EQUITY

Equity represents amount of money that would be returned to its owner (also called shareholders) if all the assets are liquidated & all of company's debt is paid off.

Equity = Total Assets − Total Liabilities.
↓

Used as capital raised by company. Investors seek out equity investment as it provides greater opportunity to share in the profits & growth of firm.

RER INSTRUMENTS :

(i) **STOCKS** : Smallest unit of equity which represent unit ownership in company (which has issued stocks)
Along with share in company (which itself holds a value), owner/holder of stock receives periodic cash payments (subjected to tax) called devident. (Generally paid at each quarter end, but for simplicity & consistency we will assume continuous yield)

Price (Worth of single stock in its domestic currency)
QUALITATIVE → Why is it not constant? Supply & Demand.
i.e. share prices rise & fall in response to investor's desire to either buy or sell.

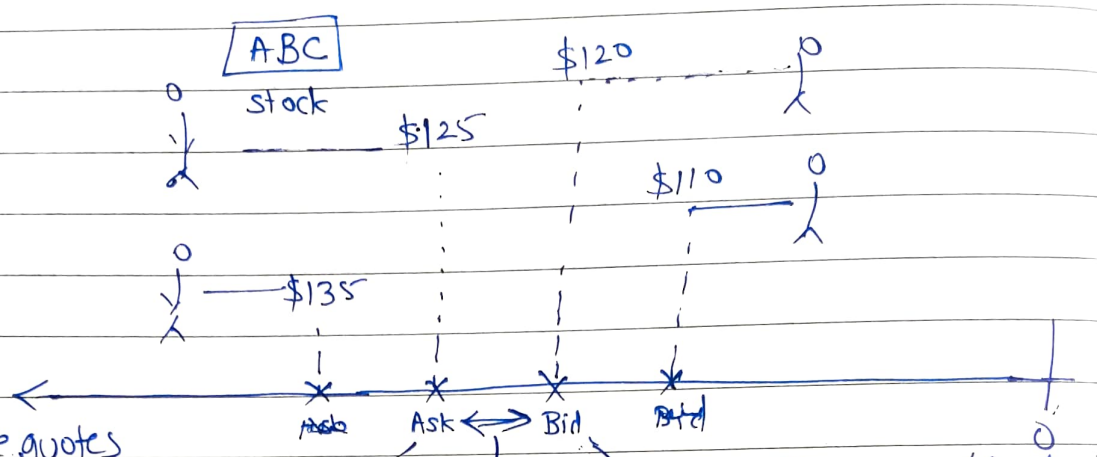
Factors that affect investor's desire:

i) Earning reports:
for example if a company misses its earning forcast then investors might selling their shares, since there will be lot of supply of stock & less demand, it will force people to sell at lower price & then prices will decrease

ii) Political/Regulatory changes:
for example introduction of tarrifs, interest rate hike

iii) News/Press/Economy etc.

Supply & demand are represented by bid & ask. in Market



price quotes (Market Makers (m.m.)) Sellers ('Ask' for price to sell)

lowest amount seller is willing to sell.

Bid-Ask Spread ~ $5

Bid / Ask

∴ Price Quote: 120/125

Buyers (traders) ('Bid' for price to buy)

highest amount bidder is willing to pay

(traders

QUANTITATIVE:

Let's represent price (worth) of a stock by S_t at time t (starting to $t=0$, representing today), the SDE that can best represents a S_t is given by

$$\frac{dS_t}{S_t} = \mu_t\,dt + \sigma_t\,dW \qquad \frac{d\tilde{S}_t}{\tilde{S}_t} = (\mu_t + \delta_t)\,dt + \sigma_t\,dW_t \quad \text{——— (1)}$$

but tradable in market is $\tilde{S}_t$

$$\frac{d\tilde{S}_t}{\tilde{S}_t} = \frac{dS_t}{S_t} + \delta_t\,dt$$

represents the growth in company

μ_t → yield

δ_t → dividend → fixed payment (continuously reinvested in buying new stocks)

σ_t → volatility → bid-ask spread

If we plot dS_t/S_t for historic t, we find distorted Normal distribution. (The reason its distorted is because it implies that μ_t, δ_t or σ_t are stochastic as well)

