

Homework #1

Devin Powers

1. Which of these sentences are propositions? What are the **truth values** of those that are propositions?

- a) Boston is the capital of Massachusetts.
- b) Miami is the capital of Florida.
- c) $2 + 3 = 5$.
- d) $5 + 7 = 10$.
- e) $x + 2 = 11$.
- f) Answer this question.

- a) Boston is the capital of Massachusetts.

proposition ~ statement
T

- b) Miami is the capital of Florida.

proposition ~ statement
F

- c) $2 + 3 = 5$.

proposition
T

- d) $5 + 7 = 10$.

proposition
F

- e) $x + 2 = 11$.

not a proposition
F

- f) Answer this question.

not a proposition
F

6. Suppose that **Smartphone A** has 256 MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP; **Smartphone B** has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and **Smartphone C** has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions. **True or False**

- a) Smartphone B has the most RAM of these three smartphones.
- b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.
- c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
- d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.
- e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

- a) Smartphone B has the most RAM of these three smartphones.

True

- b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.

True ~ Higher resolution Camera

- c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.

False

- d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.

False

- e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

False

Smartphone A

RAM: 256 MB

ROM: 32 GB

Camera: 8 MP

Smartphone B

RAM: 288 MB

ROM: 64 GB

Camera: 4 MP

Smartphone C

RAM: 128 MB

ROM: 32 GB

Camera: 5 MP

9. Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these compound propositions as an English sentence.

- a) $\neg q$
- b) $p \wedge q$
- c) $\neg p \vee q$
- d) $p \rightarrow \neg q$
- e) $\neg q \rightarrow p$
- f) $\neg p \rightarrow \neg q$
- g) $p \leftrightarrow \neg q$
- h) $\neg p \wedge (p \vee \neg q)$

$p = \text{Swimming at the New Jersey Shore is allowed}$

$q = \text{Sharks have been spotted near the shore}$

a) $\neg q$

Sharks have **not** been spotted near the shore.

b) $p \wedge q$

Swimming at the New Jersey Shore is allowed **and** Sharks have been spotted near the shore.

Connectives (some notation) to choose meaning
 - p is a well-formed formula (wff)
 - $\neg p$ is a wff
 - $p \wedge q$ is a wff
 - $p \vee q$ is a wff
 - $\neg p \wedge q$ is a wff
 - $\neg p \vee q$ is a wff
 - $p \rightarrow q$ is a wff
 - $\neg p \rightarrow q$ is a wff
 - $p \leftrightarrow q$ is a wff
 \Leftrightarrow : if and only if

c) $\neg p \vee q$

Swimming at the New Jersey Shore is **not** allowed, **or** Sharks have been spotted near the shore.

d) $p \rightarrow \neg q$

Swimming at the New Jersey Shore is allowed, **then** Sharks have **not** been spotted near the shore.

e) $\neg q \rightarrow p$

Sharks have **not** been spotted near the shore, **then** Swimming at the New Jersey Shore is allowed.

f) $\neg p \rightarrow \neg q$

Swimming at the New Jersey Shore is **not** allowed, **then** Sharks have **not** been spotted near the shore.

g) $p \leftrightarrow \neg q$

Swimming at the New Jersey Shore is allowed **if and only if** Sharks have **not** been spotted near the shore.

h) $\neg p \wedge (p \vee \neg q)$

Swimming at the New Jersey Shore is **not** allowed, **and either** Swimming at the New Jersey Shore is allowed **or** Sharks have **not** been spotted near the shore.

13. Let p and q be the propositions

p : You drive over 65 miles per hour.

q : You get a speeding ticket.

Write these propositions using p and q and logical connectives (including negations).

- You do not drive over 65 miles per hour.
- You drive over 65 miles per hour, but you do not get a speeding ticket.
- You will get a speeding ticket if you drive over 65 miles per hour.
- If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
- Driving over 65 miles per hour is sufficient for getting a speeding ticket.
- You get a speeding ticket, but you do not drive over 65 miles per hour.
- Whenever you get a speeding ticket, you are driving over 65 miles per hour.

- a) You do not drive over 65 miles per hour.

$$\neg p$$

- b) You drive over 65 miles per hour, but you do not get a speeding ticket.

$$p \wedge \neg q$$

- c) You will get a speeding ticket if you drive over 65 miles per hour.

$$p \rightarrow q$$

- d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.

$$\neg p \rightarrow \neg q$$

- e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

$$p \rightarrow q$$

- f) You get a speeding ticket, but you do not drive over 65 miles per hour.

$$q \wedge \neg p$$

- g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

$$q \rightarrow p$$

16. Determine whether these biconditionals are true or false.

- a) $2 + 2 = 4$ if and only if $1 + 1 = 2$.
- b) $1 + 1 = 2$ if and only if $2 + 3 = 4$.
- c) $1 + 1 = 3$ if and only if monkeys can fly.
- d) $0 > 1$ if and only if $2 > 1$.

