2022

Equity Option

CHEAT SHEET

RAGHAVENDRA BAZARI

Contents

European Options ($O_{k,\,T}$):

Payoff/Claim Definition

Contract that gives owner right (but no obligation) to buy/sell underlying for predetermined price, known as strike (represented by k) at some future time, known as maturity (represented by T). Depending upon right to buy or sell there are two fundamental option types:

*All definitions throughout are from buyer's perspective, to get seller's perspective just multiply everything by -1.
For example: if someone says we are short the Call or Put then all the formulas will be -1 times all formulas mentioned below & words in all definitions will change as follows

1. '*Gives right*' will change to 'Obligated to'
2. 'buy' will change to 'sell' & vice versa etc.

	Call Option ($C_{K,\,T}$)	Put Option ($P_{K,\,T}$)
Payoff/ Claim, $O_{k,\,T}$	Gives right to buy underlying at T for K price in denominated: $$C_{K,\,T} = Max\left(S_T - K,\ 0\right)$$	Gives right to sell the underlying at T for K price in denominated. $$P_{K,\,T} = Max\left(K - S_T,\ 0\right)$$

Moneyness of Options

- At any other t before T, moneyness of an Option $O_{K,\,T}$, is defined depending upon current level of underlying S_t and strike K.

		Call Option ($C_{K,\,T}$)	Put Option ($P_{K,\,T}$)
Moneyness	In-The-Money	$S_t > K$	$S_t < K$
	At-The-Money	$S_t = K$	$S_t = K$
	Out-Of-The-Money	$S_t < K$	$S_t > K$

Fair Valuation $V_{O_{k,\,T},\,t}$

Fair Valuation of Option at time t denoted by $V_{O_{K,\,T},\,t}$ is the worth of option $O_{K,\,T}$ as observed at t such that there won't exist any arbitrage.

Properties:

- At $t = 0$, Valuation $V_{O_{K,\,T},\,0}$ is called premium.
- At $t = T$, Valuation $V_{C_{K,\,T},\,T}$ becomes claim itself $(= O_{K,\,T})$

There is various way we can compute this valuation, below listed are some of the ways:

	Call Option $(C_{K,\,T})$	Put Option $(P_{K,\,T})$
Using Risk Neutral Pricings	Using Martingale Pricing Theorem/Fundamental Theorem for Asset Pricing/Risk Neutral ricing: $V_{O_{K,\,T},\,t} = B_t E\left[O_{K,\,T} \times B_T^{-1} \middle\vert F_t\right]$	
	$V_{C_{K,\,T},\,t} = S_t e^{-q\Delta}\varphi(d1) - Ke^{-r\Delta}\varphi(d1 - \sigma\Delta)$	$V_{P_{K,\,T},\,t} = -\left(S_t e^{-q\Delta}\varphi(-d1) - Ke^{-r\Delta}\varphi\left(-(d1 - \sigma\Delta)\right)\right)$
	$where,$ $d1 = \dfrac{\ln\frac{S_t}{K} + \left(\mathrm{r} - \mathrm{q} + \frac{\sigma^2}{2}\right) \times \Delta}{\sigma\sqrt{\Delta}}$ $\Delta = T - t$ $r = constant\ interest\ rate$ $\sigma = constant\ volatility$ $q = constant\ continuous\ divident\ yield\ on\ underlying$ $\varphi(x) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{x} e^{-\frac{x^2}{2}}dx\ \left(\varphi(x) + \varphi(-x) = 1\right)$	

<table>
<tr>
<td rowspan="2">Using Call Put Parity</td>
<td colspan="6">Call-Put-Parity, means long the Call and short the Put on option maturing at T with strike K is same as long the underlying and short the $\$K$,ie.,
$$C_{K,T} - P_{K,T} = S_T - K$$
If we apply the valuation (conditional expectation under risk neutral measure with filtration F_t) on both the side, then
$$V_{C_{K,T},t} - V_{P_{K,T},t} = S_t e^{-q\Delta} - Ke^{-r\Delta}$$
where,
$V_{C_{K,T},t}$ is Fair Valuation of Call Option, $C_{K,T}$ with some strike K and maturity T.
$V_{P_{K,T},t}$ is Fair Valuation of Put Option, $P_{K,T}$ (which is on same strike and maturity)</td>
</tr>
<tr>
<td colspan="3">$$V_{C_{K,T},t} = V_{P_{K,T},t} + \left(S_t e^{-q\Delta} - Ke^{-r\Delta}\right)$$</td>
<td colspan="3">$$V_{P_{K,T},t} = V_{C_{K,T},t} - \left(S_t e^{-q\Delta} - Ke^{-r\Delta}\right)$$</td>
</tr>
<tr>
<td rowspan="2">Using Intrinsic and Extrinsic Value</td>
<td colspan="6">Valuation of Option at any time t consists of two components, intrinsic and extrinsic,
$$V_{O_{K,T},t} = I_{O_{K,T},t} + E_{O_{K,T},t}$$
where,
$I_{O_{K,T},t}$ is Intrinsic Value of Option $O_{K,T}$ at time t: Value of $O_{K,T}$ if exercised immediately at t.
$E_{O,t}$: is Extrinsic Value of Option $O_{K,T}$ at time t: Time Value of Option if allowed to live till T from t. It's also called Time Premium. Mostly computed by subtracting intrinsic value from total valuation.

Properties:
• Minimum value of any Option will be its intrinsic value i.e., $\min_t V_{O_{K,T},t} = I_{O_{K,T},t}$ as $E_{O_{K,T},t} \geq 0 \bigvee t$
    ○ Such that $\min_t I_{O_{K,T},t} = 0$

*Don't memorize below table (just remember how to compute intrinsic value and then use valuation from Call-Put-Parity minus Intrinsic to get extrinsic values)</td>
</tr>
<tr>
<td></td>
<td>$\boldsymbol{I_{C_{K,T},t}}$
$= Max(S_t - K,\ 0)$</td>
<td>$\boldsymbol{E_{C_{K,T},t}}$
$= V_{C_{K,T},t} - Max(S_t - K,\ 0)$</td>
<td></td>
<td>$\boldsymbol{I_{P_{K,T},t}}$
$= Max(K - S_t,\ 0)$</td>
<td>$\boldsymbol{E_{P_{K,T},t}}$
$= V_{P_{K,T},t} - Max(K - S_T,\ 0)$</td>
</tr>
</table>

<table>
<tr>
<td rowspan="3"></td>
<td>In-The-Money
$S_t > K$</td>
<td>$S_t - \mathrm{K}$</td>
<td rowspan="2">$\left(V_{P_{K,T},t} + \left(S_t e^{-q\Delta} - K e^{-r\Delta}\right)\right) - (S_t - K)$

Sp. Case: if $r = q = 0$ then
$= V_{P_{K,T},t}$
i.e., Extrinsic value of Call Option is Put Option Price if interest rates, and dividend rates are zero (only for In-The Money or At-The-Money Call Option)</td>
<td>Out-Of-The-Money
$S_t > K$</td>
<td rowspan="2">0</td>
<td>$\left(V_{C_{K,T},t} - \left(S_t e^{-q\Delta} - K e^{-r\Delta}\right)\right) - (K - S_t$</td>
</tr>
<tr>
<td>At-The-Money
$S_t = K$</td>
<td rowspan="2">0</td>
<td>At-The-Money
$S_t = K$</td>
<td rowspan="2">$\left(V_{C_{K,T},t} - \left(S_t e^{-q\Delta} - K e^{-r\Delta}\right)\right) - (K - S_t)$