~~Solving above SDE using Euler stepper~~

This is real world measure (W_t is B.M. in R measure) but we know from financial pricing theorem (fair valuation) that it needs to be in measure Q which risk neutral & that make $Z_t = \tilde{S}_t N_t^{-1}$ a martingale. (N_t is numeraire, in which we are pricing S_t), for our case we will assume. $N_t = B_t = B_0 e^{\int_0^t r_t dt}$ (cash bond)

$$\hookrightarrow \frac{dB_t}{B_t} = r_t dt \Rightarrow B_t = B_0 e^{\int_0^t r_t dt}$$

Solving (1) to get process S_t:

we know

$$d(\ln S_t) = \frac{1}{S_t} d\tilde{S}_t - \frac{1}{S_t^2}(dS_t)^2 \quad \left(\text{substituting } \frac{dS}{S_t} \text{ from 1 in } 2^{nd} \text{ term}\right)$$

$$= \frac{d\tilde{S}_t}{\tilde{S}_t} - \frac{1}{2}\left((\mu_t + \delta_t)^2 \cancel{dt^2} + 2(\mu_t + \delta_t) dt\, \sigma\, \cancel{dW_t} + \sigma_t^2 \underbrace{dW_t^2}_{dt}\right)$$

(Ito's

$$= \frac{dS_t}{S_t} - \frac{\sigma_t^2}{2} dt$$

$$\therefore d(\ln \tilde{S}_t) = \left(\mu_t + \delta_t - \frac{1}{2}\sigma_t^2\right) dt + \sigma_t dW_t$$

integrating both side from 0 to t

$$\ln\left(\frac{\tilde{S}_t}{\tilde{S}_0}\right) = \int_0^t \left(\quad \right) dt + \int_0^t \sigma_t dW_t$$

$$\int_0^t \left(\mu_t + \delta_t - \frac{1}{2}\sigma_t^2\right) dt + \int_0^t \sigma_t \, dW_t$$

$$\tilde{S}_t = S_0 \, e^{\int_0^t \left(\mu_t + \delta_t - \frac{1}{2}\sigma_t^2 - r_t\right) dt + \int_0^t \sigma_t \, dW_t}$$

Let's call it X_t

$$Z_t = \tilde{S}_t \cdot B_t = S_0 B_0 \, e^{X_t}$$

$$dZ_t = S_0 B_0 \left(e^{X} \frac{dX}{\cancel{dt}} + \frac{1}{2} e^{X} (dX)^2\right) \quad \left(S_0 B_0 e^{X_t} = Z_t\right)$$

$$\therefore \frac{dZ_t}{Z_t} = \left(\mu_t + \delta_t - \frac{1}{2}\sigma_t^2 - r_t\right) dt + \frac{1}{2}\left(\sigma^2 \, dW^2\right) + \sigma \, dW_t$$

$$\frac{dZ_t}{Z_t} = (\mu_t + \delta_t - r_t) dt + \sigma_t \, dW_t$$

to make it martingale $\tilde{W}_t = W_t + \gamma_t t$.

Such that $\gamma_t = \dfrac{\mu_t + \delta_t - r_t}{\sigma_t}$

$$\therefore \frac{d\tilde{S}_t}{\tilde{S}_t} = (\mu_t + \delta_t) dt + \sigma_t \left(\tilde{W}_t - \left(\frac{\mu_t + \delta_t - r_t}{\sigma_t}\right) dt\right)$$

$$= r_t \, dt + \sigma_t \, d\tilde{W}_t$$

$$\therefore \boxed{\frac{dS_t}{S_t} = (r_t - \delta_t) dt + \sigma_t \, d\tilde{W}_t}$$

Stock price free from dividend (making it tradable in market)

while $\tilde{S}_t$ is process/quantity that is tradable in market as it includes dividend. In general we use dividend less stock price in all vanillas & derivatives.

$$S_t = S_0 \, e^{\int_0^t (r_t - \delta_t) - \frac{1}{2}\sigma_t^2 dt + \int_0^t \sigma_t \, d\tilde{W}_t}$$

CLAIMS/CONTRACTS

(i) FORWARD/FUTURE

Quantitative: A forward contract is an aggrement to buy or sell a security at a certain future time for certain price. At inception the forward ~~price~~ contract has zero value, i.e. it initially costs nothing to enter into a forward contract. Forward of OTC derivatives, Future is same as forward but traded in exchange (very standard).

