PROGRAMMING AND DATA STRUCTURES

BINARY TREES (BST)

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OUTLINE

- Binary Search Trees (BST)
- Properties of the BST
- Operations on the BST
- Implementation of the BST class

STUDENT LEARNING OUTCOMES

At the end of this chapter, you should be able to:

- Describe the properties of binary search trees (BST)
- Trace operations on the BST
- Implement the BST generic data structures
- Use the BST data structure
- Evaluate the complexity of the operations on the BST

Binary Search Tree (BST)

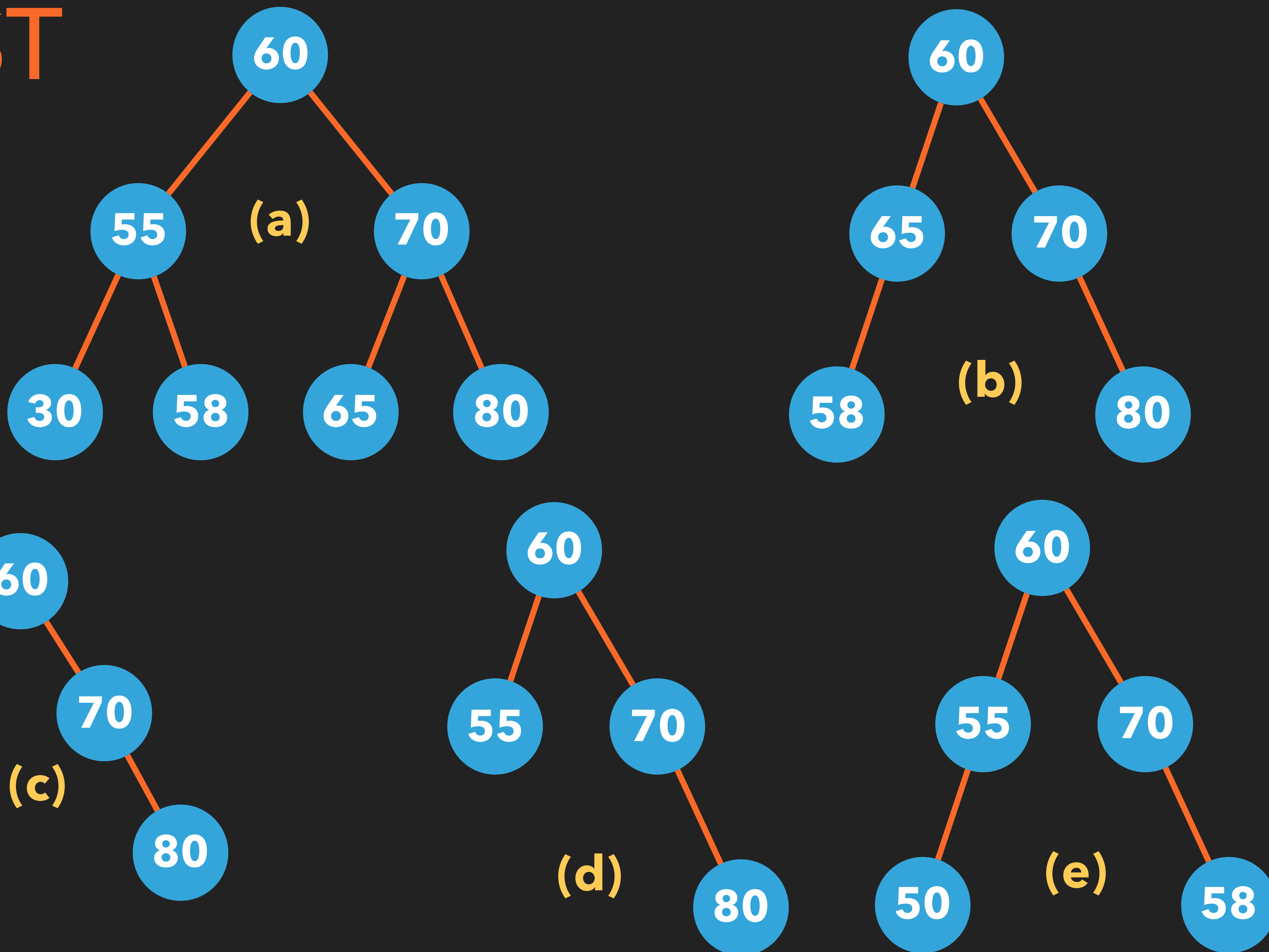
- Special binary tree
- Used for efficient search in large data sets
- Implements the binary search operation
- BST is a set (no duplicates)

BST

✦ Properties of the BST

- BST has a root, a left subtree (L) and a right subtree (R)
- The value of the root is greater than the value of every node in L
- The value of the root is less than the value of every node in R
- L and R are also BSTs

BST

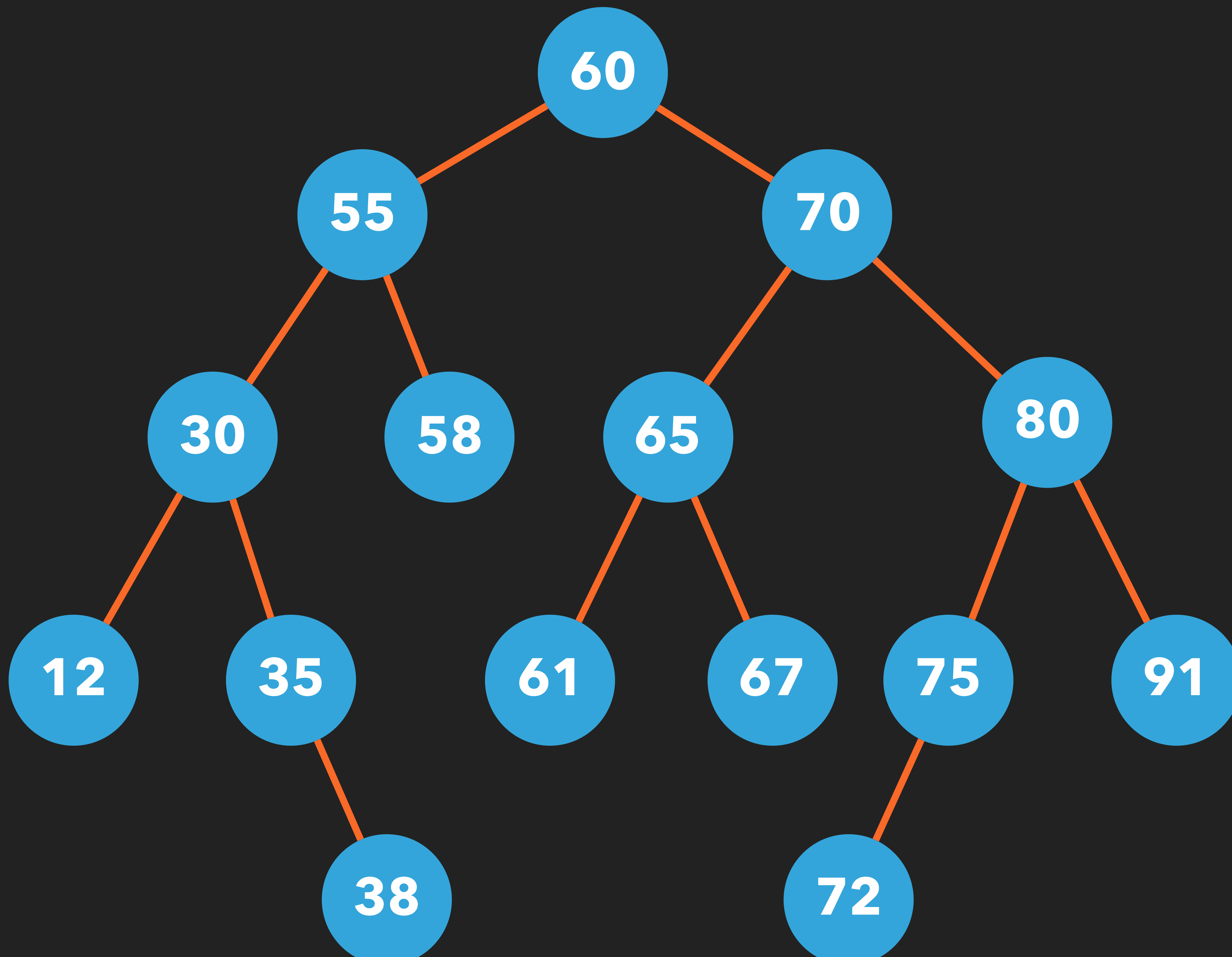

60
55
(a)
70
30
58
65
80
60
65
70
58
(b)
80
60
70
(c)
80
60
55
70
(d)
80
60
55
70
50
(e)
58

BST

- Common operations on the BST
  - Search for a specific value in the BST
  - Add a node to the BST while keeping the BST properties
  - Remove a node from the BST while keeping the BST properties
  - Traverse the BST (preorder, inorder, postorder)

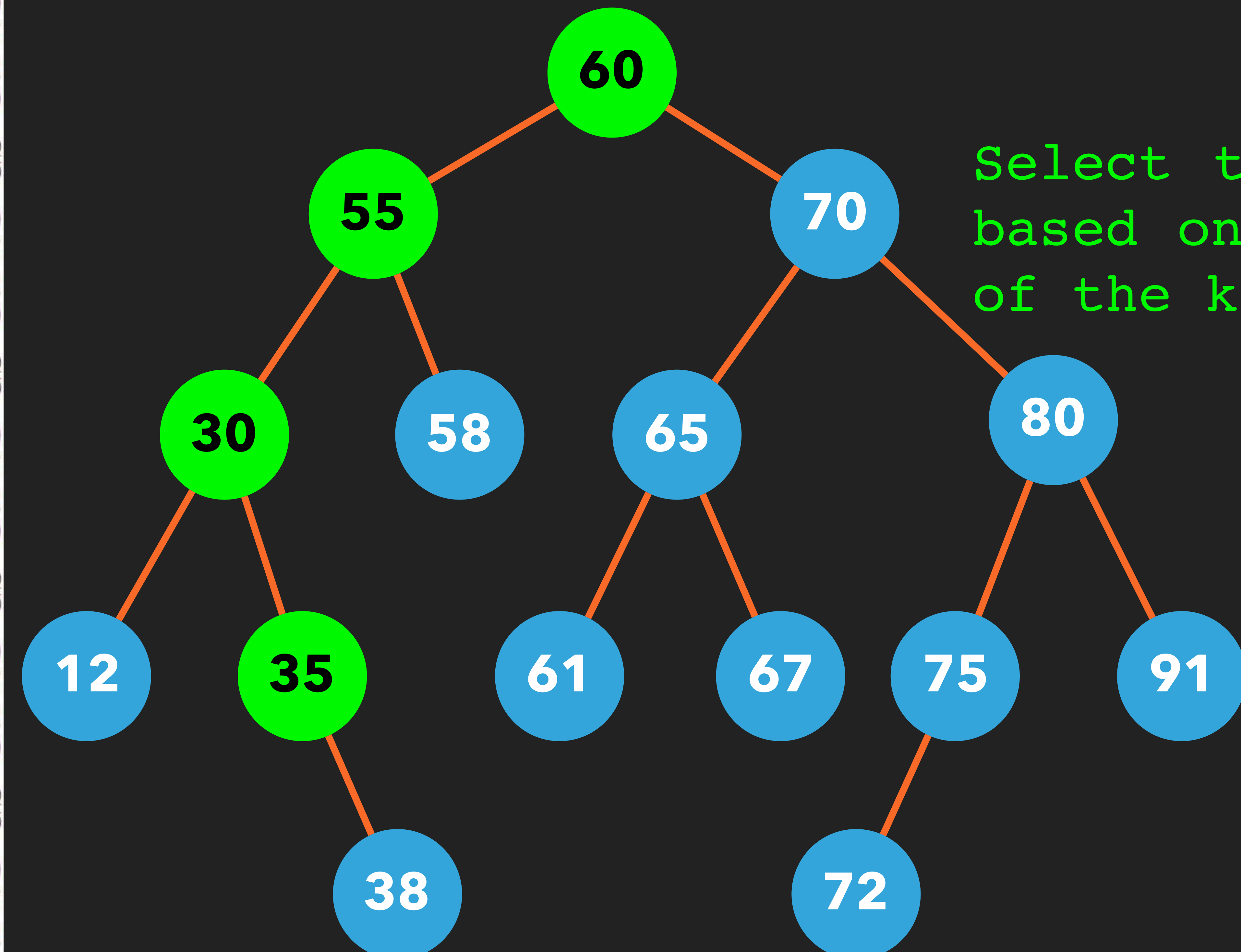
BST (Search)

