Statistical research paper

To what extent does the number of Covid-19 cases per million people affect the economic performance of different countries?
30 pages

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"*One is a tragedy, a million is a statistic.*"[1]

~ Oliver Cromwell

Introduction:

Pandemics are merciless; and so are their statistics. Covid-19, a respiratory disease first discovered in January, quickly evolved into a pandemic. To this date, it has infected 36 million and killed over one million[2]. And these statistics are not just mere numbers, they dictate the many critical decisions that governments make about the economy: They decide whether to reopen trade, schools, and factories based on these numbers, and watch attentively when the cases spike up. My interest in this topic first began when lockdowns happened in Europe during April and I started tracking the growth of cases in major European countries.

Prior to that, I have been involved with Model United Nations for 5 years and have represented team Canada on many occasions, where International economic development became my area of interest. By reaching out to the WHO, I was able to obtain their weekly report which gave an accurate, first hand count of Covid-19 data. Upon looking at these numbers and comparing them, I was fascinated by the range of different Covid-19 cases in every countries and their association with economic contraction, unemployment rate, and in short, the virus' ability to ruin a country. This leads me to consider if a nation's economic performance during the pandemic can be explained by its Covid-19 cases per million population, this is because a large number of cases will lead to lockdowns and quarantine, thus preventing the economy from functioning normally . Therefore for this Internal assessment I will be conducting a real world application investigation to detect the strength of the correlation between the number of Covid-19 cases per million population people, and the changes in the real GDP as a percentage. For this investigation, I plan to focus on understanding how statisticians can work with numbers to assist politicians to make important decisions, and applying it to a real-life, on-going pandemic situation at present.

Plan of Investigation:

[1]Quote Investigator. *Oliver Cromwell Quotes.* https://quoteinvestigator.com/2010/05/21/one-is-a-tragedy-a-million-is-a-statistic

[2] World Health Organization. *Novel Coronavirus Pandemic.* https://www.who.int/diseases/novel-coronavirus-2020 (accessed October 10th, 2020). Case count as of October 10th, 7:20pm CEST

Introduction**:**

The first step is to explore the univariate data of my exploration. Covid-19 cases (per million population) is the independent variable which will be determined for advanced and emerging economies to examine the range and the spread of the data. This will enable comparison and contrasting of the Covid-19 Cases and their respective GDP change for advanced and emerging economies by constructing parallel boxplots. In addition, the boxplot provides visuals for the five numbers summaries, which will allow me to find the spread of the of the data and test for any outliers.

Bivariate data:

Following this I will use bivariate mathematics to investigate the relationship between the two variables: cases of Covid-19 (per million people) and the change in real GDP as a percentage with both advanced and emerging economies. I plan to do this through the use of scatter diagrams, which will allow me to employ the use of Regression equations or the Perason's Product Moment Correlation to determine if there is a correlation between the data and an equation can be generated using methods such as the Least squares regression line to represent this relationship. If an equation can be formulated I plan to model the expected real GDP change in percentage of a country and compare it with the actual results to calculate the percentage error in the experiment and talk about errors and factors and their implications in the reflection.

Correlation and statistical testing

Finally, to further explore the relationship between the two variables I will combine all the data collected and generate a scatter diagram to see if there is an overall correlation between the independent and dependent variable. If the correlation obtained using regression equation is insufficient and shows little correlation. I plan to employ the use of alternative mathematics such as the the Spearman's Correlation Coefficient as it gives a clear understanding in formulating a stronger association in the form of linear regression. Upon finding an equation, I will look at how errors factors that could potentially affect the data such as an effective lockdown will cause a decrease in Covid-19 cases and attempt to locate and explain them using statistics such as the chi-squared test for statistical significance. These steps will ultimately allow me to

answer the research question of my investigation 'To what extent does the number of Covid-19 cases per million people affect the economic performance of different countries?'

Table 1: Definitions of key terms

These definitions were obtained from the World and the World Health organization, Note that terms such as "rich or poor" are no longer used by major world institutions (i.e. The UN) due to their derogatory nature and replaced by terms like "advanced and emerging economies" - World Bank[3]

Term	Definition
Advanced Economies	Advanced economies are high-income countries with a GDP per capita of $12,535 USD or more in the current fiscal year of 2021, inflation adjusted.
Emerging Economies	Emerging economies are low and middle-income countries with a GDP per capita that is lower than $12,535 USD in the current fiscal year of 2021, inflation adjusted.[4]
Nominal change in Gross Domestic Product (GDP)	The sum of gross value added by all resident producers in the economy, plus any product taxes but minus any subsidies not included in the value of the products. It is calculated without making deductions for depreciation of fabricated assets or for depletion and degradation of natural resources. Nominal GDP change is measured at current market price and thus not adjusted by inflation.
GDP deflator (Implicit price deflator)	GDP deflator is a measure of price inflation/deflation, with respect to a specific base year. Unlike the Consumer Price Index (CPI) it includes the price of all products and materials produced by a certain country instead of only the consumer goods, thus greater accuracy is ensured.
Independent Variable: Covid-19 cases (per million population)	The Covid-19 Cases per million is the total, cumulative number of people who were infected with SARS-COV2. The rate of the case per million population of a given country can be determined by the number of total cumulative cases, divided by the population of that country, and multiplied by one million.[5]
Dependent Variable: Real change in Gross Domestic Product (GDP)	The term *real* is used to denote the sum of gross value added by all resident producers in the economy, plus any product taxes but minus any subsidies not included in the value of the products, with adjustments of inflation included. Its calculation involves dividing the nominal change with the GDP deflator. The following equation is used to calculate Real GDP change: ● Real GDP = Nominal GDP / GDP deflator[6]

[3] The World Bank. *Country and Lending Groups.2021.* https://data.worldbank.org/906519-world-bank-country-and-lending-groups (Accessed October 10th, 2020.

[4] The World Bank. *Methodologies.* https://data.worldbank.org/knowledgebase/articles/906531-methodologies (accessed October 20th, 2020).

[5] The World Health Organization. *WHO Coronavirus disease dashboard.* https://covid19.who.int/ (accessed November 10th, 2020).

[6] The World Bank. *Inflation, GDP deflator (Annual %)* https://data.worldbank.org/indicator/NY.GDP.DEFL.KD.ZG (Accessed October 10th, 2020)

Obtaining data and the process of sampling

Given the rapidly changing nature of the data being investigated, for the purpose of consistency in this investigation, all Covid-19 related data[7] obtained were dated from October 10th, 2020, with the data obtained with from a range of ±7 days, given that some countries update their case numbers on a weekly basis. For economic data, this will be within a year.[8] In addition, the scope of this investigation has been limited to the 193 UN recognized countries and the sample size will be generated within the range of the 193 UN-recognized countries.