Quantitative:

$$\text{Forward Contract}(\underset{\text{maturity}}{T}, \underset{\text{strike}}{K}) = (S_T - K)$$

Price at t

$$\text{Price}_t(F) = N_t^{-1} E_Q\left[N_T^{-1}(S_T - K) \mid \mathcal{F}_t \right]$$

$$N_T^{-1} S_T = S_0 B_0 e^{\underbrace{\int_0^T \delta_t dt - \frac{1}{2}\sigma_t^2 dt + \int_0^T \sigma_t dW_t}_{X \sim N\left(\int_0^T \delta_t dt,\ \int_0^T \sigma_t^2 dt\right)}}$$ assuming dividend = 0

$$=$$

Applying Identifying Normals; $E[e^X] = e^{\mu + \frac{1}{2}\sigma^2}$

$$= e^{\int_0^t r_t dt}\left(e^{\int_0^T \delta_t dt + \frac{1}{2}\int_0^T \sigma_t^2 dt - \frac{1}{2}\sigma_t dt} - K e^{-\int_0^T r_t dt} \right)$$

Since we have filtration $\mathcal{F}_t$, lets rewrite S_T as

$$S_T = S_t e^{\int_t^T (r_s - \delta_s - \frac{1}{2}\sigma_s^2) ds + \int_t^T \sigma_s dW_s}$$

$$B_T^{-1} \cdot B_t = e^{-\int_t^T r_s ds}$$

$$\therefore \text{First term, i.e. } S_T \cdot B_T^{-1} \cdot B_t = S_t e^{\int_t^T -\delta_s - \frac{1}{2}\sigma_s^2 ds + \int_t^T \sigma_s dW_s}$$

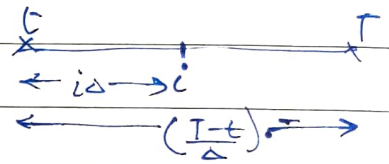
Given filteration $\mathcal{F}_t$, S_t is constant hence can be taken out.

$dW_s \sim N(0, ds)$ given $\mathcal{F}_t$ & $s > t$ (from properties of Brownian motion

$$\therefore \int_t^T \sigma_s \, dW_s \sim N\left(0, \int_t^T \sigma_s^2 \, ds\right)$$

Proof: $$\int_t^T \sigma_s \, dW_s = \lim_{\Delta \to 0} \sum_{i=t}^{t+(T-t)/\Delta} \sigma_{i\Delta} \underbrace{(W_{(i+1)\Delta} - W_{i\Delta})}_{\sigma_{i\Delta} N(0, \Delta)}$$

(by dividing line from t to T in Δ small intervals



$$\underbrace{\sum N(0, \sigma_{i\Delta}^2 \Delta)}_{\text{addition of Normal}}$$

$$\lim_{\Delta \to 0} N\left(0, \sum_{i=t}^{t+(T-t)/\Delta} \sigma_{i\Delta}^2 \Delta\right)$$

$$= N\left(0, \lim_{\Delta \to 0} \sum_{i=t}^{t+(T-t)/\Delta} \sigma_{i\Delta}^2 \Delta\right) = N\left(0, \int_t^T \sigma_s^2 \, ds\right)$$

$\therefore$ First term

$$E_Q\left[S_t e^{\int_t^T (\delta_s - \frac{1}{2}\sigma_s^2) ds + \int_t^T \sigma_s dW_s} \,\middle|\, \mathcal{F}_t\right]$$

(applying identities Normals $e^{\mu + \frac{1}{2}\sigma^2}$

$$= S_t \cdot e^{\int_t^T (\delta_t - \frac{1}{2}\sigma_s^2) ds + \frac{1}{2}\int_t^T \sigma_s^2 ds}$$

$$= S_t \cdot e^{-\int_t^T \delta_s ds}$$

Shortcut

$B_t E[B_T^{-1} \tilde{S}_T \mid \mathcal{F}_t] = B_t B_t^{-1} \tilde{S}_t$

is martingale

$= \tilde{S}_t$

$= S_t e^{-\int_t^T \delta_s ds}$

Second Term

$$E_Q\left[K e^{-\int_t^T r_s ds} \,\middle|\, \mathcal{F}_t\right] = K \cdot ZCB(t, T)$$

