. I have chosen Wikipedia only in places where it explains the theory well. I have made sure to compare and verify all the information from the Wikipedia sites with other sources. Please refer to the bibliography for all those sources.

All the theories that are relevant to my experiment are explained here. (Also more details in the Research booklet)

What is Refraction?

"Refraction is the change in direction of a wave due to a change in its medium" (Wikipedia, Refraction, 2013).

When light enters from one medium to another medium, it changes direction.

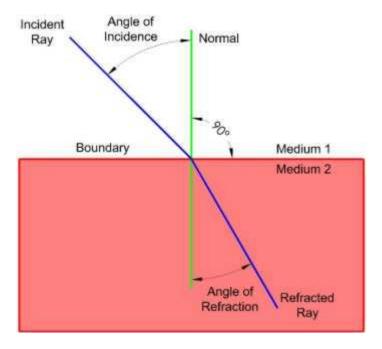


Figure 1: Refraction of light (Sabanski)

Refractive Index

The refractive index is a number without a unit, which describes how light or radiation propagates through a medium. It is a measure of the speed of light in a substance. It is defined with the equation $\mathbf{n}=\mathbf{c}/\mathbf{v}$ where c is the speed of light in vacuum and v is the phase velocity in the substance. For example, the refractive index of water for red light is 1.331, meaning that red light travels 1.331 times as

fast in vacuum as it does in water. When light travels through a dense medium the light starts to travel slower. If a ray of light passes across the boundary from a medium in which it travels fast into a medium in which travels slower, then the light ray will bend towards the normal line. (Wikipedia, Refractive Index, 2013) (Wikipedia, Phase Velocity, 2013)

What Causes Refraction?

I came across two theories for the reason for refraction.

1) Fermat's Principle of Least Time:

Fermat (year 1650) postulated that out of all possible paths that it might take to get from one point to another light takes the path which requires the **shortest time**. (Feynman) (Wikipedia, Fermat's Principle, 2013)

- a) This easily explains why light travels in a straight line in a homogenous medium.
- b) How this theory explains reflection of light

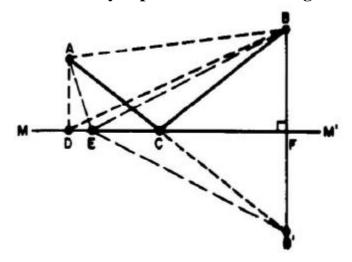


Figure 2: Fermat's Principle of Least Time (Feynman)

Consider the figure above.

MM' is a mirror. Light coming from point B gets reflected off point C and reaches A. We know that at point C the incident angle will be equal to the reflected angle. This rule will not hold good for D or E. We can prove that the path ACB is shorter than path ADB or AEB as follows. If you extend AC to B' and intersect the normal at F, BF=B'F. Therefore, travelling to B is equivalent to travelling to B'. The shortest distance between A and B' is through C since ACB' is a straight line and shorter than any other path such as AEB' or ADB'. So it proves that ACB is the shortest distance for light to travel and because it is the shortest distance, it takes the shortest time.

c) How This Theory Explains Refraction

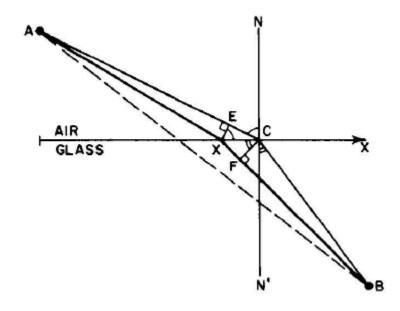


Figure 3: Fermat's Principle of Least Time (Feynman)

Consider the figure above.

The X axis is the boundary between air and glass. Light travels faster in air and slower in glass. Therefore, even if a straight line AB is the shortest distance, it might not take the shortest time because light has to travel more in glass in that case.

Let us assume that the path AXB is the optimum where it takes the shortest time to reach B. If we move to any point towards the left of X, it takes more time to reach B because more time is spent in travelling in glass. If we move to any point to the right of X it will take more time because light is travelling more time in air.

Now let us consider a point C is infinitesimally near point X. We have seen that moving leftwards or rightwards from point X increases the time to travel. Since C is infinitesimally small, time to travel through path AXB will be equal to the time to travel through the path ACB.

Now let us draw XE which is perpendicular to AC. Let us also draw CF which is perpendicular to XB. AX=AE also BF = BC.

Compare through the travels through the two paths AXB and ACB. AXB requires an extra length of travel XF through glass. ACB requires an extra travel of EC through air.

Therefore, if time to travel through AXB and ACB should be equal, then time to travel XF through glass should be equal to time to travel EC through air. Which means

$$EC \times n_1 = XF \times n_2$$
 (equation 1)

Where n_1 and n_2 are the refractive indices of air and glass.

Suppose angle ACN = θ_i = angle of incidence.

Suppose angle N'CB = θ_r = angle of refraction.

We can easily prove that

$$\frac{EC}{XC} = \sin \theta_i$$
 (equation 2).

Also

$$\frac{XF}{XC} = \sin \theta_r$$
 (equation 3).

Dividing equation 2 by equation 3, we get

$$\frac{EC}{XF} = \frac{\sin \theta_i}{\sin \theta_r}$$

From equation 1 we have

$$\frac{EC}{XF} = \frac{n_2}{n_1}$$

Substituting we get

$$\frac{n_2}{n_1} = \frac{\sin \theta_i}{\sin \theta_r} = Snell's \ Law$$

(Feynman) (Wikipedia, Fermat's Principle, 2013) (Wikipedia, Snell's Law, 2013)

2) Quantum Mechanical Explanation of Refraction

The famous scientist and Noble prize winner, Dr. Richard Feynman, in his famous textbook, Feynman's lectures on physics, gives a great explanation for refraction. In his chapter 31 on "The origin of the Refractive Index" he points out that "so far as problems involving light are concerned, the electrons (in atoms) behave as though they were held by springs" (p.31-4).

The electric field in electromagnetic radiation (in this case our light beam) accelerates the electrons held by springs in the atoms of the medium. These accelerated electrons re-radiate in all directions. The observed light is the superposition of the electric and magnetic fields of the incoming light and the re-radiation.

In the backward direction, we normally call the re-radiation "reflection", but this labelling obscures the fact that this is new light radiated by all the atoms in the glass, not old light that has magically bounced off the front surface.

In the forward direction, we speak of refraction and we say that the speed of light is slower in the medium, but in fact the speed of light does not change

in the medium. Feynman shows how the superposition of the incoming light, travelling at speed c, and the light re-radiated by the electron, travelling at speed c, shifts the phase of the radiation in the same way as it would occur if the light were to go slower than c with a shorter wavelength. (Sherwood, 2002)

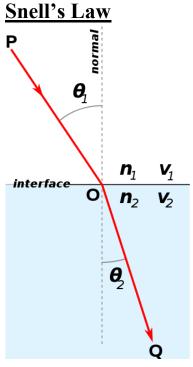


Figure 4: Snell's Law (Wikipedia, Snell's Law, 2013)

Snell's Law states that

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

Where,

 θ_1 = angle of incidence

 θ_2 = angle of refraction

 v_1 = velocity of light in medium 1

 v_2 = velocity of light in medium 2

 n_1 = refractive index of medium 1

 n_2 = refractive index of medium 2

Proof

Snell's Law can be proved using Fermat's principle of least time. Please refer to the earlier section of Fermat's principle of lease time for the proof.

Effect on Wavelength

$$v = v\lambda$$

Where v = velocity, v = frequency, and $\lambda = wavelength$.

Therefore,

$$\frac{v_1}{v_2} = \frac{v\lambda_1}{v\lambda_2}$$

The frequency v does not change when light enters from one medium to the other. Therefore,

$$\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

(Wikipedia, Snell's Law, 2013)

Cauchy's Law

Refractive index for different wavelengths of light, over the same material, is different.

