Homework #7

Section 4.1

6. Show that if a, b, c, and d are integers, where $a \neq 0$, such that $a \mid c$ and $b \mid d$, then $ab \mid cd$.

Proof

10. What are the quotient and remainder when

a) 44 is divided by 8?
b) 777 is divided by 21?
c) -123 is divided by 19?
d) -1 is divided by 23?
e) -2002 is divided by 87?
f) 0 is divided by 17?
g) 1,234,567 is divided by 1001?
h) -100 is divided by 101?

$a = dq + r$

a) 44 is divided by 8?

$44 = 8q + r$ $\quad 0 \le r < d$

$44 = 8(5) + 4$ $\quad q = 5$

$r = 4$

b) 777 is divided by 21?

$777 = 21q + r$ $\quad q = 37$

$777 = 21(37) + 0$ $\quad r = 0$

c) -123 is divided by 19?

d) -1 is divided by 23?

14. Suppose that a and b are integers, $a \equiv 11 \pmod{19}$, and $b \equiv 3 \pmod{19}$. Find the integer c with $0 \leq c \leq 18$ such that

a) $c \equiv 13a \pmod{19}$.
b) $c \equiv 8b \pmod{19}$.
c) $c \equiv a - b \pmod{19}$.
d) $c \equiv 7a + 3b \pmod{19}$.
e) $c \equiv 2a^2 + 3b^2 \pmod{19}$.
f) $c \equiv a^3 + 4b^3 \pmod{19}$.

a) $c \equiv 13a \pmod{19}$.

b) $c \equiv 8b \pmod{19}$.

28. Decide whether each of these integers is congruent to 3 modulo 7.

a) 37 **b)** 66
c) −17 **d)** −67

3 modulo 7

a) 37

$\frac{37}{7} \to 2$ $\frac{3}{7} \to 1$

remeinder remander

Not Congruent

b) 66

$\frac{66}{7} \to$ $\frac{3}{7} \to$

c) −17

d) −67

Section 4.2

2. Convert the decimal expansion of each of these integers to a binary expansion.

a) 321 **b)** 1023 **c)** 100632

6. Convert the binary expansion of each of these integers to an octal expansion.

a) $(1111\ 0111)_2$

b) $(1010\ 1010\ 1010)_2$

c) $(111\ 0111\ 0111\ 0111)_2$

d) $(101\ 0101\ 0101\ 0101)_2$

7. Convert the hexadecimal expansion of each of these integers to a binary expansion.

a) $(80E)_{16}$ **b)** $(135AB)_{16}$

c) $(ABBA)_{16}$ **d)** $(DEFACED)_{16}$

26. Use Algorithm 5 to find 11^{644} **mod** 645.

Section 4.3

4. Find the prime factorization of each of these integers.

a) 39	**b)** 81	**c)** 101
d) 143	**e)** 289	**f)** 899

24. What are the greatest common divisors of these pairs of integers?

a) $2^2 \cdot 3^3 \cdot 5^5$, $2^5 \cdot 3^3 \cdot 5^2$

b) $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$, $2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14}$

c) 17, 17^{17}

d) $2^2 \cdot 7$, $5^3 \cdot 13$

e) 0, 5

f) $2 \cdot 3 \cdot 5 \cdot 7$, $2 \cdot 3 \cdot 5 \cdot 7$

26. What is the least common multiple of each pair in Exercise 24?