HW #5

1. P -> ~Q

P	Q	~Q	**P->~Q**
T	T	F	**F**
T	F	T	**T**
F	T	F	**T**
F	F	T	**T**

Q -> ~P

P	Q	~P	**Q->~P**
T	T	F	**F**
T	F	F	**T**
F	T	T	**T**
F	F	T	**T**

P->~Q and Q->~P have the same values in the last column of their tables so they are equivalent.

P⇔~Q

P	Q	~Q	**P⇔~Q**
T	T	F	**F**
T	F	T	**T**
F	T	F	**T**
F	F	T	**F**

((P^~Q)v(~P^Q))

P	Q	~P	~Q	P^~Q	~P^Q	**(P^~Q)v(~P^Q)**
T	T	F	F	F	F	**F**
T	F	F	T	T	F	**T**
F	T	T	F	F	T	**T**
F	F	T	T	F	F	**F**

P⇔~Q and ((P^~Q)v(~P^Q)) have the same values in the last column of their tables so they are equivalent.

2. (Smoke -> Fire) -> (~Smoke -> ~Fire)

Smoke	Fire	~Smoke	~Fire	(Smoke -> Fire)	(~Smoke -> ~Fire)	**(S->F)->(~S->~F)**
T	T	F	F	T	T	**T**
T	F	F	T	F	T	**T**
F	T	T	F	T	F	**F**
F	F	T	T	T	T	**T**

It is **neither** because not all paths lead to T, which makes it not valid, but there are instances where the statement is T, so it is satisfiable.

(Smoke -> Fire) -> ((Smoke v Heat) -> Fire)

S	F	H	(S -> F)	(S v H)	(S v H) -> F	**(S -> F) -> ((S v H) -> F)**
T	T	T	T	T	T	**T**
T	T	F	T	T	T	**T**
T	F	T	F	T	F	**T**
T	F	F	F	T	F	**T**

F	T	T	T	T	T	**T**
F	T	F	T	F	T	**T**
F	F	T	T	T	F	**F**
F	F	F	T	F	T	**T**

It is **neither** because not all paths lead to T, which makes it not valid, but there are instances where the statement is T, so it is satisfiable.

((Smoke ^ Heat) -> Fire) ⇔ ((Smoke -> Fire) v (Heat -> Fire))

S	F	H	(S ^ H)	(S ^ H) -> F	(S -> F)	(H -> F)
T	T	T	T	T	T	T
T	T	F	F	T	T	T
T	F	T	T	F	F	F
T	F	F	F	T	F	T
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	F	T	T	F
F	F	F	F	T	T	T

(S -> F) v (H -> F)	**((S ^ H) -> F)⇔ ((S -> F) v (H -> F))**
T	**T**
T	**T**
F	**T**
T	**T**
T	**T**
T	**T**
T	**T**

T	**T**

It is **valid** because it is a tautology. All of the values are true.

3.

Legend: My -> mythical
Mo -> mortal
Ma -> mammal
H -> horned
Mg -> magical

a) My -> ~Mo
~My -> (Mo ^ Ma)
(~Mo v Ma) -> H
H -> Mg

b) 1. My -> ~Mo → **~My v ~Mo**
2. ~My -> (Mo ^ Ma) → My v (Mo ^ Ma) → **(My v Mo) ^ (My v Ma)**
3. (~Mo v Ma) -> H → ~(~Mo v Ma) v H → (Mo ^ ~Ma) v H → **(Mo v H) ^ (~Ma v H)**
4. H -> Mg → **~H v Mg**

c) **Proof unicorn is mythical/magical/horned**

5. (My v Mo)	Extracted from 2
6. (My v Ma)	Extracted from 2
7. (Mo v H)	Extracted from 3
8. (~Ma v H)	Extracted from 3
9. ~Ma v Mg	Resolve 4 and 8
10. ~Mo v Mg	Resolve 4 and 7
11. My v Mg	Resolve 5 and 10
12. My v H	Resolve 6 and 8
13. Ma v ~Mo	Resolve 1 and 6
14. ~My v H	Resolve 1 and 7
15. **H**	Resolve 12 and 14
16. **Mg**	Resolve 4 and 15

Shown above, I have concluded that the unicorn is horned and magical. To prove/disprove whether the unicorn is mythical, I will use proof by contradiction.

Proof by contradiction

17. ~My	Assumption
18. Mo	Resolve 5 and 17
19. Ma	Resolve 6 and 17

20. Mg	Resolve 10 and 18
21. H	Resolve 8 and 19
22. ~My	Resolve 1 and 18

I could not find any contradictions, so nothing can be concluded about whether the unicorn is mythical or not.