Sp. Case: if $r = q = 0$ then
$= V_{C_{K,T},t}$
i.e., Extrinsic value of Put Option is Call Option Price if interest rates, and dividend rates are zero (only for In-The Money or At-The-Money Put Option)</td>
</tr>
<tr>
<td>Out-Of-The-Money
$S_t < K$</td>
<td>$\left(V_{P_{K,T},t} + \left(S_t e^{-q\Delta} - K e^{-r\Delta}\right)\right)$</td>
<td>In-The-Money
$S_t < K$</td>
<td>$K - S_t$</td>
</tr>
</table>

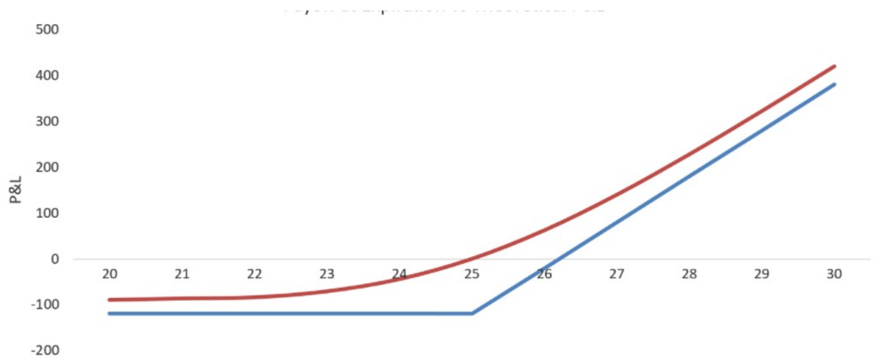
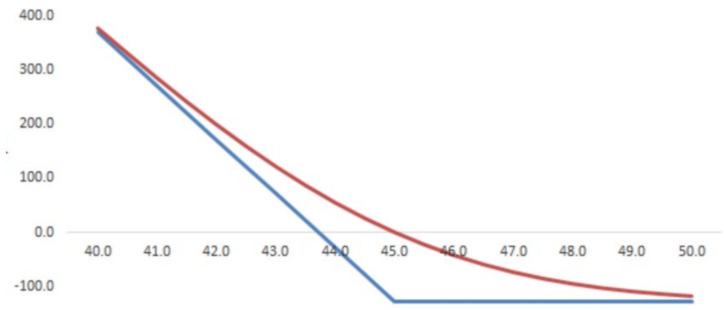
Profit and Loss (PnL)

Once we pay valuation at $t = 0$ (premium) to buy the Option then our Profit/Loss at any time t is:

$$PnL_t = V_{O_{k,T},t} - V_{O_{k,T},0} = V_{O_{k,T},t} - premium$$

Sp. Case: At $t = T$, PnL_T called Realized PnL:

$$PnL_T = V_{O_{k,T},T} - premium = O_{k,T} - premium$$

	Call Option $(C_{K,T})$	Put Option $(P_{K,T})$
Sensitivity of PnL with Spot & Maturity	$PnL_T = C_{k,T} - premium = Max(S_T - K, 0) - premium$ • Max Profit: ∞ if $S_T \gg K$ • Break Even: 0 if $S_T = K + premium$ • Max Loss: $premium$ if $S_T < K$  PnL_t PnL_T	$PnL_T = P_{k,T} - premium = Max(K - S_T, 0) - premium$ • Max Profit: $K - premium$ if $S_T < K$ • Break Even: 0 if $S_T = K - premium$ • Max Loss: $premium$ if $S_T > K$  PnL_t PnL_T

<table>
<tr>
<td rowspan="2">Sensitivity of PnL with Volatility</td>
<td colspan="2">As volatility increases, the prices of all options on that underlying - both calls and puts and at all strike prices - tend to rise. This is because the chances of all options finishing in the money likewise increase.
At one extreme if σ is very small, the option resembles a riskless bond.</td>
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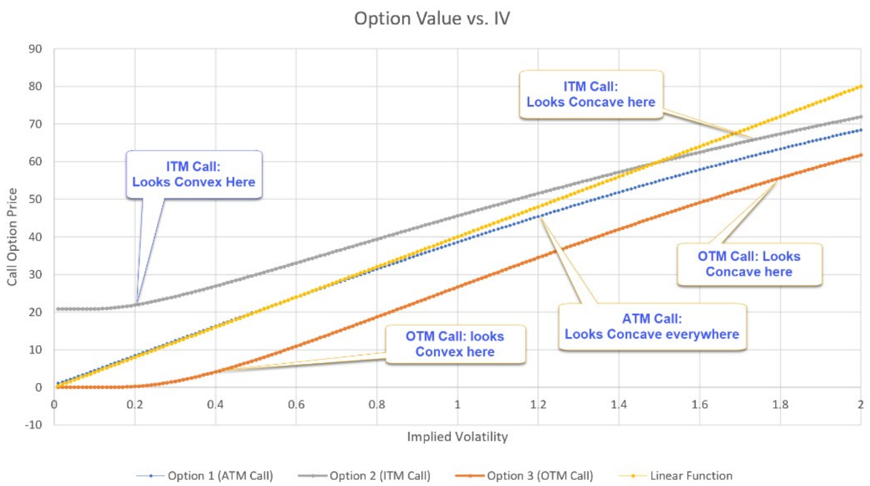
<td>

$$V_{C_{K,T},t} = S_t e^{-q\Delta}\varphi\left(d1\right) - Ke^{-r\Delta}\varphi\left(d1 - \sigma\Delta\right)$$

1. $\sigma = 0$
   a. In-The-Money or At-The-Money: $S_t \geq K \; then$
   $$V_{C_{K,T},t} = S_t e^{-q\Delta} - Ke^{-r\Delta}$$
   b. Out-Of-Money: $S_t < K \; then$
   $$V_{C_{K,T},t} = 0$$
2. $\sigma = \infty$
   $$V_{C_{K,T},t} = S_t e^{-q\Delta}$$



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<td>

$$V_{P_{K,T},t} = -\left(S_t e^{-q\Delta}\varphi\left(-d1\right) - Ke^{-r\Delta}\varphi\left(-\left(d1 - \sigma\Delta\right)\right)\right)$$

1. $\sigma = 0$
   a. In-The-Money or At-The-Money: $S_t \leq K \; then$
   $$V_{C_{K,T},t} = -\left(S_t e^{-q\Delta} - Ke^{-r\Delta}\right)$$
   b. Out-Of-Money: $S_t > K \; then$
   $$V_{C_{K,T},t} = 0$$
2. $\sigma = \infty$
   $$V_{C_{K,T},t} = S_t e^{-q\Delta}$$

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</table>

Greeks

Whenever a bank trades a derivative product, it ends up with a position that has various sources of risk. In practice, the bank does not risk manage each product independently. Instead, it adds each trade to its existing book of options and will risk manage this book globally. Indeed, some individual risks from different exotic products may offset each other. Given we know the current value of option $V_{O_{k,T},t}$ we want to figure out its constituent

Using Taylor Series, small increments in valuation $dV_{O_{k,T},t}$ can be represented as

$$dV_{O_{k,T},t} = \frac{\partial V_{O_{k,T},t}}{\partial S_t} dS_t + \frac{\partial V_{O_{k,T},t}}{\partial \sigma} d\sigma + \frac{\partial V_{O_{k,T},t}}{\partial r} dr + \frac{\partial V_{O_{k,T},t}}{\partial t} dt + \frac{1}{2} \frac{\partial^2 V_{O_{k,T},t}}{\partial S_t^2} (dS_t)^2$$