Cauchy's equation is an empirical relationship between the refractive index and wavelength of light for a particular transparent medium.

The general form of Cauchy's equation is

$$n(\lambda) = B + \frac{C}{\lambda^2} + \frac{D}{\lambda^4} + \cdots$$

Where n = the refractive index, $\lambda =$ wavelength in μm , and B, C, D, etc. = coefficients that are determined for a material by fitting the equation to measured values of refractive indices at known wavelengths.

For practical purposes, it is sufficient to use the first two terms of this equation.

$$n(\lambda) = B + \frac{c}{\lambda^2}$$

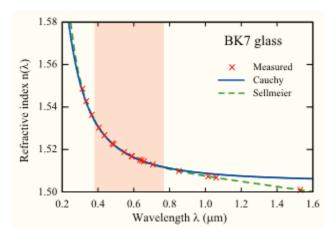


Figure 5: Refractive vs. Wavelength (Wikipedia, Cauchy's Equation, 2013)

Please note the measured values of refractive indices and their calculated values using Cauchy's equation, are matching only for a range of wavelengths. You can also note that calculating refractive indices using the Sellmeier equation is more accurate. The Sellmeier equation is explained in the next section.

(Wikipedia, Cauchy's Equation, 2013)

Sellmeier Equation

The Sellmeier Equation is more accurate than Cauchy's equation. It is also an empirical relationship between refractive index and wavelength for a particular transparent medium. It is an improvement on Cauchy's equation.

The most general form of the Sellmeier equation is

$$n^2(\lambda) = 1 + \sum_{i} \frac{B_i \lambda^2}{\lambda^2 - C_i}$$

Where n = the refractive index, $\lambda =$ wavelength in μm , and B_i and $C_i =$ different coefficients.

The usual form of the Sellmeier equation is as follows

$$n^{2}(\lambda) = 1 + \frac{B_{1}\lambda^{2}}{\lambda^{2} - C_{1}} + \frac{B_{2}\lambda^{2}}{\lambda^{2} - C_{2}} + \frac{B_{3}\lambda^{2}}{\lambda^{2} - C_{3}}$$

Where n = the refractive index, λ = wavelength in μ m, and B₁, B₂, B₃, C₁, C₂, C₃ = Sellmeier coefficients which are determined experimentally.

(Wikipedia, Sellmeier equation, 2013)

Phase Velocity

In a medium, different wavelengths travel at different speeds. The speed at which the individual crests and troughs of a plane wave (a wave with only one frequency) propagate is called the phase velocity v_p .

The largest part of the wave travels at the group velocity v_g and the front part of the wave (wave front) travels at the front velocity v_f .

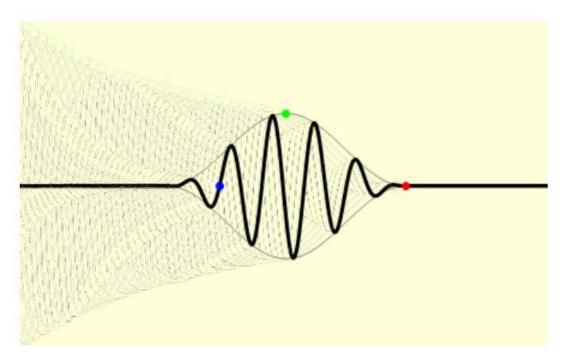


Figure 6: Phase Velocity (Wikipedia, Phase Velocity, 2013)

In the above picture, the blue dot moves at the speed of the ripples which is the phase velocity v_p . The green dot moves with the speed of the envelope which is the group velocity v_g . The red dot moves with the speed of the foremost part of the group which is the front velocity v_f .

(Wikipedia, Phase Velocity, 2013) (Wikipedia C.-B.-S.)

Dispersion

When a beam of different frequencies (white light) enters a medium, each frequency (colour) wave will have a different velocity in that medium. As a result, each of the individual frequencies gets refracted differently. Therefore, by the time these waves have travelled through the medium, each frequency is wide apart for each other. This phenomenon is called dispersion. This is the phenomenon that causes white light to split into seven colours in a prism or in the case of a rainbow.

A dispersive medium causes dispersion as said above.

Therefore, dispersion can be defined as the phenomenon in which the phase velocity of a wave depends on its frequency. Dispersion is also called chromatic dispersion or group velocity dispersion. (Wikipedia, Dispersion(optics), 2013)

Critical Angle and Total Internal Reflection

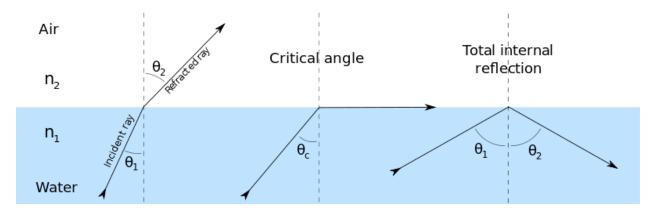


Figure 7: Critical Angle and Total Internal Reflection (Wikipedia, Refractive Index, 2013)

Consider a ray of light entering from an optically denser medium to a lighter medium. In this case, the angle of refraction is more than the angle of incidence. If we keep increasing θ_1 , θ_2 also keeps increasing and at a specific angle of θ_1 , θ_2 becomes 90° as shown in the middle ray of Figure 7.

This angle of θ_1 is called the critical angle because if θ_1 goes beyond this angle, θ_2 becomes more than 90° as shown in the third ray of Figure 7 and the ray totally returns back to the denser medium without escaping into the lighter medium.

The above phenomenon is called total internal reflection.

There are several applications for total internal reflection. For example a fibre optic wave guide uses this principle. A ray of light entering an optical fibre does not escape through its sides because of total internal reflection.

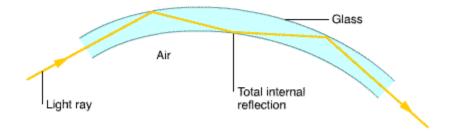


Figure 8: Total Internal Reflection in Optical Fibre (BBC, 2011)

Diamonds shimmer because of total internal reflection. A ray of light entering into it undergoes several total internal reflections, due to its high refractive index and geometry, and it finally comes out through only specific sides of the diamond. (Wikipedia, Total Internal Reflection, 2013)

What are Molar Solutions?

Before understanding what a molar solution is, we should understand what the **gram molecular weight** of a chemical is. Since I have used sodium chloride (NaCl) in my experiment, I am taking sodium chloride itself as my example. The atomic weight of sodium is 23 and the atomic weight of chlorine is 35.5. One molecule of NaCl contains one atom of sodium (Na) and one atom of chlorine (Cl). Therefore, its molecular weight is 23 + 35.5 = 58.5.

The **molar mass** is defined as the weight of one mole (or 6.02×10^{23} molecules) of any chemical compound (6.02×10^{23} is the Avogadro constant.) In simple terms, it is the molecular weight expressed in grams.

In my example, since the molecular weight of NaCl is 58.5, 1gram mole (gmol) of NaCl will be 58.5grams.

A 1 Molar Solution is a solution obtained by dissolving 1gmol of solute in 1 Litre of solvent. Therefore, a 1 Molar Solution of NaCl is 58.5grams of NaCl dissolved in 1 Litre of water.

Using the same logic, A 2 Molar Solution of NaCl is obtained by dissolving $58.5 \times 2 = 117$ grams of NaCl in 1 Litre of water. Similarly, $58.5 \times 3 = 175.5$ grams of NaCl can be dissolved in 1 Litre of water to get a 3 Molar Solution.

(Wikipedia, Molar concentration, 2013) (Bigelow, 2010)

The Experiment

In this experiment, a laser was shone at an angle through water with different salinities, and the displacements of the beam were measured. I used Water and 1, 2, 3, and 4 Molar Solutions of salt for this experiment.

