Homework #7

Section 4.1

6. Show that if a, b, c, and d are integers, where $a \neq 0$, such that $a \mid c$ and $b \mid d$, then $ab \mid cd$.

Proof

10. What are the quotient and remainder when

a) 44 is divided by 8?
b) 777 is divided by 21?
c) -123 is divided by 19?
d) -1 is divided by 23?
e) -2002 is divided by 87?
f) 0 is divided by 17?
g) 1,234,567 is divided by 1001?
h) -100 is divided by 101?

$a = dq + r$

a) 44 is divided by 8?

$44 = 8q + r \quad 0 \le r < d$
$44 = 8(5) + 4 \quad q = 5$
$r = 4$

b) 777 is divided by 21?

$777 = 21q + r \quad q = 37$
$777 = 21(37) + 0 \quad r = 0$

c) -123 is divided by 19?

$-123 = 19q + r \quad 0 \le r < d$
$-123 = 19(-7) + r \quad q = -7$
$-123 = -133 + 10 \quad r = 10$
$-123 = -123$ ✓

d) -1 is divided by 23?

$-1 = 23q + r \quad 0 \le r < d$
$-1 = 23(-1) + r \quad q = -1$
$-1 = -23 + 22 \quad r = 22$
$-1 = -1$ ✓

e) -2002 is divided by 87?

$-2002 = 87q + r \quad q = -24$
$-2002 = 87(-24) + r$
$-2002 = -2088 + r \quad r = 86$
$-2002 = -2088 + 86$
$-2002 = -2002$

f) 0 is divided by 17?

$0 = 17q + r \quad q = 0$
$0 = 17(0) + 0 \quad r = 0$

g) 1,234,567 is divided by 1001?

$1234567 = 1001q + r$
$1234567 = 1001(1233) + r$
$1234567 = 1{,}234{,}233 + r$
$1234567 = 1{,}234{,}233 + 334$
$1234567 = 1234567$

$q = 1233$
$r = 334$

h) -100 is divided by 101?

$-100 = 101q + r$
$-100 = 101(-1) + 1$
$-100 = -101 + 1$
$-100 = -100$

$q = -1$
$r = 1$

14. Suppose that a and b are integers, $a \equiv 11 \pmod{19}$, and $b \equiv 3 \pmod{19}$. Find the integer c with $0 \le c \le 18$ such that

a) $c \equiv 13a \pmod{19}$.
b) $c \equiv 8b \pmod{19}$.
c) $c \equiv a - b \pmod{19}$.
d) $c \equiv 7a + 3b \pmod{19}$.
e) $c \equiv 2a^2 + 3b^2 \pmod{19}$.
f) $c \equiv a^3 + 4b^3 \pmod{19}$.

$q = a \text{ div } d$
$r = a \bmod d$

Let m be a positive integer

If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then $a + c \equiv b + d \pmod{m}$ and $ac \equiv bd \pmod{m}$

$a = 11 \pmod{19}$ $\quad b = 3 \pmod{19}$ $\quad$ for $0 \le c \le 18$

a) $c \equiv 13a \pmod{19}$.

$c = 13a \pmod{19}$
$\equiv 13 \cdot 11 \pmod{19}$
$\equiv 143 \pmod{19}$
$\equiv 10 \pmod{19}$

remainder from 143/mod 19

$c = 10$

b) $c \equiv 8b \pmod{19}$.

$c \equiv 8b \pmod{19}$
$\equiv 8 \cdot 3 \pmod{19}$
$\equiv 24 \pmod{19}$
$\equiv 5 \pmod{19}$
$c = 5$

c) $c \equiv a - b \pmod{19}$.

$c \equiv a - b \pmod{19}$
$c \equiv 11 - 3 \pmod{19}$
$c \equiv 8 \pmod{19}$
$c = 8$

d) $c \equiv 7a + 3b \pmod{19}$.

$c \equiv 7a + 3b \pmod{19}$
$c \equiv 7(11) + 3(3) \pmod{19}$
$c \equiv 77 + 9 \pmod{19}$

$c \equiv 86 \pmod{19}$
$c = 10 \pmod{19}$

$c = 10$

e) $c \equiv 2a^2 + 3b^2 \pmod{19}$.

