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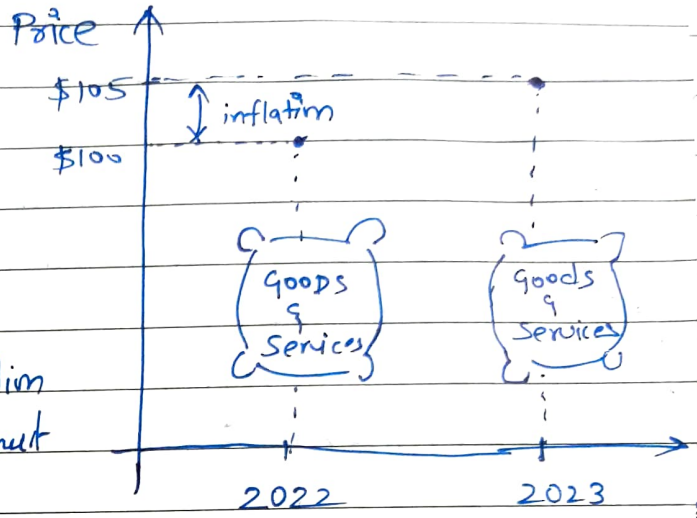
INFLATION.

Inflation is the rate at which general level of price of 'goods & services' is rising.

↓

includes diversified set of products as host of services for living comfortable life, forexample: food grain, metal, fuel, utilities like electricity/transportation & services like healthcare, entertainment & labor.



→ if prices increase then its positive inflation/ inflation otherwise negative inflation / deflation

→ With inflation, purchasing power of a currency decreases and vice versa w.r.t. deflation

→ Reasons for inflation

- Demand pull (as demand ↑↑ than production capacity)
- Cost push (Prices of raw material ↑↑ ∴ pushes price of final product)
- Built-in (ppl's expectation of inflation?)

Inflation Index — CPI (Consumer Price Index) — UK CPI, Japan CPI, Canada CPI

Inflation Index — RPI (Retail Price Index)

RPIX (Retail Price Index excluding Mortgage)

WPI (wholesale)

PPI (Producer)

instrument

Tradables

claims

Zero Coupon Inflation linked Bond

Inflation Linked Bond.

Inflation Swap. & YOY Inflation Swap

inflation option

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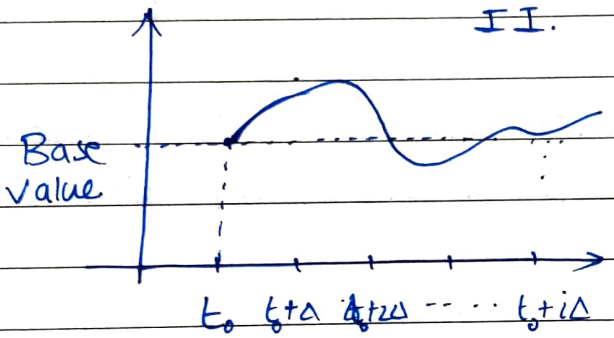
INSTRUMENTS.

INFLATION INDEX (II): Indicative measure of price of 'Goods & services' (G&S) in particular currency.

→ It is fixed to some base value ('100' unit currency) for the year-(month-date) it was first released/recognized by the country.

→ A country can have many II to indicate inflation on different sectors.



Base value: Value of II at t_0

t_0: Time of release

gap: Interval in which revised value of index will be released

Lag (L): Δ

interpolation scheme: interpolation scheme for querying II at any time t between 'gap' interval (most of time its linear)

$$II(t_0+i\Delta) = \frac{Price(G\&S)@(t_i+i\Delta)}{Price(G\&S)@t_i} \times \text{Base value.}$$

$II(t_0+i\Delta)$ → inflation index at $t_0+i\Delta$ time

$$II(t) = \text{interpolation}\left(II(t_0+j\Delta) \;\&\; II(t_0+(j+1)\Delta)\right)$$

t → any other time t between $t_0+j\Delta$ & $t_0+(j+1)\Delta$

→ Inflation Index (II_t) acts as conversion factor for real money into nominal, for example, if an investor gets \$1 at T in real denominated (taking inflation in account) then his actual claim is \$$II_T$ in nominal amount (assuming zero inflation)

TYPES OF INFLATION INDEX:

CONSUMER PRICE INDEX (CPI_t): Most popular index across countries & favoured by govt.

