① Probability — CSCI 104 - Devin C. Martin

Solution:

→ the # of ways to choose 8 diff students out of 15 for each question is $\frac{15!}{(15-8)!}$

→ the total # of ways to choose any student for each of the 8 questions is $15^8$

→ the probability that no student will have to answer more than one question is $\frac{\text{\# of ways to choose diff students}}{\text{total ways to choose any student}}$

Calculation:

$$\text{Probability} = \frac{15!/(15-8)!}{15^8}$$

Probability $= 0.1012$

Final Answer: the probabity is approx $= 0.1012$ or $10.12\%$

②

Solution:

→ the total # of 5-digit #'s where all digits are unique is $10 * 9 * 8 * 7 * 6$

→ the total # of such #'s that are even, start w/ 2 odd digits, & have all unique digits is 5 * 4 * 5 * 4 * 3 (5 options for the first odd digit, 4 for the secon 5 even options for the last digit, & 4 & 3 choices for the remaining 2 unique digits).

→ the probability of one # meeting the criteria is

$$\frac{\text{total even \#'s w/ criteria}}{\text{total 5-digit unique \#'s}}$$

→ using the binomial distribution, calculate the probability of exactly 5 out of 8 #'s meeting the criteria

Calculation:

Probability of one # meeting criteria $= \frac{5 * 4 * 5 * 4 * 3}{10 * 9 * 8 * 7 * 6}$

Probability of exactly 5 out of 8 =

binom.pmf(5, 8, Probability of one # meeting criteria)

$= 4.8801 * 10^{-6}$

Final Answer: the probability is approx $= 4.8801 \times 10^{-6}$

or 0.00048801%

③

Solution

→ Count the # of outcomes where at least 2 dice show 4 or above

→ Count the # of outcomes where all dice show the same value (6 possibilities)

→ Calculate the probability of each event

→ events A & B are indep. if $P(A \& B) = P(A) * P(B)$

Calculation:

Probabity of event A & B was calculated

Found that $P(A \& B) \neq P(A) * P(B)$

Final Answer: events A & B are not indep

④

Solution:

→ the total # of 5-card hands is $\binom{52}{5}$

→ the # of flush hands (same suit) is $\binom{13}{5}$ for each suit, multiplied by 4 for each suit

→ the probability of getting a flush in one hand is $\dfrac{\text{\# of flush hands}}{\text{total \# of 5-card hands}}$.

→ the exp. # of hands to get a flush is the reciprical of the probability of getting a flush in one hand

Calculation:

$$\text{Probability of flush} = \frac{4 \times \binom{13}{5}}{\binom{52}{5}}$$

$$\text{Exp \# of hands} = \frac{1}{\text{Probability of flush}}$$

Exp # of hands = 2019.39

Final Answer: the exp # of hands to get a flush is approx = 2019.39

5

Solution:

→ use Bayes Theorem: P(superstar played | 4 wins)

$$= \frac{P(\text{4 wins} \mid \text{superstar played}) \times (\text{Superstar played})}{P(\text{4 wins})}$$

→ Calculate P(4 wins | superster played) & P(4 wins | superster didn't play) using the binomial distribution

→ Calculate P(4 wins) as the weighted sum of these Probabilities

Calculation:

P(superstar played | 4 wins)

$$= \frac{\text{binom.pmf}(4,5,0.70) * 0.75}{\text{binom.pmf}(4,5,0.70) * \text{binom.pmf}(4,5,0.50) * 0.25}$$

$= 0.8737$

Final Answer: the probability that the superster played all 5 games given the team won 4 of them is approx $= 0.8737$ or $87.37\%$