Materials

Item	Specification	Qty
Wood (LxWxH)	9.5cm X 51cm X 2cm 61cm X 23.5cm X 2cm	1 4
	92cm X 6.3cm X 3.7cm 45cm X 6.2cm X 4cm	2 4
	2 cm	8
L Brackets	2.7 cm	24
	6.2 cm	5
	7.6 cm 2.3 cm	2 8
	2.5 CIII	8
Screws	2.5 cm	For all L- brackets
Drafting Machine	Vemco Model: 3300	1
Laser Pen	Wavelength = 650nm; power <5mW (Class IIIa FDA spec)	1
Steel Strapping	1 cm wide	1 roll
Acrylic tank with parallel sides	25.5cm X 9.5cm X 25cm	1
Grid paper	1 mm grid, A4 or letter size	1
Clipboard with frame cutout	23cm X 23cm Hardboard	1
Clips to hold grid paper to		2
clipboard		
Salt (NaCl)	Food grade	468g (8 Moles)
Water	Food grade	Up to 3L
Mirror	15cm X 18cm	1
Mirror	8cm X 8cm	1

Procedure:

Step A: Infrastructure

- 1) Use a flat wooden plank which will be used to keep the drafting machine, the tank and the graph paper on, so that they will all be in the same X-Z plane
- 2) Ensure that the wooden plank is perfectly flat by moving a spirit level across its surface in multiple directions. (It is not important that the wooden plank is perfectly horizontal for this experiment, however it is important for the wooden plank to be perfectly flat.)
- 3) Firmly mount the drafting machine on one side of the wooden plank
- 4) Firmly fix the laser on the scale of the drafting machine so that the angle of the laser will always stay the same when the scale is moved
- 5) Use the drafting machine to draw parallel lines and align the sides of the tank and the clipboard with those lines so that they are all parallel to each other
- 6) Use L-brackets to keep the tank and the clipboard fixed onto the wooden plank
- 7) Draw the axes on the graph paper and clip it onto the modified clipboard, which works as a frame for the graph paper.
- 8) For the mirror experiment connect two mirrors onto the tank using clips. (See Step G)

Step B: Initial Settings

- 1) Ensure that the sides of the tank are aligned with the lines made earlier by the drafting machine. The tank should be firmly sitting between the L-brackets without any movement.
- 2) Make sure that the plane of the graph paper is accurately aligned with the lines made earlier by the drafting machine, so that it is parallel to the sides of the tank.
- 3) Make sure that the X axis of the graph paper is parallel to the wooden plank.

- 4) Mark the side of the tank that is aligned with the L-bracket on the right. Use this mark to make sure that the tank has the same orientation every time it is removed and placed back between the L-brackets.
- 5) Mark a small square (approx. 2cm X 2cm) on the tank using a dry-erase marker. The laser beam should always pass through this square. If the laser falls on the same point on the tank every time, any fine curvature on the surface of the acrylic tank will equally apply to all experiments and therefore cancel out.

Step C: Align laser beam to normal

- 1) Loosen the protractor brake wing nut (bottom nut) and align the 0 of the vernier with the 90° mark of the circular protractor. Then tighten the nut.
- 2) Loosen the base-line wing nut (top nut) and keep the indexing thumb piece at the bottom position. Then set the scale approximately perpendicular to the tank.
- 3) Turn the laser on. Check if there is a horizontal displacement on the graph, when you remove the tank.
- 4) Place the tank back and use the top knob to adjust the angle to the opposite direction of the displacement.
- 5) Check if there is a displacement for the laser point again, when you remove the tank.
- 6) Repeat steps C-4 and C-5 until there is no displacement for the laser beam. At this position, the laser beam is perfectly perpendicular to the surface of the tank. Secure it by tightening the base-line wing nut (top nut).
 - From this position (normal), the laser can be turned to any desired angle accurately by loosening the protractor brake wing nut (bottom nut), aligning the vernier to the exact angle on the protractor, and then tightening the nut.

Step D: Inclining Laser beam to 30°

1) As explained above, loosen the protractor brake wing nut (bottom nut), align the 0 of the vernier with 60° (90°-30°=60° so it is 30°) on the protractor and tighten the nut. Now the laser beam is at an angle of 30° from the normal.

Step E: Making Molar Solutions with Salt

- 1) Take 58.5g of salt and dissolve it in water.
- 2) Once it has dissolved, add more water so that it totals to 1L.

Step F: Experiment 1 (without mirrors)

- 1) Shine the laser beam at 30° through the empty tank
- 2) Note the point, where the laser beam falls, on the graph
- 3) Remove the tank, fill it with water and place the tank back between the L-brackets
- 4) Note the point, where the laser beam falls, on the graph
- 5) Calculate the horizontal distance between the above points and record it as the displacement.
 - There could also be a small vertical displacement for the laser point. This can be ignored because the vertical displacement does not affect the horizontal displacement. All our observations and calculations are confined to the X-Z plane.
- 6) Repeat steps F-3, F-4, and F-5 for 1, 2, 3, and 4 Molar Solutions.
- 7) Calculate the angle of refraction using the equation for d₁
- 8) Calculate the refractive index using Snell's Law and record the value

Step G: Experiment 2 (with mirrors)

- 1) Attach two mirrors to each side of the tank using clips. The small mirror should face the laser beam and the large mirror should face the clipboard
- 2) To check if the laser beam only reflects once off of each mirror, move a folded piece of paper between both mirrors and check how many times a laser beam is shown on the folded paper. It should only be shown three times. (Refer to diagram with mirrors). If it is reflecting more than three times, adjust the position of the mirrors.

3) Repeat all of Step F, but use the equation for d₃ to calculate the angle of refraction

Data and Observations:

Experiment 1

	Medium									
	Air	Water	1 Molar Aqueos Solution	2 Molar Aqueos Solution	3 Molar Aqueos Solution	4 Molar Aqueos Solution				
Coordinate (x, y)	12.55, 1.15	11.05, 1.15	11.00, 1.15	10.95, 1.15	10.9, 1.15	10.85, 1.15				
X-Coordinate (of P)	12.55	11.05	11	10.95	10.9	10.85				
Horizontal Displacement (d ₁ = Q-P) (cm)	0	1.5	1.55	1.6	1.65	1.7				

Experiment 1.1

	Medium								
	Air	Water	1 Molar Aqueos Solution	2 Molar Aqueos Solution	3 Molar Aqueos Solution	4 Molar Aqueos Solution			
Coordinate (x, y)	9.0, 1.15	7.5, 1.15	7.45, 1.15	7.4, 1.15	7.35, 1.15	7.3, 1.15			
X-Coordinate (of P)	9	7.5	7.45	7.4	7.35	7.3			
Horizontal Displacement $(d_1 = Q-P)$ (cm)	0	1.5	1.55	1.6	1.65	1.7			

Experiment 1.2

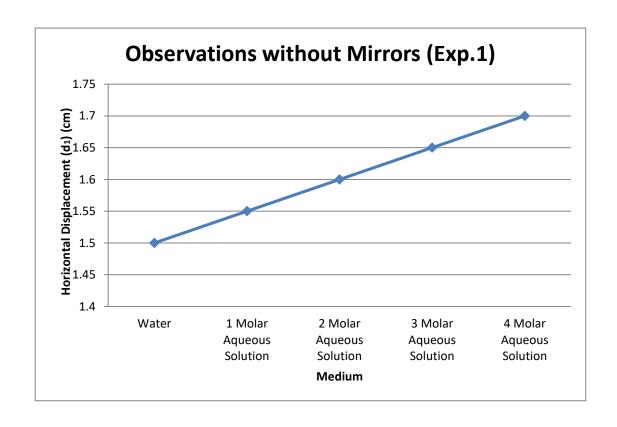
	Medium									
	Air	Water	1 Molar Aqueos Solution	2 Molar Aqueos Solution	3 Molar Aqueos Solution	4 Molar Aqueos Solution				
Coordinate (x, y)	8.25, 1.15	6.75, 1.15	6.7, 1.15	6.65, 1.15	6.6, 1.15	6.55, 1.15				
X-Coordinate (of P)	8.25	6.75	6.7	6.65	6.6	6.55				
Horizontal Displacement (d ₁ = Q-P) (cm)	0	1.5	1.55	1.6	1.65	1.7				

