Maximizing National Welfare Under Price Controls: A Tariff Optimization Model

MAT 4800: Introduction to Nonlinear Optimization

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Agenda

Background: What Are Tariffs?

- Tariffs are taxes placed on imported goods.
- Intended to protect domestic industries and generate government revenue.
- Common in international trade disputes and policy debates.
- At a glance, tariffs help the importing country and hurt the exporting one.

But too high a tariff leads to economic inefficiency and deadweight loss.

Framing the Problem

- Tariffs shift national welfare by affecting:
  - Consumer surplus
  - Producer surplus
  - Government revenue
- Over-tariffing can backfire – leading to losses in total welfare.
- This creates an optimization question:

Can we find a tariff rate that maximizes the importing country's gains while minimizing deadweight loss?

Welfare Effects of a Tariff

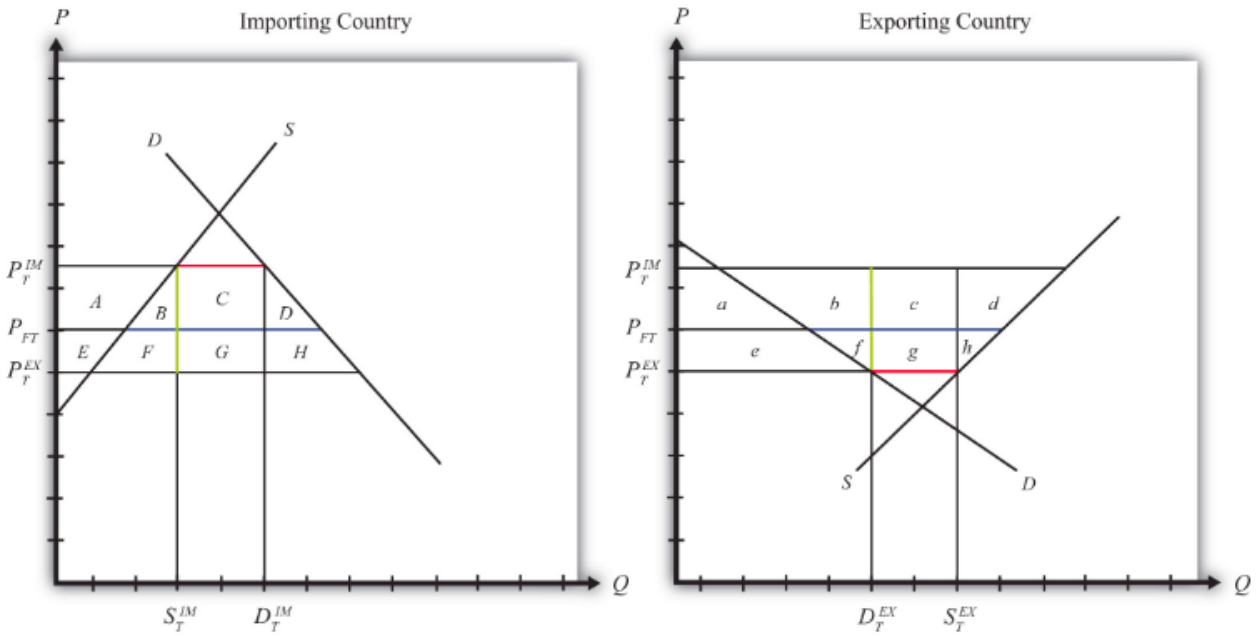


Figure: Welfare Effects of a Tariff [1]

Importing Country: Welfare Breakdown

- **Consumer Surplus:** $-(A + B + C + D)$
- **Producer Surplus:** $+A$
- **Government Revenue:** $+C + G$

Net National Welfare: $G - (B + D)$

This will be the objective function in our optimization.

Exporting Country: Welfare Loss

- Unlike importers, exporters **don't collect tariff revenue**.
- Their national welfare is just:
  - **Consumer Surplus**
  - **Producer Surplus**
- Represented by loss: $-(f + g + h)$ in the diagram.

To fully assess welfare effects, we subtract this loss from the importer's gains.

Methodology Overview

- We model the welfare change for an importing country under a tariff.
- The welfare function is derived from the area between supply and demand curves.
- We incorporate economic constraints and basic price relationships.

Welfare Function

$$W(t, p_i) = (p_f - p_e)(D(p_i) - S(p_i)) \quad \text{(G)}$$
$$- \frac{1}{2}(p_i - p_f)(S(p_i) - S(p_f)) \quad \text{(B)}$$
$$- \frac{1}{2}(p_i - p_f)(D(p_f) - D(p_i)) \quad \text{(D)}$$

Key Variables:

- t: Tariff Rate
- p_i: Price Floor in Importing Country
- p_e: Exporting Country Price
- p_f: Free Trade Price
- $S(P)$: Supply Function
- $D(P)$: Demand Function

Tariff Rate Relationship

$$t = \frac{p_i}{p_e} - 1 \quad \text{where } t \geq 0,\ p_i \geq p_f$$

- This expresses t as a function of price distortion.
- Used to substitute into the welfare function.

