ECE532 Assignment #1 – DEVIN BRESSER

a) The *decision boundary* is the set of $\{x_1, x_2\}$ that satisfy $x_1a_1 + x_2a_2 = b$. Thus, we may assign the label to the data using the sign of $y = x_1a_1 + x_2a_2 - b$ since label 1 is decided if $y > 0$ and label -1 is decided if $y < 0$. That is, the label may be obtained as sign$\{y\}$. Express y as an inner product of a vector $\boldsymbol{x}$ containing the features and $\boldsymbol{w}$ containing weights, that is, write $y = \boldsymbol{x}^T\boldsymbol{w}$.

$$y = \underline{x}^T \underline{w} = \begin{bmatrix} x_1 & x_2 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ -b \end{bmatrix} = x_1a_1 + x_2a_2 - b$$

$$\rightarrow y = x_1a_1 + x_2a_2 - b$$

b) Let x_2 be the vertical axis and x_1 be the horizontal axis. Show that the decision boundary $y = 0$ is a straight line. Find the slope and intercept with the vertical axis as a function of a_1, a_2, b.

$$y = 0$$

$$\rightarrow x_1a_1 + x_2a_2 - b = 0$$

$$\rightarrow x_2a_2 = -x_1a_1 + b$$

$$\rightarrow x_2 = -\frac{a_1}{a_2}x_1 + \frac{b}{a_2}$$

$$\rightarrow \text{Slope: } \frac{-a_1}{a_2}$$

$$\rightarrow y\text{-int: } \frac{b}{a_2}$$

c) You classify n data samples using $\text{sign}\{\boldsymbol{y}\}$ where $\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \boldsymbol{X}\boldsymbol{w}$. Suppose $n = 4$ and the features for the 4 data samples are $1 : (0, 0.4), 2 : (0.2, 0.1), 3 : (0.5, 0.6), 4 : (0.9, 0.8)$. Write out the matrix $\boldsymbol{X}$.

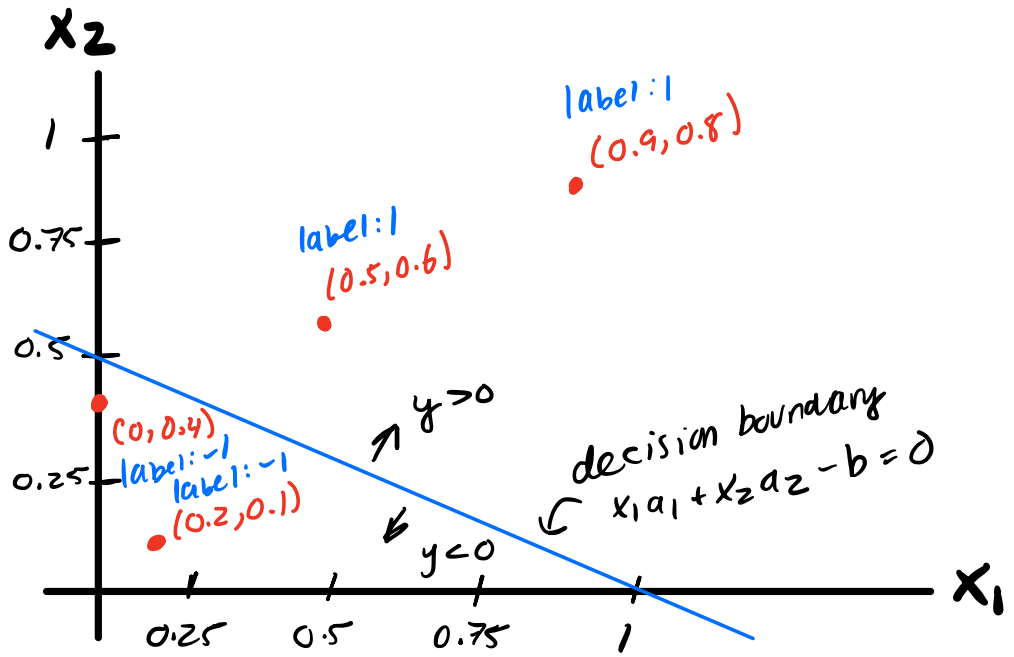
$$\underset{4\times 1}{\boldsymbol{y}} = \underbrace{\begin{bmatrix} 0 & 0.4 & 1 \\ 0.2 & 0.1 & 1 \\ 0.5 & 0.6 & 1 \\ 0.9 & 0.8 & 1 \end{bmatrix}}_{4\times 3} \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ -b \end{bmatrix}}_{3\times 1}$$