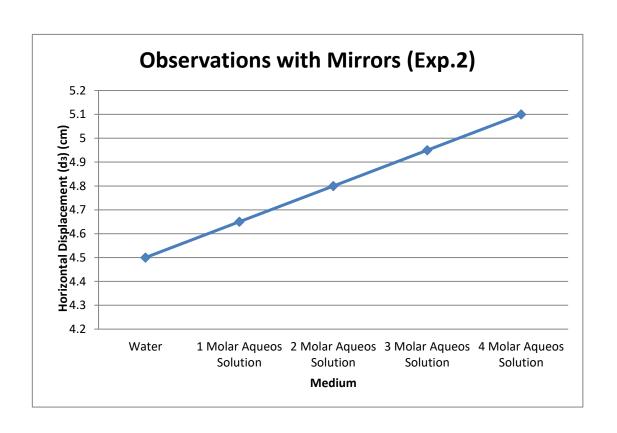
Experiment 2

	Medium							
	Air (Including Pot)	Water	1 Molar Aqueos Solution	2 Molar Aqueos Solution	3 Molar Aqueos Solution	4 Molar Aqueos Solution		
Coordinate (x, y)	15.7, 3.5	11.2, 4.3	11.05, 4.35	10.9, 5.4	10.75, 5.1	10.6, 5.0		
X-Coordinate (of T)	15.7	11.2	11.05	10.9	10.75	10.6		
Horizontal Displacement (d ₃ = U-T) (cm)	0	4.5	4.65	4.8	4.95	5.1		

Experiment 2.1

	Medium								
	Air (Including Pot)	Water	Water 1 Molar Aqueos 2 Solution 2		3 Molar Aqueos Solution	4 Molar Aqueos Solution			
Coordinate (x, y)	14.7, 3.5	10.2, 4.3	10.05, 4.4	9.9, 5.5	9.75, 5.15	9.6, 5.0			
X-Coordinate (of T)	14.7	10.2	10.05	9.9	9.75	9.6			
Horizontal Displacement $(d_3 = U-T)$ (cm)	0	4.5	4.65	4.8	4.95	5.1			



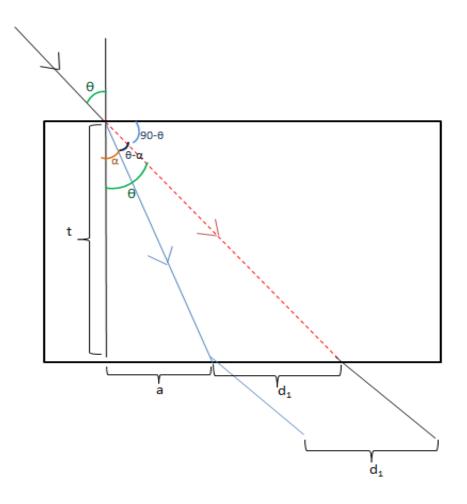












Dervation of α from d_1

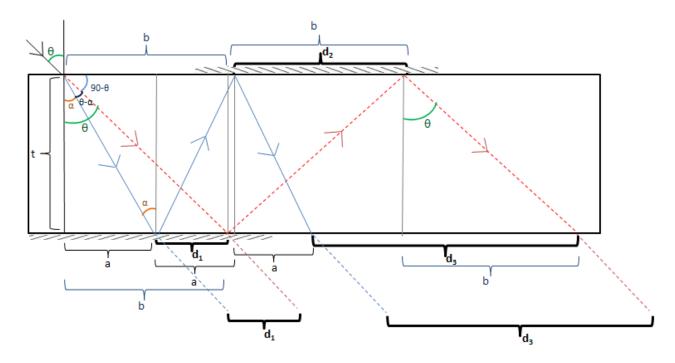
$$\tan \alpha = a/t$$
 $\tan \theta = (a+d_1)/t$
 $a = t \tan \alpha$ $a+d_1 = t \tan \theta$

$$a = t tan\alpha$$

 $a+d_1 = t tan\theta$

$$\begin{aligned} &d_1 = t \tan\theta - t \tan\alpha \\ &d_1 - t \tan\theta = - t \tan\alpha \\ &(d_1 - t \tan\theta)/-t = \tan\alpha \\ &\tan^{-1}[(d_1 - t \tan\theta)/-t] = \alpha \end{aligned}$$

$$\alpha = \tan^{-1}[(d_1 - t \tan\theta)/-t]$$



Dervation of α from d_3

```
a = t tanα
```

a+d=b

 $b = t tan\theta$

$$d_1 = b-a$$

 $d_1 = t tan\theta - t tan\alpha$

$$d_2 = 2b - 2a$$

$$d_2 = 2(b - a)$$

 $d_2 = 2(t \tan \theta - t \tan \alpha)$

$$d_3 = 3b - 3a$$

$$d_3 = 3(b - a)$$

 $d_3 = 3(t \tan \theta - t \tan \alpha)$

 $d_3 = 3t \ tan\theta - 3t \ tan\alpha$

 d_3 - 3t $tan\theta$ = -3t $tan\alpha$

 $(d_3-3t \tan\theta)/-3t = \tan\alpha$

 $tan^{-1}\left[(d_3-3t\,tan\theta)/-3t\right]=\alpha$

$\alpha = \tan^{-1} \left[(d_3 - 3t \tan \theta) / - 3t \right]$

Generalized Equation

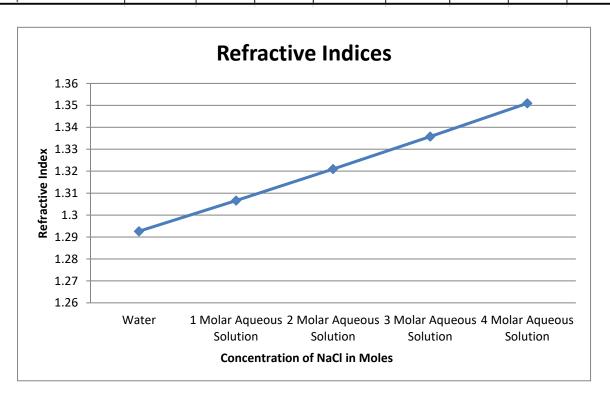
$$\overline{\alpha = tan^{-1} \left[\frac{d-nt \ tan\theta}{-nt} \right]}$$

Where n = number of straight line beams within the medium

Equation Caculating α from d ₁							
$\alpha = \tan^{-1} [(d_1 - t \tan \theta)]$	9)/-t]					
$\theta = 30^{\circ} =$	30						
t = 9.5cm =	9.5						
Medium	d ₁ =	d ₁	tanθ	t(tanθ)	d ₁ - t(tanθ)	[d ₁ - t(tanθ)]/-9.5	tan ⁻¹ {[d ₁ - t(tanθ)]/-9.5} (degrees) (α)
Water	d ₁ =	1.5	0.57735	5.484828	-3.984827557	0.419455532	22.75588264
1 Molar Aqueos Solution	d ₁ =	1.55	0.57735	5.484828	-3.934827557	0.414192374	22.4989638
2 Molar Aqueos Solution	d ₁ =	1.6	0.57735	5.484828	-3.884827557	0.408929217	22.24108706
3 Molar Aqueos Solution	d ₁ =	1.65	0.57735	5.484828	-3.834827557	0.403666059	21.98225758
4 Molar Aqueos Solution	d ₁ =	1.7	0.57735	5.484828	-3.784827557	0.398402901	21.72248068

