CS/ECE/ME 532 Sec. 004
Matrix Methods in Machine Learning

[Unit 4] Applications of Singular Value Decomposition – Overview

Learning Objectives

At the end of this module, students will be able to:
- Find the best low-dimensional representation of a data matrix using SVD (principal components analysis)
- Represent a graph using an adjacency matrix
- Find important nodes in a graph using the adjacency matrix (Page Rank algorithm)
- Find the missing entries in a low-rank matrix (matrix completion) using iterative singular value thresholding
- Express the Tikhonov-regularized least-squares problem solution using SVD
- Analyze the impact of noise in relationship to the small singular values
- Implement regularization by truncating singular values
- Use SVD to solve least-squares signal recovery problems

Significance of Unit

We looked in Unit 3 at the singular value decomposition. We defined what it meant and saw that we could take an arbitrary matrix and factor it into a product of a matrix with orthonormal columns, times a diagonal matrix, times another matrix with orthonormal columns. In this unit we are going to look at applying that decomposition to solve some important machine learning problems. We are going to look at matrix completion, which is a problem of filling in missing entries in a matrix that occurs during ratings. When some people rate products and movies, not everyone is going to rate every product. If we are willing to assume that the matrix is relatively low rank (which is often a good assumption), then we can use singular value decomposition to fill in missing values. We are also going to look at the PageRank algorithm developed by Larry Page at the beginning of Google, that was the core technology used in the search engine. What PageRank does is it represents the links in the web or the connections from one website to another in terms of a matrix problem adjacency matrix. Then we can apply a singular value decomposition to the matrix to figure out which pages or links are the most important ones. We are also going to look at transfer component analysis which is the problem of representing very high dimensional data in terms of low dimensional sub-spaces. That allows us to extract patterns as well as remove noise and other features from the data that we are not interested in. Finally, we are going to look at using singular value decomposition to analyze ill-posed inverse problems, ones where the rank of the matrix is really low. We are going to see why ridge regression or Tikhonov Regularization is effective and apply that analysis to signal recovery or data recovery in a case where our signal or data has been distorted by some operations or noise has been added. This typically happens with instrumentation when we're collecting data, singular value decomposition can help us solve that. Unit 4 will be very interesting with lots of cool things to talk about, so keep at it and we'll see you again soon.

Key Topics

1. Principal Component Analysis (PCA) and best subspace approximation
2. Page rank algorithm
3. Power iterations
4. Matrix Completion
5. Solving systems of linear equations
   - 5.1. Operator norm of pseudo-inverse and Ill-conditioning
   - 5.2. Tikhonov regularization
   - 5.3. Small singular values and noise
   - 5.4. Truncated SVD regularization
6. Signal recovery

Learning Activities

- Instructional Units 4.1, 4.2
- Activity 12
- Instructional Unit 4.3
- Activity 13
- Assignment 6
- Instructional Units 4.4, 4.5
- Activity 14
- Instructional Units 4.6, 4.7
- Activity 15
- Assignment 7
- Unit 4 Overview Quiz

Recommended Reading

- LE: Lars Elden, Matrix Methods in Data Mining and Pattern Recognition
- LE 6.4 Principal Component Analysis
- LE 12.1 Pagerank
- LE 12.3 The Power Method for Pagerank Computation
- LE 6.6 Condition Number and Perturbation Theory for the Least-Squares Problem
- LE 7.1 Truncated SVD: Principal Component Regression