$\boldsymbol{X}$

$$\underset{n\times 1}{\boldsymbol{y}} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \underset{n\times 3}{\boldsymbol{X}}\ \underset{3\times 1}{\boldsymbol{w}}$$

d) Suppose $a_1 = 1$, $a_2 = 2$, and $b = 1$. Sketch the decision boundary in the x_1-x_2 plane assuming x_2 is the vertical axis and x_1 is the horizontal axis. Graph the four data samples from the previous part and classify them.

$$x_2 = -\frac{a_1}{a_2}x_1 + \frac{b}{a_2} \rightarrow x_2 = -\frac{1}{2}x_1 + \frac{1}{2}$$



In [16]:

```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt

# number of features
p = 2

# number of examples
n = 5000

# generate matrix of n (random) examples of p features with a column of all ones
X0 = np.random.rand(n,p)
onevec = np.ones(shape = (n,1))
X = np.concatenate((X0,onevec),axis=1)

# Classifier weights
w = [[-1], [2], [-0.4]]

# Multiply feature matrix with weights  yhat = X*w
yhat = X@w

# Decide which class based on whether yhat is >< 0
# sign function returns +1 when yhat(i)>0 and -1 when yhat(i)<0
pred_label = np.sign(yhat);

plt.scatter(X[:,[0]],X[:,[1]], color = ['r' if i==-1 else 'b' for i in pred_label])

plt.xlabel("x_1")
plt.ylabel("x_2")
plt.title("Case 2")
plt.axis('square')
plt.show()


# Problem 1.e comment: The boundary is a line with y-intercept 0.2 and slope 0.5
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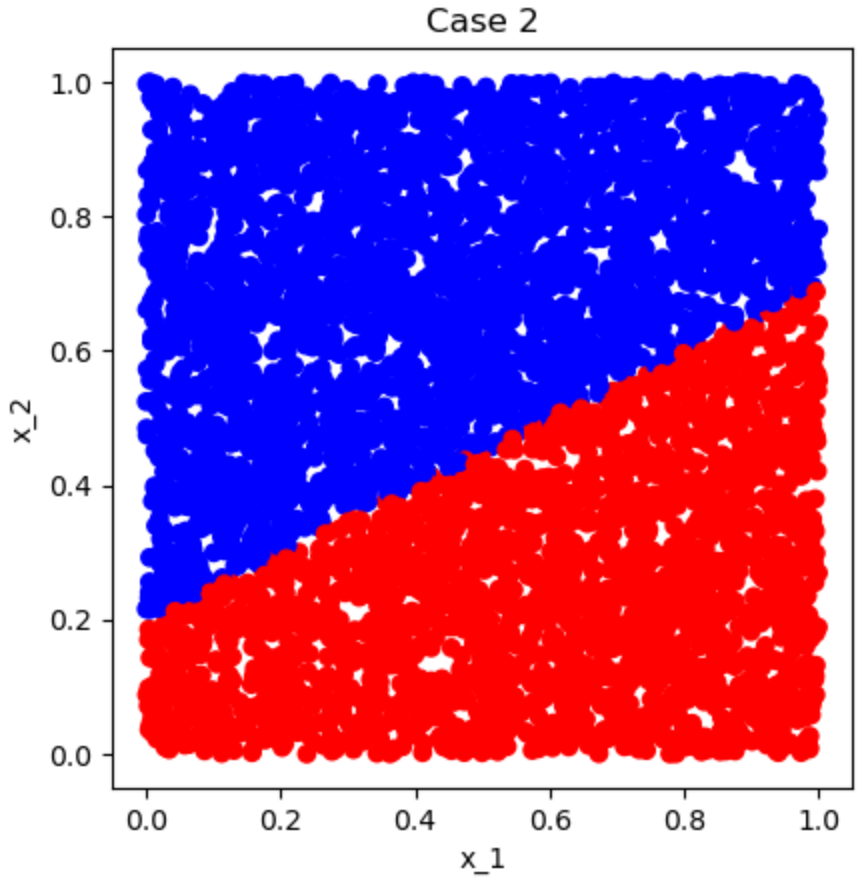

In [18]:

