Sequences, Series & Progressions

Series Name			
Taylor Series	One-dimensional function $f(x)$	$\sum_{i=0}^{\infty} \frac{f^{(i)}(a)}{i!} \times (x-a)^i$	$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(i)}(a)}{i!}(x-a)^i + \cdots + \infty$
	Maclaurin Series: Special case of Taylor with $a = 0$ for $f(x)$	$\sum_{i=0}^{\infty} \frac{f^{(i)}(0)}{i!} \times x^i$	$f(0) + \frac{f'(0)}{1!}\mathrm{x} + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(i)}(0)}{i!}\mathrm{x}^i + \cdots + \infty$
	Two-dimensional function $f(x,y)$	$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \frac{f^{(i)(j)}(a,b)}{i!\,\mathrm{j}!}(x-a)^i(y-b)^j$	$f(a,b) + \frac{f^x(a,b)}{1!}(x-a) + \frac{f^y(a,b)}{1!}(y-b) + \frac{f^{x^2}(a,b)}{2!}(x-a)^2 + \frac{f^{xy}(a)}{1!\,1!}(x-a)(y-b) + \frac{f^{y^2}(a,b)}{2!}(\mathrm{y}-\mathrm{b})^2 + \cdots + \frac{f^{(i)(j)}(a,b)}{i!\,\mathrm{j}!}(x-a)^i(y-b)^j + \cdots + \infty$
Trigonometric function	$sin(x)$	$\sum_{i=0}^{\infty} \frac{(-1)^{\mathrm{i}}}{(2\mathrm{i}+1)!} \times x^{(2i+1)}$	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \frac{x^{13}}{13!} + \cdots + \infty$
	$cos(x)$	$\sum_{i=0}^{\infty} \frac{(-1)^{\mathrm{i}}}{2\mathrm{i}!} \times x^{2i}$	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \cdots + \infty$
	$tan(x)$	NA	$x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots + \infty$
	$\frac{\sin\left(a + \frac{(n-1)h}{2}\right)\sin\left(\frac{nh}{2}\right)}{sin\left(\frac{h}{2}\right)}$	$\sum_{i=0}^{n-1} \sin\,(a+ih)$	$\sin(a) + \sin(a+h) + \sin(a+2h) + \cdots + \sin(a+(n-1)h)$
	$\frac{\cos\left(a + \frac{(n-1)h}{2}\right)\sin\left(\frac{nh}{2}\right)}{sin\left(\frac{h}{2}\right)}$	$\sum_{i=0}^{n-1} \cos\,(a+ih)$	$\cos(a) + \cos(a+h) + \cos(a+2h) + \cdots + \cos(a+(n-1)h)$

Exponential function	e^x	$\sum_{i=0}^{\infty} \frac{1}{\mathrm{i}!} \times x^i$	$1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \infty$
	$\log(1+x)$	$\sum_{i=1}^{\infty} \frac{(-1)^{\mathrm{i}}}{\mathrm{i}} \times x^i$	$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + \infty$
Algebraic functions	$\frac{n(n+1)}{2}$	$\sum_{i=0}^{n} i$	$1 + 2 + 3 + \cdots + (n-1) + n$
	$\frac{n(n+1)}{2} \times \frac{(2n+1)}{3}$	$\sum_{i=0}^{n} i^2$	$1^2 + 2^2 + 3^2 + \cdots + (n-1)^2 + n^2$
	$\left(\frac{n(n+1)}{2}\right)^2$	$\sum_{i=0}^{n} i^3$	$1^3 + 2^3 + 3^3 + \cdots + (n-1)^3 + n^3$
	$\frac{n(n+1)}{2} \times \frac{(2n+1)}{3} \times \frac{(3n^2+3n-1)}{5}$	$\sum_{i=0}^{n} i^4$	$1^4 + 2^4 + 3^4 + \cdots + (n-1)^4 + n^4$
	$\frac{(\sum_{i=0}^{n} a_i)^2 - \sum_{i=0}^{n} a_i^2}{2}$	$\sum_{i=0}^{n} \sum_{\substack{j=0 \\ i\,!=j}}^{n} a_i a_j$	$a_0a_1 + a_0a_2 + \cdots + a_1a_2 + a_1a_3 + \cdots + a_{n-1}a_n$
	$\frac{\pi^2}{6}$	$\sum_{i=1}^{\infty} \frac{1}{i^2}$	$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \cdots + \infty$
Factorial	$\mathrm{n}!$	$\prod_{i=1}^{n} i$	$1 \times 2 \times 3 \times \ldots \times (\mathrm{n}-1) \times \mathrm{n}$
	$2^{\mathrm{n}} \times \mathrm{n}!$	$\prod_{i=1}^{n} (2i)$	$\underbrace{2 \times 4 \times 6 \times \ldots \times (2\mathrm{n}-2) \times 2\mathrm{n}}_{\text{n terms}}$

