

# Geometric Manifolds and the Shape of Space

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MATH 331: Geometry

Fall 2014

# Outline

1. Geometric 2-Manifolds
2. Geometric 3-Manifolds
3. The Shape of Space

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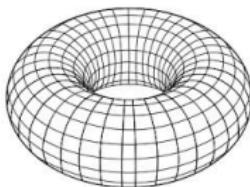
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# Introduction

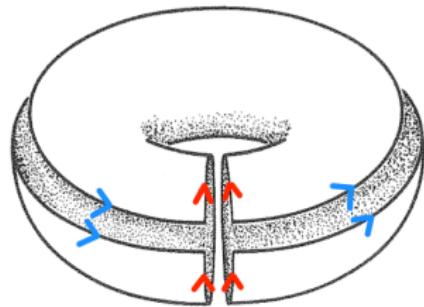
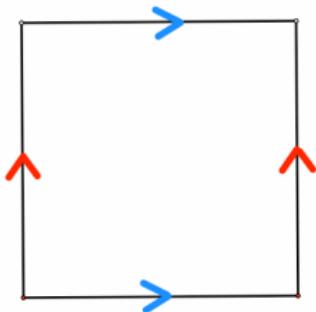
- A **Geometric 2-Manifold** is a connected surface that is locally isometric to either the Euclidean plane, hyperbolic plane, or sphere.
- Cones (excluding cone point), Cylinders, and Tori are examples of flat 2-manifolds



[http://commons.wikimedia.org/wiki/File:Simple\\_Torus.svg](http://commons.wikimedia.org/wiki/File:Simple_Torus.svg)

# Gluings

- **Gluings** are when two edges or sides of a surface are “connected” and share the same set of points.
- The orientation of the gluings determine the structure of the manifold and whether it is flat, hyperbolic, or spherical.



The Shape of Space - Figure 7.14, pg. 117

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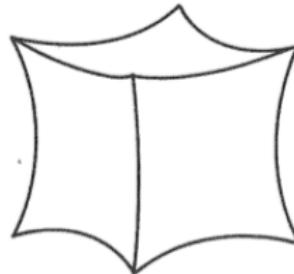
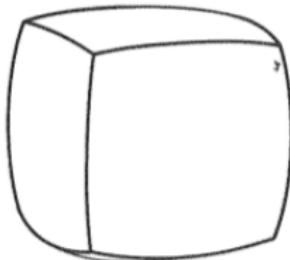
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## Geometric 3-Manifolds

- A **Geometric 3-Manifold** is a space where the region surrounding any point is isometric to Euclidean 3-space, hyperbolic 3-space, or the 3-sphere.
- **Euclidean 3-space** is formed by adding a third dimension to the plane.
- **Hyperbolic 3-space** is formed by adding a third dimension to the hyperbolic plane.
- **The 3-Sphere** is the set of points in 4-D space that are an equal distance away from an origin.

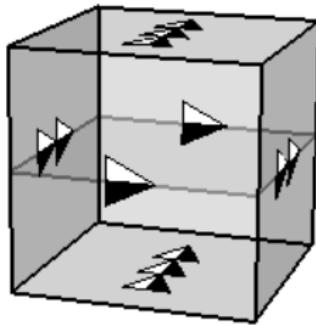


The Shape of Space - Figure 14.7, pg. 209

The Shape of Space - Figure 15.2, pg. 215

# 3-Torus

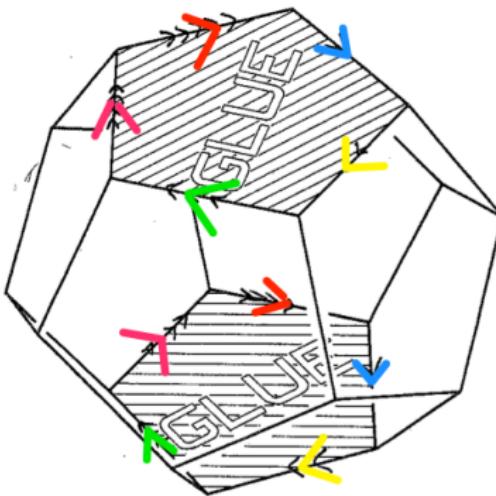
- The 3-Torus is:
  - Constructed by gluing opposite sides of a cube.
  - A geometric 3-manifold that is isometric to Euclidean 3-space.
- Cubes perfectly fill 3-space (8 cubes meet at each corner).



