Geometric Manifolds and the Shape of Space

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MATH 331: Geometry

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Outline

1. Geometric 2-Manifolds
2. Geometric 3-Manifolds
3. The Shape of Space

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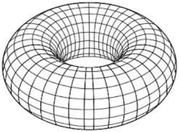
Introduction

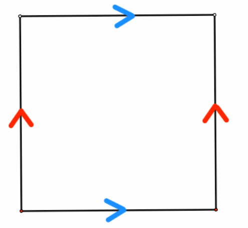
- A Geometric 2-Manifold is a connected surface that is locally isometric to either the Euclidean plane, hyperbolic plane, or sphere.
- Cones (excluding cone point), Cylinders, and Tori are examples of flat 2-manifolds

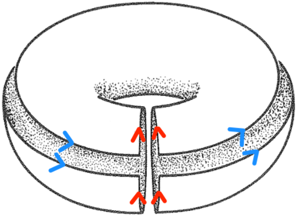


http://commons.wikimedia.org/wiki/File:Simple_Torus.svg

Gluings

- Gluings are when two edges or sides of a surface are "connected" and share the same set of points.
- The orientation of the gluings determine the structure of the manifold and whether it is flat, hyperbolic, or spherical.





The Shape of Space - Figure 7.14, pg. 117

Outline

1. Geometric 2-Manifolds

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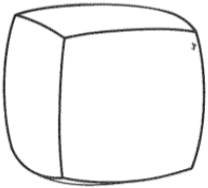
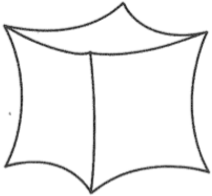
Geometric 3-Manifolds

- A Geometric 3-Manifold is a space where the region surrounding any point is isometric to Euclidean 3-space, hyperbolic 3-space, or the 3-sphere.
- Euclidean 3-space is formed by adding a third dimension to the plane.
- Hyperbolic 3-space is formed by adding a third dimension to the hyperbolic plane.
- The 3-Sphere is the set of points in 4-D space that are an equal distance away from an origin.



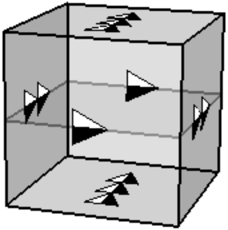
The Shape of Space - Figure 14.7, pg. 209



The Shape of Space - Figure 15.2, pg. 215

3-Torus

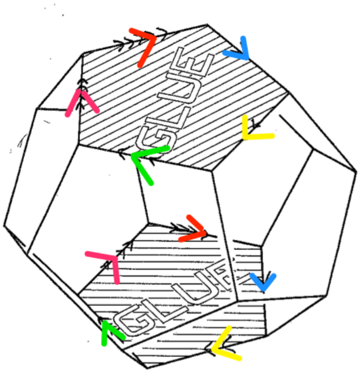
- The 3-Torus is:
  - Constructed by gluing opposite sides of a cube.
  - A geometric 3-manifold that is isometric to Euclidean 3-space.
- Cubes perfectly fill 3-space (8 cubes meet at each corner).



http://euler.slu.edu/escher/index.php/The_Three_Geometries

Poincaré Dodecahedral Space

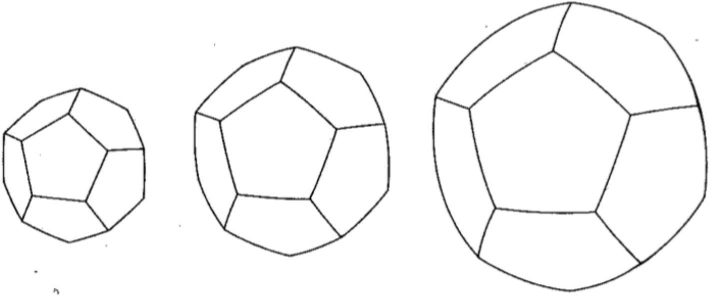
- Poincaré Dodecahedral Space is:
  - Constructed by gluing opposite sides of a dodecahedron with a clockwise $\frac{1}{10}$ rotation.
  - A geometric 3-manifold that is isometric to the 3-Sphere.



