NUMBER SYSTEMS

- KOUSTAV

CONCEPT – REMAINDERS

$$\frac{11 + 12}{10} \quad R \to 3 \qquad (1 + 2)$$

$$\frac{11 \times 12}{10} \quad R \to 2 \qquad (1 \times 2)$$

$$\frac{13 \times 15 \times 17}{11} \quad R \to \frac{48}{11} \quad R \to 4 \qquad (2 \times 4 \times 6)$$

$$\frac{14 + 15 \times 17}{12} \quad R \to \frac{17}{12} \quad R \to 5 \qquad (2 + 3 \times 5)$$

1. On dividing a number by 5, we get 3 as remainder. What will be the remainder when the square of this number is divided by 5?

A. 0 B. 1 C. 2 D. 4 ✓

$\frac{3^2}{5} = \frac{9}{5}$ R $\Rightarrow 4$

$\frac{8^2}{5} = \frac{64}{5}$ R $\Rightarrow 4$

$\frac{13^2}{5} = \frac{169}{5}$ R $\Rightarrow 4$

2. On dividing a number by 774, we get 35 as remainder. What will be the remainder when the same number is divided by 18?

A. 14 B. 17 C. 18 D. 19

$774)N(Q$

35

$N = 774 \times Q + 35$ (0 × q + 17)

18

$\Rightarrow 0 \times q + 17$

$= 0 + 17 = 17$

Smallest such num = 35

$\frac{35}{18} \Rightarrow 17$

3. What is the remainder when 2^{25} is divided by 3?

A. 2 B. 1 C. 0 D. 3

2^{25} R $\Rightarrow (-1)^{25} = -1$

$\Rightarrow 3 - 1 = 2$

3)2(0 or 3)2(1

-0 -3

2 $\longleftrightarrow$ -1

5^{44} R $\Rightarrow 2^{44}$ R $\Rightarrow (-1)^{44} = 1$

34^{34} R $\Rightarrow 6^{34}$ R $\Rightarrow (-1)^{34} = 1$

19^{19} R $\Rightarrow 4^{19}$ R $\Rightarrow (-1)^{19} = -1$

$\Rightarrow 5 - 1 = 4$

19^{19} R $\Rightarrow 1^{19} = 1$

4. What is the remainder when $(1^1 + 2^2 + 3^3 + \ldots + 100^{100})$ is divided by 4?

A. 3 B. 1 C. 2 ~~D. 0~~ ✓

$$\frac{1^1 + 2^2 + 3^3 + 4^4}{4}$$

$\xrightarrow{R}$ $1 + 0 + (-1)^3 + 0^4$

$= 1 + 0 - 1 + 0$

$= 0$

$$\frac{5^5 + 6^6 + 7^7 + 8^8}{4}$$

$\xrightarrow{R}$ $$\frac{1^5 + 2^6 + 3^7 + 0^8}{4}$$

$\xrightarrow{R}$ $1 + 0 + (-1)^7 + 0$

$= 1 + 0 - 1 + 0$

$= 0$

5. Find the remainder when 53^{12} is divided by 17.

A. 8 B. 0 C. 1 ✓ D. 16

$\frac{53^{12}}{17} \xrightarrow{R} \frac{2^{12}}{17} = \frac{(2^4)^3}{17}$

[Power of 2, nearest to 17]

$(a^m)^n = a^{mn}$

$= \frac{16^3}{17} \xrightarrow{R} (-1)^3 = -1 \Rightarrow 17-1 = 16$ //

$\frac{32^{32}}{15} \xrightarrow{R} \frac{2^{32}}{15} = \frac{(2^4)^8}{15} = \frac{16^8}{15} \xrightarrow{R} 1^8 = 1$ //

$\frac{16^{24}}{9} \xrightarrow{R} (-2)^{24} = \frac{2^{24}}{9} = \frac{(2^3)^8}{9} = \frac{8^8}{9} \xrightarrow{R} (-1)^8 = 1$ //

$\frac{20^{21}}{9} \xrightarrow{R} \frac{2^{21}}{9} = \frac{(2^3)^7}{9} = \frac{8^7}{9} \xrightarrow{R} (-1)^7 = -1$

$\Rightarrow 9-1 = 8$ //

$\frac{27^{22}}{8} \xrightarrow{R} \frac{3^{22}}{8} = \frac{(3^2)^{11}}{8} = \frac{9^{11}}{8} \xrightarrow{R} 1^{11} = 1$ //