$c \equiv 2a^2 + 3b^2 \pmod{19}$

$c \equiv 2(11^2) + 3(3)^2 \pmod{19}$
$c = 242 + 27 \pmod{19}$
$c = 269 \pmod{19}$
$c = 3 \pmod{19}$

$c = 3$

f) $c \equiv a^3 + 4b^3 \pmod{19}$.

$c \equiv a^3 + 4b^3 \pmod{19}$
$c \equiv (11)^3 + 4(3)^3 \pmod{19}$

$c \equiv 1331 + 108 \pmod{19}$
$c \equiv 1439 \pmod{19}$

$c = 14 \pmod{19}$

$c = 14$

28. Decide whether each of these integers is congruent to 3 modulo 7.

a) 37 **b)** 66

c) −17 **d)** −67

3 modulo 7

a) 37

$37 \equiv 3$ modulo 7

$7 \mid 37 - 3$

$7 \mid 34$

Not Congruent

b) 66

$66 \equiv 3$ modulo 7

$7 \mid 66 - 3$

$7 \mid 63$

Congruent

c) −17

$-17 \equiv 3$ modulo 7

$7 \mid (-17 - 3)$

$7 \mid -20$

Not Congruent × not divisible

d) −67

$7 \mid -67 - 3$

$7 \mid -70$

Congruent

Section 4.2

2. Convert the decimal expansion of each of these integers to a binary expansion.
a) 321 **b)** 1023 **c)** 100632

Division Algorithm: $a = dq + r$

Binary = 2

a) 321

$321 = 2 \cdot q + r$ ← PATTERN
$321 = 2(160) + 1$
$160 = 2(80) + 0$
$80 = 2(40) + 0$
$40 = 2(20) + 0$
$20 = 2(10) + 0$
$10 = 2(5) + 0$
$5 = 2(2) + 1$
$2 = 2(0) + 2$

Value of 2

1 0 1 0 0 0 0 0 1

$2 = 1\,0$

We need to group by 4's, add zeros if needed

$\boxed{(0001\ 0100\ 0001)_2}$

b) 1023

$1023 = 2 \cdot q + r$
$1023 = 2 \cdot (511) + 1$
$511 = 2 \cdot (255) + 1$
$255 = 2 \cdot (127) + 1$
$127 = 2(63) + 1$
$63 = 2(31) + 1$
$31 = 2(15) + 1$
$15 = 2(7) + 1$
$7 = 2(3) + 1$
$3 = 2(1) + 1$
$1 = 2(0) + 1$

1 1 1 1 1 1 1 1 1 1

$\boxed{(0011\ 1111\ 1111)_2}$

c) 100632

$100632 = 2(50316) + 0$
$50316 = 2(25158) + 0$
$25158 = 2(12579) + 0$
$12579 = 2(6289) + 1$
$6289 = 2(3144) + 1$
$3144 = 2 \cdot (1572) + 0$
$1572 = 2 \cdot (786) + 0$
$786 = 2(393) + 0$
$393 = 2(196) + 1$
$196 = 2(98) + 0$
$98 = 2(49) + 0$
$49 = 2(24) + 1$
$24 = 2(12) + 0$
$12 = 2(6) + 0$
$6 = 2(3) + 0$
$3 = 2(0) + 1$

1000 1001 0001 1000

$\boxed{(1000\ 1001\ 0001\ 1000)_2}$

6. Convert the binary expansion of each of these integers to an octal expansion.

a) $(1111\ 0111)_2$
b) $(1010\ 1010\ 1010)_2$
c) $(111\ 0111\ 0111\ 0111)_2$
d) $(101\ 0101\ 0101\ 0101)_2$

for octal:

$\frac{4}{2^2}\ \frac{2}{2^1}\ \frac{1}{2^0}$

a) $(1111\ 0111)_2$

7. Convert the hexadecimal expansion of each of these integers to a binary expansion.

a) $(80E)_{16}$ **b)** $(135AB)_{16}$

c) $(ABBA)_{16}$ **d)** $(DEFACED)_{16}$

$\boxed{\frac{8}{2^3}\ \frac{4}{2^2}\ \frac{2}{2^1}\ \frac{1}{2^0}}$

A = 10
B = 11
C = 12
D = 13
E = 14
F = 15

a) $(80E)_{16}$

$(80E)_{16}$

8	0	E = 14
1 0 0 0	0 0 0 0	1 1 1 0

$\boxed{(1000\ 0000\ 1110)_2}$

b) $(135AB)_{16}$

1	3	5	A = 10	B = 11
0 0 0 1	0 0 1 1	0 1 0 1	1 0 1 0	1 0 1 1

$\boxed{(0001\ 0011\ 0101\ 1010\ 1011)_2}$

c) $(ABBA)_{16}$

A = 10	B = 11	B = 11	A = 10
1 0 1 0	1 0 1 1	1 0 1 1	1 0 1 0

$\boxed{(1010\ 1011\ 1011\ 1010)_2}$

d) $(DEFACED)_{16}$

D = 13	E = 14	F = 15	A = 10	C = 12	E = 14	D = 13
1 1 0 1	1 1 1 0	1 1 1 1	1 0 1 0	1 1 0 0	1 1 1 0	1 1 0 1

$\boxed{(1101\ 1110\ 1111\ 1010\ 1100\ 1110\ 1101)_2}$

26. Use Algorithm 5 to find 11^{644} **mod** 645.

Section 4.3

4. Find the prime factorization of each of these integers.

a) 39 **b)** 81 **c)** 101
d) 143 **e)** 289 **f)** 899

a) 39

$39 = 3 \cdot 13$

$39 = 3^1 \cdot 13^1$

b) 81

$81 = 3 \cdot 27$

$= 3 \cdot 3 \cdot 9$

$= 3 \cdot 3 \cdot 3 \cdot 3$

$= 3^4$

c) 101

$101 = 101^1$

d) 143

$143 = 11 \cdot 13$

$143 = 11^1 \cdot 13^1$

e) 289

$289 = 17 \cdot 17$

$289 = 17^2$

f) 899

$899 = 29 \cdot 31$

$899 = 29^1 \cdot 31^1$

24. What are the greatest common divisors of these pairs of integers?

a) $2^2 \cdot 3^3 \cdot 5^5, 2^5 \cdot 3^3 \cdot 5^2$

b) $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13, 2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14}$

c) $17, 17^{17}$

d) $2^2 \cdot 7, 5^3 \cdot 13$

e) $0, 5$

f) $2 \cdot 3 \cdot 5 \cdot 7, 2 \cdot 3 \cdot 5 \cdot 7$

a) $2^2 \cdot 3^3 \cdot 5^5, 2^5 \cdot 3^3 \cdot 5^2$

$2^2 \cdot 3^3 \cdot 5^2 =$

gcd $= 4 \cdot 27 \cdot 25 = 2700$

b) $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13, 2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14}$

$2 \cdot 3 \cdot 11 = 66$

gcd $= 66$

c) $17, 17^{17}$

gcd $= 17$

d) $2^2 \cdot 7, 5^3 \cdot 13$

gcd $= 1$

e) $0, 5$

gcd $= 5$

f) $2 \cdot 3 \cdot 5 \cdot 7, 2 \cdot 3 \cdot 5 \cdot 7$

$2 \cdot 3 \cdot 5 \cdot 7$

gcd $= 210$

26. What is the least common multiple of each pair in Exercise 24?

a) $2^2 \cdot 3^3 \cdot 5^5, 2^5 \cdot 3^3 \cdot 5^2$ Look for most time exp occurs

$lcm = 2^5 \cdot 3^3 \cdot 5^5 =$

$32 \cdot 27 \cdot 3125$

$lcm = 2{,}700{,}000$

b) $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13, 2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14}$

$2^{11} \cdot 3^9 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17^{14}$

$lcm = 3.397 \times 10^{28}$

c) $17, 17^{17}$

$Lcm = 17^{17} = 8.27 \times 10^{20}$

d) $2^2 \cdot 7, 5^3 \cdot 13$

$lcm = 2^2 \cdot 7 \cdot 5^3 \cdot 13$

$lcm = 45500$

e) $0, 5$

Undefined

f) $2 \cdot 3 \cdot 5 \cdot 7, 2 \cdot 3 \cdot 5 \cdot 7$

$lcm = 2 \cdot 3 \cdot 5 \cdot 7$

$lcm = 210$