Stratified sampling was employed here to obtain data. All countries were first grouped into advanced and emerging economies. Using the two lists, a sample of 20 economies (representing the 64 advanced economies), and 40 emerging economies (representing the 129 emerging economies) were obtained. The sample size combined will roughly represent ⅓ of the 193 countries. However, given the possibility that a random sampling might not be representative of the ⅓ of the global population, thus the economies of both groups separately in groups of 3 in ascending order from most populated to least were sampled. Using this method, a sample size with similar representation of the global population as well as the number of countries can be obtained. Sampling in groups of threes also ensure that I wont have both India and China in the sample at the same time, whose combined population of 2.77 billion people is already 35.5% of the world's 7.8 billion people.

In addition, by employing the method of quota sampling, it ensures that there will be enough representation of countries from each stratum as well as population equal in proportion. The website which was used for sampling was www.random.org. The p-hat equation, $\hat{p} = \frac{x}{n}$ was used to calculate the sample size in proportion to the population as a whole. Thus, it is verified that all the samples correspond to the ⅓ representation of the whole, the representation of the sample size in proportion to the population is ≈0.310, for advanced economies, this is figure 0.3125, and 0.310 for emerging economies, with 3 significant figures. In summary, all sample groups represent roughly the equivalent of ⅓ of the global population, as well as the number of economies, within an error range of ±1.75%. The sample and the population list can be found in Appendix #1.

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[7] World Bank. *Database on country population. 2020.*https://databank.worldbank.org/data/download/POP.pdf (Accessed October 11th, 2020)
[8] Sciencing .*p hat calculation*. https://sciencing.com/calculate-phat-8384855.html (Accessed October 18th, 2020)

Stage 1: Univariate data and Statistical Distribution

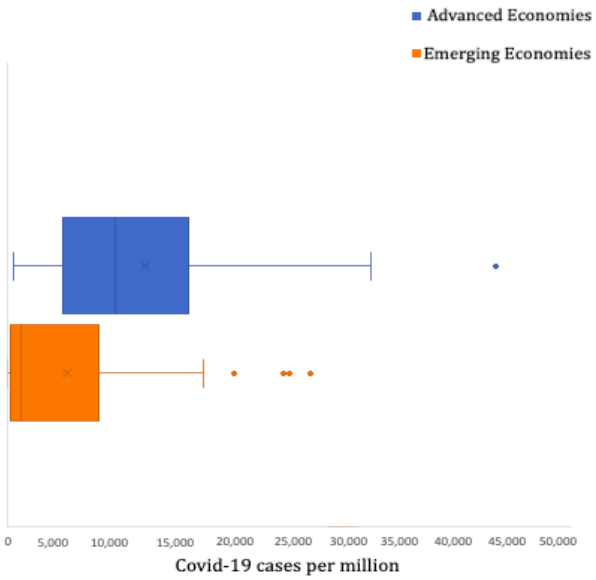
After generating a sample for the exploration, the Covid-19 cases per million from the World Health Organization's Covid-19 weekly report, published on October 11th, 2020 for table 2 was extracted.[9] And the economic data[10] was obtained from Trading Economics, an online statistical organization in partnership with the World Bank. They are used as they are considered reliable and cited by many international credible institutions; it has many authors, and conducts its studies with a variety of institutions around the world.

In order to find and compare the differences in the statistical distribution of the two groups, Microsoft Excel was used to construct the parallel boxplots which consisted of the advanced and emerging economies. Each will include the five figure summaries, the range, the Interquartile range. Outliers will be identified.

Table 2: Five figure summaries of Covid-19 cases (per million population)

	Advanced Economies	Emerging Economies
Minimum value	480	0
Quartile 1 value	5,733	301
Median value	9,809	1,196
Quartile 3 value	15,438	7,790
Maximum value	44,351	27,543

Graph 1: Parallel Box Plot of Covid-19 cases, per million population in Advanced and Emerging economies



Boxplot #1: Advanced Economies (Blue)

Range = Maximum value - Minimum value

= 44351 −480

= 43871 Cases of Covid-19 per million population

IQR = Q_3 - Q_1

= 15438 −5733

= 9705 Cases of Covid-19 per million population

Test for outliers:

Upper boundary: Upper quartile = $Q_3 + 1.5$ IQR

$= 15438 + 1.5(9705)$

$= 29995$ Covid-19 case per million population

- Two outliers are detected, as they both exceeds the upper boundary, the Covid-19 cases per million population in these two countries are: 33,055 (Israel) and 44,351 (Qatar) respectively.

Lower boundary: Lower quartile = $Q_1 - 1.5$ IQR

$= 5733 - 1.5(9705)$

$= -8824$ Covid-19 case per million population

- No outliers are detected, as no point exceeds -8,824 in the lower boundary but also because Covid-19 cases is a natural number and cannot be negative.

Implications of data:

The shape of boxplot #1 shows a positively skewed in the IQR with a median value of 9,809 cases per million population. The spread of this data has a range of 43,871 cases per million population and an IQR of 9,705 cases per million population. There are two extreme values, namely the 33,055 of Israel and 44,351 of Qatar. This is due to the unique socio-economic and political situation in both countries. In Qatar, only a partial lockdown in the Doha Industrial Area was imposed, and there were no mandatory restrictions on wearing masks. In addition, the high income inequality in Qatar means many people live in crowded areas with bad ventilation, thus helping the virus to spread. In Israel, a lockdown was opposed by the ultra-orthxodox sector of the country; thus, large public gatherings for religious purposes went uninterrupted.

[9] World Health Organization. *Covid-19 Weekly Update. 2020*. https://www.who.int/coronavirus/20201012-weeklyreport.pdf (Accessed October 15th, 2020)
[10] Trading Economics. *Annual GDP growth rate. 2020.* https://tradingeconomics.com/forecast/gdp-annual-growth-rate-2020 (Accessed October 10th, 2020)

This led to a huge spike of cases after Yom Kippur, a traditional holiday celebrated by many in the country. Therefore, these factors have rendered Qatar and Israel the two outliers in the Advanced Economies group.

Boxplot #2: Emerging Economies (Orange)

Range = $27543 - 0$

= 27,543 Covid-19 case per million population

IQR = $7790 - 301$

= 7,489 Covid-19 case per million population

Test for Outliers:

Upper boundary : = 7790 + 1.5(7489)

= 19,023 Covid-19 case per million population

- Four outliers are detected, as they all exceed the upper boundary, the Covid-19 cases per million population in these four countries are: Montenegro (20,566), Chile (25,088), Peru (25,578) and Panama (27,543) respectively.

Lower boundary = $301 - 1.5(7489)$

= -10,932 Covid-19 case per million population

- Not outliers are detected, as no point exceeds -10,932 in the lower boundary but also that Covid-19 cases, as a natural number, cannot be negative.