			$\frac{2n!}{2^{n} \times n!}$	$\prod_{i=1}^{n}(2i-1)$	$\underbrace{1 \times 3 \times 5 \times \ldots \times (2n-1)}_{\text{n terms}}$
	Binomial Expansion		$(1+x)^{\alpha}$ α can be any complex number & $\lvert x \rvert < 1$	$\sum_{i=0}^{\infty} \frac{\alpha(\alpha-1)(\alpha-2)\ldots(\alpha-i+1)}{i!} x^{i}$	$1+\frac{\alpha}{1!}+\frac{\alpha(\alpha-1)}{2!}+\frac{\alpha(\alpha-1)(\alpha-2)}{3!}+\frac{\alpha(\alpha-1)(\alpha-2)(\alpha-3)}{3!}+\cdots+\infty$
			Sp. case: α is Positive Integer, i.e. $\alpha = n \, where \, n \in N$ $(1+x)^{n}$	$\sum_{i=0}^{n} {}_{i}^{n}C x^{i}$	${}_{0}^{n}C + {}_{1}^{n}C x^{1} + {}_{2}^{n}C x^{2} + {}_{3}^{n}C x^{3} + \cdots + {}_{n-1}^{n}C x^{n-1} + {}_{n}^{n}C x^{n}$
		Examples	Substitute $x = 1$ 2^{n}	$\sum_{i=0}^{n} {}_{i}^{n}C$	${}_{0}^{n}C + {}_{1}^{n}C + {}_{2}^{n}C + {}_{3}^{n}C + \cdots + {}_{n-1}^{n}C + {}_{n}^{n}C$
			Substitute $x = -1$ 0	$\sum_{i=0}^{n} {}_{i}^{n}C(-1)^{i}$	${}_{0}^{n}C - {}_{1}^{n}C + {}_{2}^{n}C - {}_{3}^{n}C + \cdots - {}_{n-1}^{n}C + {}_{n}^{n}C$
			Add above two and divide by 2. 2^{n-1}	$\sum_{i=0}^{n/2} {}_{2i}^{n}C = \sum_{i=0}^{\frac{n-1}{2}} {}_{2i+1}^{n}C$	${}_{0}^{n}C + {}_{2}^{n}C + \cdots + {}_{n}^{n}C = {}_{1}^{n}C + {}_{3}^{n}C + \cdots + {}_{n-1}^{n}C$
			Differentiate wrt to x and substitute $x = 1$ $n \times 2^{n-1}$	$\sum_{i=0}^{n} i \times {}_{i}^{n}C$	${}_{1}^{n}C + 2\,{}_{2}^{n}C + 3\,{}_{3}^{n}C + \cdots + (n-1)\,{}_{n-1}^{n}C + n\,{}_{n}^{n}C$
			Differentiate wrt to x and substitute $x = -1$ 0	$\sum_{i=0}^{n} (-1)^{i-1} \times i \times {}_{i}^{n}C$	${}_{1}^{n}C - 2\,{}_{2}^{n}C + 3\,{}_{3}^{n}C - \cdots + (n-1)\,{}_{n-1}^{n}C + n\,{}_{n}^{n}C$
			Multiply $(1+x)^{n}$ with $(1+x)^{n}$ and find coefficient of x^{n} i.e. coefficient of x^{n} in $(1+x)^{2n}$	$\sum_{i=0}^{n} {}_{i}^{n}C^{2}$	${}_{0}^{n}C^{2} + {}_{1}^{n}C^{2} + {}_{2}^{n}C^{2} + {}_{3}^{n}C^{2} + \cdots + {}_{n-1}^{n}C^{2} + {}_{n}^{n}C^{2}$

		Sp. case: α is Negative Integer, i.e. $\alpha = -n \; where \; n \in N$ $(1+x)^{-n}$	$\sum_{i=0}^{\infty} {}^{n+i-1}_{i}C(-x)^i$ i.e. ${}^{-n}_{i}C = {}^{n+i-1}_{i}C \; where \; n \in N$	${}^{n-1}_{0}C - {}^{n}_{1}Cx + {}^{n+1}_{2}Cx^2 - {}^{n+2}_{3}Cx^3 + \cdots + {}^{n}_{n-1}Cx^{n-1} + {}^{n}_{n}Cx^n$