Search for the value 35



BST (Search)

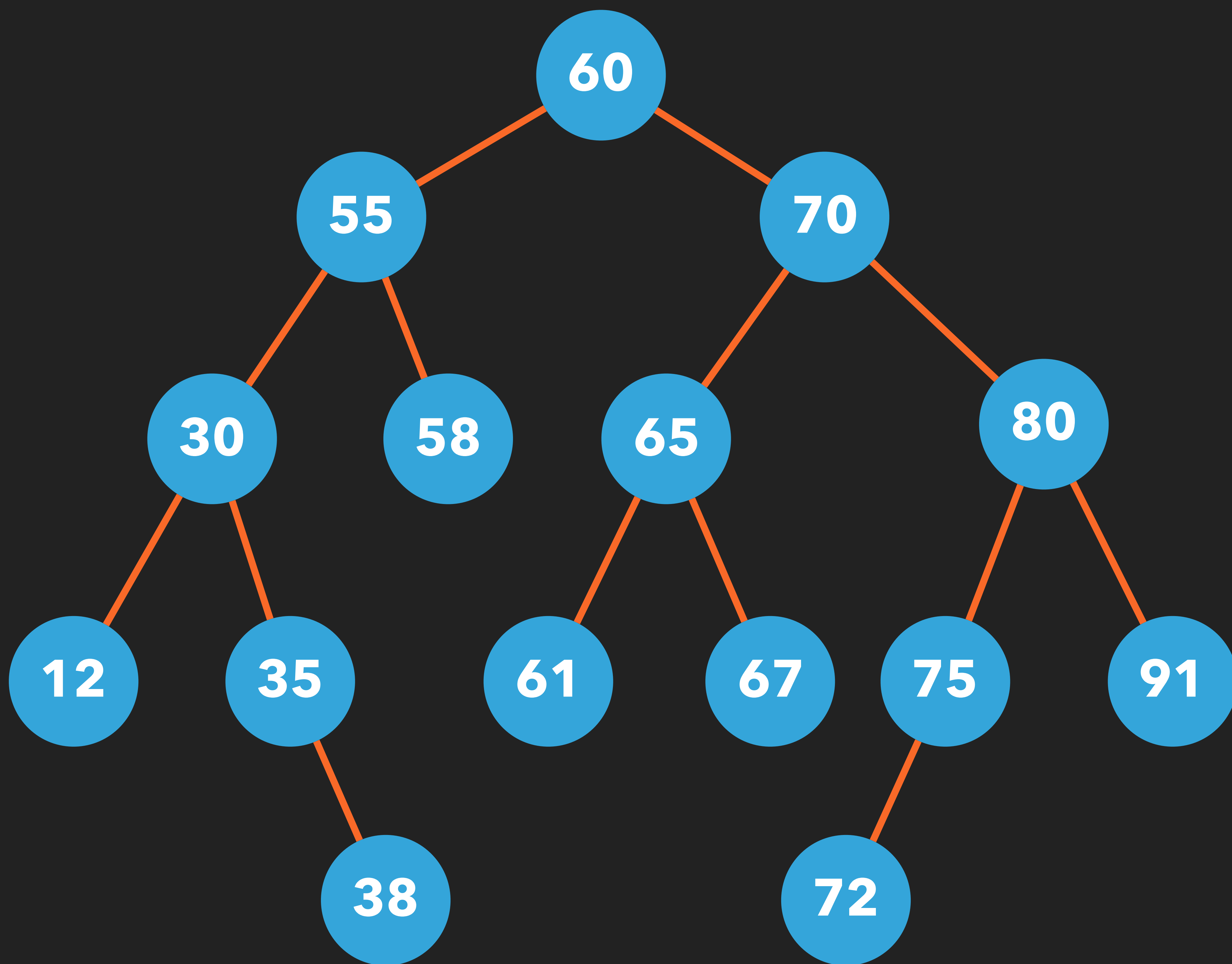
Search for the value 35



Select the subtree based on the value of the key

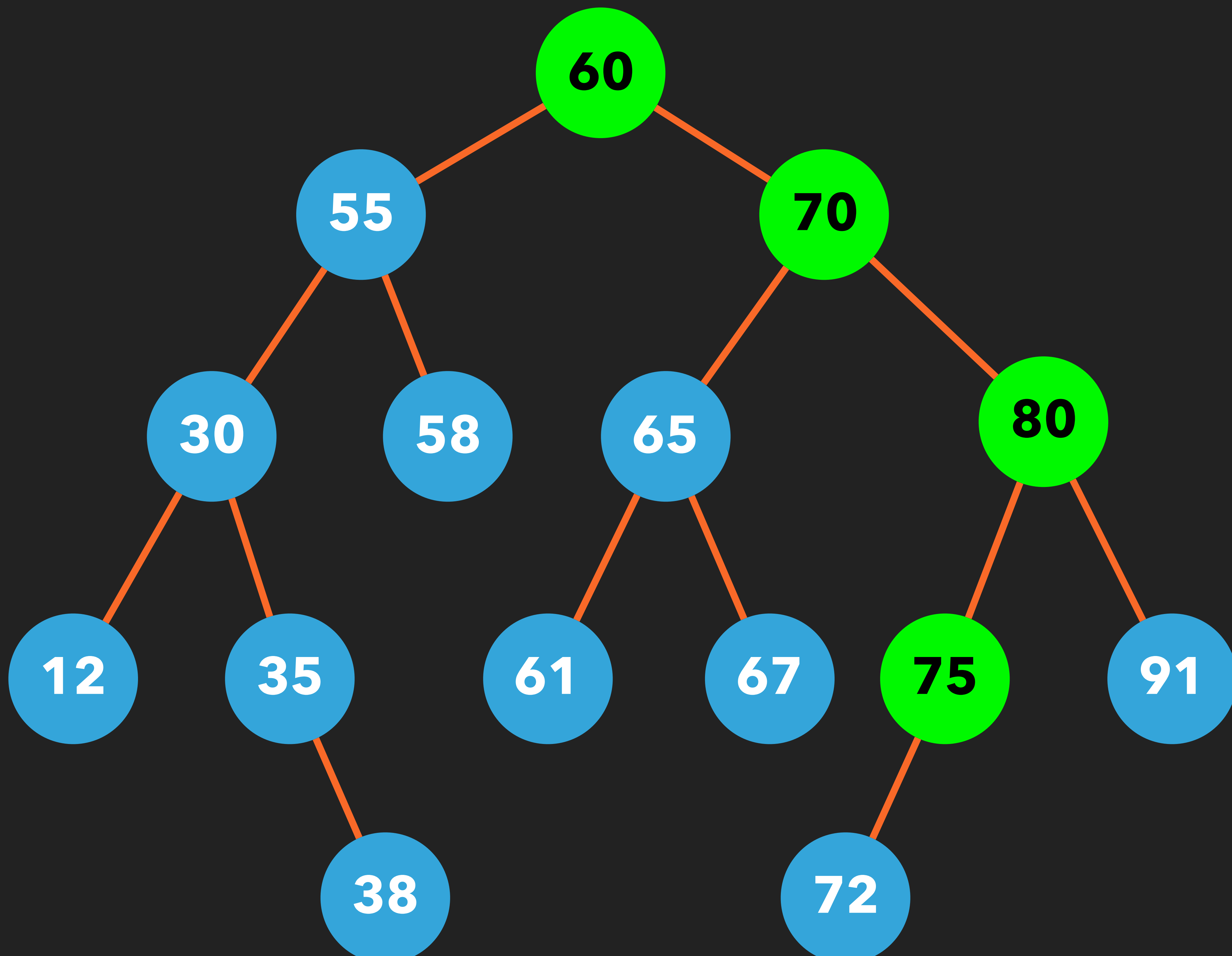
BST (Search)

Search for the value 75



BST (Search)