Welfare Function with Tariff Substitution

$$
\begin{aligned}
W(t, p_i) = (p_f - \frac{p_i}{t+1})(D(p_i) - S(p_i)) \\
- \frac{1}{2}(p_f - \frac{p_i}{t+1})(S(p_i) - S(\frac{p_i}{t+1})) \\
- \frac{1}{2}(p_f - \frac{p_i}{t+1})(D(\frac{p_i}{t+1}) - D(p_i)) \qquad (2)
\end{aligned}
$$

Supply and Demand Functions

We define linear demand and supply functions to simplify:

$$D(p) = b_d - m_d \cdot p$$
$$S(p) = b_s + m_s \cdot p$$

where:

- m_d, m_s: Slopes of demand and supply
- b_d, b_s: Y-intercepts

Real-World Data: Steel Imports

We apply the optimization model to U.S. steel import data.

- Free trade price of steel: **$833 per metric ton**
- Data from: *UN Comtrade, FocusEconomics, steel.org*

Period	Reporter	Partner	Commodity Code	Trade Value (US$)	Net Weight (kg)	Free Trade Price
2024	USA	World	7208 (steel)	$2,431,079,947	2,753,484,814	$883 per metric ton

Figure: U.S. Steel Import Data (HS Code 7208) from UN Comtrade [4]

Model Setup and Assumptions

We estimated linear demand and supply functions from market data:

$$D(p) = 8 - 0.001716 \cdot p$$
$$S(p) = 1 + 0.000189 \cdot p$$

Tariff bounds and trade price assumptions:

- Free trade price: $p_f = 883$
- Tariff rate bounds: $t_{floor} = 0$, $t_{ceiling} = 0.5$

Optimization performed using MATLAB's `fmincon` solver.

Optimal Tariff Policy (Results)

Objective: Maximize national welfare for the importing country
Optimal solution found:

- Optimal tariff rate: **50%**
- Optimal price floor: **$883**
- Resulting welfare gain: **$1482 million USD**

Conclusion: Under current market conditions, a higher tariff maximizes welfare – but only when supply and demand remain feasible.

Objective Function Extension

We can add a new condition to the objective function that takes into account the exporting country's welfare:

$$-\frac{1}{2}(p_f - p_e)\left(D(p_f) - S(p_f) + D(p_e) - S(p_e)\right)$$

This makes the new objective function:

$$\begin{aligned} W_2(t, p_i) = (p_f - \frac{p_i}{t+1})(D(p_i) - S(p_i)) \\ - \frac{1}{2}(p_f - \frac{p_i}{t+1})(S^{-1}(p_i) - S^{-1}(\frac{p_i}{t+1})) \\ - \frac{1}{2}(p_f - \frac{p_i}{t+1})(D(\frac{p_i}{t+1}) - (p_i)) \\ - \frac{1}{2}(p_f - p_e)\left(D(p_f) - S(p_f) + D(p_e) - S(p_e)\right) \quad (3) \end{aligned}$$

Optimal Results for Objective Function Extension

Objective: Maximize national welfare for both countries

Optimal solution found:

- Optimal tariff rate: **0%**
- Optimal price floor: **$883**
- Resulting welfare gain: **$0 million USD**