Equation Cacul	atir	ng o	t from (<u>d</u> ₃			
$\underline{\alpha = \tan^{-1} \left[(d_3 - 3t \tan \theta) \right]}$	θ)/-3	3t]					
θ = 30° =	30						
t = 9.5cm =	9.5						
Medium	d ₃ =	d ₃	tanθ	3t(tanθ)	d ₃ - 3t(tanθ)	[d ₃ - 3t(tanθ)]/-3(9.5)	$tan^{-1}\{[d_3 - 3t(tan\theta)]/-3(9.5)\}$ (degrees) (α)
Water	d ₃ =	4.5	0.57735	16.45448	-11.95448267	0.419455532	22.75588264
1 Molar Aqueos Solution	d ₃ =	4.65	0.57735	16.45448	-11.80448267	0.414192374	22.4989638
2 Molar Aqueos Solution	d₃=	4.8	0.57735	16.45448	-11.65448267	0.408929217	22.24108706
3 Molar Aqueos Solution	d ₃ =	4.95	0.57735	16.45448	-11.50448267	0.403666059	21.98225758
4 Molar Aqueos Solution	d ₃ =	5.1	0.57735	16.45448	-11.35448267	0.398402901	21.72248068

Refractive Index C	alculatio	<u>on</u>						
Snell's Law:								
$-\sin heta_1 v_1 n_2 -$	θ_1 = angle of	incidence	= 30°					
	$\theta_2 = \alpha = angle$	e of refract	ion					
$-\frac{\sin\theta_2}{\sin\theta_2} - \frac{1}{v_2} - \frac{1}{n_1}$	v = velocity	of light res	pective to	medium (m/	/s)			
		_	-	medium (un				
$\theta_1 = 30^{\circ} =$	30							
$\theta_2 = \alpha$								
v ₁ = velocity of light in air =	298,925,574							
n ₁ = refractive index of air =	1							
Medium	α (θ2)	sin0₁	sinθ ₂	$sin\theta_1/sin\theta_2$	V ₂	v ₁ / v ₂	n ₂	Refractive Index
Water	22.7558826	0.5	0.386806	1.29263884	231252198	1.292639	1.292639	1.292638845
1 Molar Aqueos Solution	22.4989638	0.5	0.382667	1.30662001	228777740	1.30662	1.30662	1.306620014
2 Molar Aqueos Solution	22.2410871	0.5	0.378505	1.32098778	226289431	1.320988	1.320988	1.320987784
3 Molar Aqueos Solution	21.9822576	0.5	0.374319	1.33575743	223787319	1.335757	1.335757	1.335757429
4 Molar Aqueos Solution	21.7224807	0.5	0.370111	1.35094503	221271457	1.350945	1.350945	1.350945027



Derivation of Equation for the Above Curve

The above curve is a straight line. Therefore the following equation will apply:

$$y = mx + b$$

Where,

$$m = slope$$

$$b = y$$
 intercept

x (Molar Solution)	y (refractive index)
$1(\mathbf{x}_1)$	1.292639(y ₁)
$2(\mathbf{x}_2)$	1.30662001(y ₂)

Calculation of Slope and y-intercept

$$Slope = m = \frac{\Delta y}{\Delta x}$$

$$\Delta y = y_2 - y_1$$

$$\Delta x = x_2 - x_1$$

$$\Delta y = y_2 - y_1$$

$$\Delta y = 1.30662001 - 1.292639$$

$$\Delta y = 0.013981169$$

$$\Delta x = x_2 - x_1$$

$$\Delta x = 2 - 1$$

$$\Delta x = 1$$

$$\frac{\Delta y}{\Delta x} = 0.013981169$$

m = 0.013981169

$$b = y - mx$$

$$y = 1.292639$$

$$x = 1$$

m = 0.013981169

$$b = 1.27865767586534$$

$$y = mx + b$$

y = 0.013981169 x + 1.27865767586534

Shortening it,

$$y = 0.014 x + 1.279$$

Where,

x = Concentration of NaCl in Moles

y = Refractive index

Discussion

The y-intercept =1.279 is the refractive index for zero salinity or pure water. The actual refractive index of pure H2O for red light (650nm) is 1.331 (Polyanskiy, Refractive Index Database, 2012)

Therefore, the measured value is different by 3.91%. This is mainly due to three factors:

- 1) Total Dissolved Solids (TDS) that was already present in the water
- 2) Measurement errors
- 3) The exact wavelength of the Laser could be off by up to 30 nm from the Laser pen manufacturer's specification.

Results & Conclusion

Results

According to the observations, when the amount of salt in the water increased, the refractive index of the water also increased. Also, there was a linear change in the refractive indices of each medium.

The relationship between salinity and the refractive index can be expressed as

$$y = 0.014 x + 1.279$$

Where,

x = Concentration of NaCl in Moles

y = Refractive index

Conclusion

The hypothesis was correct. The refractive index of the water increased as the salinity of the water increased. The refractive index is directly proportional to the salinity.

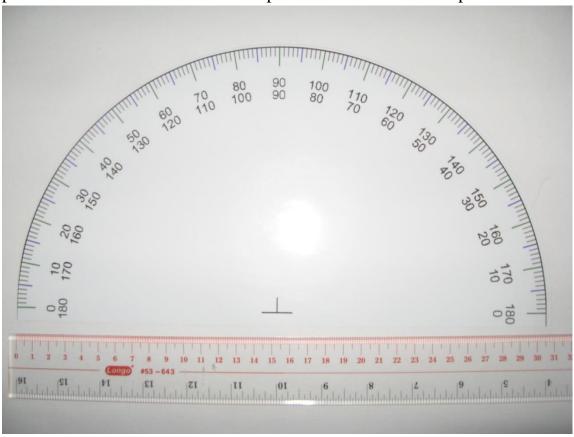
Problems Encountered

Keeping the Laser at a Specific Angle

One major issue was to set the laser beam perfectly perpendicular to the tank and then turn it to exact angles at various points. I wanted a standard method to do it so that there would not be errors from experiment to experiment.

The first thing I thought about was to use a large protractor.

Later I thought of a better method of making an enlarged printout of the image of a protractor. I made it so that I could paste it on to the wooden plank.



The first eureka moment of my project was when I got this idea of using a drafting machine. It is officially made to be set at specific angles so that when the scales are moved its angles are not changed. I bought a used Vemco drafting machine from craigslist. It even had a vernier scale for its protractor.

I could not only set accurate angles with it, but also use it to set my tank and graph paper exactly parallel to each other. It worked very well. I could easily mount my laser pen to its long scale.

Increasing the Accuracy of the Measured Displacement

The tank I used had a width of 9.5cm. This meant that the laser beam that is passing through the medium, even at an angle of 30° will not travel more than 11cm within the medium (considering a maximum possible angle of 30° the distance travelled in the medium will be $9.5/\cos 30 = 10.96$ cm. In fact, when the incident angle itself is set at 30° , the refracted angle will be less than that, further shortening this distance.)

If the beam could travel more distance through the medium, it will naturally result in bigger displacement on the graph. This in turn will increase the accuracy of my measurement (For e.g. a 1mm measuring error for a 1cm distance is 10%. A 1mm error for a 10cm distance is only 1 %.)

The obvious solution for this is to use a much wider tank. I was already using the widest possible tank.

This was the real eureka moment for me. I decided to use mirrors within the medium and bounce the beam a few times, that would double or triple the distance travelled. This is an increase of 200 % - 300 % accuracy for my experiment. It would equally increase the complexity of my geometry and therefore my calculations, but it was worth it.

As expected, there was a threefold increase in displacement when the beam was reflected twice within the medium.

Vertical Displacement

Keeping the laser beam perfectly parallel to the X-Y plane (horizontally), turned out to be difficult. The error created a vertical displacement. My first reaction was to use a polarized glass (used to watch 3D movies) so that I could polarize the laser beam vertically and eliminate the vertical displacement. It did not work.

