Homework 7 STAT 351

Chapter 7

1. Obtain the numerical value of the following critical values:

a.) $t_{0.05,17} = 1.740$

b.) $t_{0.01,60} = 2.390$

c.) $t_{0.25,7} = .2365$

d.) $z_{0.0005} = 3.291$

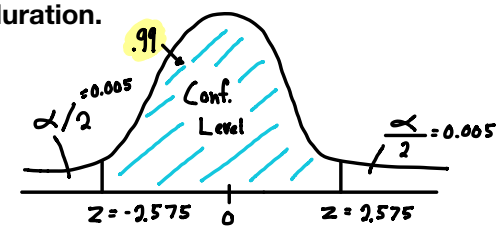
e.) $\chi^2_{.025,7} = 16.012$

f.) $\chi^2_{.99,19} = 7.632$

2. A random sample of 100 lightning flashes in a certain region resulted in a sample average radar echo duration of .81 sec and a sample standard deviation of .34 sec.

$n = 100 \quad \bar{x} = 0.81 \quad \sigma = 0.34 \quad \hat{p} = \frac{.81}{100} =$

(a) Calculate a 99% (two-sided) confidence interval for the true average echo duration.



$$\bar{x} \pm z_{critical} \frac{\sigma}{\sqrt{n}}$$

$$= 0.81 \pm (2.575) \cdot \frac{0.34}{\sqrt{100}}$$

$$= 0.72 \leq p \leq 0.90$$

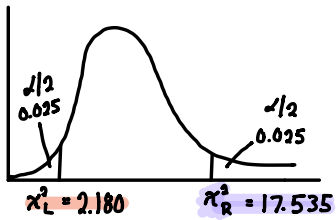
(b) If you want to cut the length of the above confidence interval in half (1/2) while keeping the confidence level the same, what sample size do you need?

$$\frac{1}{\sqrt{n}}$$

$n = 400$

3. The amount of lateral expansion (mils) was determined for a sample of size 9 pulsed- power gas metal arc welds used in LNG ship containment tanks. The resulting sample standard deviation was s = 2.80 mils. Assuming normality, derive a two-sided 95% CI for σ.

Given: $s = 2.80$ miles
$n = 9$
95% C.I. $\rightarrow \alpha = 0.05$
d.f $= n-1 = 9-1 = 8$
Variance



~ Use χ^2 Table to find χ^2_R and χ^2_L

χ^2 Distribution

$$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}} \rightarrow \sqrt{\frac{(9-1)(2.8)^2}{17.535}} < \sigma < \sqrt{\frac{(9-1)(2.8)^2}{2.180}}$$

$$\boxed{1.89 < \sigma < 5.36}$$

4. It was reported that, in a sample of 507 adult Americans, only 142 correctly described the Bill of Rights as the first ten amendments to the U.S. Constitution.

(a) Give an estimate for the proportion of all U. S. adults that could give a correct description of the Bill of Rights.

$$\hat{p} = \frac{\#\text{Correct}}{\#\text{Sample}} \qquad \hat{p} = \frac{142}{507} = 0.28 = \boxed{28\%}$$

(b) What is the margin of error (in %) of above estimate (with 95% confidence)?

$$E = z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}\cdot\hat{q}}{n}}$$

$z_{\frac{\alpha}{2}} = 1.96 \qquad n = 507$

$\hat{p} = 0.28$
$\hat{q} = 1 - \hat{p} = 1 - 0.28$
$\hat{q} = 0.72$

$$E = 1.96\sqrt{\frac{(0.28)\cdot(0.72)}{507}}$$

$$\boxed{E = \pm 0.039}$$

~ So Margin of Error is 4%

(c) Calculate a (two-sided) confidence interval using a 95% confidence level for the proportion of all U. S. adults that could give a correct description of the Bill of Rights.

0.28 ± 0.04

$$\boxed{0.24 \leq p \leq 0.32}$$

5. A study of the ability of individuals to walk in a straight line reported that accompanying data on cadence (strides per seconds) for a sample of 20 randomly selected men:

.95, .81, .93, .95, .93, .86, 1.05, .92, .85, .81 .92, .96, .92, 1.00, .78, 1.06, 1.06, .96, .85, .92

Sample mean of above data was 0.9255, and sample standard deviation was 0.0809. Normal q-q plot of the data looks like the following.

a. Is it plausible that this sample was selected from a normal population distribution?

Looks like a line!

b. Assume the normality of the sample regardless of your answer in part (a). Calculate and interpret a 99% confidence interval for a population mean cadence.

KEY!

$\bar{x} - E < \mu < \bar{x} + E$

$E = T_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$

Given: $\bar{x} = 0.9255$, $n = 20$, $s = 0.0809$

$\alpha = 0.01 \rightarrow \frac{\alpha}{2} = 0.005 \rightarrow t_{\frac{\alpha}{2}} = 2.861$

d.f. = 19

$E = 2.861 \frac{0.0809}{\sqrt{20}}$

$E = 0.05175 \longrightarrow \bar{x} - E < \mu < \bar{x} + E$

$0.9255 - 0.05175 < \mu < 0.9255 + 0.05175$

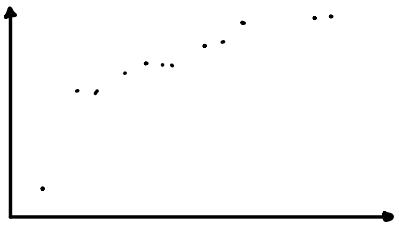
$0.8737 < \mu < 0.97725$

c. Calculate and interpret a 95% prediction interval for the cadence of a single individual randomly selected from the population.

6. The results of a Wagner turbidity test performed on 15 samples of standard Ottawa testing sand were (in microamperes)

26.9, 25.8, 24.4, 24.1, 26.4, 25.9, 24.0, 21.7, 24.9, 25.9, 27.3, 26.7, 26.9, 24.8, 27.3

Normal q-q plot of the data looks like the following.



(a) Is it plausible that this sample was selected from a normal population distribution?

Yes, normal q-q plot looks like line.

b. Calculate and (one-sided) upper confidence bound with confidence level 90% for the true mean of turbidity. This is Case 2 CI.

Given: $\bar{X} = 25.53$ and $S = 1.55$

c. Calculate an (two-sided) prediction interval with confidence level 90% for the next Wagner turbidity test.