It measures the weighted average prices of basket of goods & services consumed by household at retail level

- Food/Beverages
- Fuel, Light
- Clothing/Housing
- Footwear

- Wage contracts
- allowance of govt employees.

WHOLESALE PRICE INDEX: Measures avg price of only good at the wholesale level

PRODUCER PRICE INDEX: Measure of inflation from producer's perspective.

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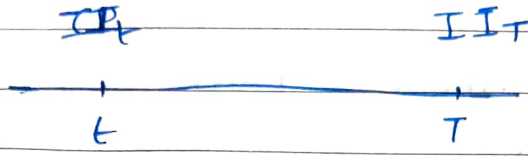
INFLATION RATE (f_t): Rate with which worth of currency changes assuming zero nominal (~~rate~~ risk-free interest) rate.



	Simple Compounded.	Continuous Compounded.
Spot Rate.	$F_s(t,T) = \frac{1}{(T-t)}\left(\frac{II_T - II_t}{II_t}\right)$	$F_c(t,T) = \frac{\ln\left(II_T / II_t\right)}{(T-t)}$

Instantaneous

$$f_t = \lim_{T \to t} \frac{\ln\left(\frac{II(T)}{II(t)}\right)}{(T-t)} = + \frac{\partial}{\partial t} \ln\left(\frac{II(t+dt)}{II(t)}\right)$$

REAL RATE (R_t): Represents combined effect of nominal interest rate (time value of money) & inflation interest rate (buying power of money).

$$r_t = n_t - f_t \leftarrow \text{instantaneous inflation rate.}$$

r_t ↑ instantaneous real rate

n_t → instantaneous nominal rate

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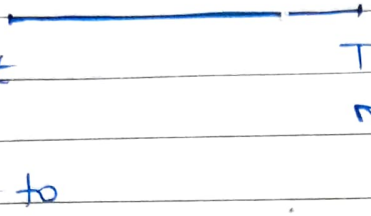
ZERO COUPON INFLATION-LINKED BOND

(Real Zero Coupon bond) = $\hat{P}(t,T)$



Amount to be invested at t i.e. $\hat{P}(t,T)$

such that it grows to $1 at T accounting the inflation

Or in other words its worth of $1 @ T observed at t with inflati

$$\hat{P}(t,T) = \hat{B}_t E_{Q^R}\left[\frac{\hat{B}_T^{-1}}{} \cdot 1 \,|\, \mathcal{F}_t\right]$$

Q^R is real-risk neutral measur

where $\hat{B}_t$ is $\left(e^{\int_0^t r_t dt}\right)$ i.e. real cash bond, is the numeraire