Implications of data:

Boxplot #2 shows a very positively skewed distribution with a median value of 1,196 and a mean of 5,409. Four outliers were detected. Three are Latin American and one is European. All these economies share the similarities of locking down late[11] and imposing little restrictions such as mandatory mask wearing and contact tracing. Unlike advanced economies, emerging economies lack the funding and often times inable to enforce lockdowns[12] due to factors such corruption, this has caused the above four emerging economies to be rendered as outliers.

[11]World Health Organization. Pandemic lockdown table. https://www.who.int/emergencies/Covid-19/pandemic-lockdown-table (Accessed November 1st, 2020)
[12]Wall street Journal. Peru and Brazil: Stay home and starve. https://www.wsj.com/articles/stay-home-and-starve (Accessed November 1st, 2020)

Graph 2: Real GDP change in Advanced and emerging economies, in percentages (%)

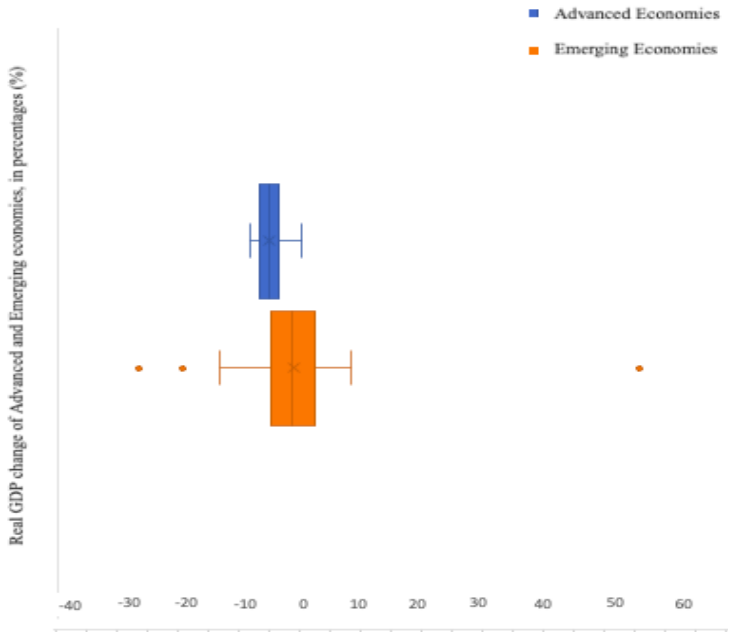


Table 3: Five sigure summaries of Real GDP change of Advanced and Emerging economies, in percentage points (%)

	Advanced Economies	Emerging Economies
Minimum value	-9.3	-27.2
Quartile 1 value	-7.6	-5.85
Median value	-6.35	-2.5
Quartile 3 value	-4.8	0.925
Maximum value	-1.2	52.8

Boxplot #1: Advanced Economies

Range: 8.1% **IQR:** 2.8%

Test for Outliers: Upper Boundary: -0.6% Lower Boundary: -11.8%

- No outliers were identified in the Upper Boundary.

Boxplot #2: Emerging Economies

Range: 80% **IQR:** -4.93%

Test for Outliers: Upper Boundary: 11.1% Lower Boundary: -15.7%

- Three outliers are identified, Guyana (52.8%) in the Upper Boundary; Venezuela (-20%) and Peru (-27.2%) in the Lower Boundary.

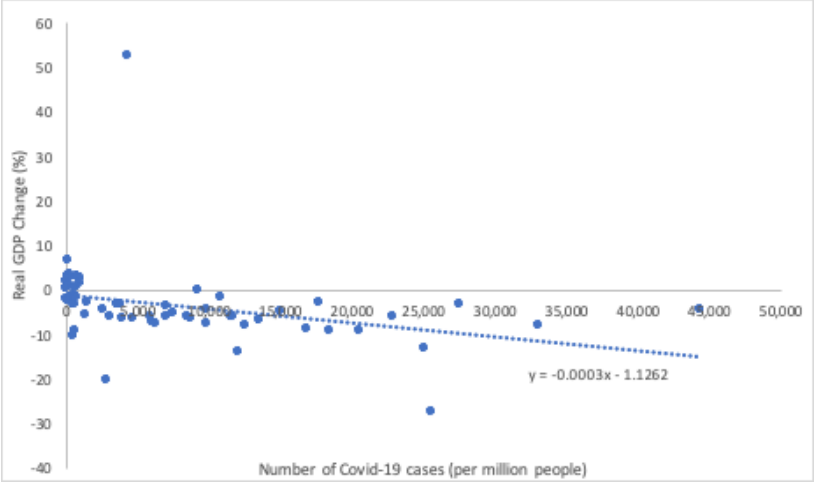
The voxplot of Advanced Economies shows an approximately symmetrically distributed boxplot with a median value of -6.35%. The shape of the Emerging Economies also shows a fairly asymmetrical distribution with a median value of -2.5%. For the three economies that were rendered as outliers, each has a unique socio-economic situation. Guyana was able to grow by 52.8% this year thanks to the discovery of large oil fields in its water, which has brought the country billions of dollars in revenue and allowed its economy to grow at skyrocketing rate even during a pandemic. In contrast, it's neighbour, Venezuela, despite having 20% of the world's oil fields, saw its GDP contrast by -20% in real terms this year, because of the sanctions imposed by the West in the previous years, as well as top of civil unrest. Venezuela's oil industry, which makes up 70% of the Country's export had already collapsed before the pandemic,[13] which has made it one of the hardest hit countries in the world. Peru, already facing debt with an increasingly high government budget deficit,[14] was unable to roll out any economic plans to stimulate the economy and resorted to printing large amounts of money. This has led to widespread inflation and massive unemployment, which suggested reasons for a drop in the export of copper, gold and zinc, which accounts roughly 40.5% Peru's economy.[15] Thus, the following factors have made the three Latin American economies outliers in this investigation.

Stage _2: Bivariate data and statistical testing

The second stage of this exploration was to investigate the relationship between the number of Covid-19 cases (per 1 million people) and Real GDP change (in percentages). Therefore, a scatter plot showing the two variables and the correlation was produced using Pearson's product moment correlation. When a correlation was obtained, it was used to generate an equation on the relationship found between the two variables.