```
# Classifier weights
w = [[1.6], [2], [-1.6]]

# Multiply feature matrix with weights  yhat = X*w
yhat = X@w

# Decide which class based on whether yhat is >< 0
# sign function returns +1 when yhat(i)>0 and -1 when yhat(i)<0
pred_label = np.sign(yhat);

plt.scatter(X[:,[0]],X[:,[1]], color = ['r' if i==-1 else 'b' for i in pred_label])

plt.xlabel("x_1")
plt.ylabel("x_2")
plt.title("Case 2")
plt.axis('square')
plt.show()

# Problem 1.f comment: The new weights adjusted the slope and y-intercept of the
# decision boundary. Now, the boundary has y-intercept 0.8 and slope -0.8
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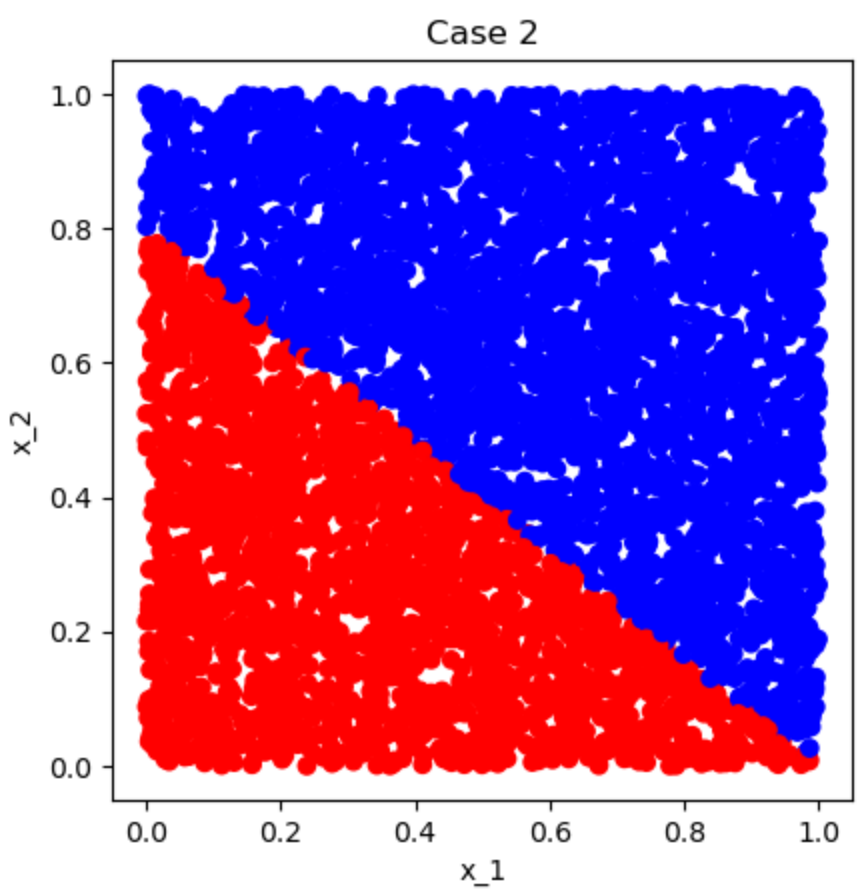

In [ ]: