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Section 1A

HW #2

3.

a. Base Case: First, we check if **isEven (double (S^0 Z))** evaluates to True. **isEven (double (S^0 Z))** is the same as **isEven (double Z)**. **double Z** evaluates to Z via the first branch in the case statement, thus the statement becomes **isEven Z**. This last statement then evaluates to True via the first branch in the case statement. Thus, **isEven (double (S^0 Z))** is True and the statement holds for n = 0.

b. Inductive Step: We need to show that **isEven (double (S^(n+1) Z))** evaluates to True. We assume that **isEven (double (S^n Z))** evaluates to True. Notice that **isEven (double (S^(n+1) Z))** can be represented as **isEven (double (S (S^n Z)))**, so by the second branch of the case statement in the double function, this will evaluate to **isEven (S (S (double (S^n Z))))**. However in the assumption, we have already established the fact that **(double (S^n Z))** is even. We just need to show that the rest of **(S (S (double (S^n Z))))** is even. We show this by using the second branch of the case statement in the isEven function. n' is **(S (double (S^n Z)))** and n'' is **(double (S^n Z)).** As already established, n'' is even. Therefore **isEven (double (S^(n+1) Z))** evaluates to True and the **isEven (double (S^n Z))** evaluates to True for all natural numbers n. ☐