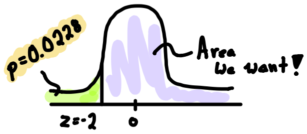
Homework 4 STAT 351

1. If X is a continuous random variable, and c is any number, then P(X = c) =

$$\boxed{P(X=c) = 0}$$

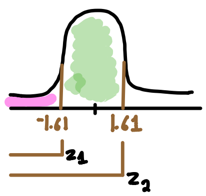
2. If Z is a standard normal random variable, then P (Z <= -2.0) =



$$\boxed{P(z \le -2) = 1 - 0.0228}$$

Less than or Equal to

3. If Z is a standard normal random variable, then P(-1.61 <= Z <= 1.61) =



$$P(z(1.61)) - P(z(-1.61)) = 0.9463 - 0.0537$$

$$\boxed{P(-1.61 \le z \le 1.61) = 0.8926}$$

4. The 90th percentile of the standard normal distribution is

$$\boxed{Z_{score} = 1.28 = 0.8997}$$

5. In your textbook, the author used the notation z_α $Z_{.07}$ is identical to the __________ percentile of standard normal.

$$\boxed{93^{rd} \text{ Percentile}}$$

6. Numerical value of $z_{.17}$ is :

$$\boxed{\Phi(0.95) = 0.83}$$

7. If the population distribution of a variable is approximately normal, then about ________ % of the values are within one standard deviation of the mean.

$$\boxed{68\%}$$

8. The expected value of a random variable X having a gamma distribution with parameters α =.3 and β = 2 is:

$$\alpha \cdot \beta = (0.3)\cdot(2) \boxed{= 0.6}$$

9. The variance of a random variable X having a gamma distribution with parameters α = .3 and β = 2 is:

$$\alpha \cdot \beta^2 = (0.3)(2)^2 \boxed{= 1.2}$$

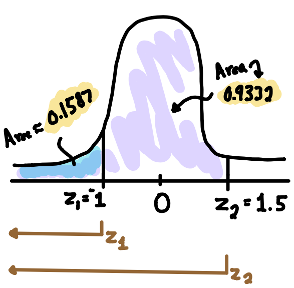
10. The mean of a random variable X having the chi-squared distribution with 3 degrees of freedom is

?

11. The variance of a random variable X having the chi-squared distribution with 5 degrees of freedom is

?

12. If X is a normally distributed random variable with a mean of 10 and standard deviation of 4, then the probability that X is between 6 and 16 is



$\mu = 10$
$\sigma = 4$

$P(6 \le x \le 16)$

$z_2 = \dfrac{16 - 10}{4}$

$z_2 = \frac{3}{2}$

$z_1 = \dfrac{6 - 10}{4} = \frac{-4}{4} = -1$

$z_1 = -1$ Find Z-Score Probs.

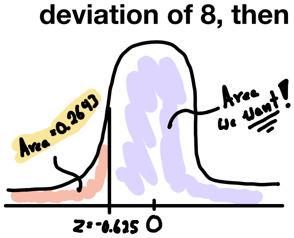
Continuous random Variable

$Z = \dfrac{X - \mu}{\sigma}$

$P(6 \le x \le 16) = \text{Area}2 - \text{Area}1 = 0.9332 - 0.1587$

$\boxed{P(6 \le x \le 16) = 0.7745}$

13. If X is a normally distributed random variable with a mean of 25 and a standard deviation of 8, then the probability that X exceeds 20 is approximately :



$\mu = 25$
$\sigma = 8$

$P(X > 20)$

$Z = \dfrac{X - \mu}{\sigma} = \dfrac{20 - 25}{8} = \dfrac{-5}{8} = -0.625$

$Z = -0.625$

$P(X > 20) = 1 - 0.2643$

$\boxed{P(X > 20) = 0.7357}$

14. If X is a normally distributed random variable with a mean of 80 and a standard deviation of 12, then 80th percentile of X is

$$\frac{X - 80}{12} = 0.80$$

$$\boxed{X = 90.08}$$

15. If the probability density function of a continuous random variable X is

$$f(x) = \begin{cases} kx & 0 \le x \le 2 \\ 0 & \text{Otherwise} \end{cases}$$

a) Determine the value of k

$$K \left. \frac{x^2}{2} \right|_0^2 = 1$$

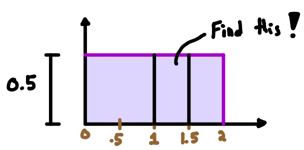
$$\frac{2^2}{2} K = 1 \rightarrow 2k = 1$$

$$k = \frac{1}{2}$$

Since pdf is intergrate up to 1

$$\int_0^2 kx\,dx = 1 \longrightarrow 2k = 1 \rightarrow \boxed{K = \frac{1}{2}}$$

b) Calculate P(1 <= x <= 1.5)



$$\int_1^{1.5} \underbrace{\tfrac{1}{2}}_{\text{Constant}} x\,dx \longrightarrow \frac{1}{2}\int_1^{1.5} x\,dx \rightarrow \left. \frac{1}{4}x^2 \right|_1^{1.5} \longrightarrow \frac{1}{4}(1.5)^2 - \frac{1}{4}(1)^2 \boxed{= 0.3125}$$

c) Obtain CDF F(x)

$$\boxed{F(x) = \int_0^x kt\,dt = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x^2}{4} & \text{if } 0 \le x \le 2 \\ 1 & \text{if } 2 < x \end{cases}}$$

d) Calculate E(X)

$$x^2$$

$$E(x) = \int_0^2 x \underbrace{k}_{\frac{1}{2}} x\,dx = \frac{1}{2}\int_0^2 x^2\,dx \longrightarrow \left. \frac{1}{2} \cdot \frac{x^3}{3} \right|_0^2 \longrightarrow \left. \frac{1}{6}x^3 \right|_0^2 \rightarrow \left[\frac{1}{6}(2)^3 - \cancel{\frac{1}{6}(0)^3}\right] \rightarrow \boxed{\frac{8}{6}}$$

e) Calculate V(X)

$$E(x^2) = \int_0^2 x^2 kx\,dx = 2$$

$$V(x) = 2 - \left(\frac{8}{6}\right)^2$$

$$\boxed{V(x) = 0.222}$$

f) Calculate 55th percentile of X

16. The time X (minutes) for a lab assistant to prepare the equipment for a certain experiment is believed to have a uniform distribution with A = 20 and B = 30.

(a) Write the pdf of X and sketch its graph.

(b) What is the probability that preparation time exceeds 27 minutes?

$$P(x \geq 27) = \int_{27}^{30} \frac{1}{10}\,dx \longrightarrow \frac{1}{10}x \Big|_{27}^{30} \longrightarrow \frac{1}{10}(30) - \frac{1}{10}(27) = \frac{3}{10}$$

(c) Find the preparation mean time, then calculate the probability that preparation is within 2 minutes of the mean time?

(d) For any a such that 20 < a < a + 2 < 30, what is the probability that preparation Time is between a and a + 2 minutes?