[13]IMF. *IMF World Economic Outlook. October* https://www.imf.org/en/Publications/WEO/weo-database/2020/October/weo-report-Venezuela (Accessed November 5th, 2020)
[14] Ceidata. *Peru national government debt*.https://www.ceicdata.com/indicator/peru/national-debt. (Accessed November 5th, 2020)
[15] Observatory of Economic Complexity. *Peru Country profile* https://oec.world/en/profile/country/per/ (Accessed November 7th, 2020)

Graph 3: Scatter diagram of all economies (advanced & emerging) and their respective Covid-19 cases per million population in relation to real GDP changes (%)



Pearson's product moment correlation coefficient:

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{(n\sum x^2 - (\sum x)^2)(n\sum y^2 - (\sum y)^2)}}$$

r= Pearson correlation coefficient,

n =the number of terms in the dataset

x = independent variable (Covid-19 cases per million population)

y= dependent variable (Real GDP change in percentage points).

Finding the r-value of Graph 3

Utilizing this formula, the following result may be found (note that the excel software did not rounded to ensure the accuracy of results, thus the actual result shown the sample calculation will differ slightly from the numbers found in table 2 of the appendix) The values of X, Y, X squared, Y squared, XY were also found on table 2 in the appendix. The calculation of r goes as follows:

$$r = \frac{60(-3250815.06) - (466780)(-210.48)}{\sqrt{60(8901017088) - (466780^2)(60(5775.7904) - (-210.48^2)}}$$

$$r = \frac{-195048903.6 - (-98247854.4)}{\sqrt{316177456880 \times (302245.6)}}$$

r = -0.313

- The Pearson's product moment correlation r shows the strength of the linear regression, the value r = -0.313 shows that there is a weak negative correlation existing between Covid-19 cases per million population and Real GDP change (%).

r^2 coefficient of determination = 0.0981

- This is an indication that around 9% of the variation in Real GDP change can be explained by Covid-19 cases per million population in a country. This also means that 90.2% of the variations in Real GDP (in %) is attributed to other factors.

Least Squares Regression Line (determined using excel graphing feature)

Real GDP change (%) = m×Covid-19 cases per million population + b

= (-0.0003×Covid-19 cases per million population) − 1.1.3

$\frac{rise}{run} = \frac{-0.0003}{1} \times \frac{1\,000}{1\,000}$

= -3

Summary:

It is deduced that for every 1,000 increase of Covid-19 cases per million population, the Real GDP change in percentages will decrease by -0.3%.

Regression equation testing #1

The linear regression line can be tested by selecting one country from each advanced and emerging economies group, and then substituting the Covid-19 cases per million population to find the expected Real GDP change in percentage. The following two countries were selected using www.random.org[16]. Outlier economies identified in stage 1 were excluded for the purpose of yielding more accurate estimates.

United Kingdom

- Is a high income country located in Europe; rendered as an advanced economy
- UK has a population of approximately 66.8 million people according to the 2020 World Bank population by countries[17]

[16]Random generator. *Random.org.* www.random.org. (Accessed Nov, 10th, 2020)

[17] World Bank. *Database on country population. 2020.*https://databank.worldbank.org/data/download/POP.pdf (Accessed October 29th, 2020)

Real GDP change (%) = -0.0003 ×(UK's Covid-19 cases per million population) − 1.13

= (-0.0003×8704) − 1.13

= -3.74

Findings : According to the least squares regression line derived from Graph 3, UK's expected Real GDP change in percentage is around -3.74%.

However, the real value from table 2 in the appendix is actually -6.50%. The percentage of error for UK's results can be determined using:

$$Percentage\ error = \frac{estimated\ value - real\ value}{real\ value} \times \frac{100}{1}$$

$$Percentage\ error = \frac{-3.74 - (-6.50)}{-6.50} \times \frac{100}{1}$$

Percentage error = 42.5.%

Chile

- Is a middle income country located in the Latin America; it is rendered as an emerging economy as its GDP per capita that is lower than $12,535 USD, as defined by table 1.
- Chile has a population of 18.7 million people[18]

Real GDP change (%) = (-0.0003 ×Chile's Covid-19 cases per million population − 1.13)

Substitute the Covid-19 cases per million population of Chile

- Real GDP change = (-0.0003×25088) − 1.13

= -8.66%

Findings : According to the least squares regression line derived from Graph 3, the expected value of Real GDP change for Chile is around -8.66% . However, the observed real GDP change from table 2 in the appendix is -13.2% for Chile.

$$Percentage\ error = \frac{-8.66 - (-13.2)}{-13.2} \times \frac{100}{1}$$

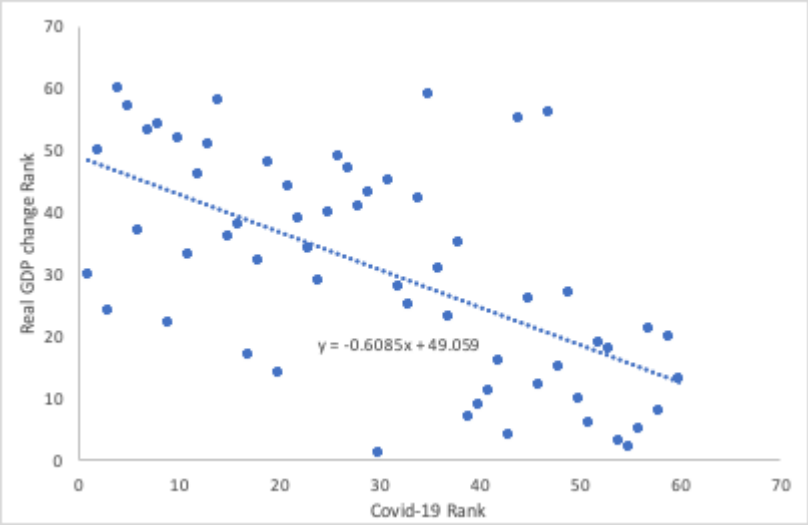
Percentage error = 34.4%

[18] ibid, 11.

After the generation of this graph and determination of its correlation, it is evident that although a relationship between these variables exists but it cannot be best represented by a linear equation due to the existence of data in a largely non-linear fashion, which is no longer explainable by the linear regression equation. The correlation coefficient is relatively weak with a value of r = -0.313. After considering the option of continuing this investigation or stop here and conclude there is a weak negative correlation, and the data is largely non monotonic, meaning that the numbers go both ways as the independent variable increases. It is decided that perhaps using Spearman's ranking system would support the research aim proving a monotonic correlation exists in association with the two variables and generate a linear graph. A linear line would also not fit best because as seen in Graph 3, the majority of the data is clustered around from 0 - 10,000 of the x value. The data is also moving in different directions, which a linear equation cannot represent because it only moves in one direction.

Spearman's ranking system test

Graph 4: Scatter diagram showing the ranked data for Covid-19 cases per million population and Real GDP change in percentages



Ranking key:

Covid-19 ranking x-axis: The Covid-19 caes per million population is ranked in order from the highest to lowest, this means that economies with high Covid-19 cases per million population will have a lower

ranking. An example of this is the country of Qatar; it has the highest Covid-19 cases per million population in the set of investigated data (44,351 cases of Covid-19 per million population), thus it is ranked first for Covid-19 cases per million population rankings.

