SURDS & INDICES

- KOUSTAV

CONCEPT

$(1024)^{-\frac{1}{10}} = \left(\frac{1}{1024}\right)^{\frac{1}{10}} = \frac{1}{2}$

1. Laws of Indices:

i. $a^m \times a^n = a^{m+n}$

ii. $\frac{a^m}{a^n} = a^{m-n}$

iii. $(a^m)^n = a^{mn}$

iv. $(ab)^n = a^n b^n$

v. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

vi. $a^0 = 1$ $\quad a^{-1} = \frac{1}{a}$

2. Surds:

Let a be rational number and n be a positive integer such that $a^{(1/n)} = \sqrt[n]{a}$

Then, $\sqrt[n]{a}$ is called a surd of order n.

3. Laws of Surds:

i. $\sqrt[n]{a} = a^{(1/n)}$

ii. $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$

iii. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

iv. $(\sqrt[n]{a})^n = a$

v. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

vi. $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

1. $(17)^{3.5} \times (17)^{?} = 17^{8}$

A. 2.29

B. 2.75

C. 4.25

✓ D. 4.5

$17^{3.5+x} = 17^{8}$

$x = 8 - 3.5$

$= 4.5$

2. If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$, then the value of x is:

A. $\frac{1}{2}$

B. 1

✓ C. 2

D. $\frac{7}{2}$

$\left(\frac{a}{b}\right)^{x-1} = \left(\frac{a}{b}\right)^{-(x-3)}$

$x - 1 = -x + 3$

$2x = 4$

$x = 2$

3. Given that $10^{0.48} = x$, $10^{0.70} = y$ and $x^z = y^2$, then the value of z is close to:

A. 1.45

B. 1.88

C. 2.9

D. 3.7

$10^{0.48z} = 10^{0.70\times 2}$

$0.48z = 1.4$

$z = \frac{1.4}{0.48} = \frac{140}{48} = \frac{35}{12} \sim 3$

4. If $5^a = 3125$, then the value of $5^{(a-3)}$ is:

A. 25

B. 125

C. 625

D. 1625

$a = 5$

$5^{5-3} = 5^2 = 25$

5. If $3^{(x-y)} = 27$ and $3^{(x+y)} = 243$, then x is equal to:

A. 0

B. 2

✓ C. 4

D. 6

$x - y = 3$

$x + y = 5$

$+$

$2x = 8$

$x = 4$

6. $(256)^{0.16} \times (256)^{0.09} = ?$

✓ A. 4

B. 16

C. 64

D. 256.25

$256^{0.25}$

$256^{1/4} = 2^{8 \times \frac{1}{4}} = 2^2$

7. The value of $[(10)^{150} \div (10)^{146}]$

A. 1000

B. 10000

C. 100000

D. 10^6

8. $$\frac{1}{1 + x^{(b-a)} + x^{(c-a)}} + \frac{1}{1 + x^{(a-b)} + x^{(c-b)}} + \frac{1}{1 + x^{(b-c)} + x^{(a-c)}} = ?$$

A. 0

B. 1

C. x^{a-b-c}

D. None of these

$$\frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}} + \frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}}$$

$$\frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^b+x^a+x^c} + \frac{x^c}{x^c+x^b+x^a} = \frac{x^a+x^b+x^c}{x^a+x^b+x^c} = 1$$

9. $(25)^{7.5} \times (5)^{2.5} \div (125)^{1.5} = 5^{?}$

A. 8.5

B. 13

C. 16

D. 17.5

E. None of these

$5^{2\times7.5} \times 5^{2.5} \div 5^{3\times1.5}$

$5^{15+2.5-4.5}$

5^{13}

10. $(0.04)^{-1.5} = ?$

A. 25

B. 125

C. 250

D. 625

$\left(\frac{4}{100}\right)^{-\frac{15}{10}}$

$\left(\frac{1}{25}\right)^{-\frac{3}{2}} = 25^{3/2} = 5^{2\times3/2}$

$= 125$

11. $\dfrac{(243)^{n/5} \times 3^{2n+1}}{9^n \times 3^{n-1}} = ?$

A. 1

B. 2

C. 9

D. 3^n

$$\frac{3^{5\times\frac{n}{5}} \times 3^{2n+1}}{3^{2n} \times 3^{n-1}} = 3^{n+2n+1-2n-(n-1)}$$

$$= 3^{n+1-n+1}$$

$$= 3^2$$

$$= 9$$

12. $\dfrac{1}{1+a^{(n-m)}} + \dfrac{1}{1+a^{(m-n)}} = ?$

A. 0

B. $\frac{1}{2}$

C. 1

D. a^{m+n}

$$\frac{1}{1+\frac{a^n}{a^m}} + \frac{1}{1+\frac{a^m}{a^n}}$$

$$\frac{a^m}{a^m+a^n} + \frac{a^n}{a^n+a^m}$$

$$= 1$$

13. If m and n are whole numbers such that $m^n = 121$, the value of $(m - 1)^{n+1}$ is:

A. 1

B. 10

C. 121

D. 1000

11^2 $m = 11$ $n = 2$

10^3

14. $\left(\frac{x^b}{x^c}\right)^{(b+c-a)} \cdot \left(\frac{x^c}{x^a}\right)^{(c+a-b)} \cdot \left(\frac{x^a}{x^b}\right)^{(a+b-c)} = ?$

A. x^{abc}

B. 1

C. $x^{ab+bc+ca}$

D. x^{a+b+c}

$$\frac{x^{b^2+bc-ba+c^2+ca-cb+a^2+ab-ac}}{x^{cb+c^2-ac+ac+a^2-ab+ba+b^2-bc}}$$

$$\frac{x^{a^2+b^2+c^2}}{x^{a^2+b^2+c^2}} = 1$$

ANSWER KEY

QUESTION	ANSWER	QUESTION	ANSWER
1	D	8	B
2	C	9	B
3	C	10	B
4	A	11	C
5	C	12	C
6	A	13	D
7	B	14	B