So, we can replace all the increments in valuation (price changes) by measuring all partial derivatives (called Greeks) and then trade corresponding multipliers, some of the famous Greeks:

Greek	***Symbol***	***Measures***	***Definition***
Delta	$\Delta = \frac{\partial V}{\partial S}$	Equity Exposure	Change in option price due to spot
Gamma	$\Gamma = \frac{\partial^2 V}{\partial S^2}$	Payout Convexity	Change in delta due to spot
Theta	$\Theta = \frac{\partial V}{\partial t}$	Time Decay	Change in option price due to time passing
Vega	$\upsilon = \frac{\partial V}{\partial \sigma}$	Volatility Exposure	Change in option price due to volatility
Rho	$\rho = \frac{\partial V}{\partial r}$	Interest Rate Exposure	Change in option price due to interest rates
Volga	$\frac{\partial^2 V}{\partial \sigma^2}$	Vol of Vol Exposure	Change in vega due to volatility
Vanna	$\frac{\partial^2 V}{\partial S \partial \sigma}$	Skew	Change in vega due to spot OR change in delta due to volatility
Charm	$\frac{\partial^2 V}{\partial S \partial t}$		Change in delta due to time passing

Delta $\delta_{O_{K,T},t}$

The delta of an option is the sensitivity of an option price to changes in the price of an underlying asset. Delta will tell us how much the price of the option (or option strategy) will change as the underlying changes

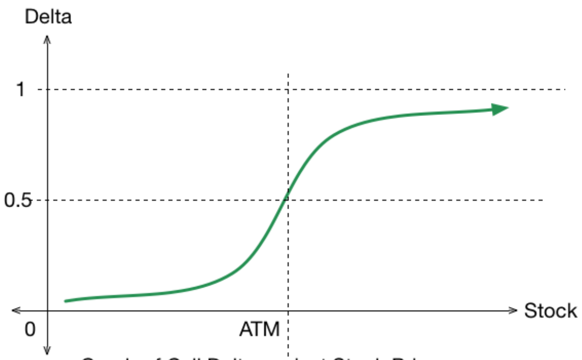
$$\delta_{O_{K,T,t}} = \frac{\partial V_{O_{k,T},t}}{\partial S_t}$$

<table>
<tr><td></td><td colspan="2">Call Option $(C_{K,T})$</td><td>Put Option $(P_{K,T})$</td></tr>
<tr><td rowspan="2">Utility</td><td rowspan="2">Hedge Ratio</td><td colspan="2">That is, if we buy or sell a certain number of options, how many futures must we buy/sell so that we are delta neutral? Hedging options with the correct number of underlying contracts allows us to establish a position which is indifferent to directional moves in the underlying.
Number of underlying's one need to sell is $\delta_{O_{K,T,t}}$ to be delta neutral.
$$Delta\ Hedge = -\delta_{O_{K,T,t}} \times dS_t = -\frac{\partial V_{O_{k,T},t}}{\partial S_t} dS_t$$
Negative sign is required to offset any change in PnL.</td></tr>
<tr><td>(Buy Call) Sell Delta on Spot increase

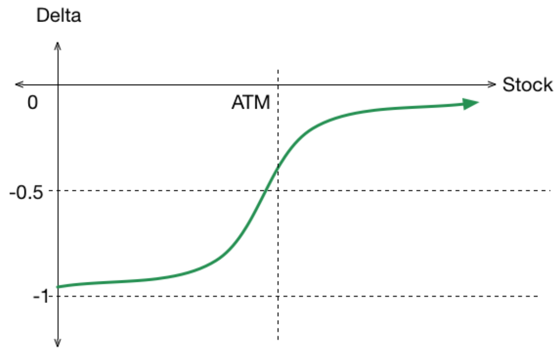
If S_t increases ($dS_t > 0$), then we need to buy $\delta_{O_{K,T},t}$ times underlying to replicate $V_{O_{k,T},t}$, i.e., to selling will offset the same valuation</td><td>(Buy Put) Sell Delta, but Delta is negative for Put option hence buy Delta on Spot increase

If S_t increases, then we need to sell $|\delta_{P_{K,T},t}|$ times underlying to replicate $V_{P_{k,T},t}$, i.e., to buying will offset the same valuation.</td></tr>
</table>

	Moneyness Of Option	Delta also be thought as the probability that an option will expire ITM. e.g., a 0.4 (or 40) delta call is considered to have a 40% chance of finishing ITM. Mathematically least accurate definition.	
Value 0f Delta, $\delta_{O_{K,T}}$	Using derivative	$\delta_{C_{K,T}} = e^{-q\Delta}\varphi\left(d1\right)$ Sp. Case, if $q = 0$ $\delta_{C_{K,T}} = \varphi\left(d1\right)$	$\delta_{P_{K,T}} = -e^{-q\Delta}(\varphi\left(-d1\right))$ Sp. Case, if $q = 0$ $\delta_{C_{K,T}} = \varphi\left(d1\right) - 1$
		where $d1 = \dfrac{\ln\frac{S_t}{K} + \left(\mathrm{r} - \mathrm{q} + \frac{\sigma^2}{2}\right) \times \Delta}{\sigma\sqrt{\Delta}}$	
	Using Call-Put-Parity	From Call-Put-Parity, we know $V_{C_{K,T},t} - V_{P_{K,T},t} = S_t e^{-q\Delta} - Ke^{-r\Delta}$ differentiating both side w.r.t. S_t, $\delta_{C_{K,T}} - \delta_{P_{K,T}} = e^{-q\Delta}$	
		$\delta_{P_{K,T}} + e^{-q\Delta}$ Sp. Case, if $q = 0$ $\delta_{P_{K,T}} + 1$	$\delta_{C_{K,T}} - e^{-q\Delta}$ Sp. Case, if $q = 0$ $\delta_{C_{K,T}} - 1$

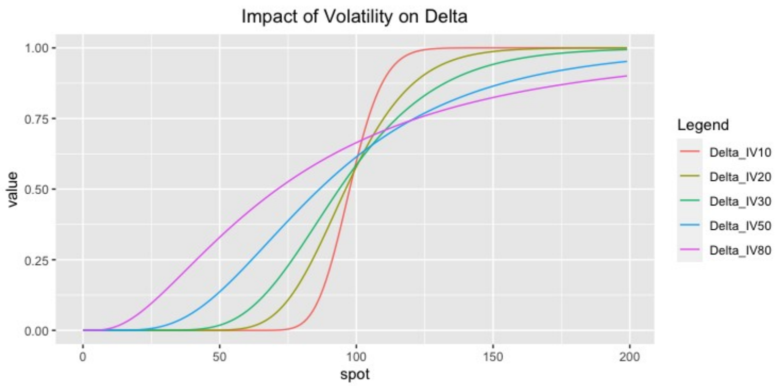
<table>
<tr>
<td>Sensitivity of Delta</td>
<td>Impact of S_t</td>
<td>