The Shape of Space - Figure 16.3, pg. 222

Poincaré Dodecahedral Space, cont.

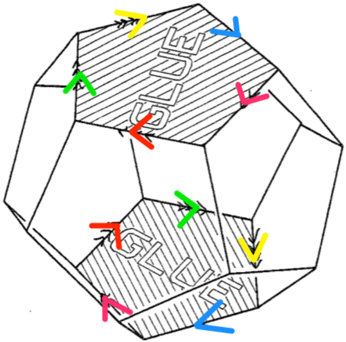
- Four dodecahedrons meet at each vertex, and the angles are too small to perfectly fill 3-Space.
- In the 3-Sphere, as the dodecahedrons expand, the angles increase and eventually fit together in groups of four.



The Shape of Space - Figure 16.4, pg. 223

Seifert-Weber Space

- Seifert-Weber Space is:
  - Constructed by gluing opposite sides of a dodecahedron with a clockwise $\frac{3}{10}$ rotation.
  - A geometric 3-manifold that is isometric to Hyperbolic 3-Space.



The Shape of Space - Figure 16.1, pg. 220

Seifert-Weber Space, cont.

- 20 dodecahedrons meet at each vertex, and the angles are too large to perfectly fill 3-Space.
- In Hyperbolic 3-Space, as the dodecahedrons expand, the angles decrease and eventually fit together in groups of twenty.

The Shape of Space - Figure 16.2, pg. 221

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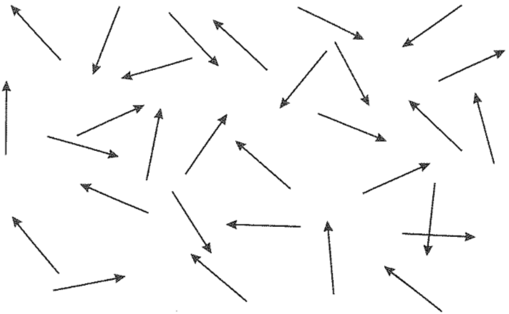
Cosmic Microwave Background

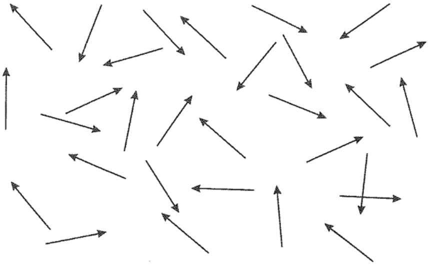
- Following the Big Bang, radiation scattered throughout the Universe.
- As the Universe expanded overtime, this radiation has cooled to microwaves known as Cosmic Microwave Background (CMB).



The Shape of Space - Figure 22.5, pg. 300

Cosmic Microwave Background, cont.

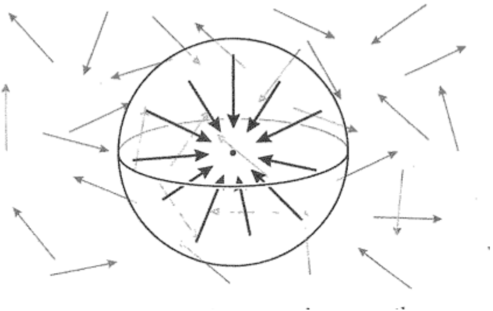
- Microwaves are nearly uniform in temperature
- Small fluctuations in temperature due to the density of the region where they formed.
- Microwaves travel in all directions.
- First observed in 1965 by Arno Penzias and Robert Wilson of Bell Labs.



