

## CS/ECE/ME532 Classroom Activity

1. Let  $\mathbf{z} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$  and  $\mathbf{w} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ .
  - a) Sketch the subspace spanned by  $\mathbf{z}$  in  $\mathbb{R}^2$ .
  - b) Sketch the subspace spanned by  $\mathbf{w}$  in  $\mathbb{R}^2$ .
  - c) Sketch span  $\{\mathbf{z}, \mathbf{w}\}$  in  $\mathbb{R}^2$ .
  - d) Are  $\mathbf{z}$  and  $\mathbf{w}$  orthogonal? Why or why not?
  - e) Do  $\{\mathbf{z}, \mathbf{w}\}$  form an orthonormal basis? Why or why not? If not, can you modify  $\mathbf{z}$  and  $\mathbf{w}$  to form an orthonormal basis?
  
2. Consider the line in  $\mathbb{R}^2$  defined by the equation  $x_2 = x_1 + 1$ .
  - a) Sketch the line in  $\mathbb{R}^2$ .
  - b) Does this line define a subspace of  $\mathbb{R}^2$ ? Why or why not?

3. You collect ratings of three space-related science fiction movies and two romance movies from seven friends on a scale of 1-10.

Movie	Jake	Jennifer	Jada	Theo	Ioan	Bo	Juanita
Star Trek	4	7	2	8	7	4	2
Pride and Prejudice	9	3	5	6	10	5	5
The Martian	4	8	3	7	6	4	1
Sense and Sensibility	9	2	6	5	9	5	4
Star Wars: Empire Strikes	4	9	2	8	7	4	1

You put this data into a matrix  $\mathbf{X}$  (available in the file `movie.mat`) and decide to model (approximate) as the product of a rank- $r$  taste matrix with orthonormal columns and a weight matrix. That is,  $\mathbf{X} \approx \mathbf{TW}$ .

- a) What is the rank of  $\mathbf{X}$ ? Relevant Python commands are `numpy.linalg.matrix_rank()`.
- b) What are the dimensions of  $\mathbf{T}$  and  $\mathbf{W}$  (in terms of  $r$ )?

- c) You know that each user's ratings have an average value that is greater than zero because the scale is 1-10. And you suspect the baseline (average) rating may differ from user to user. To account for this you decide your first basis vector in the taste matrix should be

$$\mathbf{t}_1 = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

Choose  $w_{1j}$  so that each element of the vector  $\mathbf{t}_1 w_{1j}$  equals the average value  $j^{th}$  column of  $\mathbf{X}$ , denoted as  $\mathbf{X}_{:,j}$ . Find an expression for  $w_{1j}$  that depends on  $\mathbf{t}_1$  and  $\mathbf{X}_{:,j}$ .

- d) Define  $\mathbf{w}_1^T = [w_{11} \ w_{12} \ \cdots \ w_{17}]$  and find the rank-1 approximation to  $\mathbf{X}$  that reflects the baseline ratings of each friend,  $\mathbf{t}_1 \mathbf{w}_1^T$ .
- e) Which friend has the highest baseline rating? Which friend has the lowest baseline rating?
- f) Find the residual not modeled by  $\mathbf{t}_1 \mathbf{w}_1^T$ , that is,  $\mathbf{X} - \mathbf{t}_1 \mathbf{w}_1^T$ . Do you see any patterns in the residual? Briefly describe them qualitatively.

This problem is continued in a homework assignment.