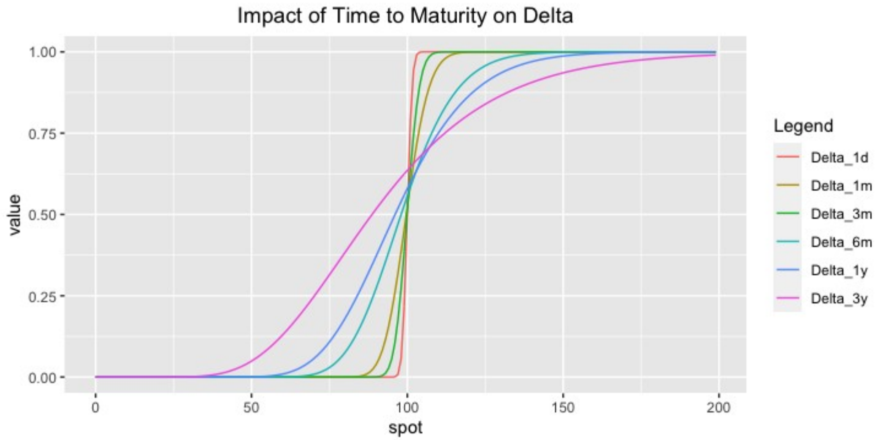
Graph of Call Delta against Stock Price

Assuming zero dividend ($q = 0$): $0 \leq \delta_{C_{K,T}} \leq 1$
1. if $S_t = 0, \delta_{C_{K,T}} = 0$
2. At $S_t = K$,
$$\delta_{C_{K,T}} = \varphi\left(\frac{\left(r - q + \frac{\sigma^2}{2}\right) \times \Delta}{\sigma\sqrt{\Delta}}\right)$$ ~0.5 for small values of r σ.
3. At $S_t = \infty, \delta_{C_{K,T}} = 1$
</td>
<td>


Graph of Put Delta against Stock Price

Assuming zero dividend ($q = 0$): -1 $\leq \delta_{C_{K,T}} \leq 0$
1. if $S_t = 0, \delta_{C_{K,T}} = -1$
2. At $S_t = K, \delta_{C_{K,T}} = -(\varphi\left(\frac{\left(r - q + \frac{\sigma^2}{2}\right) \times \Delta}{\sigma\sqrt{\Delta}}\right) \sim -0.5$
for small values of r σ.
3. At $S_t = \infty, \delta_{C_{K,T}} = 0$
</td>
</tr>
</table>

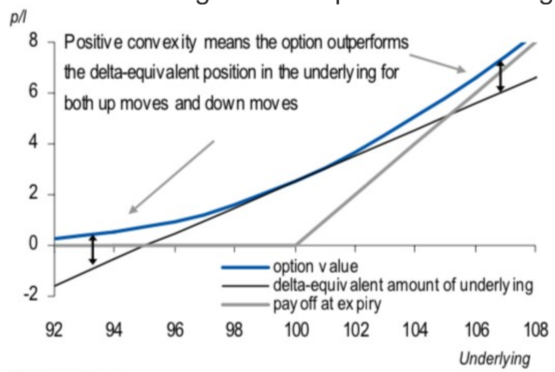
Impact of Volatility σ	 More volatile stocks therefore have a less pronounced delta, i.e., ○ A higher volatility increases the delta for OTM options. The more volatility, the less OTM an OTM option really is. ○ A higher volatility decreases the delta for ITM options. The more volatility, the less ITM an ITM option really is.

	Impact of Time to Maturity $t \rightarrow T$	 At maturity, delta has a digital shape around the strike. Once we move ourselves away from maturity, the delta becomes much smoother shaped. The further we are from maturity, the flatter the curve looks. The delta converges faster and faster to its intrinsic value as time to maturity goes to zero. The option picks its direction. That is true, except for ATM option, where the delta remains flatter. Main variation from the delta is always located in the ATM range. The closer we are to maturity, the tighter the range is where the delta changes from small value to full value.

Gamma, $\gamma_{O_{K,T},t}$

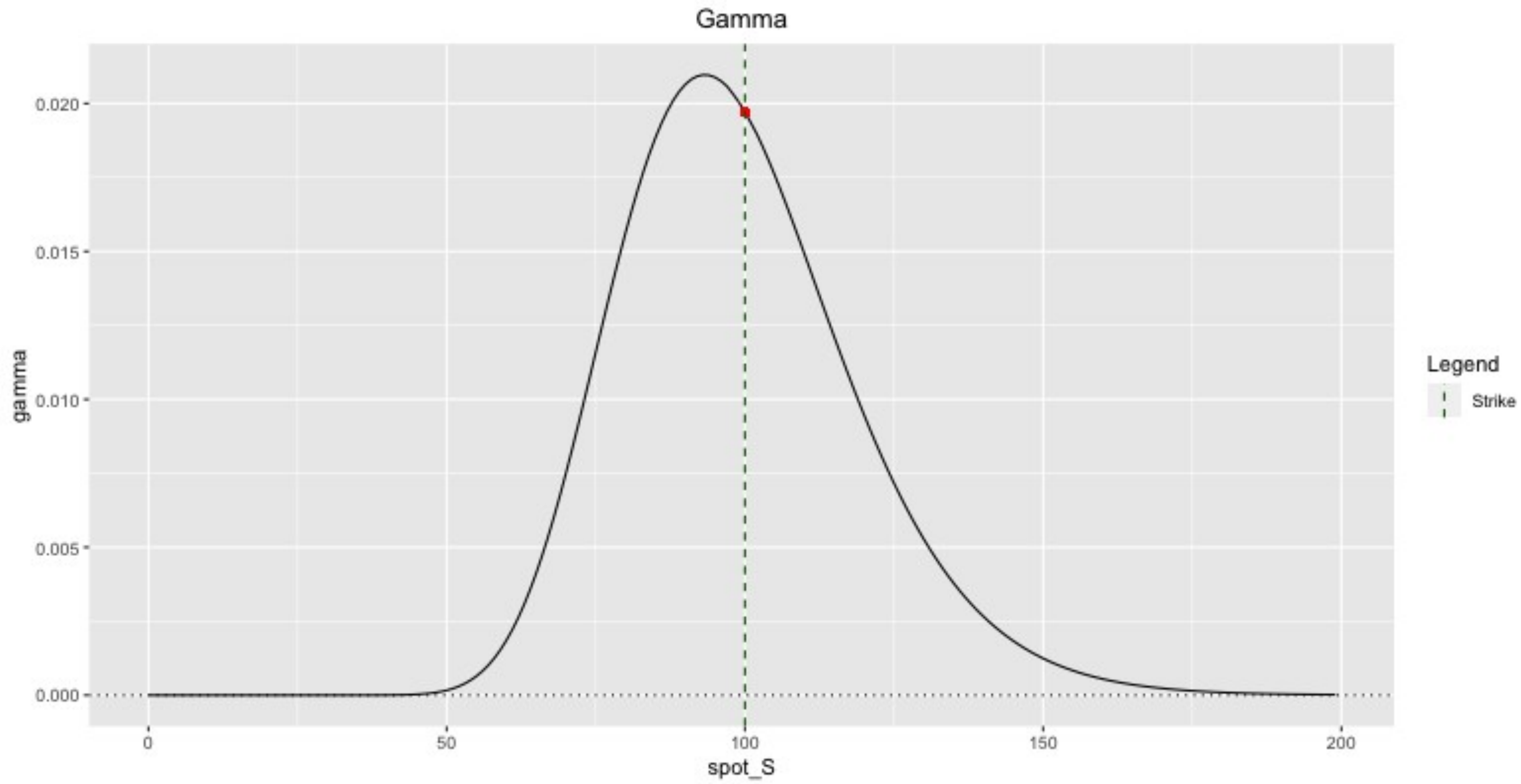
We know the delta of an option for a certain underlying price. However, option deltas change as the underlying price changes. The rate at which deltas change with changes in the underlying is usually expressed per 1-point move in the underlying. This rate of change in delta is known as Gamma