$\frac{32^{33}}{15} \xrightarrow{R} \frac{2^{33}}{15} = \frac{(2^4)^8 \times 2^1}{15} = \frac{16^8 \times 2}{15} \xrightarrow{R} 1^8 \times 2$

$= 1 \times 2$

$= 2$ //

$\frac{20^{23}}{9} \xrightarrow{R} \frac{2^{23}}{9} = \frac{(2^3)^7 \times 2^2}{9} = \frac{8^7 \times 4}{9} \xrightarrow{R} (-1)^7 \times 4$

$= -1 \times 4 = -4 \Rightarrow 9-4 = 5$ //

6. The remainder when $(7^{21}+7^{22}+7^{23}+7^{24})$ is divided by 25:

A. 1 B. 24 C. 0 D. 12

$$7^{21} + 7^{22} + 7^{23} + 7^{24}$$

$$= 7^{20}\left(7 + 7^2 + 7^3 + 7^4\right)$$

$$= \frac{\left(7^2\right)^{10}\left(7 + 7^2 + 7^2 \times 7 + \left(7^2\right)^2\right)}{25}$$

$\frac{7^2}{25}$ R $\Rightarrow -1$

R $\Rightarrow (-1)^{10}\left(7 + (-1) + (-1) \times 7 + (-1)^2\right)$

$= 1(7 - 1 - 7 + 1)$

$= 0$

7. $P = (1!)^2 + (2!)^2 + (3!)^2 + \ldots + (100!)^2$.
The remainder when 5^{2P} is divided by 13 is:

A. 1 B. 12 C. 0 D. 2

$$\frac{5^{2P}}{13} = \frac{25^{P}}{13} \xrightarrow{R} (-1)^{P}$$

If P = even, Ans = 1

If P = odd, Ans = 12

$$P = (1!)^2 + \underbrace{(2!)^2 + (3!)^2 + \ldots + (100!)^2}_{\text{even}}$$

$(1!)^2 \rightarrow$ odd

$= \text{odd} + \text{even}$

$P = \text{odd}$

$$\text{Ans} = (-1)^{P} = (-1)^{\text{odd}} = -1$$

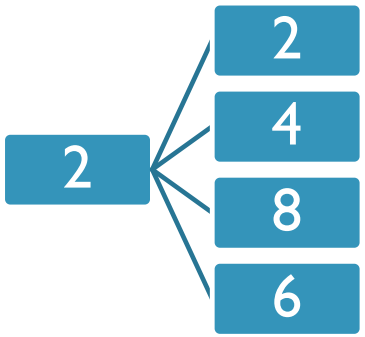
$\Rightarrow 13 - 1 = 12$

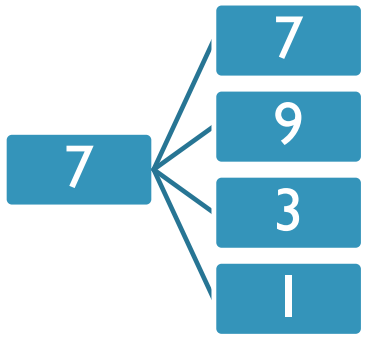
If $n \geq 2$, $n! = \text{even}$

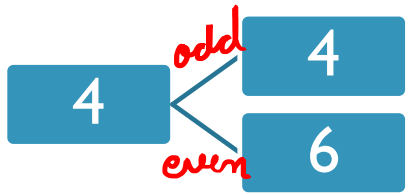
odd × even = even

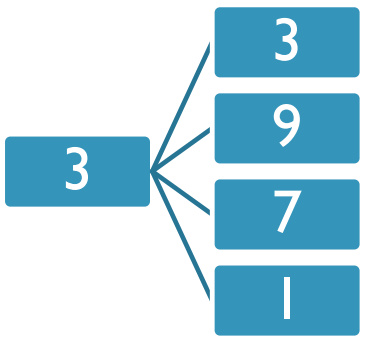
odd + even = odd

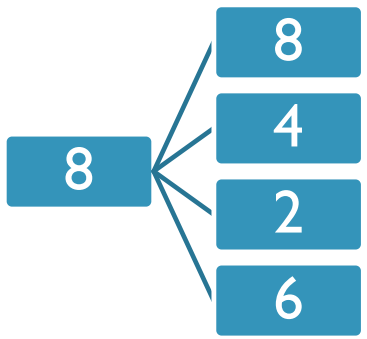
CONCEPT – CYCLICITY (UNIT'S PLACE)