Real GDP change ranked y-axis: The Real GDP change in percentage points is also ranked in order from the highest to lowest, meaning that countries with a lower/negative GDP change will have a higher ranking value. One example of this Peru, which is projected to have its GDP contract by 27.2% in real terms. Hence, Peru's ranking is ranked 60th in the dataset of Real GDP change in percentages.

$P = 1 - \frac{6\sum_{i=0}^{n} d_i^2}{n(n^2-1)}$ This correlation coefficient (P),is used to measured the strength of association between two ranked variables in a monotonic function.

where p represents the spearman correlation, d_i is the difference between the two ranks of each observation and n is the number of observations and n is the number of terms. The ranking table can be found under Table 4 in the appendix.

$P = 1 - \frac{6(57890)}{60(60^2-1)} = 1 - \frac{347340}{215940} = 1 - 1.608502362 = -0.608$

Spearman's Ranking Correlation Coefficient

$r = -0.608$

This value is an indication that a moderately strong negative correlation exists between the Covid-19 cases per million population and the Real GDP change as a percentage. In addition the r^2 value of 0.369 indicates that 36.9% of the variation in GDP can be explained by the variation in Covid-19 cases. Furthermore, it is found that the critical value for a correlation with 59 degrees of freedom and a level of confidence of 0.005 (see table of critical values in appendix), is 0.216. Thus it is deduced that if the absolute value of r is more than 0.216, then the statistical significance can be proved. It is found that the absolute value of r obtained from this investigation is 0.608, which is higher than the critical value of $a = 0.216$. Therefore it can be concluded that the research aim of finding the correlation between Covid-19 cases per million population

having a negative correlation with the Real GDP change in terms of percentages of a country is valid and statistically significant.

Least squares regression line for ranked data (determined using excel graphing feature in the form of y=mx+b)

Ranked Real GDP change (in percentages) = -0.608 (Covid-19 cases per million population) + 49.1

Ranked regression equation testing #2

United Kingdom

- 8704 cases of Covid-19 per million of its population
- *Ranked Real GDP Change (%) = -0.608 (Ranked Covid-19 cases per million population) + 49.1*

$$= -0.608(21) + 49.1$$

$$= 61.9$$

$$\text{Percentage of error} = \frac{36.3-44}{44} \times \frac{100}{1}$$

$$= |-17.5|\%$$

Chile

- 25088 cases of Covid-19 per million of its population

Ranked Real GDP Change (%) = -0.608 (Ranked Covid-19 cases per million population) + 49.1

$$= -0.608(5) + 49.1$$

$$= 46.1$$

$$\text{Percentage of error} = \frac{46.1-57}{57} \times \frac{100}{1}$$

$$= |-19.2|\%$$

Reflection of results

One strength of this investigation was the ranking of data. It was observed that the percentage of error obtained using Spearman's ranking coefficient was lower than the percentage error obtained from using Pearson's correlation coefficient by 25% and 19.2%, as s from regression equation testing #1 and 2. This implies that relationship between the two is correlated and can be used to extrapolate the Real GDP change of other economies. The other strength of this investigation was that the factor inflation was taken into

account by collecting the GDP deflator of every country which prevented inaccuracy of data caused by a sudden increase in monetary value.

Regardless, there are some limitations of this investigation that can be improved on. However, the sample size of countries used were too small, as only UK and Chile was chosen to be representative of the entire population. This can be addressed by conducting more interpolations using the regression equations to gauze more insights test the reliability of this equation. The lack of controlled variables also had incorporated greater amount of uncertainty into the results. For example, it was noticed from table 2 in the appendix that when comparing the results of North and South Korea, the north was in a much better shape than the South. South Korea has 480 cases of Covid-19 per million of its population, while the north has none. South Korea's GDP contracted by 1.8% in real terms this year and North Korea's was growing by 0.4%. Therefore theoretically it can be concluded that North Korea is handling the pandemic better, but after conducting some researches, it was discovered that North Korea responded to the pandemic by imposing one of the harshest lockdowns in the world, by completely shutting itself from the rest of the world.[19] As a result, it is experiencing one of the worst humanitarian crisis in decades where 60% of the population is estimated to be malnourished.[20] The contrasting differences between what the data from this investigation implies and reality were inaccurate as a lot of control factors were neglected. For future researches, this limitation can be improved by keeping control factors such as inequality, governmental policies and country demographics constant. Focusing on more specific groups of economies with similarities will allow higher correlations as more variables are being controlled, though this may limit the available sample size.

Separating larger nations into their constituent district/province or state (i.e. The United States can be separated into their respective 50 state plus the overseas territories. Whereas United Kingdom will be separated into England, Scotland, Wales, and Ireland. This method can prevent certain outliers and ensures that local data has strong correlation instead of a national data with weak or no correlation as different parts of a country can be affected differently. However, this method will dramatically increase the sample size

[19]Human Rights Watch. *North Korea famine 2020*hmttps://www.hrw.org/world-report/2020/country-chapters/north-korea (Accessed Nov 11th, 2020)

[20]Noland, Markus. *Famine and reform in North Korea.* https://www.mitpressjournals.org/doi/abs/10.1162/1535351044193411 (Accessed Nov 11th, 2020)

being investigated and thus it is needed to knowing what to separate the constituents, such as having one constituent every 10 million population.

Conclusion of findings

Overall, throughout this investigation it was found that the Real GDP change of a country, has to some extent dependent upon the number of Covid-19 cases per million population in a country. This can be seen in Graph 4:"Scatter diagram of the ranked data for Covid-19 cases per million population and Real GDP changes in percentage" as the ranked correlation coefficient was found to be r = -0.608, demonstrating a moderate negative correlation exists between the two variables [20] - table from IB textbook.

In addition, it was found that there is a significant difference in the number of Covid-19 cases between advanced and emerging economies, which was found in the parallel boxplot and the bivariate data. It was observed during the five figure summary tables that emerging economies have a covid-19 case median of 1,196 per million population, while the figure is 9,809 for advanced economies, almost nine times higher. While the figures for Real GDP change were -2.5% and -6.35%. This could be attributed to the difference of sectors that makes up the economy, where most emerging economies are rural, Advanced economies are mostly urban, meaning a pandemic will affect the modern sectors such as service, manufacture and such due to social-distancing. Emerging economies typically have less Covid-19 cases because not a lot of the population travels. Thus some communities in emerging economies were able to continue their economic activities such as mining or agriculture, which ensures a stable economy and low employment rate. The extent to which Covid-19 affects GDP in advanced and emerging economies can be better understood and explained if each has a separate linear regression line.