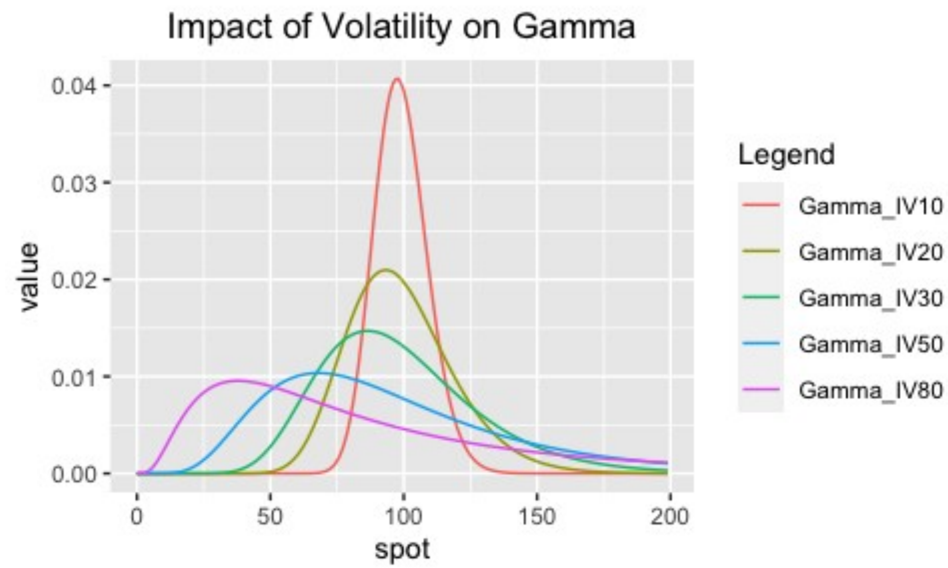
$$\gamma_{O_{K,T},t} = \frac{\partial \delta_{O_{K,T},t}}{\partial S_t} = \frac{\partial^2 V_{O_{k,T},t}}{\partial S_t^2}$$

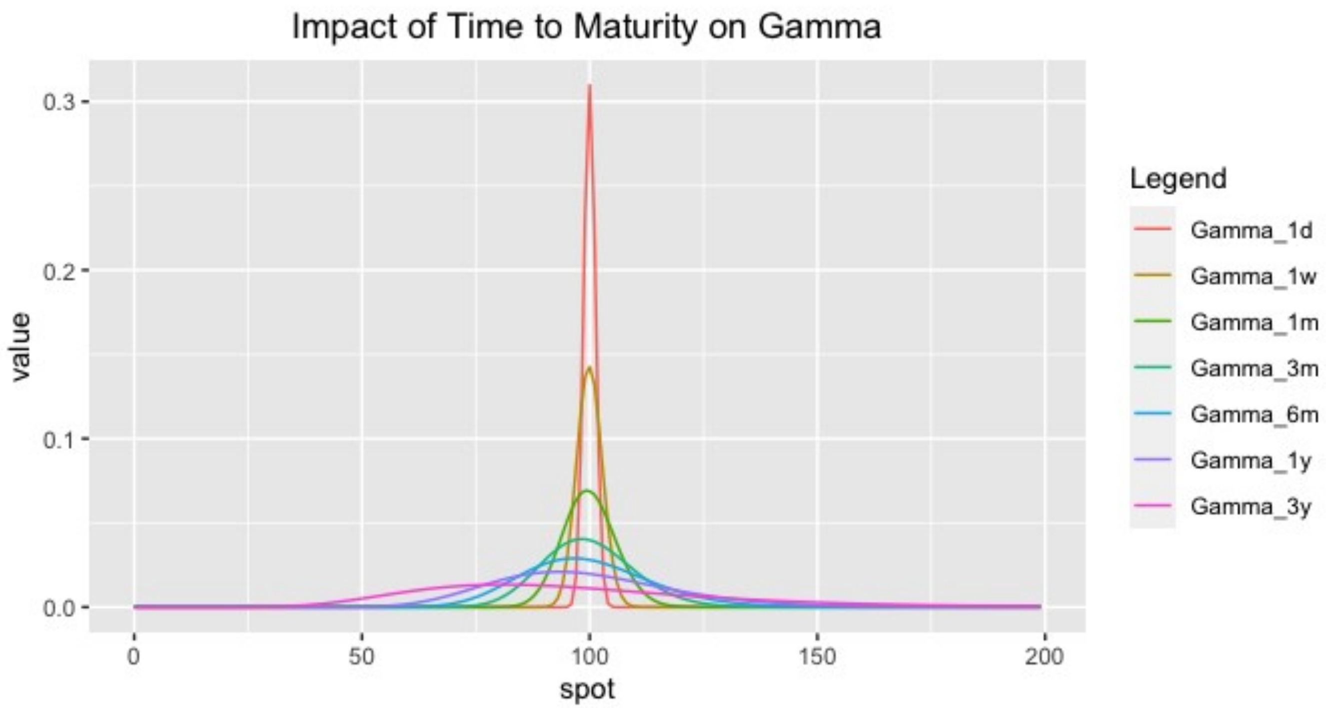
<table>
<tr><th></th><th colspan="2">Call Option $(C_{K,T})$</th><th>Put Option $(P_{K,T})$</th></tr>
<tr><td>Utility</td><td>Hedging</td><td colspan="2">To hedge this gamma, we need to trade other convex instruments such as other European options so that gammas cancel out. A lower gamma means that we will have a lower need for large and frequent rebalancing of delta.


Source : JPMorgan
Gamma Scalping: Large Gamma means, for long position, delta will increase as spot increases by large amount, so we would need to sell large increased Delta to be again Delta Neutral, but if spot decreases, we would have to buy underlying (as delta would have decreased). If we think market is moving up and then again down, doing Delta hedging will give us profit of maintaining portfolio (as we would have sold at increased spot and bought them back at lower price.</td></tr>
<tr><td></td><td>Moneyness</td><td colspan="2">Gamma tells us how much delta will move if the underlying moves. Gamma generally has its peak value close to ATM and decreases as the option goes deeper ITM or OTM. Options that are deeply ITM/OTM have gamma close to zero.</td></tr>
</table>

<table>
<tr>
<td rowspan="4">Value 0f Gamma, $\gamma_{O_{K,T},t}$</td>
<td rowspan="2">Using derivative</td>
<td>$$\gamma_{C_{K,T},t} = \frac{e^{-q\Delta}\varphi'(d1)}{S_t\sigma\sqrt{\Delta}}$$
Sp. Case, if $q = 0$
$$\gamma_{C_{K,T},t} = \frac{\varphi'(d1)}{S_t\sigma\sqrt{\Delta}}$$</td>
<td>$$\gamma_{P_{K,T},t} = \frac{e^{-q\Delta}\varphi'(d1)}{S_t\sigma\sqrt{\Delta}}$$
Sp. Case, if $q = 0$
$$\gamma_{P_{K,T},t} = \frac{\varphi'(d1)}{S_t\sigma\sqrt{\Delta}}$$</td>
</tr>
<tr>
<td colspan="2">where $d1 = \dfrac{\ln\frac{S_t}{K} + \left(\mathrm{r} - \mathrm{q} + \frac{\sigma^2}{2}\right) \times \Delta}{\sigma\sqrt{\Delta}}$</td>
</tr>
<tr>
<td rowspan="2">Using Call-Put-Parity</td>
<td colspan="2">From Call-Put-Parity for Delta, we know
$$\delta_{C_{K,T}} - \delta_{P_{K,T}} = e^{-q\Delta}$$
differentiating both side w.r.t. S_t,
$$\gamma_{C_{K,T},t} = \gamma_{P_{K,T},t}$$</td>
</tr>
<tr>
<td>$\gamma_{P_{K,T},t}$</td>
<td>$\gamma_{C_{K,T},t}$</td>
</tr>
</table>

Sensitivity of Gamma	Impact of S_t	 ○ $0 \leq \gamma_{C_{K,T},t}$ for both Call and Put ○ Since, $\delta_{C_{K,T}}$ moves lot more around ATM, hence its derivative which is gamma, $\gamma_{C_{K,T},t}$ is high around atm.