Search for the value 75

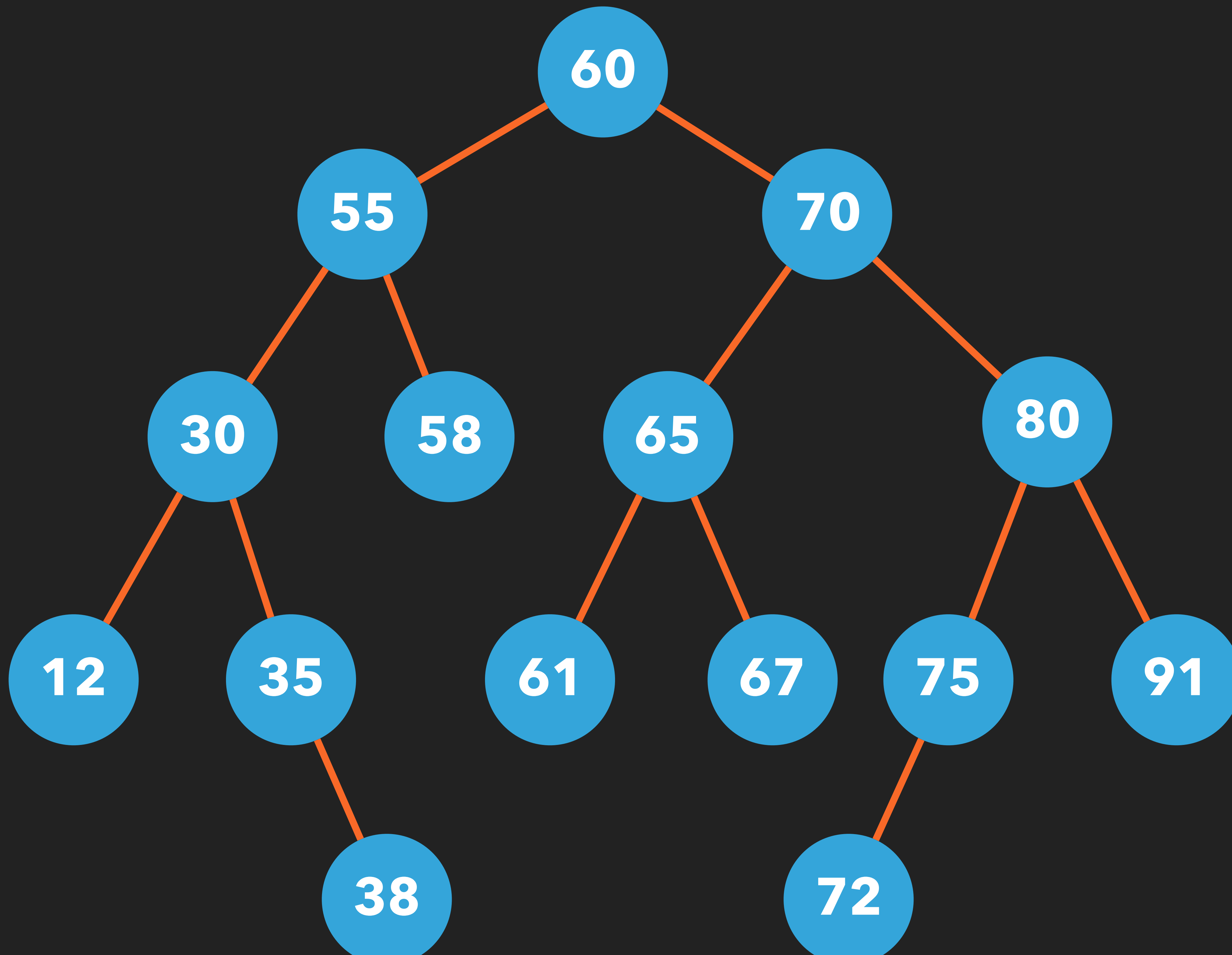


BST (Search)

```
boolean contains (value)

   current node = root // start from the root
   while(current node is not null){
      if(the value of the current node == value)
         return true
      else if (value of the current node > value)
         current node is set to its left child
      else
         current node is set to its right child
   }
   return false

end contains
```

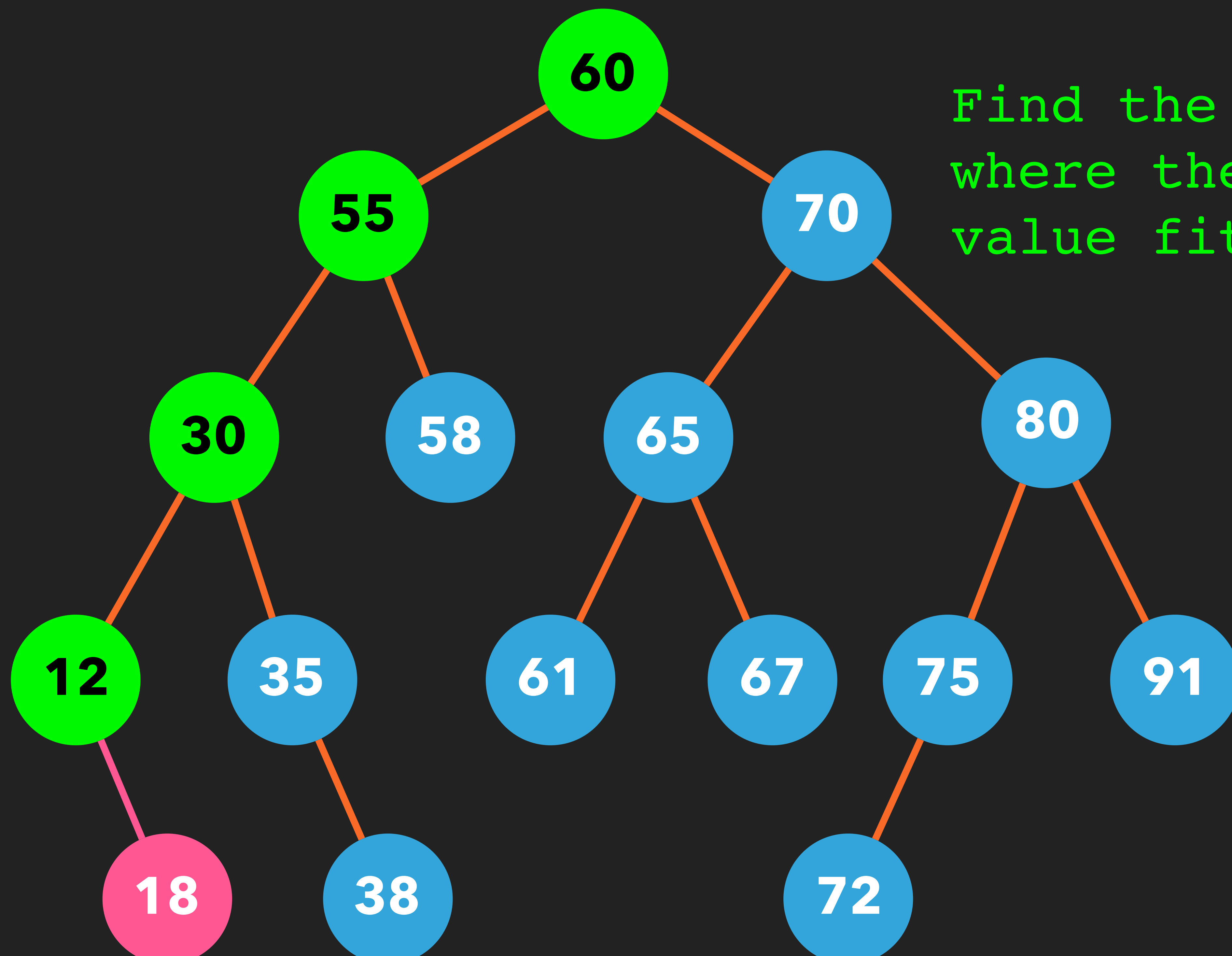
BST (Add)

Add the value 18



BST (Add)

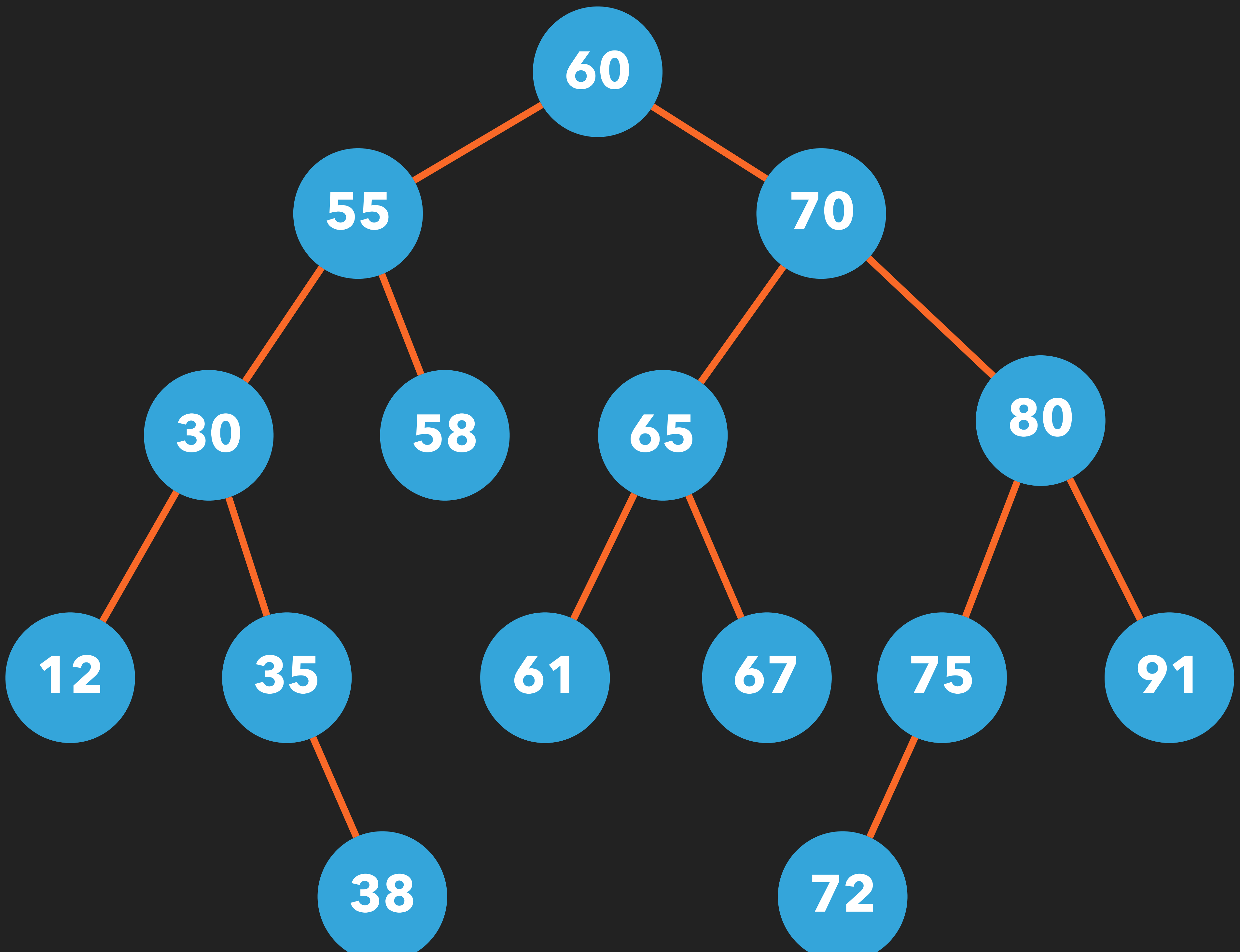
Add the value 18



Find the subtree where the new value fits

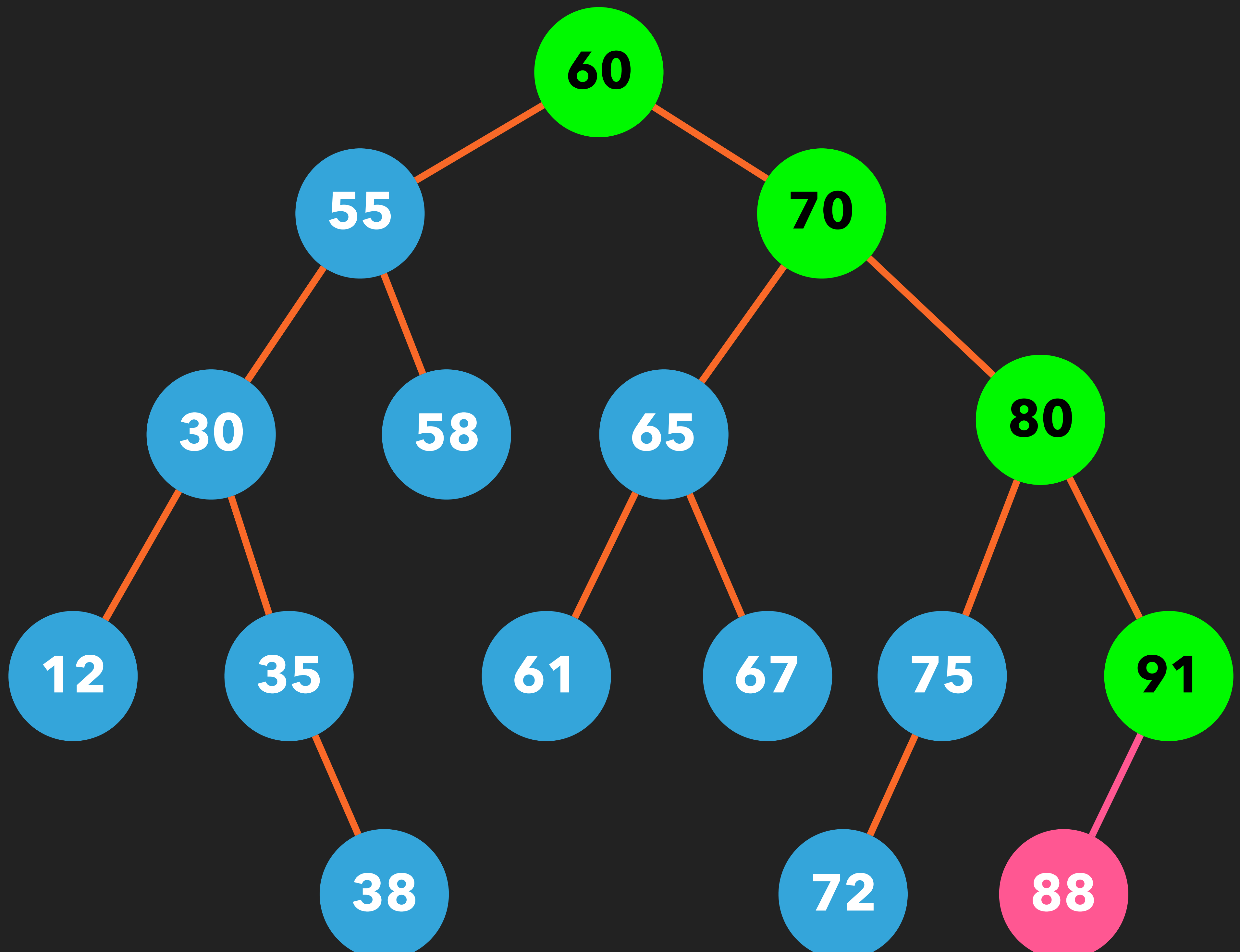
BST (Add)