It was also noticed that governments played a much more important role in the Real GDP change than Covid-19 cases. This also explains why it was hard to obtain a monotonic correlation in spearman's ranking test. The difference in GDP change between Venezuela and Guyana, both oil states with large reserves, has implied that decisions which governments make play any important role in controlling the

pandemic and determining the economy of a country. If measures are ineffective, a country can have both high Covid-19 cases and high economical contraction.

Nonetheless, throughout this exploration it was found the real GDP change of economies has an effect to an extent upon the number of Covid-19 cases per million population. However, it was also found that emerging economies are impacted less by this pandemic and during a certain point, the Covid-19 cases will not cause a significant decrease in the real GDP change of a country.

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Appendix 1:

The countries The World Bank and WHO provided data on for Covid-19 cases per million population and Real GDP change in percentages. This list was categorized into Advanced and Emerging economies based on the definitions provided by The Word Bank and then a sample size was obtained

List of Advanced Economies - By the World Bank

1. Andorra
2. Antigua and Barbuda
3. Argentina
4. Aruba
5. Australia
6. Austria
7. The Bahamas
8. Bahrain
9. Barbados
10. Belgium
11. Brunei
12. Canada
13. Chille
14. Croatia
15. Cyprus
16. Czech Republic
17. Denmark
18. Estonia
19. Finland
20. France
21. Germany
22. Greece
23. Hungary
24. Iceland
25. Ireland
26. Israel
27. Italy
28. Japan
29. South Korea
30. Kuwait
31. Latvia
32. Liechtenstein
33. Lithuania
34. Luxembourg
35. Malta
36. Mauritius
37. Monaco
38. Nauru

39. Netherlands
40. New Zealand
41. Norway
42. Oman
43. Palau
44. Panama
45. Poland
46. Portugal
47. Romania
48. Qatar
49. Saint Kitts and Nevis
50. Samoa
51. San Marino
52. Saudia Arabia
53. Seychelles
54. Singapore
55. Slovakia
56. Slovenia
57. Spain
58. Sweden
59. Switzerland
60. Trinidad and Tobago
61. United Arab Emirates
62. United Kingdom
63. United States
64. Uruguay

List of Emerging Economies - By the World Bank

1. Venezuela
2. Pakistan
3. Sri Lanka
4. Kyrgyz Republic
5. Malawi
6. São Tomé and Principe
7. Iraq
8. Korea, Dem Rep.
9. Libya
10. Angola
11. Mexico
12. Chad
13. Zimbabwe
14. China
15. Philippines
16. Armenia
17. Mali
18. Cambodia
19. Afghanistan
20. Chile
21. Botswana
22. Panama
23. Uganda
24. Palestine

25. Madagascar
26. Congo, Dem. Rep
27. Nigeria
28. South Africa
29. Benin
30. Tunisia
31. Peru
32. Bosnia and Herzegovina
33. Syrian Arab Republic
34. Nepal
35. Sierra Leone
36. Cameroon
37. Iran, Islamic Rep.
38. Fiji
39. Indonesia
40. Cape Verde
41. Uzbekistan
42. Sudan
43. Bangladesh
44. Guyana
45. Tajikistan
46. Azerbaijan
47. Ethiopia
48. Timor-Leste
49. Vanuatu
50. Somalia
51. Congo, Rep.
52. Honduras
53. Macedonia, FYR
54. Bolivia
55. Tonga
56. Bulgaria
57. Algeria
58. Senegal
59. Brazil
60. Eritrea
61. Serbia
62. Swaziland
63. India
64. Turkmenistan
65. Suriname
66. Belarus
67. Gambia
68. Dominican Republic
69. Papua New Guinea
70. Belize
71. Central African Republic
72. Lao PDR
73. Latvia
74. Solomon Islands
75. Moldova
76. Georgia
77. Dominica

78. Tanzania
79. Colombia
80. El Salvador
81. Micronesia, Fed. Sts.
82. Ukraine
83. Haiti
84. Djibouti
85. Marshall Islands
86. Lebanon
87. Guinea-Bisau
88. Cuba
89. Malaysia
90. Niger
91. Palau
92. Namibia
93. Ecuador
94. Ghana
95. Morocco
96. Tuvalu
97. Lesotho
98. Nicaragua
99. Vietnam
100. Butan
101. Rwanda
102. Turkey
103. Russian Federation
104. Guinea
105. Mozambique
106. Togo
107. Jordan
108. Albania
109. Kiribati
110. Liberia
111. Burkina Faso
112. Grenada
113. Paraguay
114. Kenya
115. Comoros
116. Mauritania
117. Egypt, Arab Rep.
118. Montenegro
119. Mongolia
120. Jamaica
121. Kazakhstan
122. Thailand
123. Burundi
124. Yemen, Rep.
125. Guatemala
126. Myanmar

127. Côte d'Ivoire

128. Maldives

129. Gabon

This statistical report by the world bank is excluding: Taiwan, New caledonia,

Appendix Table 1 : The sample populations of 20 advanced economies and 40 emerging economies

Advanced Economies	**Emerging Economies (1)**	**Emerging Economies (2)**
1. Hungary	1. Malaysia	21. Cambodia
2. Luxembourg	2. Venezuela	22. Afghanistan
3. Japan	3. Pakistan	23. Chile
4. Finland	4. Sri Lanka	24. Botswana
5. Korea	5. Kyrgyz Republic	25. Panama
6. Israel	6. Malawi	26. Uganda
7. United Kingdom	7. São Tomé and Principe	27. Palestine
8. Slovak Republic	8. Guyana	28. Madagascar
9. Costa Rica	9. Korea, Dem Rep.	29. Congo, Dem. Rep
10. Singapore	10. Egypt	30. Nigeria
11. Austria	11. Montenegro	31. South Africa
12. Switzerland	12. Ethiopia	32. Benin
13. USA	13. Uruguay	33. Tunisia
14. Latvia	14. Mexico	34. Peru
15. Spain	15. Moldova	35. Bosnia and Herzegovina
16. Bahamas	16. Zimbabwe	36. Haiti
17. Netherlands	17. China	37. Nepal
18. Canada	18. Philippines	38. Sierra Leone
19. Belgium	19. Armenia	39. Cameroon
20. Qatar	20. Mali	40. Iran, Islamic Rep.