Impact of Volatility σ	 s We have seen that volatility attacked the delta, so it is not surprising it weakened the gamma too. Since a higher volatility induces a less pronounced S-shape delta curve, it also induces a less pronounced/wider more stable bell-curved gamma.

	Impact of Time to Maturity $t \to T$	 We have seen that delta get smoother for larger time to maturity. So, we can expect a smoother gamma for a longer maturity. As the time to expiration draws nearer, gamma of ATM options increases while gamma of ITM/OTM options decreases.

Vega $\nu_{O_{K,T},t}$

The Vega of an option is the change in options value with a change in implied volatility. Specifically, Vega is defined as the option value price change per 1-point change in implied volatility.

$$\nu_{O_{K,T,t}} = \frac{\partial V_{O_{k,T},t}}{\partial \sigma}$$

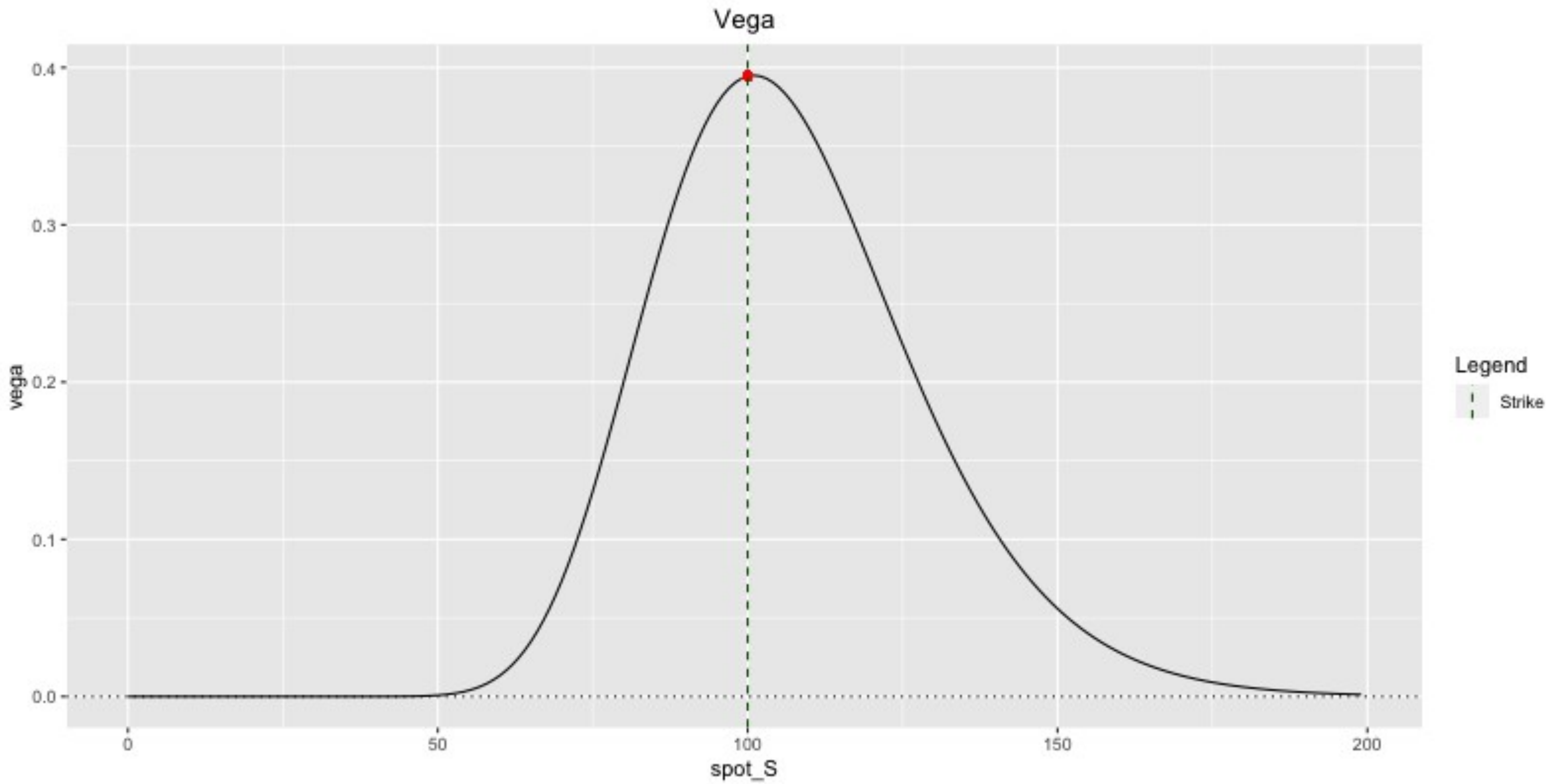
		Call Option $(C_{K,T})$	Put Option $(P_{K,T})$
Utility	Hedge Ratio	As for gamma, Vega is greatest for ATM options and decays exponentially on both sides. In fact, Gamma and Vega play a similar role to the options price. Gamma is crucial to understanding how much money the hedge is making/losing. It was directly related to the realised volatility. The Vega is in a way the forward measure for this.	
	Moneyness Of Option	When ATM, a change in the volatility can send the option either ITM or OTM thus the large effect on the price. The Vega sensitivity becomes small when the option is either far ITM or far OTM because these options have already picked their direction in a way. A larger volatility won't make that much difference to the price of the option. These kinds of options are quite easy to hedge, at least to the extent that when the market stays in the same regime, the hedge adjustments are minor over the lifetime of the option.	
Value Of Delta, $\delta_{O_{K,T}}$	Using derivative	$\nu_{C_{K,T}} = S_t e^{-q\Delta}\sqrt{\Delta}\varphi'(d1)$ Sp. Case, if $q = 0$ $\nu_{C_{K,T}} = S_t\sqrt{\Delta}\varphi'(d1)$	$\nu_{P_{K,T}} = S_t e^{-q\Delta}\sqrt{\Delta}\varphi'(d1)$ Sp. Case, if $q = 0$ $\nu_{P_{K,T}} = S_t\sqrt{\Delta}\varphi'(d1)$
		where $d1 = \dfrac{\ln\frac{S_t}{K} + \left(\mathrm{r} - \mathrm{q} + \frac{\sigma^2}{2}\right) \times \Delta}{\sigma\sqrt{\Delta}}$	

<table>
<tr><td rowspan="2"></td><td rowspan="2">Using Call-Put-Parity</td><td colspan="2">From Call-Put-Parity, we know