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$\therefore$ Price of forward at t expiring at T $= S_t e^{-\int_t^T \delta_s ds} - K\,ZCB(t,T)$

sp. case, at $t=0$ $\Rightarrow$ $Price(0) = S_0 - K\,ZCB(0,T)$

but we know forwards have zero price at $t=0$

$$\therefore \quad K = \frac{S_0}{ZCB(0,T)}$$

$ZCB(0,T)$ is defined at value of \$1 paid at T as of today (its known in the market & drives a short rate which constant

$$ZCB(0,T) = e^{-\int_0^T r_s ds} = \text{know time value of money}$$

So forwar price$_t$ $= S_t e^{-\int_t^T \delta_s ds} - \frac{S_0 ZCB(t,T)}{ZCB(0,T)}$

Note: all above derivation r_t & σ_t are still not stochastic otherwise we would need to take their joint probability distribution. While compute forwar we assume r_t & σ_t to be constant $\therefore K = S_0 e^{r\Delta}$

$\therefore$ Forwar price$_t$ $= S_t e^{-\int_t^T \delta_s ds} - S_0 e^{r\Delta}$ where $\Delta = T-t$

OPTIONS.

Quantitative: Contracts that gives owner right (but no obligation) to buy/sell underlying for predetermined price (also called strike.). Depending upon right to buy or sell there are two fundamental options type.

1. Call Options.

Definition

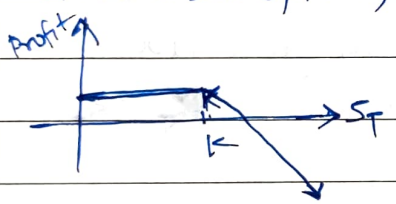
Seller $\xrightarrow{\text{sells}}$ $\boxed{\text{call option}}$ $\xrightarrow{\text{to}}$ Buyer.

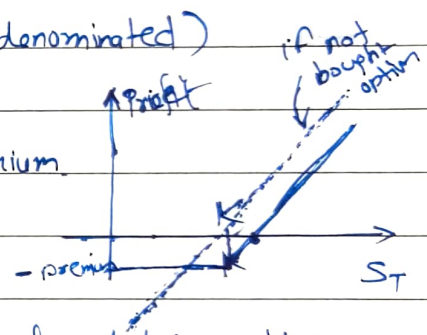
(T, K)

(Obligation to sell underlying at T for K if buyer executes the option)

in exchange of fair value of trade as of date of purchase ($t=0$) also called premium

(Gives right to buy underlying at T for K price in denominated)





Intentions (Profits)

1. Bets that price of underlying will not go above K (Bearish) and earns a premium at $t=0$.
   ∴ Max Profit = Premium.

1. Bets that price of underlying will go above K. (Bullish) & pays premium.
2. Protects buying power of underlying, i.e. incase prices of underlying shoots up by $t=T$, buyer wants to ensure he can still buy it at K.
   Max Profit = $(S_T - K)$ - premium

Losses if things go south (Loss)

If bearish position didn't work out & prices went up still seller has to sell same underlying to buyer as agreed at K & bears the loss of $(S_T - K)$
∴ Max Loss = $(S_T - K)$ - premium

In case price S_T didn't go above K at T then buyer will just let option expire (obv not going to buy at large K if market price is low)

Max loss = premium

* Based different strikes, call options can have diff worth.

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2. Put Options

Seller —sells.→ [Put option] —to→ Buyer

obligation to buy underlying if buyer choose to at T at K.

in exchange of premium

gives buy right to (but no obligation) to sell the underlying at T time at K.

Intensions (Profits)

Seller thinks underlying prices will go up (bullish)

max profit = premium.

1) Bets on prices going down (bearish) & allows selling at higher price K to buyer

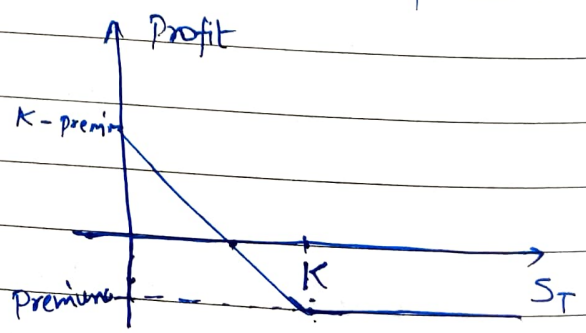
2.) Wants to protects selling power of underlying. (if he wants to ensure he can sell underlying at K even if market goes down)

Max profit = $(K_T - S_T)$ - premium

Losses

if underlying goes down than has to buy same underlying at higher cost.

max loss = $(K_T - S_T)$ + premium

in case price S_T does not go down then only premium is lost

max loss = premium.