$$= E_{Q^R}\left[e^{-\int_t^T r_t dt} \,|\, \mathcal{F}_t\right]$$

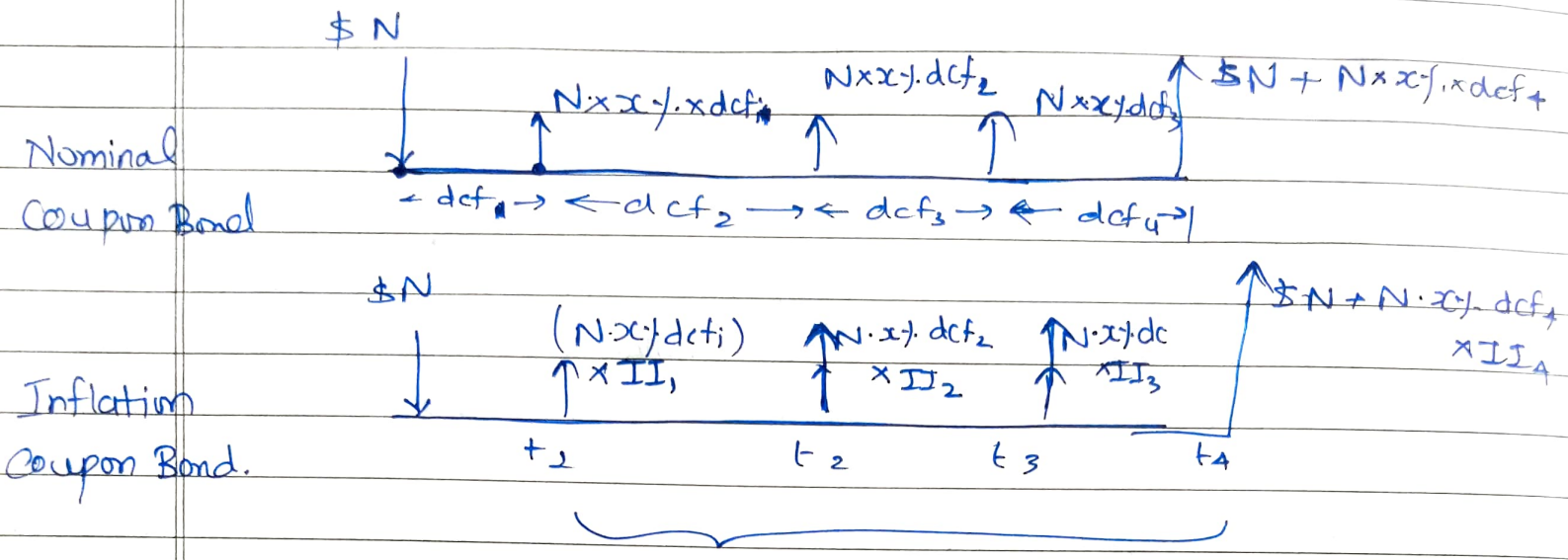
in terms of inflation index (II) we can re-write same in Q^N nominal-risk neutral measure

$$= B_t E_{Q^N}\left[\frac{II_T}{II_t} \cdot B_T^{-1} \,|\, \mathcal{F}_t\right] \quad \text{where } B_t = e^{\int_0^t r_t dt}$$

$$= E_{Q^N}\left[\frac{II_T}{II_t} \cdot e^{-\int_t^T r_t dt} \,|\, \mathcal{F}_t\right]$$

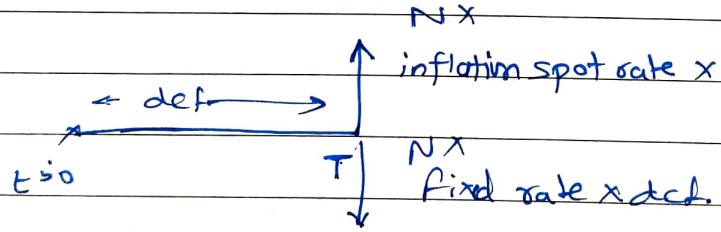
& $II_t B_t^{-1}$ is martingale

INFLATION LINKED BOND / COUPON I.L. BOND.



cashflows are increasing as inflation increases.

INFLATION SWAP : swap of fixed rate with inflation (zero coupon) spot rate



$$\text{Claim} = \frac{N}{dcf_i}\left(\underbrace{\frac{II_T - II_0}{II_0}}_{\text{inf. rate}} - \underbrace{\text{fixed rate}}_{\text{Fixed}}\right) \times dcf_i$$

$$\text{Price} = N \cdot E_Q\left[\frac{1}{dcf_i}\left(\frac{II_T}{II_0} - 1\right) - \text{fixed rate}\right] \times dcf_i \times e^{\int_0^t r_s ds}$$