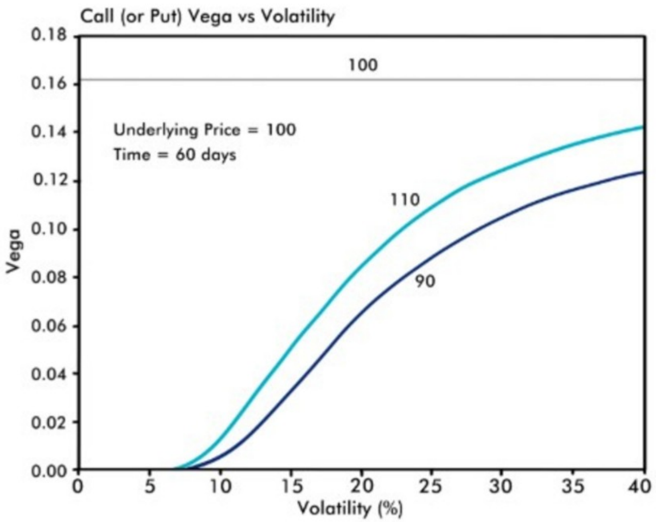
$$V_{C_{K,T},t} - V_{P_{K,T},t} = S_t e^{-q\Delta} - Ke^{-r\Delta}$$

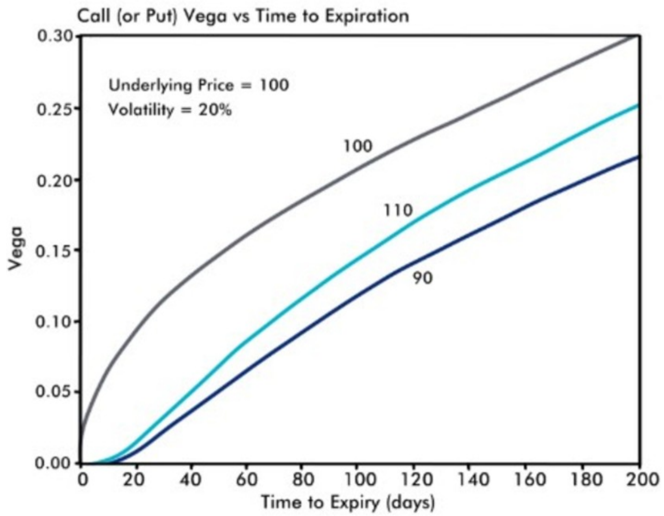
differentiating both side w.r.t.σ,

$$\nu_{C_{K,T}} = \nu_{P_{K,T}}$$</td></tr>
<tr><td>$\nu_{P_{K,T}}$</td><td>$\nu_{C_{K,T}}$</td></tr>
</table>

<table>
<tr>
<td>Sensitivity of Delta</td>
<td>Impact of S_t</td>
<td>s


The bell-shaped curve of Vega makes sense intuitively.
Of course, there is a turning point, where the impact becomes stronger again. As soon as volatility is above this level, one could say the Vega becomes meaningful, or the option is ATM from a Vega point of perspective.
The shape of the curve is very similar to the shape of the normal density function, but within the expression of Vega, there is an extra S factor that gives a twitch of bias.</td>
</tr>
</table>

Impact of Volatility σ	 Outright options have a positive Vega, so a buyer of an outright option (call or put) will make money if the implied volatility goes higher and lose money if implied volatility goes lower. Using the example above, the buyer of the call or put would profit as the price of the options increases with increasing implied volatility. As noted in our example above, calls and puts on the same strike (line) have the same Vega.

	Impact of Time to Maturity $t \to T$	 At maturity, delta has a digital shape around the strike. Once we move ourselves away from maturity, the delta becomes much smoother shaped. The further we are from maturity, the flatter the curve looks. The delta converges faster and faster to its intrinsic value as time to maturity goes to zero. The option picks its direction. That is true, except for ATM option, where the delta remains flatter. Main variation from the delta is always located in the ATM range. The closer we are to maturity, the tighter the range is where the delta changes from small value to full value.

Theta, $\theta_{O_{K,T},t}$

Theta is defined as the rate of change of the option price respected to the passage of time

$$\theta_{O_{K,T},t} = \frac{\partial V_{O_{k,T},t}}{\partial t}$$

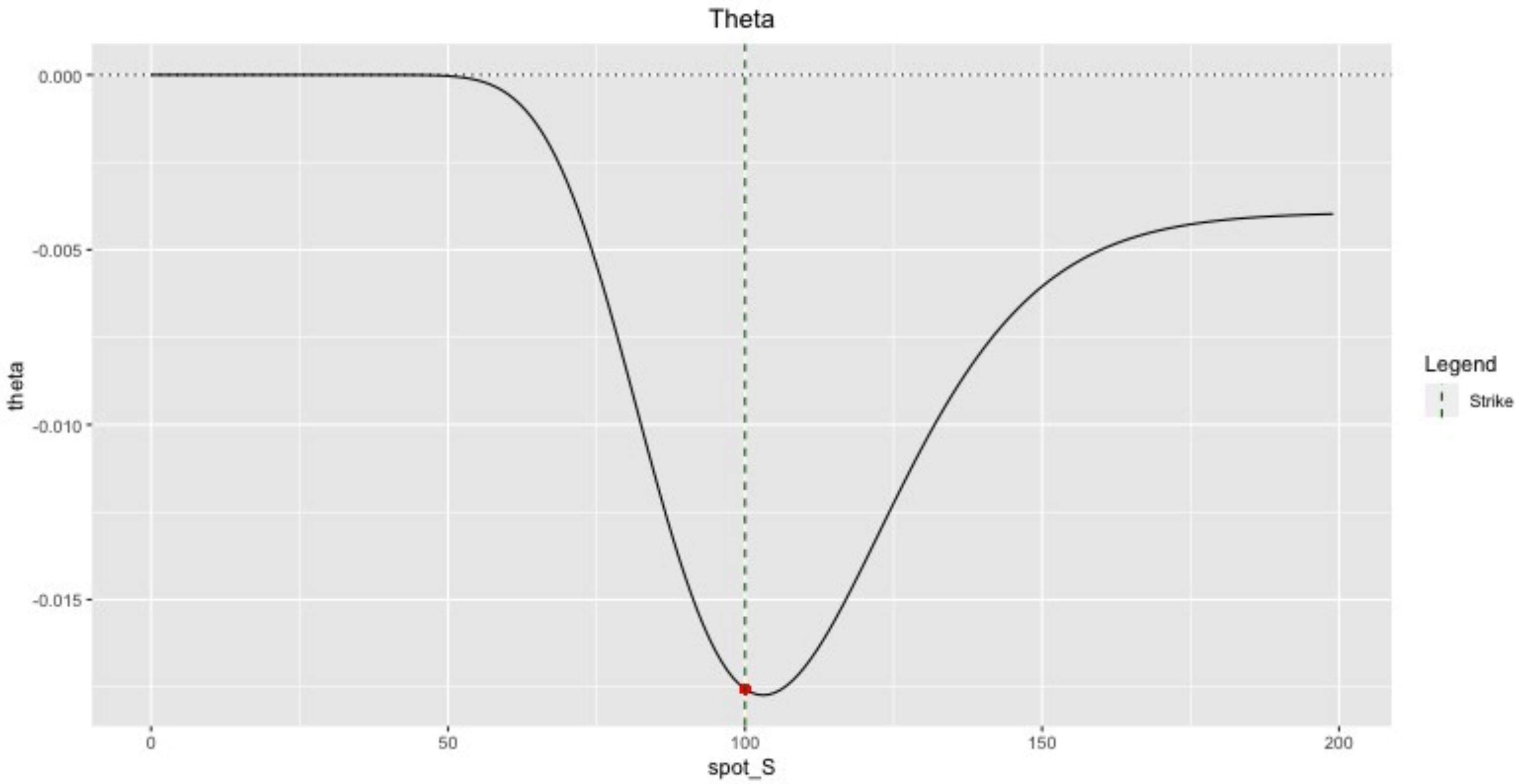
<table>
<tr><td></td><td></td><td>Call Option $(C_{K,T})$</td><td>Put Option $(P_{K,T})$</td></tr>
<tr><td>Utility</td><td>Hedging</td><td colspan="2">A trader that bought an option paid the intrinsic value (if any) and the time value. He knows the time value will be lost during the lifetime of the option. The only thing he does not know is how big the future daily losses/gains will be. However, the sum of all these will have to make up for the extrinsic premium paid. He will have to work for his money by actively delta hedging and locking in the gamma profits.

