Homework 3 STAT 351

1. Let X be a discrete random variable with V (X) = 8.6, then V (3X+5.6) is

2. Let X be a discrete random variable with E(X^2) = 19.75 and V (X) = 16.3, then E(X)

$E(x) = \sqrt{19.75 - 16.3}$ $\boxed{= 1.83}$

3. If random variable X has distribution Bin (10,.75), V (X) is

$V(x) = np(1-p) = 10(.75)(1-0.75)$

$\boxed{V(x) = 1.875}$

4. If random variable X has distribution Bin (10,.75), P (X = 3) is

$b(x;n,p) = \binom{n}{x} p^x (1-p)^{1-x}, \; x = 0, 1, \ldots, n$ $\quad \binom{10}{3} \cdot 0.75^3 \cdot (1-0.75)^{1-3} = 0.0031$

5. If the expected value of a discrete random variable X is E(X) = 5, then E(2X + 3) is

If $E(x) = 5$, then $E(2x+3) = E(2(5)+3) = \boxed{E(13)}$

6. The probability mass function of a discrete random variable X is defined as p(x) = x/10 for x = 0,1,2,3,4. Then, the value of the cumulative distribution function F(x) at x = 3 is

$F(3) = p(0) + p(1) + p(2) + p(3)$ $\quad p(x) = \frac{x}{10}$

$= \frac{0}{10} + \frac{1}{10} + \frac{2}{10} + \frac{3}{10}$

$= 0 + 0.1 + 0.2 + 0.3$ $\quad \boxed{F(3) = 0.6}$

7. If random variable X has distribution Bin(6, .3), E(X) is

$Bin(n,p)$

$E(x) = (6)(0.3)$

$\boxed{E(x) = 1.8}$

8. The mean of the hypergeometric random variable X with parameters n=10, M = 50, and N = 100 is

9. The cumulative distribution function F (x) of a discrete random variable X is: F (1) = .4, F (2) = .7, F (3) =.9, and F (4) = 1, then the value of the probability mass function p(x) at X = 3 is

$p(3) = F(3) - F(2)$

$p(3) = 0.9 - 0.7$

$\boxed{p(3) = 0.2}$

or

x	P(x)	Cum. Prob
1	0.4	0.4
2	0.3	0.7
3	0.2	0.9
4	0.1	1

$.7 - .4 = .3$

$.9 - .7 = .2$

$1 - 0.9 = 0.1$

$\sum p(x) = 1$

How to find p(x)?

10. The expected value of the negative binomial random variable X with parameters r = 5 and p = .8 is

$r = 5$
$P = 0.8$

$E(x) = \frac{r(1-p)}{p} = \frac{5(1-.8)}{.8}$

$\boxed{E(x) = \frac{5}{4}}$

11. The pmf for $X =$ the number of major defects on a randomly selected gas stove of a certain type is

x	0	1	2	3	4
P(x)	.10	.15	.45	.25	.05

$\sum P(x) = 1.00$

Compute the following:

A) E(x)

E(x) = Expected Value

$E(x) = \sum x \cdot P(x)$ ΣQ

$= (0)(0.10) + (1)(0.15) + (2)(0.45) + (3)(0.25) + 4(0.05)$

$\boxed{E(x) = 2}$

$q = 1 - p$

B) V(x) Variance $= \sigma^2 = \sum [x^2 \cdot P(x)] - \mu^2$

Variance

$\sigma^2 = 5 - 2^2 = 1$

$\boxed{\sigma^2 = 1}$

x	P(x)	P(x)·x	x^2	$x^2 \cdot P(x)$
0	0.10	0	0	0
1	0.15	.15	1	0.15
2	0.45	.80	4	1.8
3	0.25	.75	9	2.25
4	0.05	.20	16	0.8

$\sum P(x) \cdot x = 2$

$\sum = 5$

C) The standard Deviation

$S.D = \sqrt{V(x)} = \sqrt{1} \rightarrow \boxed{\sigma = 1}$

A particular telephone number is used to receive both voice calls and fax messages. Suppose that 25% of the incoming calls involve fax messages, and consider a sample of 25 incoming calls. What is the probability that

$p(\text{fax}) = 0.25$

$n = 25$

a. At most 6 of the calls involve a fax message?

Could be 1,2,3,4,5, or 6 Calls

$$\boxed{P(x \leq 6) = 0.561}$$

Use Binomal Table

b. Exactly 6 of the calls involve a fax message?

$P(x=6)$

c. At least 6 of the calls involve a fax message?

$P(x \geq 6)$

d. More than 6 of the calls involve a fax message?

An ordinance requiring that a smoke detector be installed in all previously constructed houses has been in effect in a particular city for 1 year. The fire department is concerned that many houses remain without detectors. Let p = the true proportion of such houses having detectors, and suppose that a random sample of 25 homes is inspected. If the sample strongly indicates that fewer than 80% of all houses have a detector, the fire department will campaign for a mandatory inspection program. Because of the costliness of the program, the department prefers not to call for such inspections unless sample evidence strongly argues for their necessity. Let X denote the number of homes with detectors among the 25 sampled. Consider rejecting the claim that $p \geq .8$ if $x \leq 15$.

a. What is the probability that the claim is rejected when the actual value of p is .8?

b. What is the probability of not rejecting the claim when $p = .7$? When $p = .6$?

c. How do the "error probabilities" of parts (a) and (b) change if the value 15 in the decision rule is replaced by 14?

The number of people arriving for treatment at an emergency room can be modeled by a Poisson process with a rate parameter of five per hour.

a. What is the probability that exactly four arrivals occur during a particular hour?

$$P(x=4) = \frac{e^{-5} \cdot 5^4}{4!} = 0.175$$

b. What is the probability that at least four people arrive during a particular hour?

c. How many people do you expect to arrive during a 45-min period?