[http://euler.slu.edu/escher/index.php/The\\_Three\\_Geometries](http://euler.slu.edu/escher/index.php/The_Three_Geometries)

# Poincaré Dodecahedral Space

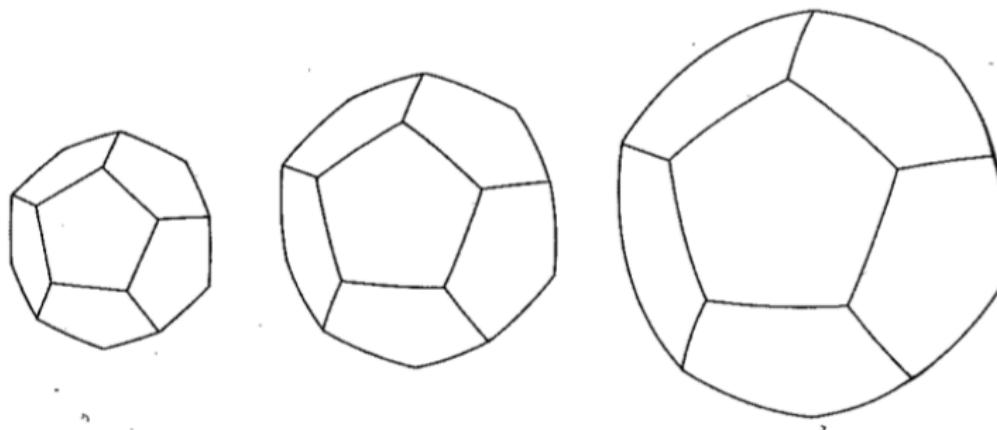
- Poincaré Dodecahedral Space is:
  - Constructed by gluing opposite sides of a dodecahedron with a clockwise  $\frac{1}{10}$  rotation.
  - A geometric 3-manifold that is isometric to the 3-Sphere.



The Shape of Space - Figure 16.3, pg. 222

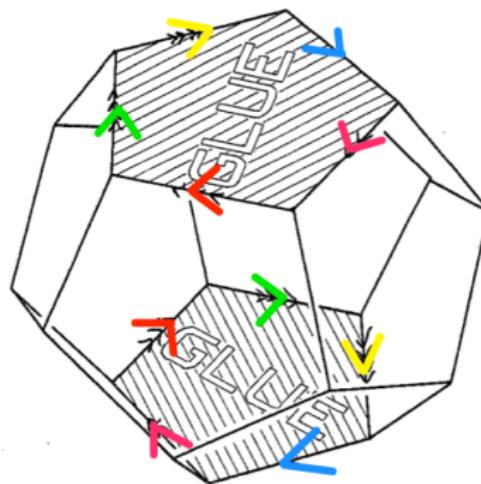
# Poincaré Dodecahedral Space, cont.

- Four dodecahedrons meet at each vertex, and the angles are too small to perfectly fill 3-Space.
- In the 3-Sphere, as the dodecahedrons expand, the angles increase and eventually fit together in groups of four.



# Seifert-Weber Space

- Seifert-Weber Space is:
  - Constructed by gluing opposite sides of a dodecahedron with a clockwise  $\frac{3}{10}$  rotation.
  - A geometric 3-manifold that is isometric to Hyperbolic 3-Space.



The Shape of Space - Figure 16.1, pg. 220

## Seifert-Weber Space, cont.

- 20 dodecahedrons meet at each vertex, and the angles are too large to perfectly fill 3-Space.
- In Hyperbolic 3-Space, as the dodecahedrons expand, the angles decrease and eventually fit together in groups of twenty.