Every single day the trader who owns the option tries to fight the theta by:
• Rebalancing his portfolio at appropriate times.
• Leaving the position open if he expects the stock to keep on moving in the same direction.
By waiting, he gets to lock even more profit in one single rebalance but he is taking a risk.</td></tr>
<tr><td></td><td>Moneyness</td><td colspan="2">NA</td></tr>
<tr><td>Value 0f Gamma, $\gamma_{O_{K,T},t}$</td><td>Using derivative</td><td>Sp. Case, if $q = 0$

$$\theta_{C_{K,T},t} = -\frac{S_t\sigma}{2\sqrt{\Delta}}\varphi'(d_1) - rKe^{-r\Delta}\varphi(d2)$$</td><td>Sp. Case, if $q = 0$

$$\theta_{P_{K,T},t} = -\frac{S_t\sigma}{2\sqrt{\Delta}}\varphi'(d_1) + rKe^{-r\Delta}\varphi(-d2)$$</td></tr>
</table>

		where $d1 = \dfrac{\ln\frac{S_t}{K} + \left(\mathrm{r} - \mathrm{q} + \frac{\sigma^2}{2}\right) \times \Delta}{\sigma\sqrt{\Delta}}$	
	Using Call-Put-Parity	From Call-Put-Parity, we know $$V_{C_{K,T},t} - V_{P_{K,T},t} = S_t e^{-q\Delta} - Ke^{-r\Delta}$$ differentiating both side w.r.t. t, $$\theta_{C_{K,T}} - \theta_{P_{K,T}} = \ ???$$	

Sensitivity of Theta	Impact of S_t	
	Impact of Volatility σ	NA

	Impact of Time to Maturity $t \to T$	NA

Combinational European Options

10 Options Strategies Every Investor Should Know (investopedia.com)

2.3 Video - Lecture 2: Theos & Combination Spreads (16:00) | Akuna Capital (teachable.com)

References

1. Chapter 5 The Greeks | The Derivatives Academy (bookdown.org)

Call Delta Calls Puts Put Delta

Theta	Gamma	Vol	QBidO	cVolum	C Delta	Bid Qty	BidPx	Theo	Ask Px	Ask Qty	Strike	T Pos	Bid Qty	BidPx	Theo	Ask Px	Ask Qt	P Delta	pVolume	QBidO p
-0.002	0.0001	0.245	0.155		1.00			109.02			260	0	0	0	0.02	0.125	25	0.00		0.048
-0.005	0.0003	0.245	0.155		1.00			99.06			270	0	0	0	0.06	0.125	23	0.00		0.048
-0.010	0.0006	0.245	0.154		0.99			89.16			280	0	25	0.13	0.16	0.250	22	-0.01		0.048
-0.018	0.0011	0.245	0.154		0.98			79.36			290	0	21	0.250	0.36	0.375	21	-0.02		0.049
-0.027	0.0018	0.238	0.154		0.97			69.64			300	0	20	0.625	0.64	0.750	20	-0.04		0.052
-0.039	0.0027	0.231	0.152		0.94			60.09			310	0	19	1.000	1.09	1.125	19	-0.06		0.055
-0.053	0.0040	0.224	0.148		0.91			50.82			320	0	18	1.750	1.82	1.875	18	-0.09		0.058
-0.071	0.0055	0.220	0.145		0.86			42.05			330	0	16	3.000	3.05	3.125	16	-0.14		0.062
-0.089	0.0072	0.216	0.140		0.79			33.87			340	0	15	4.750	4.87	4.875	15	-0.21		0.074
-0.106	0.0088	0.213	0.135		0.71			26.55			350	0	14	7.500	7.55	7.625	14	-0.29		0.091
-0.118	0.0098	0.212	0.124		0.61	11	20.250	20.27	20.375	11	360	0	13	11.250	11.27	11.375	13	-0.39		0.108
-0.122	0.0103	0.211	0.112		0.51	11	14.875	15.00	15.000	11	370	0	12	15.875	16.00	16.000	12	-0.49		0.112
-0.120	0.0099	0.213	0.108		0.41	12	10.875	10.94	11.000	12	380	0	11	21.875	21.94	22.000	11	-0.59		0.124
-0.112	0.0091	0.215	0.098		0.32	13	7.750	7.81	7.875	13	390	0			28.81			-0.68		0.135
-0.099	0.0079	0.217	0.089		0.25	14	5.375	5.49	5.500	14	400	0			36.49			-0.76		0.140
-0.085	0.0065	0.221	0.080		0.18	15	3.750	3.86	3.875	15	410	0			44.86			-0.82		0.145
-0.071	0.0053	0.225	0.073		0.14	16	2.625	2.70	2.750	16	420	0			53.70			-0.86		0.148
-0.058	0.0042	0.229	0.067		0.10	18	1.875	1.89	2.000	18	430	0			62.89			-0.90		0.150
-0.047	0.0032	0.234	0.062		0.07	19	1.250	1.34	1.375	19	440	0			72.34			-0.93		0.150
-0.037	0.0025	0.238	0.058		0.05	20	0.875	0.93	1.000	20	450	0			81.93			-0.95		0.150
-0.028	0.0018	0.241	0.055		0.04	21	0.625	0.63	0.750	21	460	0			91.63			-0.96		0.150
-0.024	0.0014	0.248	0.053		0.03	22	0.375	0.49	0.500	22	470	0			101.49			-0.97		0.150
-0.021	0.0012	0.258	0.052		0.02	23	0.375	0.40	0.500	23	480	0			111.40			-0.98		0.150
-0.015	0.0008	0.258	0.050		0.02	25	0.250	0.26	0.375	25	490	0			121.26			-0.98		0.150
-0.010	0.0006	0.258	0.049		0.01	25	0.125	0.16	0.250	25	500	0			131.16			-0.99		0.155
-0.007	0.0004	0.258	0.049		0.01	0	0.000	0.10	0.125	25	510	0			141.10			-0.99		0.155
-0.005	0.0003	0.258	0.048		0.01	0	0.000	0.06	0.125	25	520	0			151.06			-1.00		0.155
-0.003	0.0002	0.258	0.048		0.00	0	0.000	0.04	0.125	25	530	0			161.04			-1.00		0.155
-0.002	0.0001	0.258	0.048		0.00	0	0.000	0.02	0.125	25	540	0			171.02			-1.00		0.155
-0.001	0.0001	0.258	0.048		0.00	0	0.000	0.01	0.125	25	550	0			181.01			-1.00		0.155
-0.001	0.0001	0.258	0.048		0.00	0	0.000	0.01	0.125	25	560	0			191.01			-1.00		0.155

Call options are assigned a delta between 0 and 1

Put options are assigned a delta betwee 0 and -1

Sometimes expressed as a number between 0 and 100 or 0 and -100