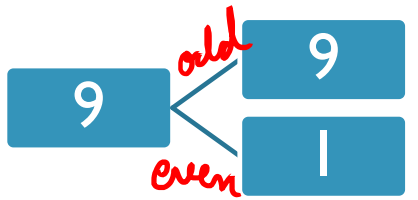


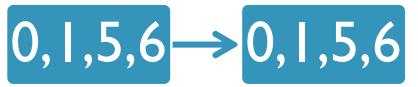












1. What is the last digit of the following expressions:

1.a) 2^5

2

1.b) 2^{25}

$\frac{25}{4}$ R ⇒ 1

$2^1 = 2$

2^{35}

$\frac{35}{4}$ R ⇒ 3

$2^3 = 8$

2, 4, 8, 6

1.c) 2^{125}

$\frac{125}{4} \Rightarrow \frac{25}{4}$ R ⇒ 1

$2^1 = 2$

1.d) 432^{1234}

$\Rightarrow 2^{1234}$

$\frac{1234}{4} \Rightarrow \frac{34}{4}$ R ⇒ 2

$\Rightarrow 2^2 = 4$

[∵ Divisibility of 4 ⇒ Last 2 digits]

2. What is the last digit of the expression 777^{777}?

A. 3 B. 1 C. 7 D. 9

777^{777}

$\Rightarrow 7^{777}$

$\frac{777}{4} \Rightarrow \frac{77}{4}$ R $\Rightarrow 1$

$\Rightarrow 7^1 = 7$

7
9
3
1

77

40 37

-4 R ↓ + ↓ R

0 + 1

1

3. The unit's digit of the product $3^{1001} \times 7^{22002} \times 13^{333003}$ is:

A. 3 B. 1 C. 5 D. 9 ✓

$3^{01} \times 7^{02} \times 3^{03}$

$3 \times 9 \times 7$

9

4. The unit's digit of the sum $22^{222} + 33^{333} + 44^{444}$ is:

A. 3 B. 1 C. 5 D. 9

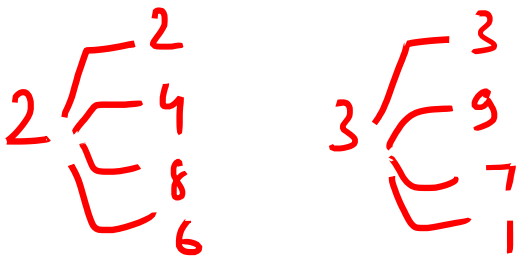
$2^{22} + 3^{33} + 4^{\text{even}}$

$\frac{22}{4}$ R $\Rightarrow$ 2 $\quad\quad$ $\frac{33}{4}$ R $\Rightarrow$ 1

$2^2 + 3^1 + 6$

$4 + 3 + 6$

$= 3$



5. N = 1! + 2! + 3! +……+ 2010!. What is the digit in the unit's place of N?

A. 3 B. 2 C. 1 D. 0

$N = 1! + 2! + 3! + 4! + 5! + \underbrace{- - - - + 2010!}_{0}$

$1 + 2 + 6 + 4 + 0$

$= 3$

$n \geq 5$

$n!$ ends with '0'

6. The unit's place of the product $34^{123!} \times 3456^{123456!}$ is:

A. 4 B. 8 C. 1 ✓~~D. 6~~

$4^{123!} \times 6$

$\Rightarrow 4^{\text{even}} \times 6$

$= 6 \times 6$

$= \underline{\underline{6}}$

$4 \begin{cases} 4 \\ 6 \end{cases}$

$6 \rightarrow 6$

$\left[\begin{array}{l} n \geq 2 \\ n! = \text{even} \end{array}\right]$

7. $1234567890^{1234567890}$

ANSWER KEY – NUMBER SYSTEMS

REMAINDERS		CYCLICITY	
QUESTION	ANSWER	QUESTION	ANSWER
1	D	1	2, 2, 2, 4
2	B	2	C
3	A	3	D
4	D	4	A
5	D	5	A
6	C	6	D
7	B	7	-