The Shape of Space - Figure 22.5, pg. 300

Last Scattering Surface

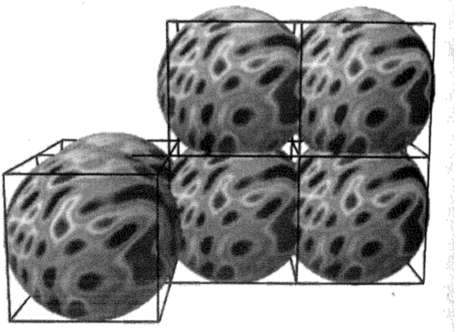
- Microwaves that we observe from Earth have travelled the same distance from their starting position to the time we record them.
- All microwaves observed at one time originated on the surface of the Last Scattering Sphere (LSS).



The Shape of Space - Figure 22.6, pg. 301

The Shape of Space

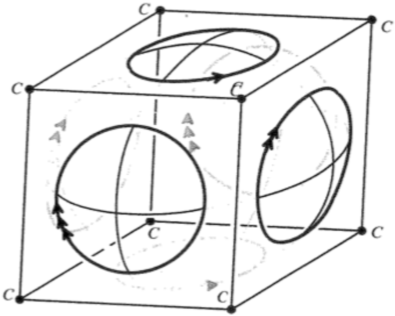
- Assuming the Universe is a 3-Manifold, there exists reoccuring images of the Earth at the center of its LSS.
- If the LSS is larger than a single image of the Universe, then it overlaps itself.



The Shape of Space - Figure 22.8, pg. 304

The Shape of Space, cont.

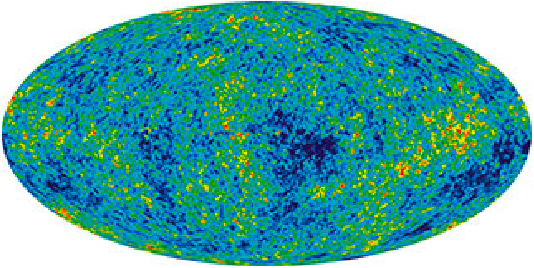
- In the case of the 3-Torus Universe, circles form along each side of the cube where the LSS overlaps itself.
- The circles on opposite sides contain the same set of CMB, which produce the same temperature pattern.



Experiencing Geometry- Figure 24.8, pg. 359

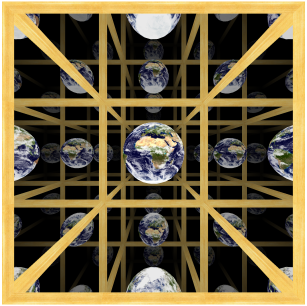
Conclusion

- The orientation of the temperature patterns of these sets hint at the gluings of the 3-Manifold Universe, and ultimately the geometric structure.
- NASA's Wilkinson Microwave Anisotropy Probe (WMAP) mapped out the CMB and its temperature patterns.
- Results showed that Universe should hold properties similar to Euclidean Geometry

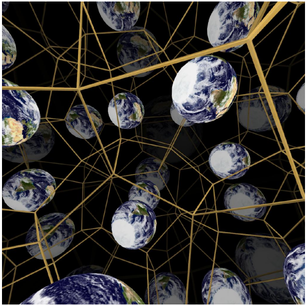


http://map.gsfc.nasa.gov/media/121238/index.html

Questions?



http://web.math.princeton.edu/conference/Thurston60th/lectures.html



http://quibb.blogspot.com/2011/05/manifolds-shape-of-universe-iii.html

References I

- Britt Griswold.
  National Aeronautics and Space Administration: Wilkinson Microwave Anisotropy Probe.
  http://map.gsfc.nasa.gov, July 2013.
- David W. Henderson, and Daina Taimina.
  Experiencing Geometry: Euclidean and Non-Euclidean with History.
  Prentice Education, Inc., Upper Saddle River, NJ, 2005.
- Jean-Pierre Luminet.
  The Wraparound Universe.
  A K Peters, Ltd., Wellesley, MA, 2008.
- Jeffrey R. Weeks.
  The Shape of Space.
  Taylor & Francis Group, LLC, Boca Raton, FL, 2002.

References II

- Jeffrey R. Weeks.
  Curved Spaces.
  http://www.geometrygames.org/CurvedSpaces/index.html, 2014.