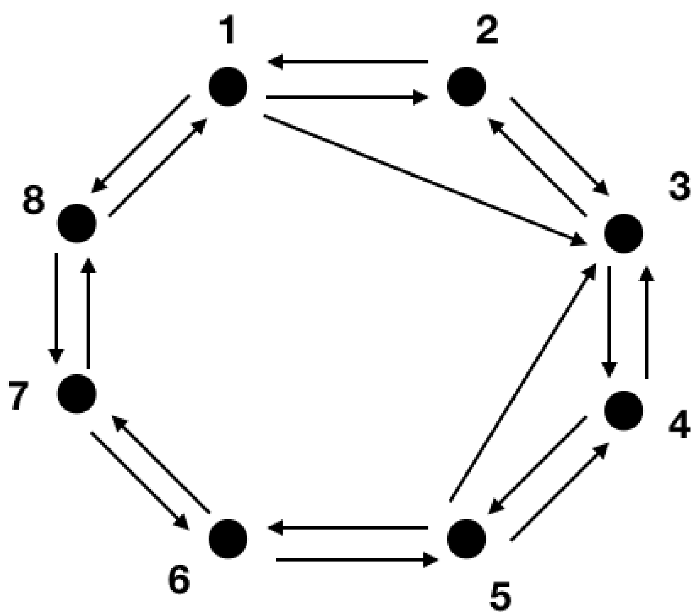
CS/ECE/ME532 Activity 14

Estimated time: 20 *minutes for Q1,* 20 *minutes for Q2 and* 10 *minutes for Q3.*

1. A ring-like network of links between web pages is shown below. Assume all feasible transitions are equally likely.



a) Find the unweighted adjacency matrix for this network.

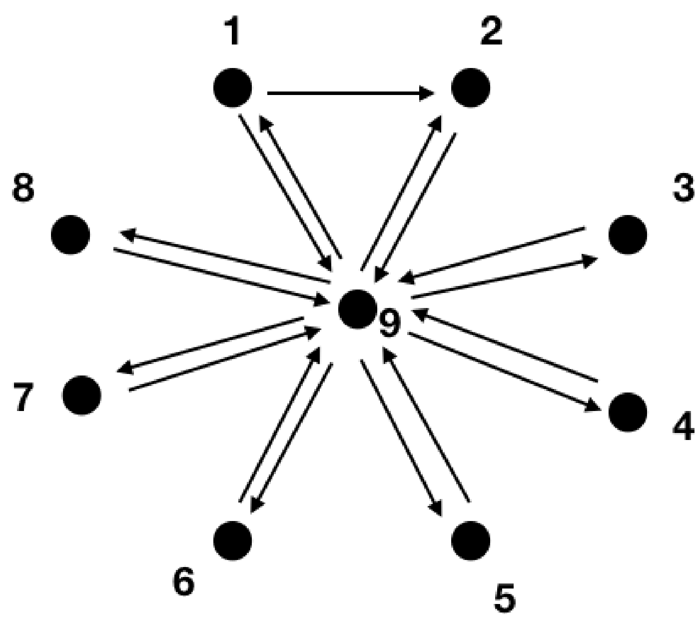
b) Find the weighted adjacency matrix for this network. Note that the entries in each column of the weighted adjacency matrix are nonnegative and sum to one. Such a matrix is called a column stochastic matrix.

c) Suppose the entries of a vector $\boldsymbol{b}$ sum to one. It is easy to show that the entries of $\boldsymbol{Ab}$ also sum to one since each column of the weighted adjacency matrix $\boldsymbol{A}$ sums to one. The PageRank algorithm thus uses the power method without normalizing the length of the vector at each iteration. Each iteration gives a new vector with positive entries that sum to one. Find the estimate of the PageRank vector after one iteration using an initial vector $\boldsymbol{b}_0 = \frac{1}{8}\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$. The initial vector $\boldsymbol{b}_0$ gives equal importance to all pages.

d) Find the estimate of the PageRank vector after 1000 iterations of the power method using an initial vector $\boldsymbol{b}_0 = \frac{1}{8}\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$. A skeleton script is available. You will need to enter the adjacency matrix into the code.

e) Do any nodes seem to be more important than other nodes? Explain.

2. A hub-like network of links between web pages is shown below. Assume all feasible transitions are equally likely.



a) Find the unweighted adjacency matrix for this network.

b) Find the weighted adjacency matrix for this network.

c) Find the estimate of the PageRank vector after one iteration using an initial vector $\boldsymbol{b}_0 = \frac{1}{9}\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$.

d) Find the estimate of the PageRank vector after 1000 iterations of the power method using an initial vector $\boldsymbol{b}_0 = \frac{1}{9}\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$.

e) Are any nodes more important than other nodes? Explain.

f) Experiment with the number of iterations of the power method that are needed to find an answer that is correct to three decimal places.

3. Consider expressing the SVD of a rank-r matrix $\boldsymbol{X}$ as

$$\boldsymbol{X} = \sum_{i=1}^{r} \sigma_i \boldsymbol{u}_i \boldsymbol{v}_i^T$$

where σ_i is the ith singular value with left singular vector $\boldsymbol{u}_i$ and right singular vector $\boldsymbol{v}_i$. Is the sign of the singular vectors unique? Why or why not? *Hint:* Consider replacing $\boldsymbol{u}_1$ with $\tilde{\boldsymbol{u}}_1 = -\boldsymbol{u}_1$.