Counting Rules Chapter 6.1-6.4

6.1 The Basics of Counting

Product Rule

When a procedure can be broken into a sequence of tasks where the number of outcomes of each task is expressed as n1, n2, n3….. etc, then there are n1 * n2 * n3 *…*nk different ways to perform the procedure.

If I have 4 different t-shirts and 3 different pairs of shorts, how many different outfits do I have?

Tree Diagrams

A visual strategy we can use to represent counting problems.

If I have 4 different t-shirts and 3 different pairs of shorts how many different outfits do I have?

Sum Rule

If a task can be performed in one of n, ways or one of n2 ways then there are n1 + n2 ways to perform the task.

I want to take a trip to the beach. I can travel to one of 37 international beaches or one of 14 domestic beaches. How many beach vacations choices do I have?

Subtraction Rules

If a task can be done in either one of n, ways or one of n2 ways, then the total number of ways to do the task is n1 + n2 minus the number of ways that are common to the two different ways

Principle of Inclusion- Exclusion

How many bitstrings of length 7 either start with 1 bit or end with the 3 bit 000 ?

Division Rule

There are n/d ways to do a task if It can be done using a procedure that can be carried out in n ways, where there are d corresponding outcomes per group.

How many ways can I sit 6 people around a circular table where two seatings are considered the same when each person has the same left and right neighbor?

6.2 The Pigeonhole Principle

The Pigeonhole Principle

Pigeonhole Principle: which states that if there are more pigeons than pigeonholes then there must be at least one pigeonhole with at least two pigeons in it.

If k is positive integer and k + 1 or more objects are placed into k boxes, then there is at least one box containing two or more of the objects

Example 1

Among any group of 367 people, there must be at least two with the same birthday, because there are only 366 possible birthdays.

Example 2

In any group of 27 English words, there must be at least two that begin with the same letter, because there are 26 letters in the English alphabet.

Example 3

How many students must be in a class to guarantee that at least two students receive the same score on the final exam, if the exam is graded on a scale from 0 to 100 points?

Example 4

Show that for every integer n there is a multiple of n that has only 0s and 1s in its decimal expansion.

The Generalized Pigeonhole Principle

If N objects are placed into k boxes, then there is at least one box containing at least objects

Example 5

Among 100 people there are at least $\lceil 100/12 \rceil = 9$ who were born in the same month.

Example 6

What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D, and F?

Example 7

a) How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen?
b) How many must be selected to guarantee that at least three hearts are selected?

Permutations and Combinations

Permutation

An Ordered arrangement of distinct objects. An r-permutation is the arrangement of r-elements of a set?

Combinations:

An unordered arrangement of elements of a set. An r-combination is a subset of the set with r elements.

Let   Find all 2-combinations. Relate this to the number of 2-permutations

How many poker hands of 5 cards can be dealt from a standard deck of 52 cards?

In how many ways can 100 marathon runners place in 1st, 2nd, and 3rd ?

Counting Rules Practice

How many bit strings of length 6 are there?

How many bit strings of length 6 start with 1 or end with 1?

How many bit strings of length n are there, not counting the empty string?

Residents of Douglas County, NE have license plate beginning with 3 letters and ending with 3 digits. How many unique license plates are there in Douglas County?

If zeros are no longer allowed as the first of the three digits, how many unique license plates are there?

Use the product rule to show that the number of different subsets of a finite set S is 2

If and , how many elements are in the Cartesian product ?

A computer password can be 5-6 characters. The password must contain at least 1 digit. All the other characters must either be digits or lower-case letters. How many unique passwords exist?

In how many ways can a photographer at a wedding arrange the bride, groom, and all 6 members of the wedding party if the bride and groom must be next to one another?

How many arrangements if each wedding party couple, those that walked together, must be next to one another, and the bride and groom must stand together with the bride to the left of the groom

How many 4-permutations of the positive integers not exceeding 100 contain 3 consecutive integers in the correct order in consecutive positions in the permutation.

If the math faculty of a large University consists of 5 men and 6 women. How many committees of size 4 can be formed if the committee must contain at least one member of each gender?

How many diagonals does a convex hexagon have?

How many diagonals does a convex n-gon have?

The Binomial Theorem

Binomial Expansion

Find the binomial expansion of (3x+2) .

What if I asked for the coefficient of the term containing ?

Binomial Theorem:

Let x and y be variables, and let n be nonnegative integer. Then:

What is the binomial expansion of ?

Binomial Expansion

Find the binomial expansion of .

What if I asked for the coefficient of the erm containing ?

What if I asked for the Coefficient of the term containing

Pascals Identity

Let n, k be positive integers with n >= k. Then

Proof: Let T be a set where there are

Subsets of T containing k elements. Each of these subsets either:

Contains a with k-1 other elements or

Contains k elements of S and not a

Pascals Triangle