Appendix Table 2: The Covid-19 cases (per million) and Real GDP change (%) of sampled advanced and emerging economies

Advanced Economies	Covid-19 cases (per million)	Real GDP Change (%)	Emerging Economies	Covid-19 Cases (per million)	Real GDP Change (%)	Emerging Economies	Covid-19 Cases (per million)	Real GDP Change (%)
Hungary	3,788	-3.3	Malaysia	466	-3	Cambodia	18	-2
Luxembourg	14,953	-4.9	Venezuela	2,873	-20	Afghanistan	1,022	2.8
Advanced Economies	**Covid-19 cases (per million)**	**Real GDP Change (%)**	**Emerging Economies**	**Covid-19 Cases (per million)**	**Real GDP Change (%)**	**Emerging Economies**	**Covid-19 Cases (per million)**	**Real GDP Change (%)**
Japan	678	-9.3	Pakistan	1,444	-2.8	Chile	25,088	-13.2
Finland	11,580	-6	Sri Lanka	216	-1.6	Botswana	1,369	-5.4
Korea, R.O.	480	-1.2	Kyrgyz Republic	7,546	-5.2	Panama	27,543	-3
Israel	33,055	-7.9	Malawi	304	1	Uganda	209	3.5
United Kingdom	8,704	-6.5	São Tomé and Principe	8,525	-6	Palestine	10,808	-1.6
Slovak Republic	3,970	-6.5	Guyana	4,269	52.8	Madagascar	603	0.4
Costa Rica	16,893	-8.6	Korea, Dem Rep.	0	0.4	Congo, Dem. Rep	121	-2.2
Singapore	9,891	-4.5	Egypt	1,020	1.7	Nigeria	292	2.9
Austria	6,071	-7	Montenegro	20,566	-9	South Africa	11,649	-5.8
Switzerland	6,959	-6	Ethiopia	726	3.2	Benin	199	6.7
USA	22,911	-5.9	Uruguay	648	-3	Tunisia	2,645	-4.3
Latvia	7,059	-3.5	Mexico	6,280	-7.7	Peru	25,578	-27.2
Spain	18,418	-9	Moldova	12,098	-14	Bosnia	9,249	-0.1
Bahamas	12,600	-8	Zimbabwe	538	-10.4	Haiti	777	-1.5
Netherlands	9,809	-7.5	China	62	2.1	Nepal	3,627	-3
Canada	4,719	-6.2	Philippine	3,075	-6.1	Sierra Leone	288	-2
Belgium	13,533	-6.9	Armenia	17,750	-2.8	Cameroon	797	0.9
Qatar	44,351	-4.3	Mali	162	3.02	Iran, Islamic Rep.	5908	-6

Appendix table 3: Pearson's product moment correlation test on the results from table 2

N	X	Y	XY	X^2	Y^2
1	4,269	52.8	225403.2	18224361	2787.84
2	199	6.7	1333.3	39601	44.89
3	209	3.5	731.5	43681	12.25
N	X	Y	XY	X^2	Y^2
4	726	3.2	2323.2	527076	10.24
5	162	3.02	489.24	26244	9.1204
6	292	2.9	846.8	85264	8.41
7	1,022	2.8	2861.6	1044484	7.84
8	62	2.1	130.2	3844	4.41
9	1,020	1.7	1734	1040400	2.89
10	304	1	304	92416	1
11	797	0.9	717.3	635209	0.81
12	603	0.4	241.2	363609	0.16
13	0	0.4	0	0	0.16
14	9,249	-0.1	-924.9	85544001	0.01
15	480	-1.2	-576	230400	1.44
16	777	-1.5	-1165.5	603729	2.25
17	10,808	-1.6	-17292.8	116812864	2.56
18	216	-1.6	-345.6	46656	2.56
19	288	-2	-576	82944	4
20	18	-2	-36	324	4
21	121	-2.2	-266.2	14641	4.84
22	17,750	-2.8	-49700	315062500	7.84
23	1,444	-2.8	-4043.2	2085136	7.84
24	27,543	-3	-82629	758616849	9
25	3,627	-3	-10881	13155129	9
26	648	-3	-1944	419904	9
27	466	-3	-1398	217156	9
28	3,788	-3.3	-12500.4	14348944	10.89
29	7,059	-3.5	-24706.5	49829481	12.25
30	44,351	-4.3	-190709.3	1967011201	18.49
31	2,645	-4.3	-11373.5	6996025	18.49
32	9,891	-4.5	-44509.5	97831881	20.25
33	14,953	-4.9	-73269.7	223592209	24.01

34	7,546	-5.2	-39239.2	56942116	27.04
35	1,369	-5.4	-7392.6	1874161	29.16
36	11,649	-5.8	-67564.2	135699201	33.64
37	22,911	-5.9	-135174.9	524913921	34.81
38	11,580	-6	-69480	134096400	36
N	X	Y	XY	X^2	Y^2
39	8,525	-6	-51150	72675625	36
40	6,959	-6	-41754	48427681	36
41	5908	-6	-35448	34904464	36
42	3,075	-6.1	-18757.5	9455625	37.21
43	4,719	-6.2	-29257.8	22268961	38.44
44	8,704	-6.5	-56576	75759616	42.25
45	3,970	-6.5	-25805	15760900	42.25
46	13,533	-6.9	-93377.7	183142089	47.61
47	6,071	-7	-42497	36857041	49
48	9,809	-7.5	-73567.5	96216481	56.25
49	6,280	-7.7	-48356	39438400	59.29
50	33,055	-7.9	-261134.5	1092633025	62.41
51	12,600	-8	-100800	158760000	64
52	16,893	-8.6	-145279.8	285373449	73.96
53	20,566	-9	-185094	422960356	81
54	18,418	-9	-165762	339222724	81
55	678	-9.3	-6305.4	459684	86.49
56	538	-10.4	-5595.2	289444	108.16
57	25,088	-13.2	-331161.6	629407744	174.24
58	12,098	-14	-169372	146361604	196
59	2,873	-20	-57460	8254129	400
60	25,578	-27.2	-695721.6	654234084	739.84
Sum	466,780	-210.48	-3250815.06	8901017088	5775.7904

Appendix table 4: Spearman Rank Correlation Test on the results from table 2 of the Covid 19 cases per million people and the Real GDP change as a percentage in selected countries

Covid cases	GDP Change	Covid Rank	GDP Rank	Diff	Diff^2
4,269	52.8	30	1	29	841
199	6.7	55	2	53	2,809

