

HW #1

1. $\text{let } x = 4 + 5 \text{ in } (3 * x)$ Given
 $\Rightarrow \text{let } x = 9 \text{ in } (3 * x)$ Arithmetic
 $\Rightarrow (3 * 9)$ Let Reduction
 $\Rightarrow 27$ Arithmetic

2. $(\lambda x \rightarrow 3 * x) (4 + 5)$ Given
 $\Rightarrow (\lambda x \rightarrow 3 * x) (9)$ Arithmetic
 $\Rightarrow 3 * 9$ Lambda Reduction
 $\Rightarrow 27$ Arithmetic

3. $((\lambda x \rightarrow (\lambda y \rightarrow x + (3 * y))) 4) 1$ Given
 $\Rightarrow (\lambda y \rightarrow 4 + (3 * y)) 1$ Lambda Reduction
 $\Rightarrow 4 + (3 * 1)$ Lambda Reduction
 $\Rightarrow 4 + 3$ Arithmetic
 $\Rightarrow 7$ Arithmetic

4. $\text{let } x = 4 \text{ in } (\text{let } y = 1 \text{ in } (x + (3 * y)))$ Given
 $\Rightarrow \text{let } x = 4 \text{ in } (x + (3 * 1))$ Let Reduction
 $\Rightarrow 4 + (3 * 1)$ Let Reduction
 $\Rightarrow 4 + 3$ Arithmetic
 $\Rightarrow 7$ Arithmetic

5. $\text{let } x = 4 \text{ in } (\text{let } y = 1 + x \text{ in } (x + (3 * y)))$ Given
 $\Rightarrow \text{let } x = 4 \text{ in } (x + (3 * (1 + x)))$ Let Reduction
 $\Rightarrow 4 + (3 * (1 + 4))$ Let Reduction
 $\Rightarrow 4 + (3 * 5)$ Arithmetic
 $\Rightarrow 4 + 15$ Arithmetic
 $\Rightarrow 19$ Arithmetic

6. $((\lambda x \rightarrow (\lambda y \rightarrow x + (3 * x))) 4) 1$ Given
 $\Rightarrow (\lambda y \rightarrow 4 + (3 * 4)) 1$ Lambda Reduction
 $\Rightarrow 4 + (3 * 4)$ Lambda Reduction (No free variables)
 $\Rightarrow 4 + 12$ Arithmetic
 $\Rightarrow 16$ Arithmetic

7. $((\lambda x \rightarrow (\lambda y \rightarrow y + (3 * y))) 4) 1$ Given
 $\Rightarrow (\lambda y \rightarrow y + (3 * y)) 1$ Lambda Reduction (No free variables)
 $\Rightarrow 1 + (3 * 1)$ Lambda Reduction
 $\Rightarrow 1 + 3$ Arithmetic

$\Rightarrow 4$ Arithmetic

8. $(\lambda y \rightarrow y + ((\lambda y \rightarrow 3 * y) 4)) 5$ Given
 $\Rightarrow (\lambda y \rightarrow y + (3 * 4)) 5$ Lambda Reduction
 $\Rightarrow 5 + (3 * 4)$ Lambda Reduction
 $\Rightarrow 5 + 12$ Arithmetic
 $\Rightarrow 17$ Arithmetic

9. $(\lambda y \rightarrow ((\lambda y \rightarrow 3 * y) 4) + y) 5$ Given
 $\Rightarrow (\lambda y \rightarrow (3 * 4) + y) 5$ Lambda Reduction
 $\Rightarrow (3 * 4) + 5$ Lambda Reduction
 $\Rightarrow 12 + 5$ Arithmetic
 $\Rightarrow 17$ Arithmetic

10. $(\lambda x \rightarrow x * (\text{let } x = 3 * 2 \text{ in } (x + 7)) + x) 4$ Given
 $\Rightarrow (\lambda x \rightarrow x * (\text{let } x = 6 \text{ in } (x + 7)) + x) 4$ Arithmetic
 $\Rightarrow (\lambda x \rightarrow x * (6 + 7) + x) 4$ Let Reduction
 $\Rightarrow 4 * (6 + 7) + 4$ Lambda Reduction
 $\Rightarrow 4 * 13 + 4$ Arithmetic
 $\Rightarrow 52 + 4$ Arithmetic
 $\Rightarrow 56$ Arithmetic

11. $g ((\text{let } x = 4 \text{ in } (\lambda y \rightarrow x + y)) 2)$ Given
 $\Rightarrow (\lambda z \rightarrow z + 4) ((\text{let } x = 4 \text{ in } (\lambda y \rightarrow x + y)) 2)$ Substitution from file
 $\Rightarrow (\lambda z \rightarrow z + 4) ((\lambda y \rightarrow 4 + y) 2)$ Let Reduction
 $\Rightarrow (\lambda z \rightarrow z + 4) (4 + 2)$ Lambda Reduction
 $\Rightarrow (\lambda z \rightarrow z + 4) (6)$ Arithmetic
 $\Rightarrow 6 + 4$ Lambda Reduction
 $\Rightarrow 10$ Arithmetic

12. $\text{let } x = 5 \text{ in } (\lambda z \rightarrow x * z)$ Given
 $\Rightarrow (\lambda z \rightarrow 5 * z)$ Let Reduction

*This line results in an error because the lambda function has no value for lambda reduction.