Add the value 88



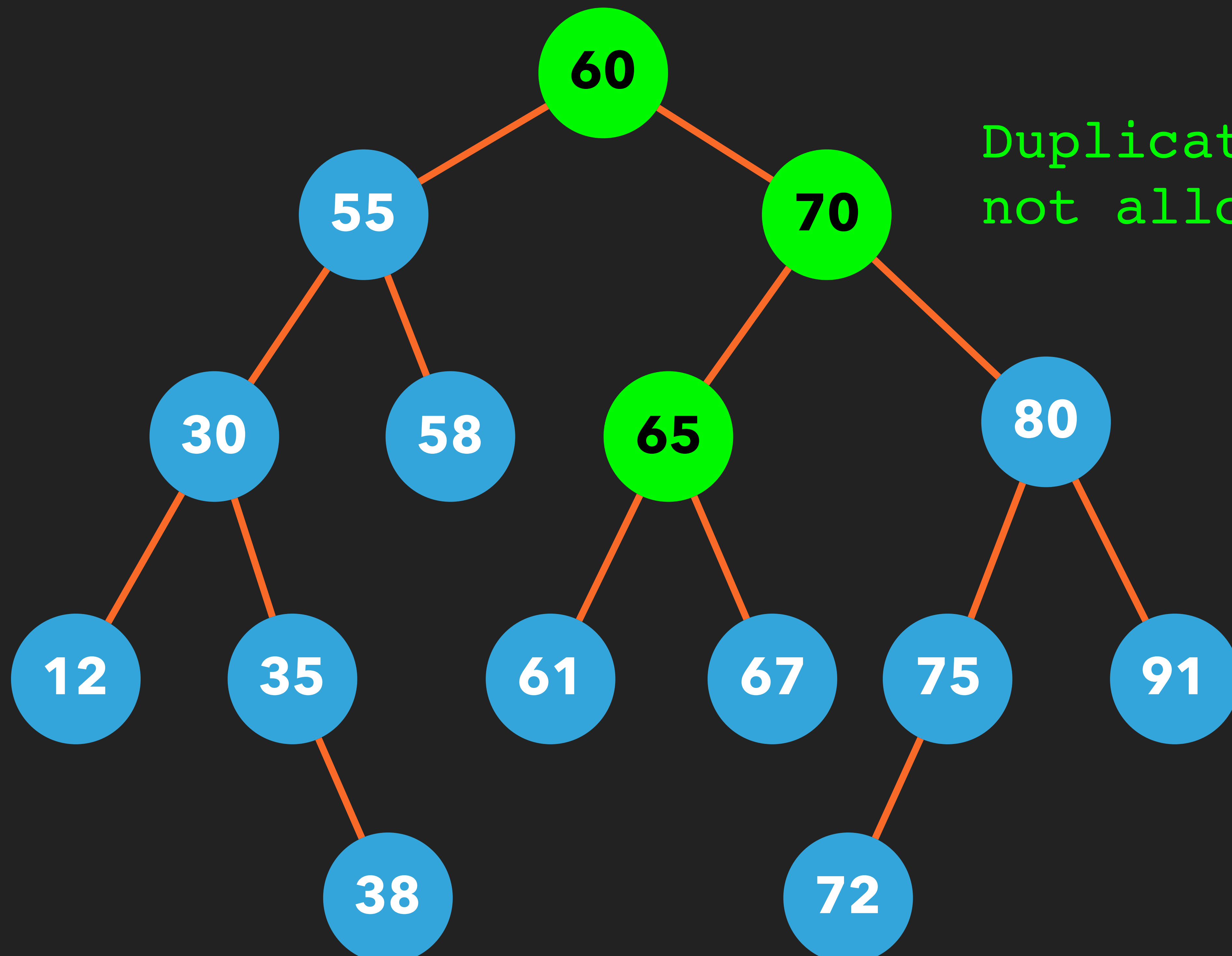
BST (Add)

Add the value 88



BST (Add)

Add the value 65



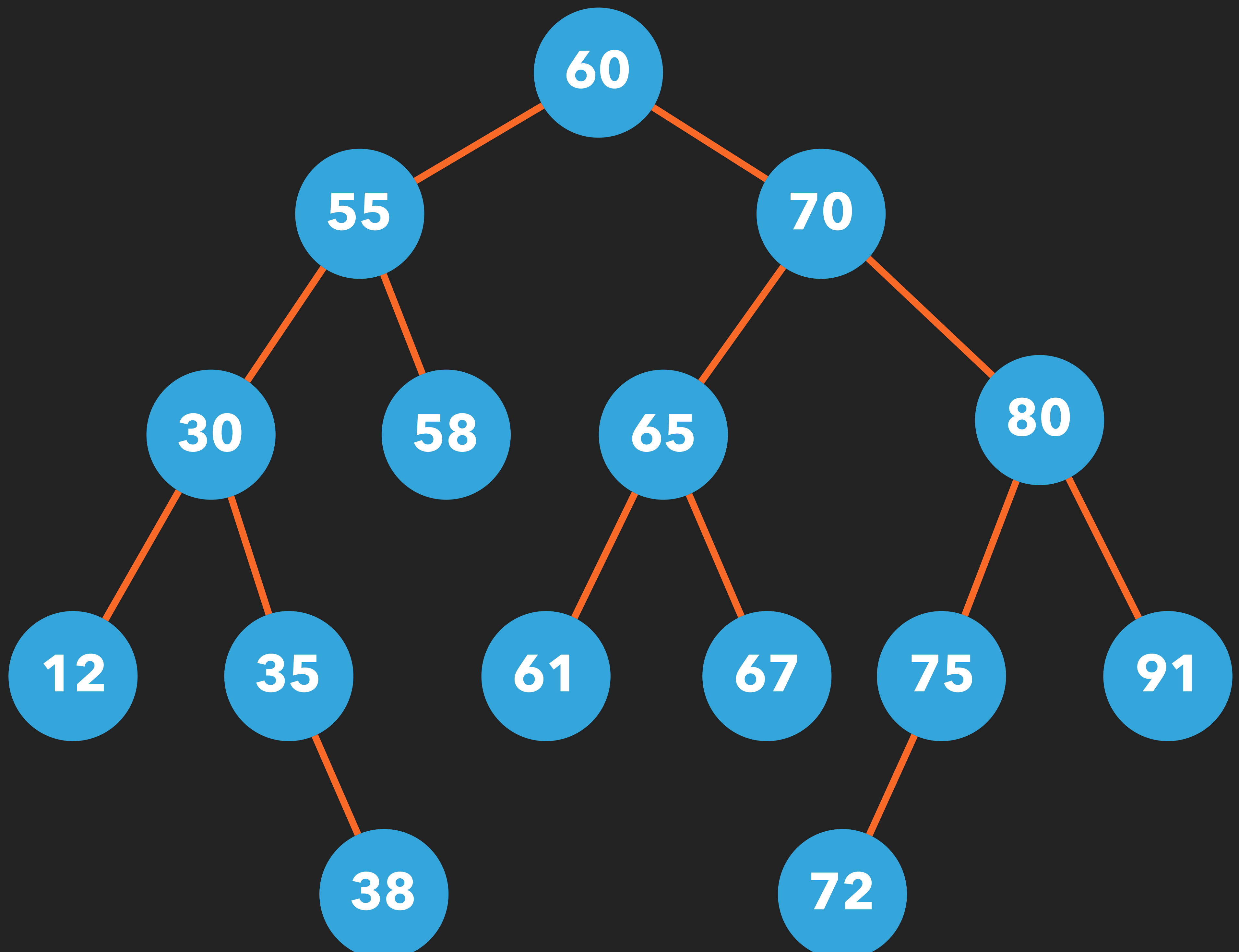
Duplicates are not allowed

BST (Add)

```
boolean add (value)
   currentNode = root
   while(currentNode is not null){
     parentNode = currentNode
     if(the value of currentNode == value)
        return false (duplicates are not allowed)
     else if (value of currentNode > value)
        currentNode is set to its left child
     else
        currentNode is set to its right child
   }
   if (the value of parentNode > value)
      Add a left child with value to parentNode
   else
      Add a right child with value to parentNode
   end if
   return true
end add
```