# Outline

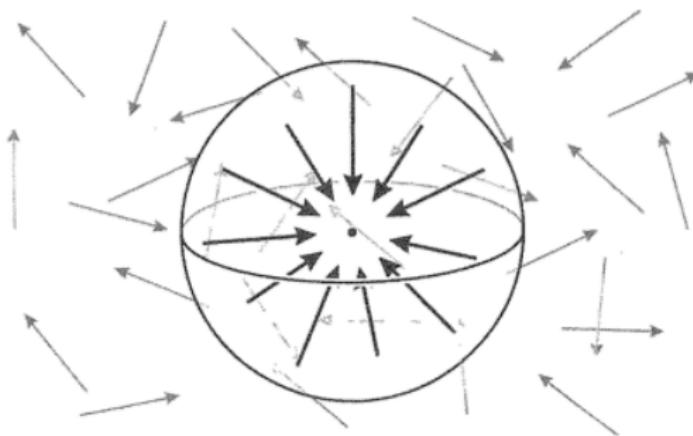
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# Cosmic Microwave Background

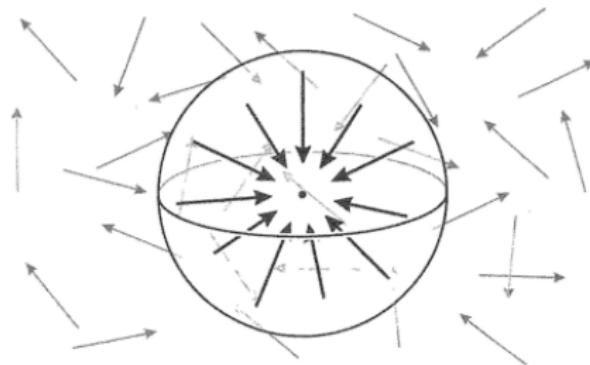
- Following the Big Bang, radiation scattered throughout the Universe.
- As the Universe expanded overtime, this radiation has cooled to microwaves known as **Cosmic Microwave Background (CMB)**.



The Shape of Space - Figure 22.5, pg. 300

## Cosmic Microwave Background, cont.

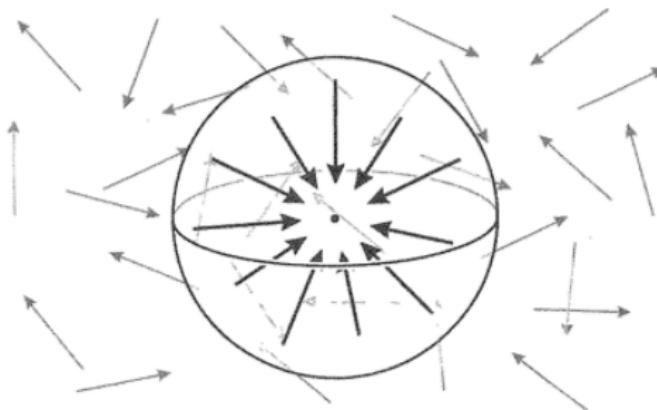
- Microwaves are nearly uniform in temperature
- Small fluctuations in temperature due to the density of the region where they formed.
- Microwaves travel in all directions.
- First observed in 1965 by Arno Penzias and Robert Wilson of Bell Labs.



The Shape of Space - Figure 22.5, pg. 300

# Last Scattering Surface

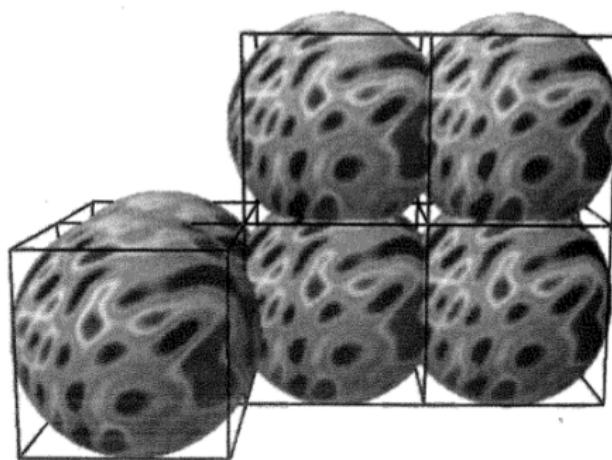
- Microwaves that we observe from Earth have travelled the same distance from their starting position to the time we record them.
- All microwaves observed at one time originated on the surface of the **Last Scattering Sphere (LSS)**.