13. $f ((\lambda fn \rightarrow fn \text{ Rock}) (\lambda x \rightarrow \text{whatItBeats } x))$ Given
 $\Rightarrow f ((\lambda x \rightarrow \text{whatItBeats } x) \text{ Rock})$ Lambda Reduction
 $\Rightarrow f (\text{whatItBeats } \text{Rock})$ Lambda Reduction
 $\Rightarrow f ((\lambda s \rightarrow \text{case } s \text{ of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\}) \text{Rock})$
Substitution from file
 $\Rightarrow f (\text{case } \text{Rock} \text{ of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\})$

- $\Rightarrow f \text{ Scissors}$ Lambda Reduction
 $\Rightarrow (\lambda s \rightarrow \text{case } s \text{ of } \{\text{Rock} \rightarrow 334; \text{Paper} \rightarrow 138; \text{Scissors} \rightarrow 99\}) \text{ Scissors}$ Case Reduction
 $\Rightarrow \text{case Scissors of } \{\text{Rock} \rightarrow 334; \text{Paper} \rightarrow 138; \text{Scissors} \rightarrow 99\}$ Substitution from file
 $\Rightarrow 99$ Lambda Reduction
Case Reduction
14. $((\lambda f \rightarrow (\lambda x \rightarrow f (f x))) \text{ whatItBeats}) \text{ Paper}$ Given
 $\Rightarrow (\lambda x \rightarrow \text{whatItBeats } (\text{whatItBeats } x)) \text{ Paper}$ Lambda Reduction
 $\Rightarrow \text{whatItBeats } (\text{whatItBeats } \text{Paper})$ Lambda Reduction
 $\Rightarrow \text{whatItBeats } ((\lambda s \rightarrow \text{case } s \text{ of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\}) \text{ Paper})$ Substitution from file
 $\Rightarrow \text{whatItBeats } (\text{case Paper of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\})$
 $\Rightarrow \text{whatItBeats Rock}$ Lambda Reduction
 $\Rightarrow (\lambda s \rightarrow \text{case } s \text{ of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\}) \text{ Rock}$ Case Reduction
 $\Rightarrow \text{case Rock of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\}$ Substitution from file
 $\Rightarrow \text{Scissors}$ Lambda Reduction
Case Reduction
15. $\text{whatItBeats } (\text{case Paper of } \{\text{Rock} \rightarrow \text{Paper}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Scissors}\})$ Given
 $\Rightarrow \text{whatItBeats Rock}$ Case Reduction
 $\Rightarrow (\lambda s \rightarrow \text{case } s \text{ of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\}) \text{ Rock}$ Substitution from file
 $\Rightarrow \text{case Rock of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\}$ Lambda Reduction
 $\Rightarrow \text{Scissors}$ Case Reduction
16. $(\text{case } (\text{Win Rock}) \text{ of } \{\text{Draw} \rightarrow \text{whatItBeats}; \text{Win } z \rightarrow (\lambda s \rightarrow \text{Scissors})\}) \text{ Paper}$ Given
 $\Rightarrow (\lambda s \rightarrow \text{Scissors}) \text{ Paper}$ Case Reduction
 $\Rightarrow \text{Scissors}$ Lambda Reduction (No free variables)
17. $\text{case } (\text{Win } (\text{whatItBeats Rock})) \text{ of } \{\text{Draw} \rightarrow n; \text{Win } x \rightarrow (n + f x)\}$ Given
 $\Rightarrow \text{case } (\text{Win } ((\lambda s \rightarrow \text{case } s \text{ of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\}) \text{ Rock})) \text{ of } \{\text{Draw} \rightarrow n; \text{Win } x \rightarrow (n + f x)\}$ Substitution from file
 $\Rightarrow \text{case } (\text{Win } (\text{case Rock of } \{\text{Rock} \rightarrow \text{Scissors}; \text{Paper} \rightarrow \text{Rock}; \text{Scissors} \rightarrow \text{Paper}\})) \text{ of } \{\text{Draw} \rightarrow n; \text{Win } x \rightarrow (n + f x)\}$

Paper}))) of {Draw -> n; Win x -> (n + f x)}

=> case (Win Scissors) of {Draw -> n; Win x -> (n + f x)}	Lambda Reduction
=> (n + f Scissors)	Case Reduction
=> (1 + f Scissors)	Case Reduction
=> (1 + (\s -> case s of {Rock -> 334; Paper -> 138; Scissors -> 99}) Scissors)	Substitution from file
=> (1 + case Scissors of {Rock -> 334; Paper -> 138; Scissors -> 99})	Substitution from file
=> 1 + 99	Lambda Reduction
=> 100	Case Reduction
	Arithmetic

18. let y = 2 in (case (Win (whatItBeats Rock)) of {Draw -> n; Win y -> (n + f y)} + y)

	Given
=> let y = 2 in (case (Win ((\s -> case s of {Rock -> Scissors; Paper -> Rock; Scissors -> Paper})) Rock)) of {Draw -> n; Win y -> (n + f y)} + y)	Substitution from file
=> let y = 2 in (case (Win (case Rock of {Rock -> Scissors; Paper -> Rock; Scissors -> Paper})) of {Draw -> n; Win y -> (n + f y)} + y)	Lambda Reduction
=> let y = 2 in (case (Win Scissors) of {Draw -> n; Win y -> (n + f y)} + y)	Case Reduction
=> let y = 2 in (n + f Scissors + y)	Case Reduction
=> let y = 2 in (1 + f Scissors + y)	Substitution from file
=> let y = 2 in (1 + (\s -> case s of {Rock -> 334; Paper -> 138; Scissors -> 99}) Scissors + y)	Substitution from file
=> let y = 2 in (1 + (case Scissors of {Rock -> 334; Paper -> 138; Scissors -> 99}) + y)	Lambda Reduction
=> let y = 2 in (1 + 99 + y)	Case Reduction
=> (1 + 99 + 2)	Let Reduction
=> 100 + 2	Arithmetic
=> 102	Arithmetic