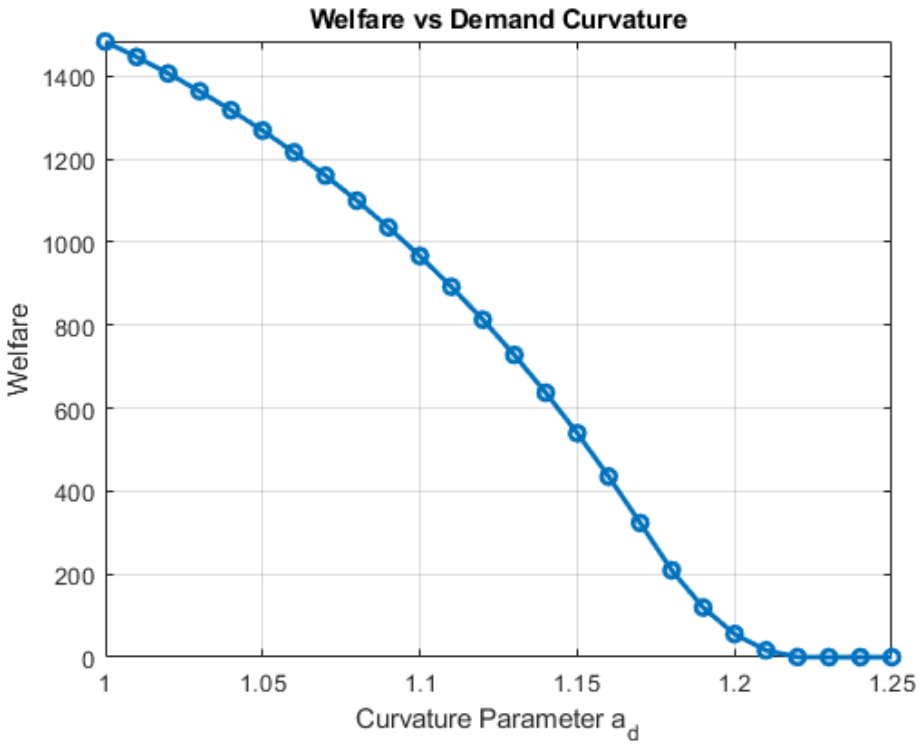
Demand Function Extension

We can add a curvature parameter to the demand function:

$$D(p) = b_d - m_d \cdot p^{a_d}$$

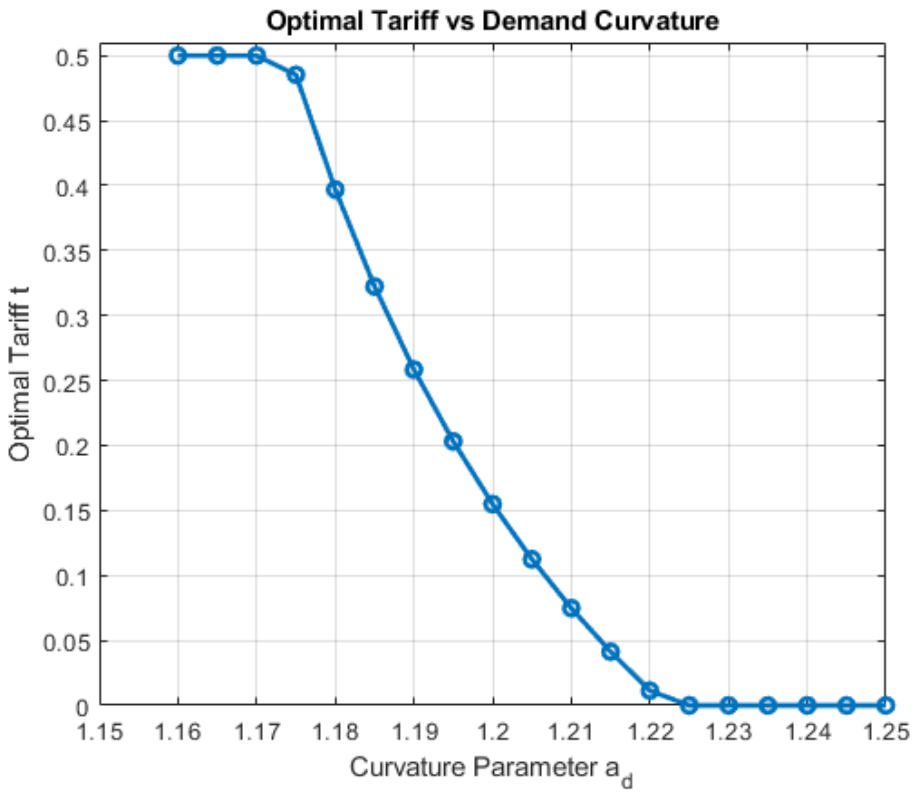
Modifying the demand function will make customers more sensitive to increases in prices, meaning the demand will now decrease much quicker when a_d increases.

Demand Function Extension



Welfare vs a_d

Demand Function Extension



Tariff rate vs a_d

Extension: Market Responsiveness Scenarios

Supply-Constrained Market

- Supply is less responsive (very inelastic).
- Imports remain high, boosting government tariff revenue.
- **Optimal Price:** \$826
  Welfare: \$2452.43 Million

Elastic Demand Market

- Demand is highly sensitive to price changes.
- Even a small price increase causes a large drop in imports.
- **Optimal Price:** \$847
  Welfare: \$1654.47 Million

Insight: Welfare gains vary widely based on market behavior. Inelastic supply supports higher welfare under tariffs, while elastic demand reduces the importer's advantage.

Conclusion

- We developed an optimization model to determine the tariff rate that maximizes an importing country's national welfare.
- A secondary model considers trade relationships by comparing gains for the importer against losses for the exporter.
- Results show two extremes as optimal outcomes: **no tariff** or the **maximum allowable tariff**.
- This model assumes perfect market conditions – a useful simplification, but one that overlooks economic realities.
- Future directions include:
  - Modeling with imperfect or dynamic markets
  - Using GDP change as a more holistic objective function
  - Evaluating whether real-world tariffs are economically justified

References

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Questions

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