However, on analysis my method of working turned out to be helpful. Using the drafting machine, I was only setting the horizontal angle which is technically the angle between the beam and the normal in the X-Z plane. There could also be a vertical angle for the beam in the vertical plane (Y-Z plane). The final angle is actually the resultant of these two angles.

Consequently, the displacement of the point on the graph sheet can also be resolved into a horizontal component (X direction) and a vertical component (Y direction).

From the above, it follows that, since I am considering only the horizontal angle of the beam, we need only consider the horizontal displacement on the graph sheet.

Fineness of the Laser Point

My laser pointer worked well however the laser beam turned out to be too wide as it made a big spot on my graph paper. This created an accuracy problem so I used a piece of tinfoil with a pinhole, to make the laser beam narrow until I got a spot that was less than a millimeter in size on my graph paper.

Temperature Dependence of Refractive Index

Refractive index changes with temperature. It ensued that I should accurately measure the temperature of the medium and incorporate it into my experiment. To begin with, I measured the refractive index of very cold water and that of very hot water. However, I did not get an observable difference. On researching scientific literature, I found out that the variation of refractive index with temperature is very small. Since I was measuring everything at room temperature, the range of temperature variation itself will be within 5°C and therefore the change in refractive index would be negligibly small.

Therefore, I did not monitor the temperature.

Further Improvements Possible

Vertical Slit

Instead of using a single dot of a laser ray, I would modify it to have a vertical line of laser. It can be made thinner. Since I am not measuring vertical displacements, a vertical slit would be ideal for accurate measurements.

Overcome the Opacity of the Liquid

As the solution becomes more concentrated, its opacity also increases. This poses a problem because the laser ray gets attenuated too much during its travel in the liquid that it does not reach the screen.

There are two ways to counter this

a) Increase the intensity of the laser

This is a simple solution

b) Reduce the travel distance inside the solution

This is a better solution; however it is a little complex. When the travel distance is reduced in a liquid, the attenuation can be reduced but it also reduces the displacement which in turn reduces the accuracy. The solution for this is to use more precise measuring methods such as using a microscope to measure the displacement.

In fact, **electronic microscopes** (refer to figure 9) are available that can be connected a computer through the USB port. It will help to get a magnified picture of the graph and the displacement on a laptop screen. Please refer to the next section - "Extended Experiment"

Refining the Equipment

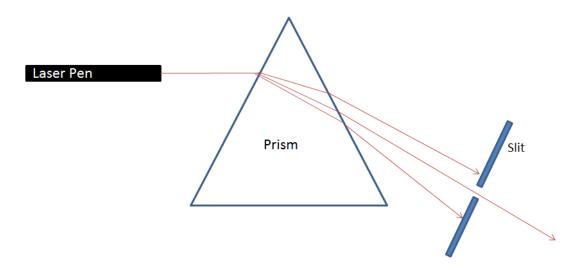
I will make my device more compact so that it will be portable. Incorporating the above improvements and adding the USB microscope with a laptop will improve the accuracy of measurement. The USB microscopes that I have looked at have magnification factors around 200X. This can make my equipment professional enough to be used for several practical applications.



Figure 9: USB microscope (PClaunches, 2008; PClaunches, 2008)

Use of a Prism at Source

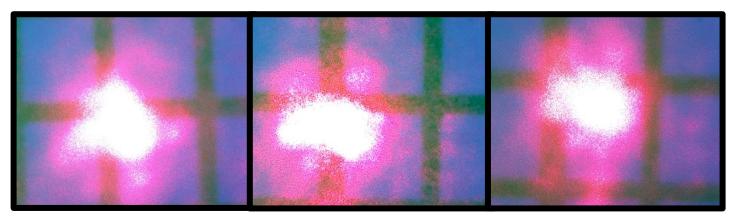
Since I had used a commercial laser pen, I found it to undergoing slight dispersion in the medium. This proves that the laser beam was not perfectly monochromatic. The specifications of the laser beam manufacturer said that the wavelength could vary from 630-680nm. My method of overcoming this will be to use a prism in front of the laser beam. The prism would disperse the all the wavelengths. I can then use a slit after the prism and choose the right wavelength to pass through the slit as shown below.



Extended Experiment

Improving Accuracy Using a Microscope

In order to improve the accuracy, I decided to use a microscope to measure the displacement on the graph. The most practical way to do this is to use a USB microscope which can be mounted horizontally. It will give a magnified picture of the object on the computer screen. I used a Celestron USB microscope which has a maximum magnification factor of 150X. I chose this because it also came with a software that had a measuring feature which could measure up to 0.01mm. I also fabricated a wooden stand that can hold the microscope steadily against the graph sheet.

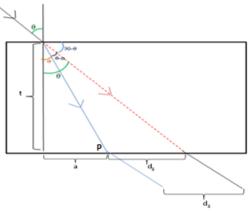


I took the coarse readings (up to the nearest millimeter which is the grid size of the graph sheet) without the microscope. Then I used the microscope to take the fine readings. As said above, it measured the distance between the right edge of the laser point and the nearest gridline. Due to time constraints, I was only able do the experiment for plain water and 1 Molar Solution of NaCl. The readings and calculations are given below.

Observations With Microscope (Experiment 3)

May 3, 2013





Medium - Water								
	Reading with Air (mm)			Readir	ng with r	nedium	Disabassassas	
	Course	Fine	Final (F1)	Course	Fine	Final (F2)	Displacement for medium (mm) (F1-F2)	
X-Coordinate (of P)	113	0.20	113.20	97	-0.16	96.84	16.36	

Medium - 1 Molar Aqueous Solution of NaCl								
	Reading with Air (mm)			Reading with medium			Di1	
	Course	Fine	Final (F1)	Course	Fine	Final (F2)	Displacement for medium (mm) (F1-F2)	
X-Coordinate (of P)	114	0.53	114.53	97	0.60	97.60	16.93	

Refraction due to Pot

The refraction due to the pot is constant for every observation. Therefore, it cancels out when we take the difference between two observations.

Note:

Displacement along the Y-axis is ignored for all observations since:

- 1) it does not affect the displacement on the X-axis
- 2) All angles are measured along the X-Z plane

Equation Caculating α from d₁

$\alpha = \tan^{-1}[(d_1 - t \tan\theta)/-t]$

θ = 30° =	30
t = 9.5cm =	9.5

Medium	d ₁ =	d1 (mm)	d ₁	tanθ	t(tanθ)	d ₁ - t(tanθ)	[d ₁ - t(tanθ)]/-9.5	$tan^{-1}\{[d_1 - t(tan\theta)]/-9.5\}$ (degrees) (α)
Water	d ₁ =	16.36	1.64	0.57735	5.48483	-3.84882756	0.405139743	22.05482557
1 Molar Aqueos Solution	d ₁ =	16.93	1.69	0.57735	5.48483	-3.79182756	0.399139743	21.75890628

Refractive Index Calculation

Snell's Law:

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

 θ_1 = angle of incidence = 30°

 $\theta_2 = \alpha$ = angle of refraction

v = velocity of light respective to medium (m/s)

n = refractive index respective to medium (unitless)

$\theta_1 = 30^{\circ} =$	30
$\theta_2 = \alpha$	
v ₁ = velocity of light in air =	298,925,574
n ₁ = refractive index of air =	1

Medium	α (θ ₂)	sinθ ₁	sinθ₂	$sin\theta_1/sin\theta_2$	V ₂	v ₁ /v ₂	n ₂	Refractive Index
Water	22.0548256	0.5	0.375494	1.33158051	224489299	1.331581	1.331581	1.331580506
1 Molar Aqueos Solution	21.7589063	0.5	0.370702	1.34879298	221624503	1.348793	1.348793	1.34879298

The same method can be directly applied to measure displacement for higher concentrations as well as displacement for the experiments with the mirrors. With the microscope method, I was able to get the refractive index of water to be 1.3316.