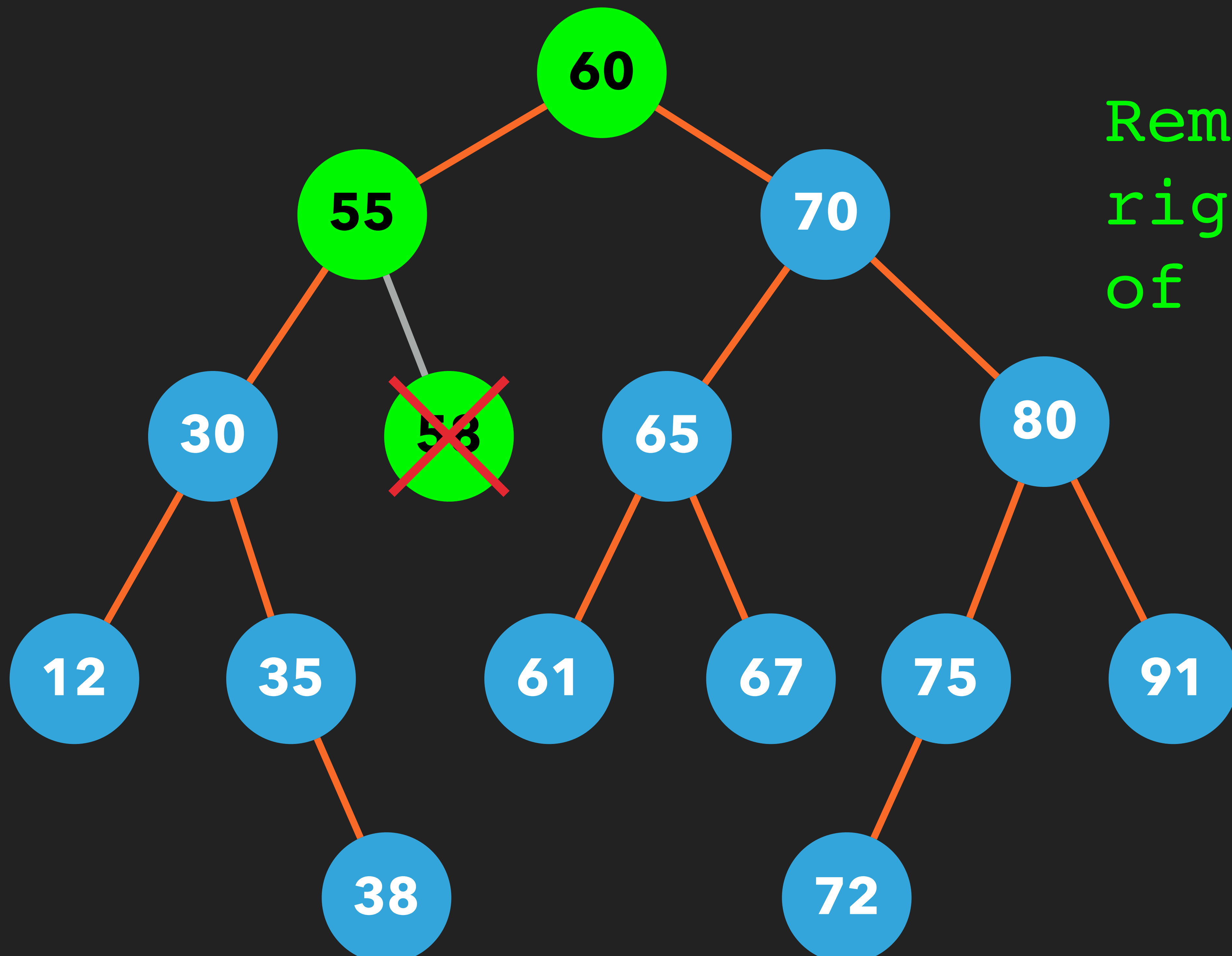
BST (Remove)

Remove the value 58 (Leaf)



BST -(Remove)

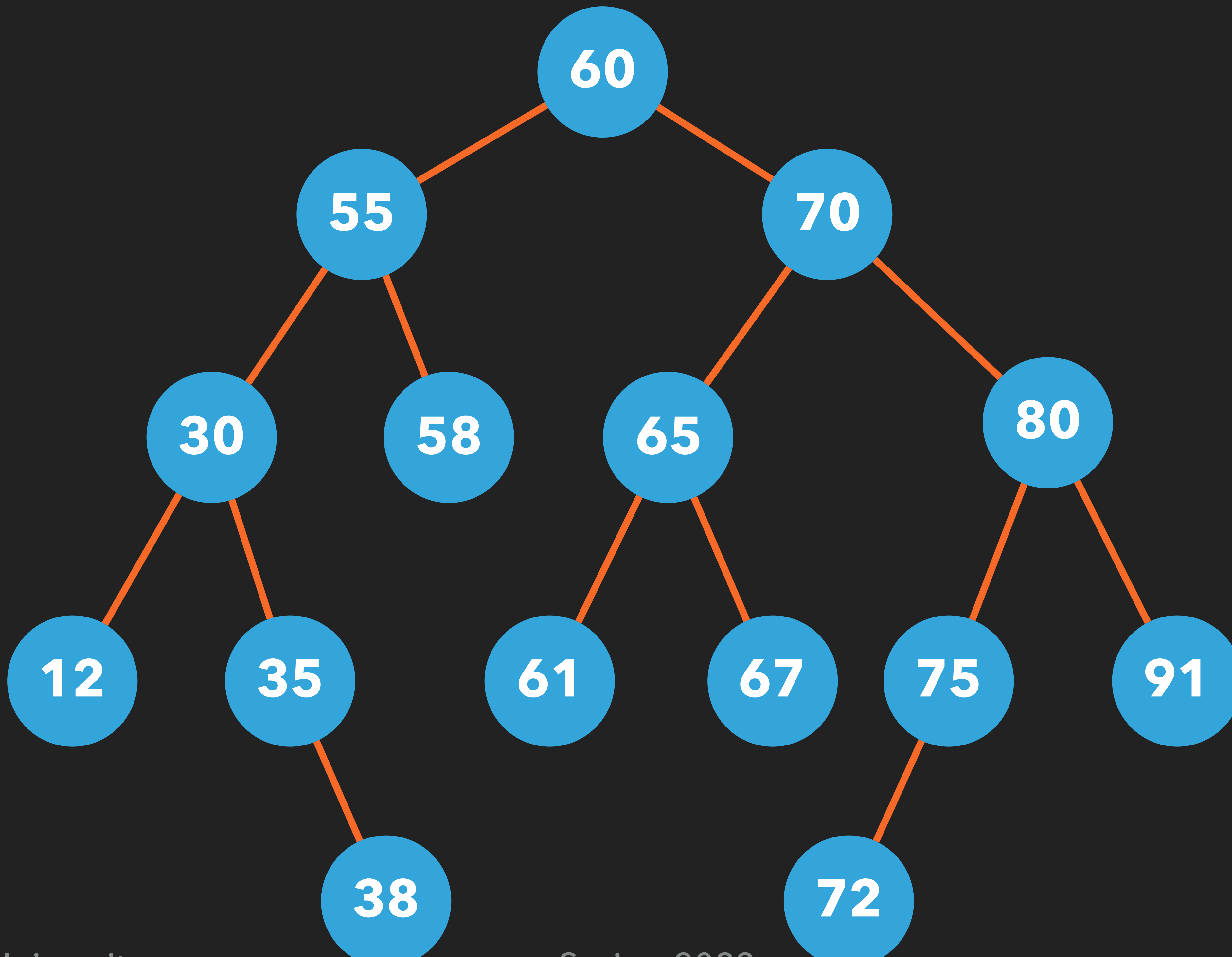
Delete the value 58 (Leaf)



Remove the right child of 55

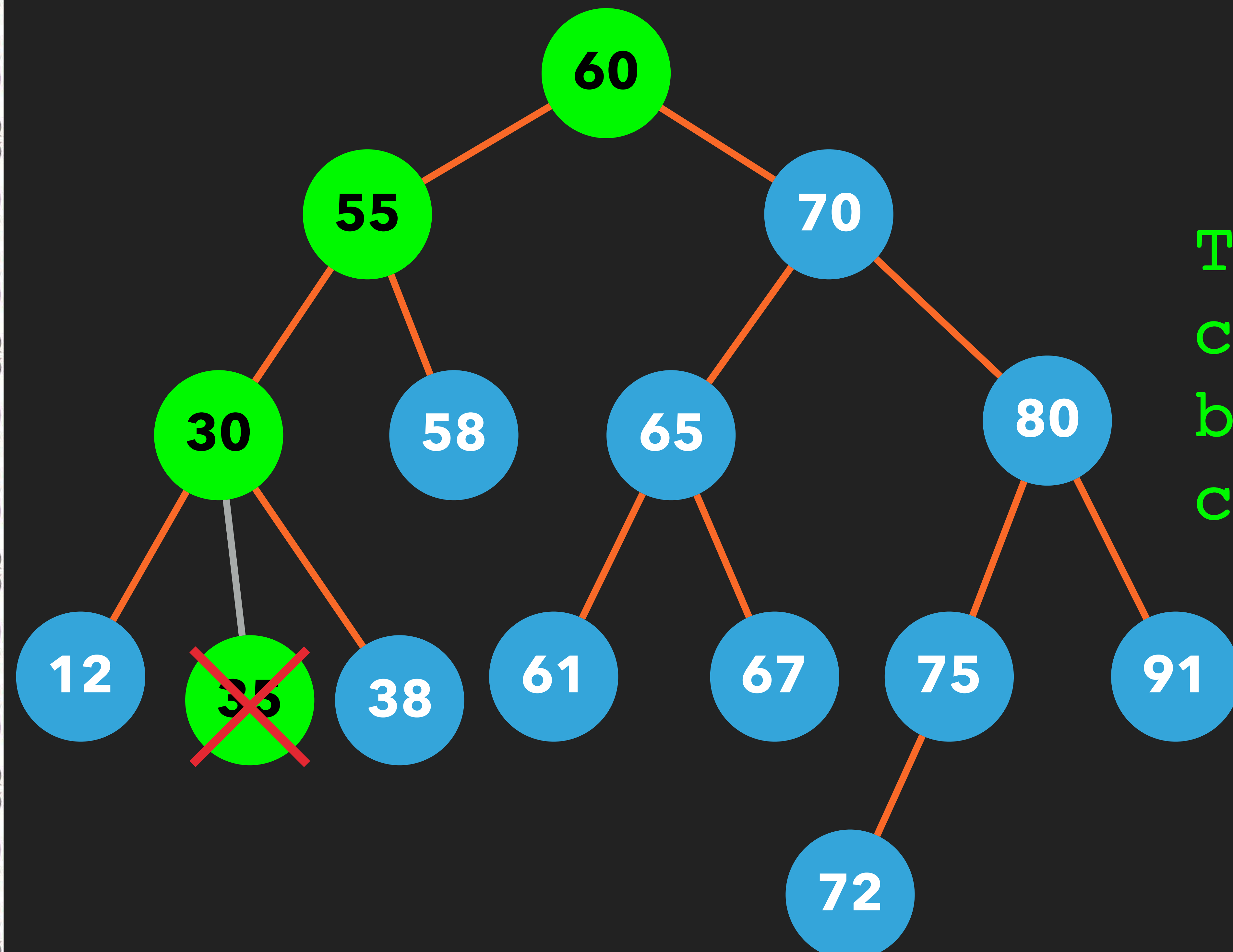
BST (Remove)

Delete the value 35 (one child)



BST (Remove)