- a) $2 + 2 = 4$ if and only if $1 + 1 = 2$.

True

Biconditional

- b) $1 + 1 = 2$ if and only if $2 + 3 = 4$.

False

- c) $1 + 1 = 3$ if and only if monkeys can fly.

True

- d) $0 > 1$ if and only if $2 > 1$.

False

P	q	$P \leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

if p and q
are the same
Value, then it's
TRUE

18. Determine whether each of these conditional statements is true or false.

- a) If $1 + 1 = 3$, then unicorns exist.
- b) If $1 + 1 = 3$, then dogs can fly.
- c) If $1 + 1 = 2$, then dogs can fly.
- d) If $2 + 2 = 4$, then $1 + 2 = 3$.

- a) If $1 + 1 = 3$, then unicorns exist.

$$F \rightarrow F$$

True

- b) If $1 + 1 = 3$, then dogs can fly.

$$F \rightarrow F$$

True

- c) If $1 + 1 = 2$, then dogs can fly.

$$T \rightarrow F$$

False

- d) If $2 + 2 = 4$, then $1 + 2 = 3$.

$$T \rightarrow T$$

True

Conditional

P	Q	$P \rightarrow Q$
1	1	1
1	0	0
0	1	1
0	0	1

if P then Q

it is only false if P is true
and Q is false, the rest of the
times it will be true

20. For each of these sentences, determine whether an **inclusive** or, or an **exclusive** or, is intended. Explain your answer.

- Experience with C++ or Java is required.
- Lunch includes soup or salad.
- To enter the country you need a passport or a voter registration card.
- Publish or perish.

- a) Experience with C++ or Java is required.

experience with C++ ✓
experience with Java ✓

You need to know at least one of
the languages!

Inclusive

- b) Lunch includes soup or salad.

Exclusive

-you want get both

- c) To enter the country you need a passport or a voter registration card.

Inclusive

- d) Publish or perish.

Inclusive

Exclusive

Or (\oplus, \vee)

P	q	$P \oplus q$
1	1	0
1	0	1
0	1	1
0	0	0

opposite of biconditional

$P \neq q$ then $P \oplus q = 1$

Inclusive
Disjunction ($\vee, +$)

Disjunction ~ "or"



P	q	$P \vee q$
1	1	1
1	0	1
0	1	1
0	0	0

Mathematically ↗

$$P \vee q = \max(P, q)$$

P and q are true if at least
one of P, q are true.
← neither P or q are true so $P \vee q$ is false.

23. Write each of these statements in the form "if p , then q " in English. [Hint: Refer to the list of common ways to express conditional statements.]

- a) It snows whenever the wind blows from the northeast.
- b) The apple trees will bloom if it stays warm for a week.
- c) That the Pistons win the championship implies that they beat the Lakers.
- d) It is necessary to walk 8 miles to get to the top of Long's Peak.
- e) To get tenure as a professor, it is sufficient to be world-famous.
- f) If you drive more than 400 miles, you will need to buy gasoline.
- g) Your guarantee is good only if you bought your CD player less than 90 days ago.
- h) Jan will go swimming unless the water is too cold.

Let p and q be propositions. The conditional statement $p \rightarrow q$ is the proposition "if p , then q ." The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise. In the conditional statement $p \rightarrow q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

$$P \rightarrow q$$

"if p , then q "	" p implies q "
"if p, q "	" p only if q "
" p is sufficient for q "	"a sufficient condition for q is p "
" q if p "	" q whenever p "
" q when p "	" q is necessary for p "
"a necessary condition for p is q "	" q follows from p "
" q unless $\neg p$ "	

- a) It snows whenever the wind blows from the northeast.

If the wind blows from the Northeast, then it snows.

- b) The apple trees will bloom if it stays warm for a week.

If it stays warm for a week, then the apple trees will bloom.

- c) That the Pistons win the championship implies that they beat the Lakers.

If the Pistons win the championship, then they beat the Lakers.

- d) It is necessary to walk 8 miles to get to the top of Long's Peak.

If you get to the top of Long's Peak, then you must have walked 8 miles.

- e) To get tenure as a professor, it is sufficient to be world-famous.

If you're world famous, then you will get tenure as a professor.

- f) If you drive more than 400 miles, you will need to buy gasoline.

If you drive more than 400 miles, then you will need to buy gasoline.

- g) Your guarantee is good only if you bought your CD player less than 90 days ago.

If your guarantee is good, then you must have bought your CD player 90 days ago.

- h) Jan will go swimming unless the water is too cold.

If the water is not too cold, then Jan will go swimming.

2004 CHAMPS!
Mr. Big Shot!