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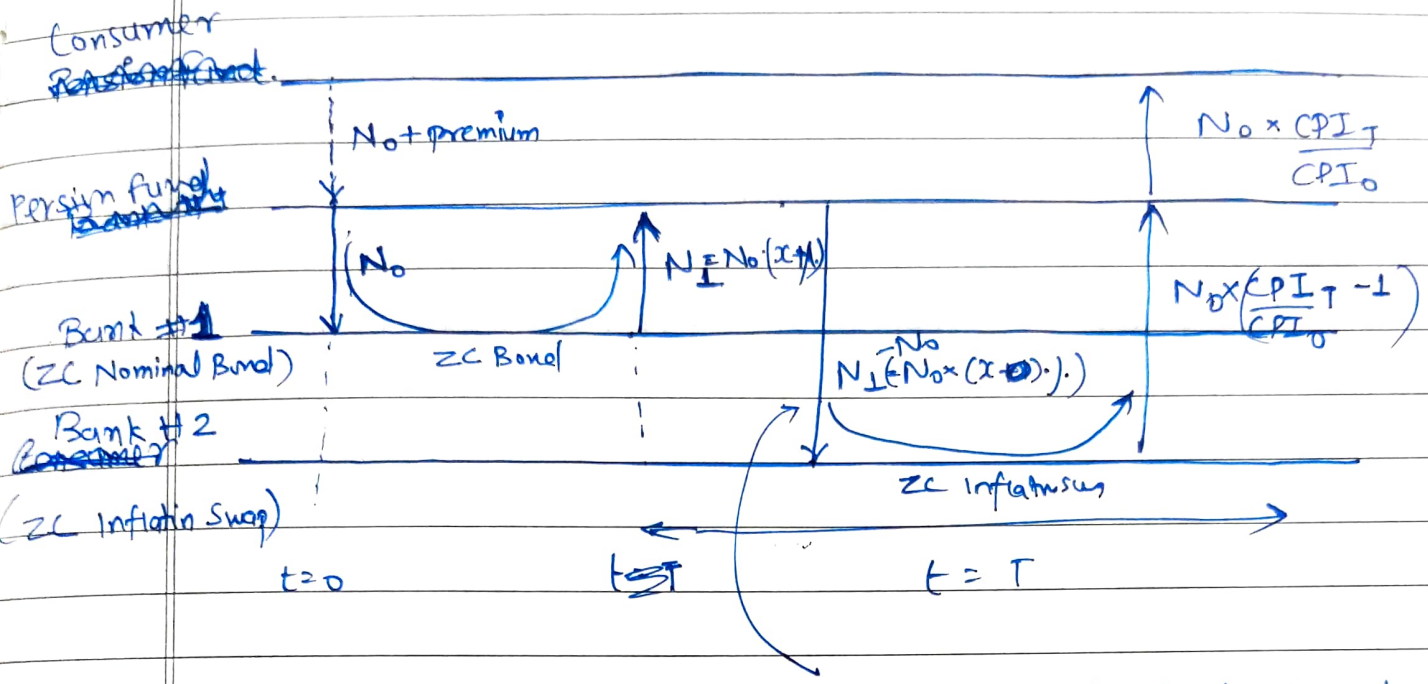
Application.

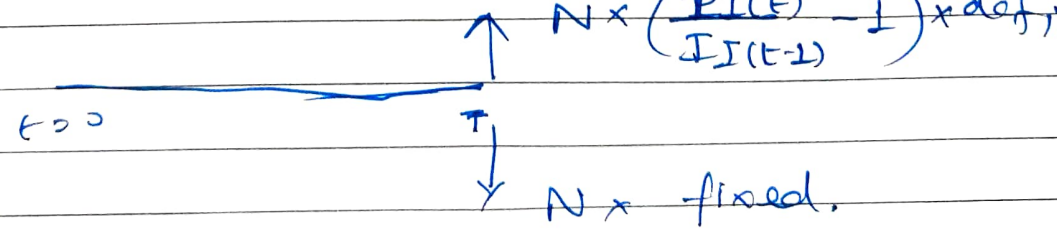


1) Here pension fund only invest $N_0 \times x\%$ in ZC inflation swap & keep N_0 with itself

2) Even though claim is executed at T; its pre purchased at $t = 0$ against some premium

YEAR-ON YEAR INFLATION SWAP. (YOY)

YoY swap consists of annual swap of YOY increase in flation against a fixed rate.



INFLATION OPTION

CAPLET: Max ($F_S(t,T)$ - strike)

$F_S(t,T)$ ↑ Spot inflation rate (realiz

Floorlet: Max (strike − $F_S(t,T)$)

t can be ~~to~~ or previous year.