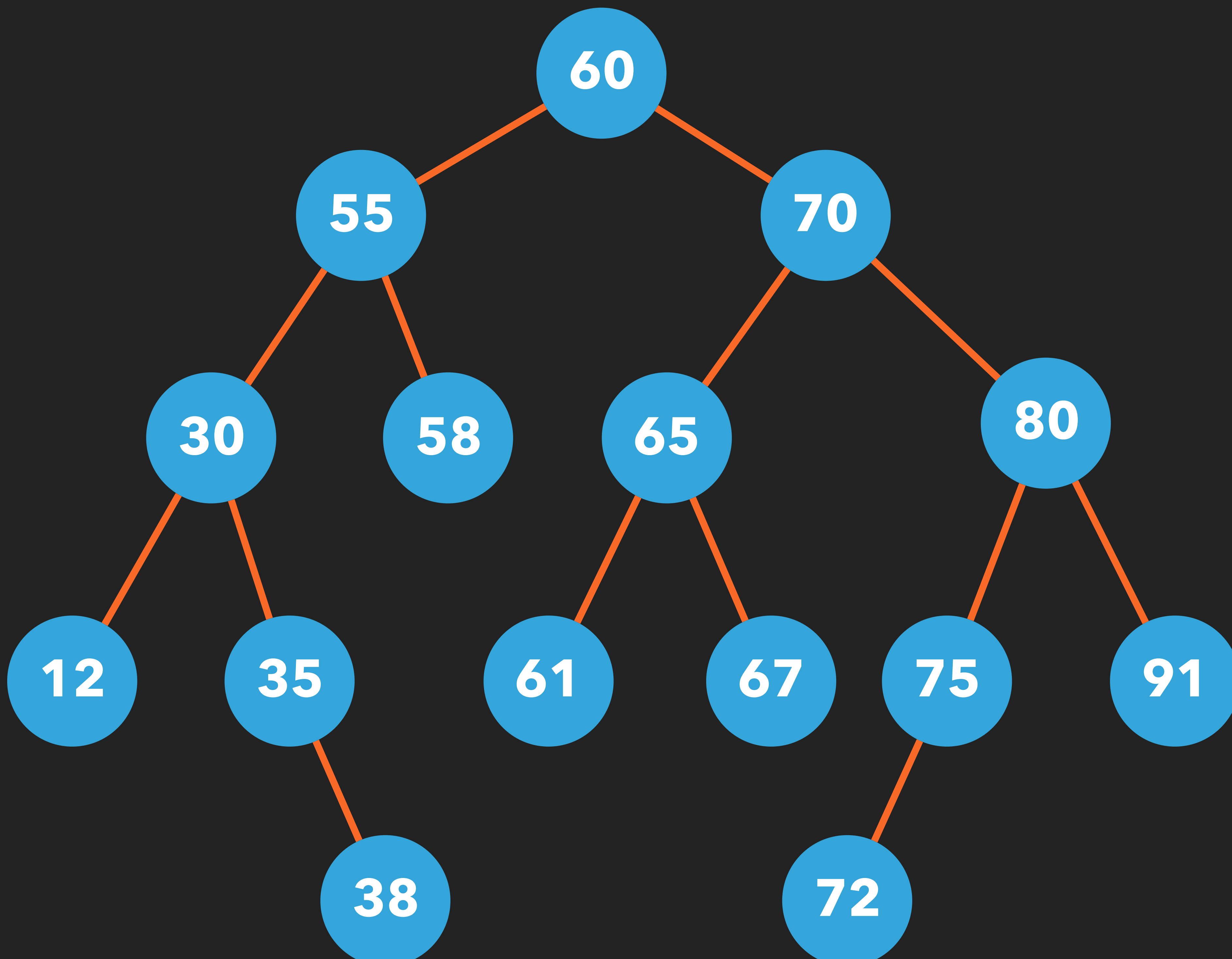
Delete the value 35 (one child)



The only child of 35 becomes the child of 30

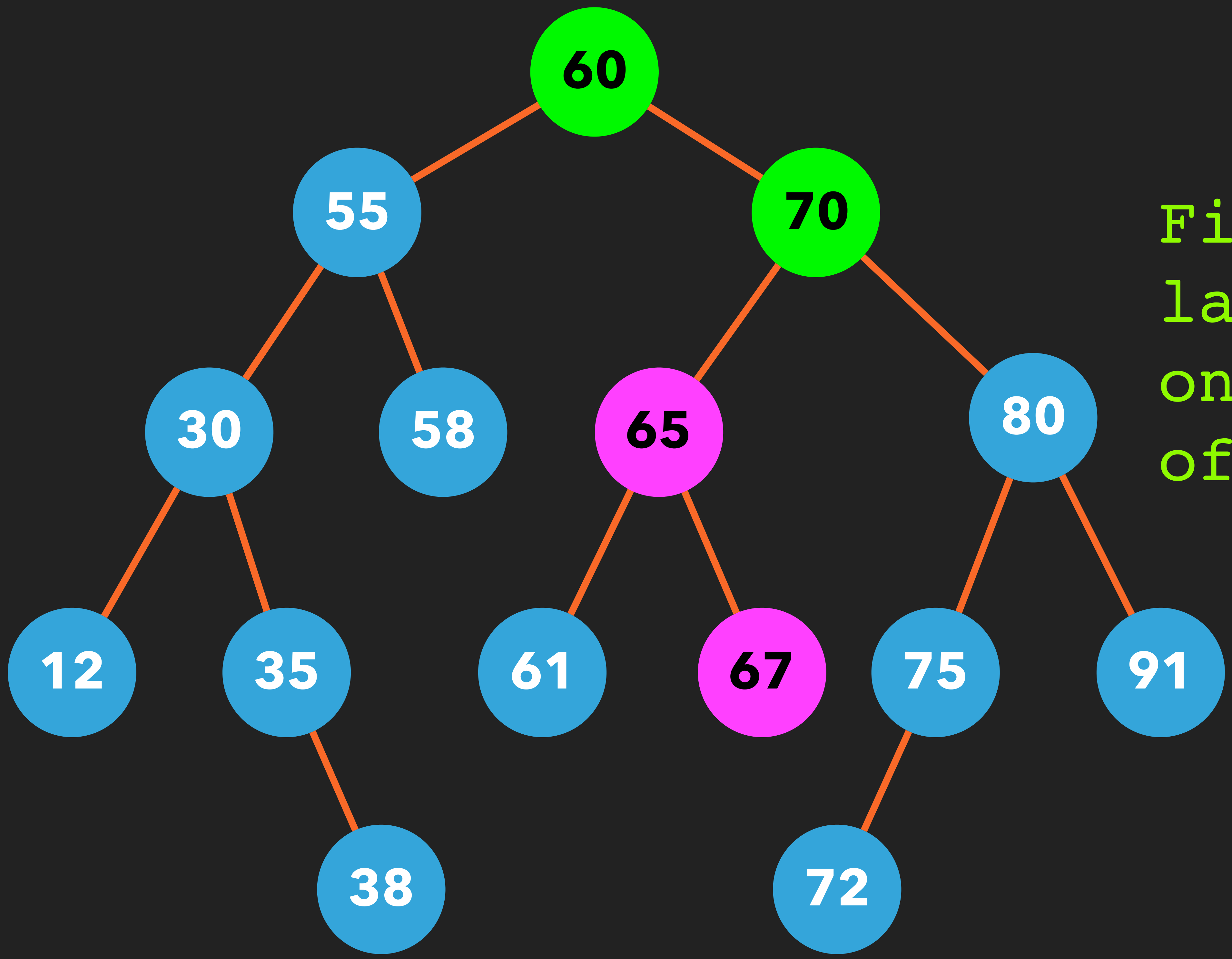
BST (Remove)

Delete the value 70 (two children)



BST (Remove)

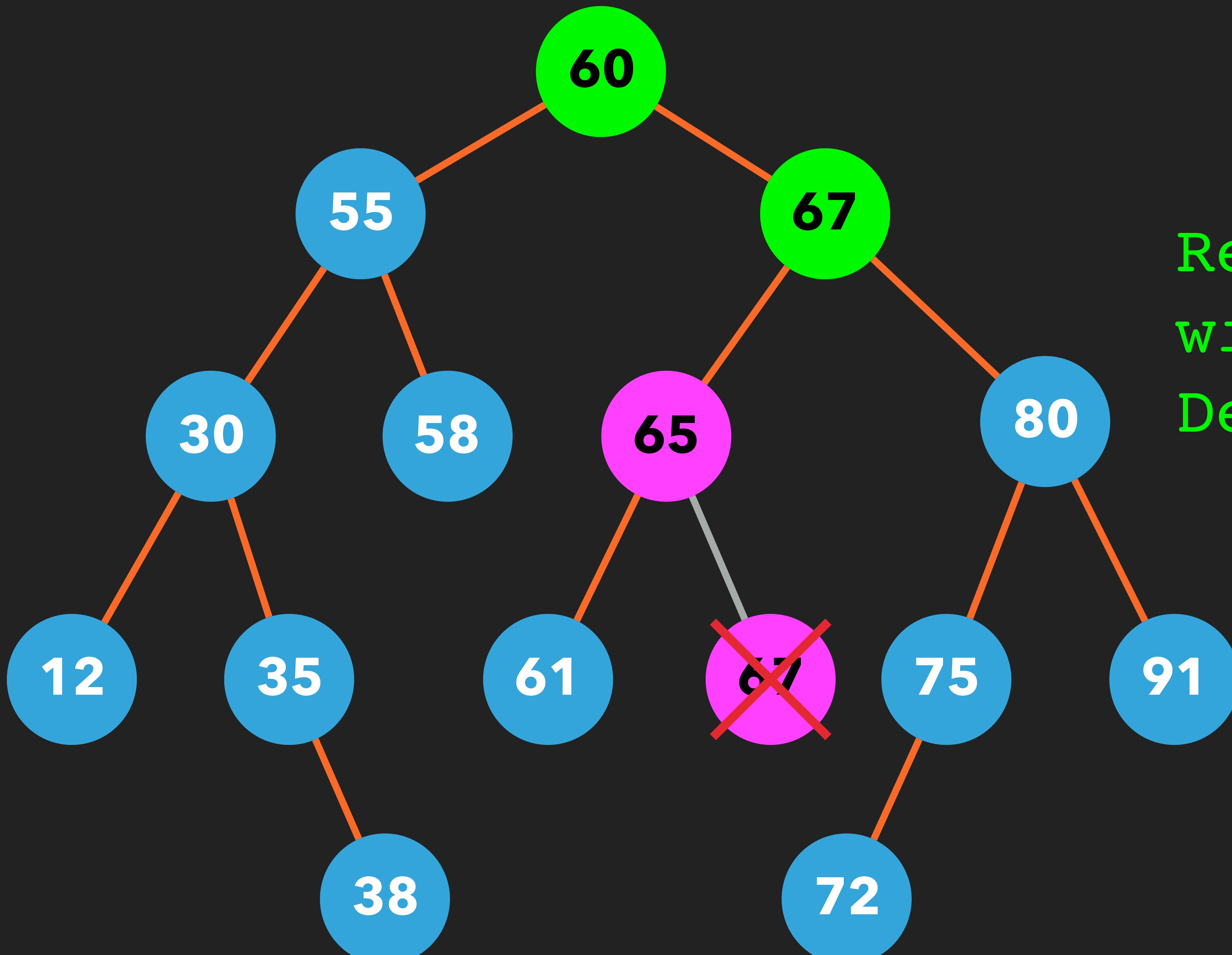
Delete the value 70 (two children)




```
Find the
largest node
on the left
of 70
```

BST (Remove)

Delete the value 70 (two children)



Replace 70 with 67 - Delete 67

BST (Remove)

