

Figure 16.3 In the Poincaré dodecahedral space every face of a dodecahedron is glued to the opposite face with a one-tenth clockwise turn.

have a Euclidean geometry for essentially the same reason that the *first* surface in Figure 11.1 failed to have one: the dodecahedron's twenty corners come together in five groups of four corners each, and they are a little too skinny to fit together properly. The solution is the same as in Chapter 11. Put the dodecahedron in a hypersphere and let it expand until its corners are fat enough that they do fit together (Figure 16.4). A Poincaré dodecahedral space made from the appropriate dodecahedron has the homogeneous

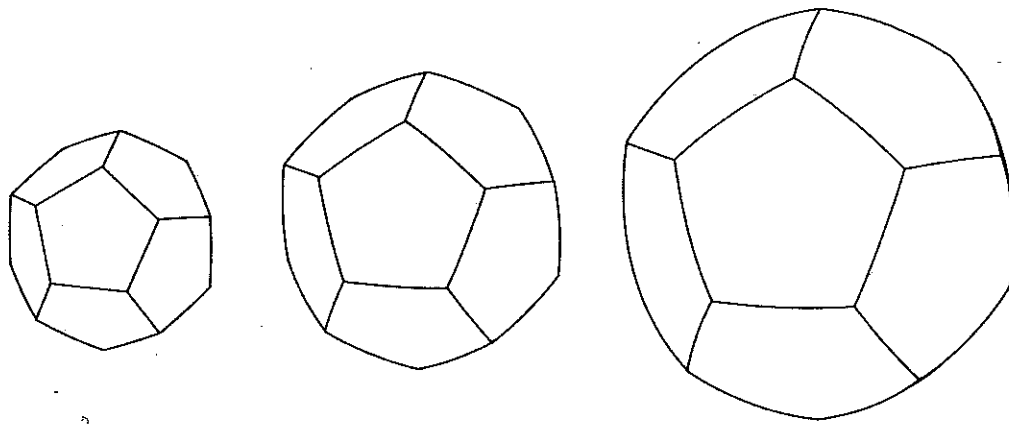


Figure 16.4 Let a dodecahedron expand in a three-sphere until its corners are the right size to fit together in groups of four. The angles shown here are accurate, but in the three-sphere itself the dodecahedron's faces do not bulge outward.