209	3.5	54	3	51	2,601
726	3.2	43	4	39	1,521
162	3.02	56	5	51	2,601
292	2.9	51	6	45	2,025
1,022	2.8	39	7	32	1,024
Covid cases	GDP Change	Covid Rank	GDP Rank	Diff	Diff^2
62	2.1	58	8	50	2,500
1,020	1.7	40	9	31	961
304	1	50	10	40	1,600
797	0.9	41	11	30	900
603	0.4	46	12	34	1,156
0	0.4	60	13	47	2,209
9,249	-0.1	20	14	6	36
480	-1.2	48	15	33	1,089
777	-1.5	42	16	26	676
10,808	-1.6	17	17	0	0
216	-1.6	53	18	35	1,225
288	-2	52	19	33	1,089
18	-2	59	20	39	1,521
121	-2.2	57	21	36	1,296
17,750	-2.8	9	22	-13	169
1,444	-2.8	37	23	14	196
27,543	-3	3	24	-21	441
3,627	-3	33	25	8	64
648	-3	45	26	19	361
466	-3	49	27	22	484
3,788	-3.3	32	28	4	16
7,059	-3.5	24	29	-5	25
44,351	-4.3	1	30	-29	841
2,645	-4.3	36	31	5	25
9,891	-4.5	18	32	-14	196
14,953	-4.9	11	33	-22	484
7,546	-5.2	23	34	-11	121
1,369	-5.4	38	35	3	9
11,649	-5.8	15	36	-21	441
22,911	-5.9	6	37	-31	961

11,580	-6	16	38	-22	484
8,525	-6	22	39	-17	289
6,959	-6	25	40	-15	225
5908	-6	28	41	-13	169
3,075	-6.1	34	42	-8	64
Covid cases	GDP Change	Covid Rank	GDP Rank	Diff	Diff^2
4,719	-6.2	29	43	-14	196
8,704	-6.5	21	44	-23	529
3,970	-6.5	31	45	-14	196
13,533	-6.9	12	46	-34	1,156
6,071	-7	27	47	-20	400
9,809	-7.5	19	48	-29	841
6,280	-7.7	26	49	-23	529
33,055	-7.9	2	50	-48	2,304
12,600	-8	13	51	-38	1,444
16,893	-8.6	10	52	-42	1,764
20,566	-9	7	53	-46	2,116
18,418	-9	8	54	-46	2,116
678	-9.3	44	55	-11	121
538	-10.4	47	56	-9	81
25,088	-13.2	5	57	-52	2,704
12,098	-14	14	58	-44	1,936
2,873	-20	35	59	-24	576
25,578	-27.2	4	60	-56	3,136

Appendix table 5: r-value and correlation table from the Oxford IB SL math textbook

r-value	Correlation
$0 < r \leq 0.25$	Very weak
$0.25 < r \leq 0.5$	Weak
$0.5 < r \leq 0.75$	Moderate
$0.74 < r \leq 1$	Strong

Appendix table 6: Table of critical values for Spearman's Ranked Correlation Coefficient from Zar

α(2):	0.50	0.20	0.10	0.05	0.02	0.01	0.005	0.002	0.001
α(1):	0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
n									
4	0.600	1.000	1.000						
5	0.500	0.800	0.900	1.000	1.000				
6	0.371	0.657	0.829	0.886	0.943	1.000	1.000		
7	0.321	0.571	0.714	0.786	0.893	0.929	0.964	1.000	1.000
8	0.310	0.524	0.643	0.738	0.833	0.881	0.905	0.952	0.976
9	0.267	0.483	0.600	0.700	0.783	0.833	0.867	0.917	0.933
10	0.248	0.455	0.564	0.648	0.745	0.794	0.830	0.879	0.903
11	0.236	0.427	0.536	0.618	0.709	0.755	0.800	0.845	0.873
12	0.217	0.406	0.503	0.587	0.678	0.727	0.769	0.818	0.846
13	0.209	0.385	0.484	0.560	0.648	0.703	0.747	0.791	0.824
14	0.200	0.367	0.464	0.538	0.626	0.679	0.723	0.771	0.802
15	0.189	0.354	0.446	0.521	0.604	0.654	0.700	0.750	0.779
16	0.182	0.341	0.429	0.503	0.582	0.635	0.679	0.729	0.762
17	0.176	0.328	0.414	0.485	0.566	0.615	0.662	0.713	0.748
18	0.170	0.317	0.401	0.472	0.550	0.600	0.643	0.695	0.728
19	0.165	0.309	0.391	0.460	0.535	0.584	0.628	0.677	0.712
20	0.161	0.299	0.380	0.447	0.520	0.570	0.612	0.662	0.696
21	0.156	0.292	0.370	0.435	0.508	0.556	0.599	0.648	0.681
22	0.152	0.284	0.361	0.425	0.496	0.544	0.586	0.634	0.667
23	0.148	0.278	0.353	0.415	0.486	0.532	0.573	0.622	0.654
24	0.144	0.271	0.344	0.406	0.476	0.521	0.562	0.610	0.642
25	0.142	0.265	0.337	0.398	0.466	0.511	0.551	0.598	0.630
26	0.138	0.259	0.331	0.390	0.457	0.501	0.541	0.587	0.619
27	0.136	0.255	0.324	0.382	0.448	0.491	0.531	0.577	0.608
28	0.133	0.250	0.317	0.375	0.440	0.483	0.522	0.567	0.598
29	0.130	0.245	0.312	0.368	0.433	0.475	0.513	0.558	0.589
30	0.128	0.240	0.306	0.362	0.425	0.467	0.504	0.549	0.580
31	0.126	0.236	0.301	0.356	0.418	0.459	0.496	0.541	0.571
32	0.124	0.232	0.296	0.350	0.412	0.452	0.489	0.533	0.563
33	0.121	0.229	0.291	0.345	0.405	0.446	0.482	0.525	0.554
34	0.120	0.225	0.287	0.340	0.399	0.439	0.475	0.517	0.547
35	0.118	0.222	0.283	0.335	0.394	0.433	0.468	0.510	0.539
36	0.116	0.219	0.279	0.330	0.388	0.427	0.462	0.504	0.533
37	0.114	0.216	0.275	0.325	0.383	0.421	0.456	0.497	0.526
38	0.113	0.212	0.271	0.321	0.378	0.415	0.450	0.491	0.519
39	0.111	0.210	0.267	0.317	0.373	0.410	0.444	0.485	0.513
40	0.110	0.207	0.264	0.313	0.368	0.405	0.439	0.479	0.507
41	0.108	0.204	0.261	0.309	0.364	0.400	0.433	0.473	0.501
42	0.107	0.202	0.257	0.305	0.359	0.395	0.428	0.468	0.495
43	0.105	0.199	0.254	0.301	0.355	0.391	0.423	0.463	0.490
44	0.104	0.197	0.251	0.298	0.351	0.386	0.419	0.458	0.484
45	0.103	0.194	0.248	0.294	0.347	0.382	0.414	0.453	0.479
46	0.102	0.192	0.246	0.291	0.343	0.378	0.410	0.448	0.474
47	0.101	0.190	0.243	0.288	0.340	0.374	0.405	0.443	0.469
48	0.100	0.188	0.240	0.285	0.336	0.370	0.401	0.439	0.465
49	0.098	0.186	0.238	0.282	0.333	0.366	0.397	0.434	0.460
50	0.097	0.184	0.235	0.279	0.329	0.363	0.393	0.430	0.456