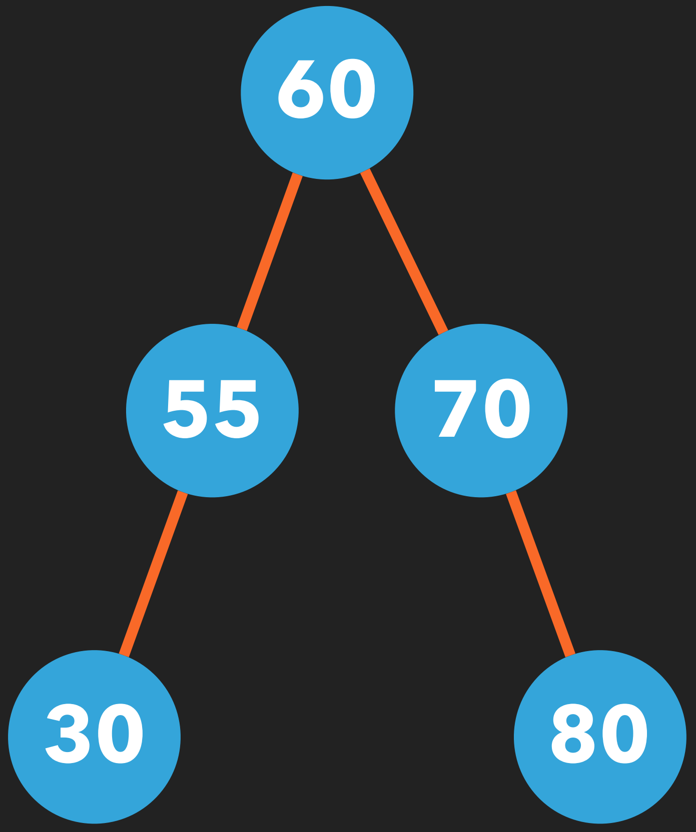
```
boolean remove (value)
  node = search(value) // find node with value first
  if (node == null)
     return false (value not found in the BST)
  else
   if (node has no children)
      remove link to node (parent points to null)
   else if (node has one child)
      replace node with its child
   else if (node has two children)
      find the largest node on the left subtree of node
      copy the value of the largest node to node
      remove the largest node
   end if
  end if
  return true
end remove
```

Traversals (Preorder)

```
preorder(){
  preorder(root)
}
preorder(node){
  if(node not null){
   print node
   preorder(left child of node)
   preorder(right child of node)
  }
}
```

Traversals (Preorder)

```
preorder(){
  preorder(60)
}
preorder(60){
   print 60                                                      60
   preorder(55) —> preorder(55){
                     print  55                                   55
                     preorder(30)  —> preorder(30){
                                       print 30                  30
                                       preorder(null)
                                       preorder(null)
                                      }
                     preorder(null)
                   }
   preorder(70) —> preorder(70){
                    print 70                                     70
                    preorder(null)
                    preorder(80)   —> preorder(80){
                                       print 80                  80
                                       preorder(null)
                                       preorder(null)
                                     }
                   }
}
```

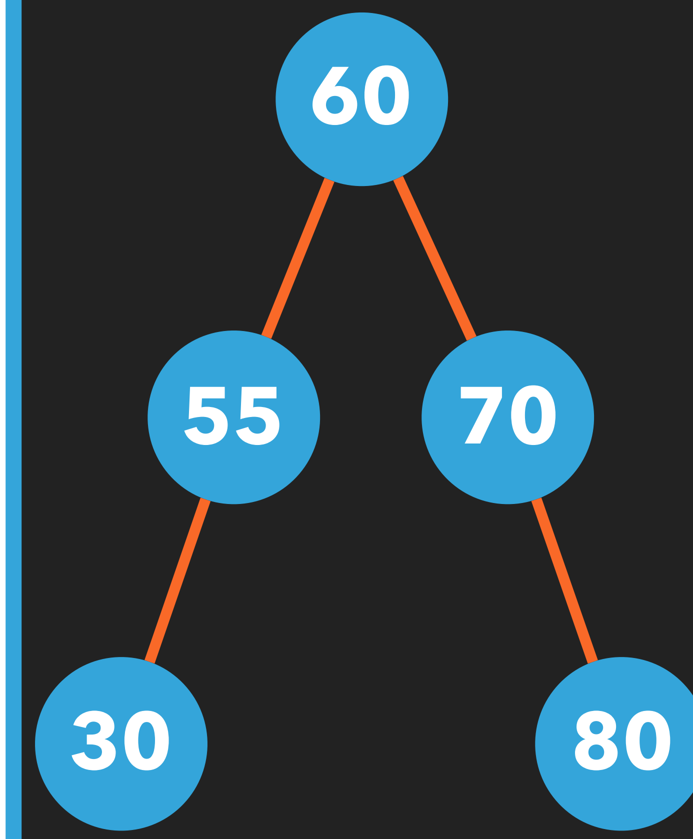


Traversals (Inorder)

```
inorder(){
  inorder(root)
}
inorder(node){
  if(node not null){
   inorder(left child of node)
   print node
   inorder(right child of node)
  }
}
```

Traversals (Inorder)

```
inorder(){
  inorder(60)
}
inorder(60){
   preorder(55) —> preorder(55){
                      preorder(30)  —> preorder(30){
                                         preorder(null)
                                         print 30          30
                                         preorder(null)
                                        }
                      print  55                            55
                      preorder(null)
                    }
    print 60                                               60
   preorder(70) —> preorder(70){
                     preorder(null)
                     print 70                              70
                     preorder(80)   —> preorder(80){
                                         preorder(null)
                                         print 80          80
                                         preorder(null)
                                       }
                    }
}
```

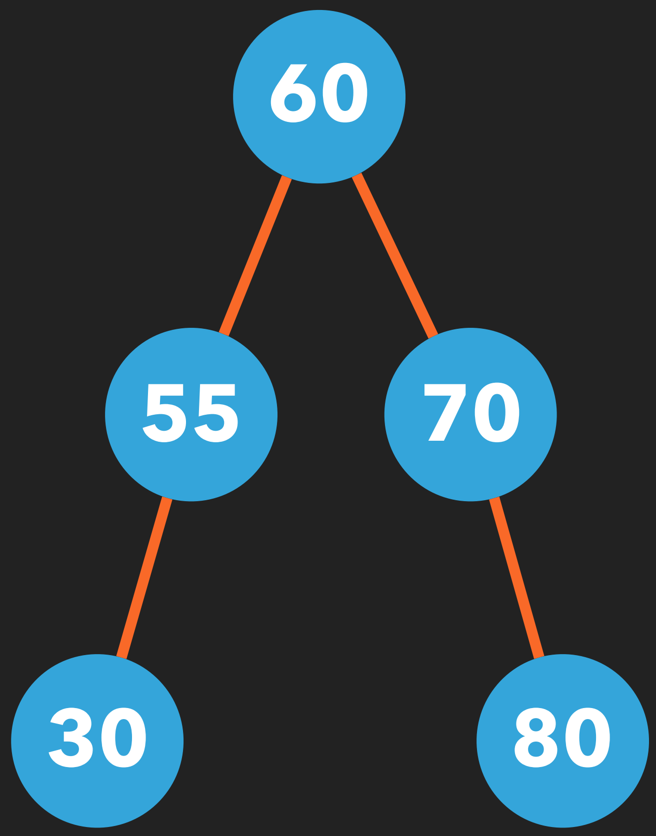


Traversals (Postorder)

```
postorder(){
  postorder(root)
}
postorder(node){
  if(node not null){
    postorder(left child of node)
    postorder(right child of node)
    print node

  }
}
```

Traversals (Preorder)

```
postorder(){
  postorder(60)
}
postorder(60){
   postorder(55) —> postorder(55){
                     postorder(30)  —> postorder(30){
                                        postorder(null)
                                        postorder(null)
                                        print 30            30
                                       }

                      postorder(null)
                      print  55                             55
                   }

   postorder(70) —> postorder(70){
                     postorder(null)
                     postorder(80)   —> postorder(80){
                                        postorder(null)
                                        postorder(null)
                                        print 80            80
                                       }
                   }
                   print 70                                 70
   print 60                                                 60
}
```



BST implementation

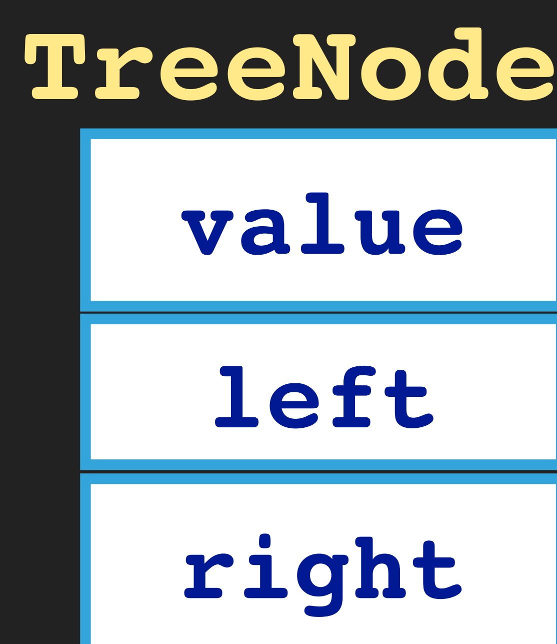
- BST may be implemented in two ways
  - Array Based BST
  - Linked BST

BST implementation

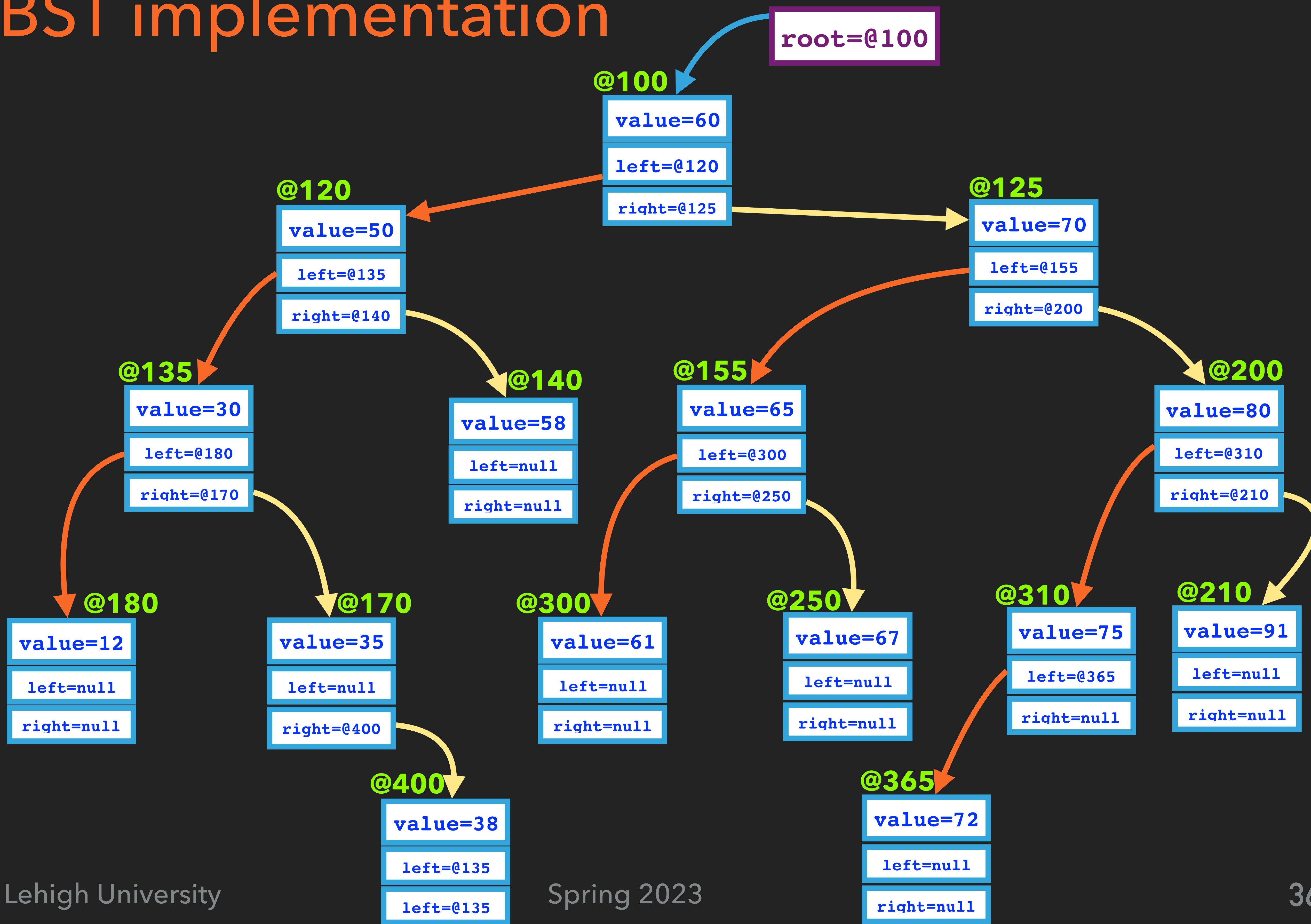
- Nodes of the tree are stored in an array
- Children of a node follow the node (at specific indices)
- Waste of space if the BST is not full