The Shape of Space - Figure 22.6, pg. 301

# The Shape of Space

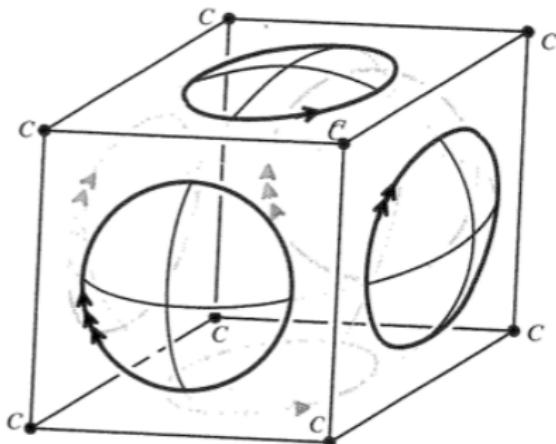
- Assuming the Universe is a 3-Manifold, there exists reoccurring images of the Earth at the center of its LSS.
- If the LSS is larger than a single image of the Universe, then it overlaps itself.



The Shape of Space - Figure 22.8, pg. 304

## The Shape of Space, cont.

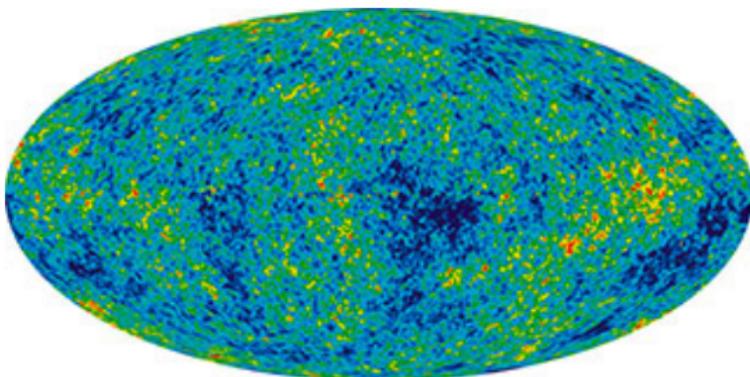
- In the case of the 3-Torus Universe, circles form along each side of the cube where the LSS overlaps itself.
- The circles on opposite sides contain the same set of CMB, which produce the same temperature pattern.



Experiencing Geometry- Figure ??, pg. 359

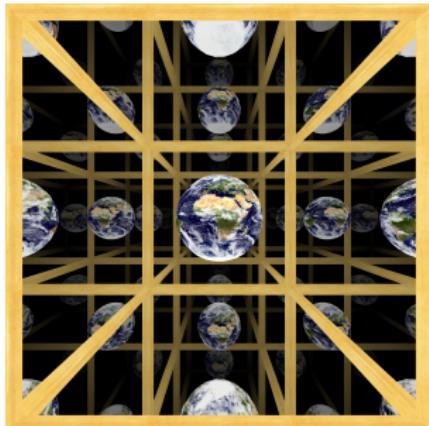
# Conclusion

- The orientation of the temperature patterns of these sets hint at the gluings of the 3-Manifold Universe, and ultimately the geometric structure.
- NASA's Wilkinson Microwave Anisotropy Probe (WMAP) mapped out the CMB and its temperature patterns.
- Results showed that Universe should hold properties similar to Euclidean Geometry

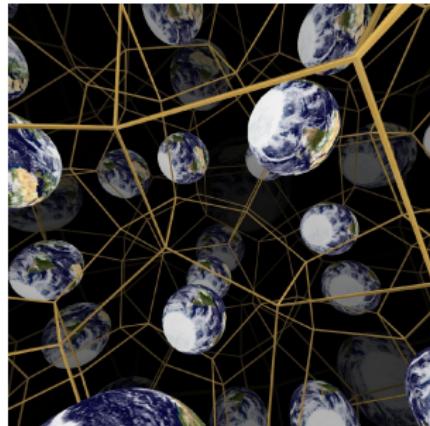


<http://map.gsfc.nasa.gov/media/121238/index.html>

# Questions?



[http://web.math.princeton.edu/  
conference/Thurston60th/lectures.html](http://web.math.princeton.edu/conference/Thurston60th/lectures.html)



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