Error Calculation

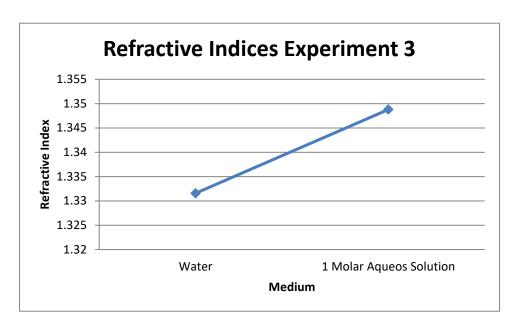
Measured Value of refractive index as calculated above = 1.3316

Actual refractive index of water for red light (650nm) = 1.331 (Polyanskiy, Refractive Index Database, 2012)

Error = 0.0006

Error (%) = $0.0006/1.331 \times 100 = 0.045\%$

Note: The error has reduced considerably from the previous value which was 3.91%



Derivation of Equation for the Above Curve

The above curve is a straight line. Therefore the following equation will apply:

$$y = mx + b$$

Where,

$$m = slope$$

$$b = y$$
 intercept

x (Medium)	y (refractive index)
$Water(x_1)$	1.331580506 (y ₁)
1 Molar Solution of NaCl(x ₂)	1.34879298 (y ₂)

Calculation of Slope and y-intercept

$$Slope = m = \frac{\Delta y}{\Delta x}$$

$$\Delta y = y_2 - y_1$$

$$\Delta x = x_2 - x_1$$

$$\Delta y = y_2 - y_1$$

$$\Delta y = 1.34879298 - 1.331580506$$

$$\Delta y = 0.017212474$$

$$\Delta x = x_2 - x_1$$

$$\Delta x = 1 - 0$$

$$\Delta x = 1$$

$$\frac{\Delta y}{\Delta x} = 0.017212474$$

$$m = 0.017212474$$

$$b = y - mx$$

$$y = 1.331580506$$

$$x = 0$$

$$m = 0.017212474$$

$$b = 1.331580506$$

$$y = mx + b$$

y = 0.017212474x + 1.331580506

Shortening it,

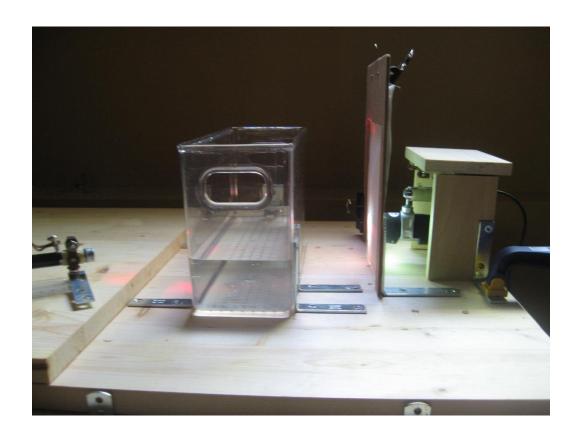
y = 0.017 x + 1.332

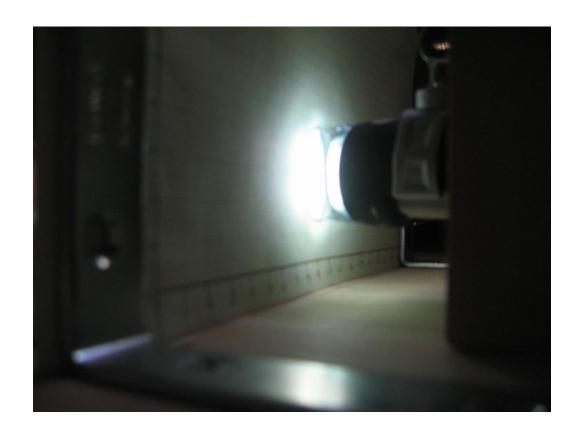
Where,

x = Concentration of NaCl in Moles

y = Refractive index







Practical Applications

Measurement of Salinity of Water

On reviewing scientific literature, I learned that refraction method is already used to measure the salinity of sea water. However for low salinities, such as 1% salt and less, the refraction method is still not used. (UniversityOfAlberta) We can use my method of measuring refraction for this.

With my innovation of using multiple mirrors within the solution, we can increase the displacement of the beam by several times. In the experiment that I did in this project, I had only used two mirrors. This, alone, increased the distance travelled by the beam, within the liquid, by three times. The corresponding increase in accuracy was 300%. We can easily design a gadget with more mirrors and increase the accuracy accordingly.

My next innovation was to use a microscope to further increase the accuracy of measurement. In my experiment, I used a cheap USB microscope with a maximum magnification factor of 150X. Combined with the accompanying software, the smallest distance it could measure was 0.01mm. Using a better microscope will further improve this accuracy.

By incorporating the above two improvements, my system can be used to measure very low levels of salinity.

The advantage of using the refraction method to measure Total Dissolved Solids (TDS)

The conventional method of measuring TDS is using a TDS meter which measures the conductivity of a solution. The problem with this method is that only ionized solids, such as salts and minerals, contribute to the conductivity of a solution. Dissolved organic solids such as sugars do not contribute to increase the conductivity in any significant manner. Therefore the TDS meter can only measure the mineral content. (Holmes-Farley, 2008)

Also, conductivity is much more temperature dependant than refractive index. (Alexey N. Bashkatov) (HAYASHI, 2003)

However, the refractive index is affected by minerals as well as sugars. Therefore my method of measurement will encompass minerals as well as sugars. As said above, its temperature variants are much less.

LOG

Date	Start Time	End Time	Activity
Feb - 16 - 2013	10:00:00 AM	5:00:00 PM	Made 1st Laser Mounting Machine
Feb - 16 - 2013	6:00:00 PM	7:00:00 PM	Tested Polarizing Lens (3D Lens)
Feb - 22 - 2013	4:00:00 PM	6:00:00 PM	Derived the equation to calculate the angle of refraction from d ₁
Feb - 24 - 2013	10:00:00 AM		Bought the drafting machine
Feb - 24 - 2013	12:00:00 PM		Made the Laser Mounting Machine
Feb - 26 - 2013	4:00:00 PM	6:00:00 PM	Found out that vertical displacement need not be measured and used a plank of wood to keep all materials in the same X-Z plane
Feb - 27 -2013	6:30:00 PM	7:00:00 PM	Found out that the glass of the frame around the grid paper will refract the ray
Mar - 5 -2013	4:00:00 PM	5:00:00 PM	Found how many grams of salt is in 1 Mole of salt
Mar - 5 -2013	5:00:00 PM	7:00:00 PM	Got salt for the experiment and measured 58.5g for several containers at pharmasave
Mar - 7 - 2013	5:00:00 PM	8:00:00 PM	Tested the equation using 2 Molar solution of NaCl and water
Mar - 11 - 2013	4:00:00 PM	6:00:00 PM	Wrote abstract for experiment
Mar - 11 - 2013	6:00:00 PM	7:00:00 PM	Found out that a mirror can be used for a larger displacement
Mar - 12 - 2013	4:00:00 PM	4:30:00 PM	Decreased the size of the laser beam using tinfoil
Mar - 12 - 2013	4:30:00 PM	10:00:00 PM	Did the experiment with 0-4 Molar solution of NaCl and water
Mar - 13 - 2013	5:00:00 PM	6:00:00 PM	Bought mirrors for the pot
Mar - 14 - 2013	5:00:00 PM	8:00:00 PM	Derived the equation to calculate the angle of refraction from d₃
Mar - 17 - 2013			Found out that a small mirror should be used and bought a small mirror
Mar - 17 - 2013			Tested the experiment with the mirrors
Mar - 18 - 2013			Typed, edited and made the good copy of the Procedure
Mar - 19 - 2013			Made the good copies of the equations and diagrams
Mar - 20 - 2013			Repeated Experiment 1 and recorded observations
Mar - 21 - 2013			Made the tables of the observations on Excel
Mar - 22 - 2013			Repeated Experiment 2 and recorded observations
Mar - 23 - 2013			Made the tables of the observations on Excel
Mar - 24 - 2013			Calculated the angles of refraction for all displacements of Experiment 1
Mar - 25 - 2013	12:00:00 PM	5:00:00 PM	Calculated the angles of refraction for all displacements of Experiment 2
Mar - 26 -2013	11:30:00 AM		Calculated the refractive index of 1-4 Molar solutions using Snell's Law
Mar - 27 - 2013	12:00:00 PM		Made graphs for the displacements and refractive index
Mar - 28 - 2013			Derived an equation to calculate the refractive index
Mar - 29 - 2013			Wrote the results and conclusion of the experiment
			Found out about TDS and researched on the topic
Mar - 30 - 2013			Made the booklet on the experiment and researched many topics
Mar - 31 - 2013			Further improved booklet and learned much of the theory with my Dad
	10:00:00 AM		Wrote about theory in the research booklet
Apr - 2 - 2013	7:00:00 PM		Wrote about theory in the research booklet
Apr - 4 - 2013	6:00:00 PM		Improved theory and learned more about the theory
Apr - 6 - 2013	1:00:00 PM		Proofread the booklet
Apr - 7 - 2013	3:00:00 PM		Proofread and printed both booklets
Apr - 8 - 2013			Made the science fair poster board
Apr - 9 - 2013	4:30:00 PM		Made the small model of machine for the science fair
Apr- 9 - 2013			Prepared speaking for the science fair Prepared speaking for the science fair
Apr- 10 - 2013			
Apr - 18 - 2013			Bought USB microscope
Apr - 18 - 2013			Tested Microscope
May - 3 - 2013			Did Experiment 3 with microscope
May - 5 - 2013			Worked on Booklet
May - 5 - 2013			Worked on Booklet
May - 5 - 2013	10:30:00 PM	11:30:00 PM	Worked on Booklet

Acknowledgements

- 1) Sunny Prabhakar (my dad) gave general guidance and helped with fabrication
- 2) Hanna from Pharmasave helped me to weigh exact quantities of salt using her precision weighing scale
- 3) Joe Francis helped print out the enlarged image of the protractor
- 4) Mr. Williamson, my science teacher, was my sponsor teacher

Bibliography

- Alexey N. Bashkatov, E. A. (n.d.). Water refractive index in dependence on temperature and wavelength: a simple approximation. Retrieved May 6, 2013, from optics.sgu.ru: http://optics.sgu.ru/_media/optics/staff/bashkatov/bashkatov_spie_03_5068_393.pdf
- BBC. (2011). *Optical fibres*. Retrieved March 31, 2013, from bbc.co.uk: http://www.bbc.co.uk/schools/gcsebitesize/science/add_ocr_pre_2011/wave _model/waveinformationrev2.shtml
- Bigelow, P. (2010). *How to make standard Solutions for chemistry*. Retrieved March 31, 2013, from scn.org: http://www.scn.org/~bh162/making_molar_&_normal_solutions.pdf
- Encyclopedia, B. (n.d.). *Encyclopedia Britannica*. Retrieved March 31, 2013, from Refractive index: http://www.britannica.com/EBchecked/topic/495677/refractive-index
- Feynman, R. (n.d.). *Optics: The Principle of Least Time*. Retrieved March 31, 2013, from physics.uci.edu:
 http://www.physics.uci.edu/~demos/pdf/optics/feynman_fermat_least_time.
 pdf
- HAYASHI, M. (2003, June 23). TEMPERATURE-ELECTRICAL

 CONDUCTIVITY RELATION OF WATER FOR ENVIRONMENTAL

 MONITORING AND GEOPHYSICAL DATA INVERSION. Retrieved May 6,
 2013, from bgs.ucalgary.ca:

 http://bgs.ucalgary.ca/files/bgs/hayashi_2004.pdf
- Holmes-Farley, R. (2008). *What is TDS?* Retrieved May 6, 2013, from reefkeeping.com: http://www.reefkeeping.com/issues/2004-04/rhf/feature/

- Madigan, J. (2011, November 15). What Wavelength Goes With a Color?

 Retrieved March 31, 2013, from science-edu.larc.nasa.gov: http://science-edu.larc.nasa.gov/EDDOCS/Wavelengths_for_Colors.html#yellow
- Muldoon. (2001, August 22). *Definition and Properties of Laser Light*. Retrieved April 1, 2013, from muldoonshealthphysicspage.com: http://muldoonshealthphysicspage.com/definition_and_properties_of_las.ht m
- PClaunches. (2008, October 25). *USB Microscope lets you take 200x object pictures*. Retrieved April 1, 2013, from pclaunches.com: http://www.pclaunches.com/other-stuff/usb_microscope_lets_you_take_200x_object_pictures.php
- Polyanskiy, M. (2012). *Refractive index database*. Retrieved April 1, 2013, from refractive index.info: http://refractiveindex.info/?group=LIQUIDS&material=Water
- Polyanskiy, M. (2012). *Refractive Index Database*. Retrieved April 1, 2013, from /refractiveindex.info: http://refractiveindex.info/?group=LIQUIDS&material=Water
- Sabanski, C. (n.d.). *Refraction of light*. Retrieved March 31, 2013, from The Sundial Primer: http://www.mysundial.ca/tsp/refraction_of_light.html
- Sherwood, B. A. (2002). *Refraction*. Retrieved March 31, 2013, from http://matterandinteractions.org:
 http://matterandinteractions.org/Content/Articles/Refraction.pdf
- UniversityOfAlberta. (n.d.). *Total Dissolved Solids*. Retrieved May 6, 2013, from ualberta.ca: http://www.ualberta.ca/~edtechpd/documents/activity_probes_procedurediss olvedsolids_boora_v1.pdf
- Wikipedia. (2013, March 6). *Cauchy's Equation*. Retrieved March 31, 2013, from wikipedia.ca: http://en.wikipedia.org/wiki/Cauchy's_equation
- Wikipedia. (2013, March 23). *Dispersion(optics)*. Retrieved March 31, 2013, from wikipedia.ca: http://en.wikipedia.org/wiki/Dispersion_(optics)

- Wikipedia. (2013, March 22). *Fermat's Principle*. Retrieved March 31, 2013, from wikipedia.org: http://en.wikipedia.org/wiki/Fermat's_principle
- Wikipedia. (2013, March 28). *Molar concentration*. Retrieved March 31, 2013, from wikipedia.org: http://en.wikipedia.org/wiki/Molar_concentration
- Wikipedia. (2013, March 25). *Phase Velocity*. Retrieved March 31, 2013, from wikipedia.org: http://en.wikipedia.org/wiki/Phase_velocity
- Wikipedia. (2013, March 23). *Refraction*. Retrieved March 31, 2013, from Wikipedia: http://en.wikipedia.org/wiki/Refraction
- Wikipedia. (2013, March 28). *Refractive Index*. Retrieved March 31, 2013, from wikipedia.org: http://en.wikipedia.org/wiki/Refractive_index
- Wikipedia. (2013, February 28). *Sellmeier equation*. Retrieved March 31, 2013, from wikipedia.ca: http://en.wikipedia.org/wiki/Sellmeier_equation
- Wikipedia. (2013, March 22). *Snell's Law*. Retrieved March 31, 2013, from wikipedia.org: http://en.wikipedia.org/wiki/Snell's_law
- Wikipedia. (2013, March 21). *Total Internal Reflection*. Retrieved March 31, 2013, from wikipedia.ca: http://en.wikipedia.org/wiki/Total_internal_reflection
- Wikipedia, C.-B.-S. (n.d.). *Phase velocity*. Retrieved March 31, 2013, from princeton.edu:
 http://www.princeton.edu/~achaney/tmve/wiki100k/docs/Phase_velocity.ht ml