BST implementation

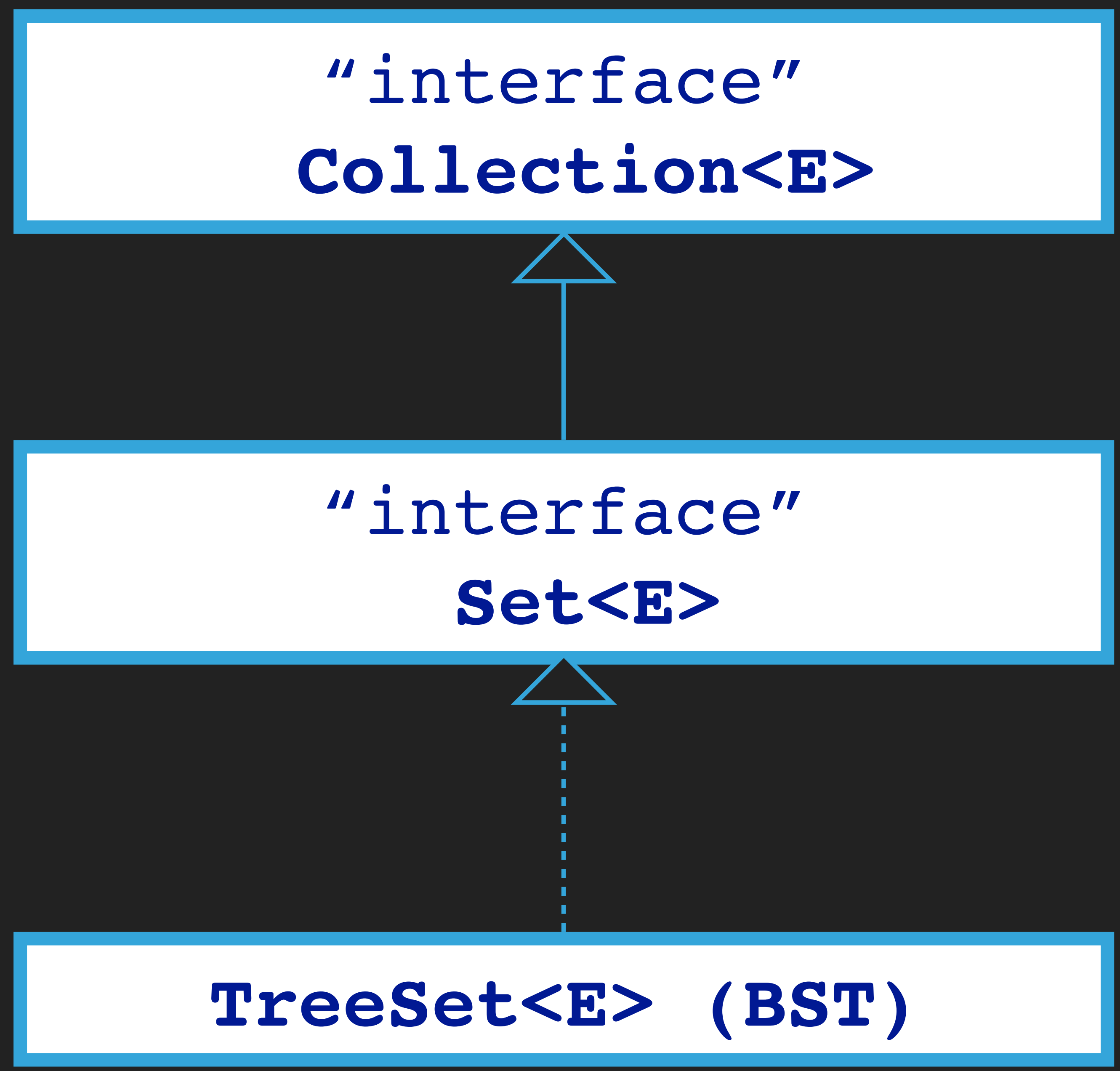
- Nodes of the tree are linked
- Every node has a value and two references, one to the left child and one to the right child



BST implementation



BST implementation



BST implementation

has

BST<E extends Comparable<E>>

```
-root: TreeNode
-size: int
```

```
+BST()
+size(): int
+isEmpty(): boolean
+clear(): void
+contains(E): boolean
+add(E): boolean
+remove(E): boolean
+inorder(): void
+preorder(): void
+postorder(): void
```

TreeNode

```
value: E
Left: TreeNode
Right: TreeNode
```

```
TreeNode(E val)
```

BST implementation

BST.java

```
public class BST<E extends Comparable<E>> {
    private TreeNode root;
    private int size;
    private class TreeNode{
        E value;
        TreeNode left;
        TreeNode right;
        TreeNode(E val){
            value = val;
            left = right = null;
        }
    }
    BST(){
        root = null;
        size = 0;
    }
```

BST implementation

BST.java

```
public int size() {
        return size;
    }
    public boolean isEmpty() {
        return (size == 0);
    }
    public void clear() {
        root = null;
        size = 0;
    }
    // Search method
    public boolean contains(E value) {
        TreeNode node = root;
        while (node != null) {
            if( value.compareTo(node.value) < 0)
                node = node.left;
            else if (value.compareTo(node.value)> 0)
                node = node.right;
            else
                return true;
        }
        return false;
    }
```

BST implementation

BST.java

```
// Method add()
    public boolean add(E value) {
        if (root == null) // first node to be inserted
            root = new TreeNode(value);
        else {
            TreeNode parent, node;
            parent = null; node = root;
            while (node != null) {// Looking for a leaf node
                parent = node;
                if(value.compareTo(node.value) < 0) {
                    node = node.left;
                }
                else if (value.compareTo(node.value) > 0) {
                    node = node.right;
                }
                else
                    return false; // duplicates are not allowed
            }
            if (value.compareTo(parent.value)< 0)
                parent.left = new TreeNode(value);
            else
                parent.right = new TreeNode(value);
        }
        size++;
        return true;
    }
```

BST implementation

BST.java

```
// Method remove()
    public boolean remove(E value) {
        TreeNode parent, node;
        parent = null; node = root;
        // Find value first
        while (node != null) {
            if (value.compareTo(node.value) < 0) {
                parent = node;
                node = node.left;
            }
            else if (value.compareTo(node.value) > 0) {
                parent = node;
                node = node.right;
            }
            else
                break; // value found
        }
        if (node == null) // value not in the tree
            return false;
```

BST implementation

BST.java

```
// Case 1: node has no children
if(node.left == null && node.right == null){
    if(parent == null) // delete root
        root = null;
    else
        changeChild(parent, node, null);
}
else if(node.left == null){
    //case 2: node has one right child
    if (parent == null) // delete root
        root = node.right;
    else
        changeChild(parent, node, node.right);
}
else if(node.right == null){
    //case 2: node has one left child
    if (parent == null) // delete root
        root = node.left;
    else
        changeChild(parent, node, node.left);
}
```

BST implementation

BST.java

```
        else { // case 3: node has two children
            TreeNode rightMostParent = node;
            TreeNode rightMost = node.left;
            // go right on the left subtree
            while (rightMost.right != null) {
                rightMostParent = rightMost;
                rightMost = rightMost.right;
            }
            // copy the value of rigthMost to node
            node.value = rightMost.value;
            //delete rigthMost
            changeChild(rightMostParent,rightMost,
                        rightMost.left);
        }
        size--;
        return true;
    }
```

BST implementation

BST.java

```
private void changeChild(TreeNode parent,
                         TreeNode node,
                         TreeNode newChild){
    if(parent.left == node)
        parent.left = newChild;
    else
        parent.right = newChild;
}
```

BST implementation

BST.java

```
// Recursive method inorder()
    public void inorder() {
        inorder(root);
    }
    private void inorder(TreeNode node) {
        if (node != null) {
            inorder(node.left);
            System.out.print(node.value + " ");
            inorder(node.right);
        }
    }
    // Recursive method preorder()
    public void preorder() {
        preorder(root);
    }
    private void preorder(TreeNode node) {
        if (node != null) {
            System.out.print(node.value + " ");
            preorder(node.left);
            preorder(node.right);
        }
    }
```

BST implementation

BST.java

```
    // Recursive method postorder()
    public void postorder() {
        postorder(root);
    }
    private void postorder(TreeNode node)  {
        if (node != null) {
            postorder(node.left);
            postorder(node.right);
            System.out.print(node.value + " ");
        }
    }
}
```

BST Testing

Test.java

```
public class Test {
    public static void main(String[] args) {
        BST<String> bst = new BST<>();
        bst.add("Kiwi");
        bst.add("Strawberry");
        bst.add("Apple");
        bst.add("Banana");
        bst.add("Orange");
        bst.add("Lemon");
        bst.add("Watermelon");
        System.out.print("BST:  ");
        bst.inorder();
        System.out.println();
        System.out.println("BST contains Banana? " + bst.contains("Banana"));
        bst.remove("Banana");
        System.out.println("BST contains Banana? " + bst.contains("Banana"));
        System.out.print("BST:  ");
        bst.inorder();
        System.out.println();
        bst.remove("Orange");
        System.out.print("BST:  ");
        bst.inorder();
        System.out.println();
        bst.remove("Kiwi");
        System.out.print("BST:  ");
        bst.inorder();
        System.out.println();
    }
}
```

BST Testing

The order in which the values are added to the BST affects its balance (shape)

```
public class Test {
    public static void main(String[] args) {
        BST<String> bst = new BST<>();
        bst.add("Apple");
        bst.add("Banana");
        bst.add("Kiwi");
        bst.add("Lemon");
        bst.add("Orange");
        bst.add("Strawberry");
        bst.add("Watermelon");
        System.out.print("BST: ");
        bst.inorder();
    }
}
```

BST

- Performance of the operations

Method	Complexity	Method	Complexity
BST()	**O(1)**	remove(E)	**O(log n)** **O(n)**
size()	**O(1)**	contains(E)	**O(log n)** **O(n)**
clear()	**O(1)**	inorder()	**O(n)**
isEmpty()	**O(1)**	**preorder()**	**O(n)**
add(E)	**O(log n)** **O(n)**	**postorder()**	**O(n)**

Summary

- Binary Search Tree
- Operations: Search, Add, Remove, Traversals
- Implementation - Linked Nodes
- Order in which data is added has an effect on the shape of the BST (balance)
- AVL BSTs to keep the shape balanced