

HW#7

1.

Product Rule

1. $\Pr(A, B | K) = \Pr(A | B, K) \Pr(B | K)$

Given

2. $\Pr(A, B, K) / \Pr(K) = (\Pr(A, B, K) / \Pr(B, K)) * (\Pr(B, K) / \Pr(K))$

Convert
using conditional
probability eq. in
textbook pg. 485

3. $\Pr(A, B, K) / \Pr(K) = \Pr(A, B, K) / \Pr(K)$

Simplify #2

Bayes' Rule

1. $\Pr(A | B, K) = \Pr(B | A, K) \Pr(A | K) / \Pr(B | K)$

Given

2. $\Pr(A, B, K) / \Pr(B, K) = ((\Pr(A, B, K) / \Pr(A, K)) * (\Pr(A, K) / \Pr(K))) / (\Pr(B, K) / \Pr(K))$

Convert using conditional probability eq. in textbook pg. 485

3. $\Pr(A, B, K) / \Pr(B, K) = (\Pr(A, B, K) / \Pr(K)) / (\Pr(B, K) / \Pr(K))$

Simplify #2

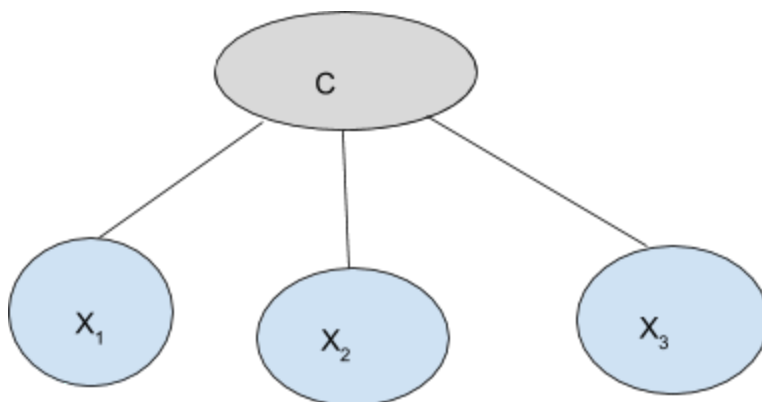
4. $\Pr(A, B, K) / \Pr(B, K) = (\Pr(A, B, K) / \Pr(K)) * (\Pr(K) / \Pr(B, K))$

Invert
the denominator in #3

5. $\Pr(A, B, K) / \Pr(B, K) = \Pr(A, B, K) / \Pr(B, K)$

Simplify #4

2.



C

C	Pr(C)
a	1/3
b	1/3
c	1/3

X_1

C	Pr(H)	Pr(T)
a	0.2	0.8
b	0.6	0.4
c	0.8	0.2

X_2

C	Pr(H)	Pr(T)
a	0.2	0.8
b	0.6	0.4
c	0.8	0.2

X_3

C	Pr(H)	Pr(T)
a	0.2	0.8
b	0.6	0.4
c	0.8	0.2

3.

	Square	Square	!Square	!Square
	One	!One	One	!One
Black	2/13	4/13	1/13	2/13
!Black	1/13	1/13	1/13	1/13

a1 = object is black

$$\Pr(a1) = 2/13 + 4/13 + 1/13 + 2/13 = \mathbf{9/13}$$

a2 = object is square

$$\Pr(a2) = 2/13 + 4/13 + 1/13 + 1/13 = \mathbf{8/13}$$

a3 = if the object is one or black, then it is also a square

a3 = square | one OR black

$$\begin{aligned} \Pr(a3) &= \Pr(\text{square, one OR black}) / \Pr(\text{one OR black}) \\ &= (2/13 + 4/13 + 1/13) / (2/13 + 4/13 + 1/13 + 1/13 + 2/13 + 1/13) \\ &= (7/13) / (11/13) \\ &= 7/13 * 13/11 \\ &= \mathbf{7/11} \end{aligned}$$

Independent Sets

Alpha: !Square Beta: Black Gamma: One

Alpha: !Square Beta: !Black Gamma: One

4.

- a) I(A, {}, {B, E})
- I(B, {}, {A, C})
- I(C, A, {D, B, E})
- I(D, {A, B}, {E, C})
- I(E, B, {A, D, C, F, G})
- I(F, {C, D}, {A, B, E})
- I(G, F, {H, C, D, E, A, B})
- I(H, {F, E}, {G, C, D, A, B})

- b) Source: <http://bayes.cs.ucla.edu/BOOK-2K/d-sep.html>

d_separated (A, BH, E)

False, Path ADFHE is not blocked. The path does not have to go through B. In addition, the H node is opened because it is a collider in the set {B,H}.

d separated (G, D, E)

True, The path from A to E has to either go through D or H. D is blocked due to it being in the set {D}. H is blocked because it is a collider.

d separated (AB, F, GH)

False, The paths ADBEH and BEH are not blocked. The paths from A to G and the path from B to G is blocked because they have to go through the node F which is in set {F}.

c)
$$\Pr(a, b, c, d, e, f, g, h) = \Pr(a) * \Pr(b) * \Pr(c | a) * \Pr(d | a, b) * \Pr(e | b) * \Pr(f | c, d) * \Pr(g | f) * \Pr(h | e, f)$$

d)
$$\begin{aligned}\Pr(A = 0, B = 0) &= \Pr(A = 0) * \Pr(B = 0) \\ &= 0.8 * 0.3 \\ &= \mathbf{0.24}\end{aligned}$$

$\Pr(E = 1 | A = 1) = \Pr(E = 1)$ since E and A are independent according to the tables.

$$\begin{aligned}\Pr(E = 1) &= \Pr(E = 1 | B = 0) + \Pr(E = 1 | B = 1) \\ &= \Pr(E = 1 | B = 0) * \Pr(B = 0) + \Pr(E = 1 | B = 1) * \Pr(B = 1) \\ &= (0.9 * 0.3) + (0.1 * 0.7) \\ &= \mathbf{0.34}\end{aligned}$$