MPCS 50103 Discrete Mathematics—Autumn 2019

Homework 1. This problem set is due Monday October 7 at 11:59 pm.

Reading: Rosen 7e, chapter 1, sections 1.7–1.8; chapter 5, sections 5.1–5.2.

Homework assignment

- "DO" exercises are strongly recommended to check your understanding of the concepts. **Do them but do not submit them**.
- **Problems labeled "HW" are homework problems that you are required to submit**.
- You are responsible for the material covered in **both** "DO" exercises and HW problems.

"DO" Exercises (solve but do *not* submit):

1. **"DO"** Rosen 7e, section 1.7, exercises 8, 9, 25, 29, 30, 35, and 41, on pages 91–92.

2. **"DO"** Rosen 7e, section 1.8, exercises 8, 16, 27, 41, and 46, on pages 108–109.

3. **"DO"** Rosen 7e, section 5.1, exercises 3, 5, 19, 25, 49, and 53, on pages 329–331.

4. **"DO"** Rosen 7e, section 5.2, exercises 3, 10, 15, and 35 on pages 341–344.

Homework Problems: DUE Monday October 7 at 11:59 pm

- **Collaboration policy: There is no penalty for acknowledged collaboration. To acknowledge collaboration, give the names of students with whom you worked at the beginning of your homework submission and mark any solution that relies on your collaborators' ideas. Note: you must work out and write up each homework solution by yourself without assistance. DO NOT COPY or rephrase someone else's solution. The same requirement applies to books and other written sources: you should acknowledge all sources that contributed to your solution of a homework problem. Acknowledge specific ideas you learned from the source. DO NOT COPY or rephrase solutions from written sources.**
- **Internet policy: Looking for solutions to homework problems on the internet, even when acknowledged, is STRONGLY DISCOURAGED. If you find a solution to a homework problem on the internet, do not not copy it. Close the website, work out and write up your solution by yourself, and cite the url of website in your writeup. Acknowledge specific ideas you learned from the website.**
- **Copied solutions obtained from a written source, from the internet, or from another person will receive ZERO credit and will be flagged to the attention of the instructor.**
- Write out your work for every problem. If you just write your answer without showing your work, you will not receive credit.

1. **HW** Alice has a special (cubic) die. The values on its faces are the distinct integers from 1 to 6, but they are not arranged as in a standard die. When Alice first tosses the die, the sum of the values on the four side faces is 15. In her second toss, the sum of the values on the four side faces is 12. Find what value appears in the face opposite 6 on Alice's special die. Prove your answer. What proof technique did you use? (3 points)

2. **HW** Twenty-five families live in the squares of a 5×5 chessboard, each occupying one square. Each family thinks that their neighbors, i.e., those families living in the squares with which their square shares a side, have better living units. At a town meeting a resolution is passed to make everyone happier by moving the 25 families around so that each family ends up in a square of one of their former neighbors. Unfortunately, the families cannot figure out any relocation scheme to carry out this resolution. Give a very simple reason explaining why this resolution was doomed to fail. Explain. (5 points)

3. **HW** A 3×3 table is filled with numbers. It is permitted to increase each number in any 2×2 square by 1, or decrease all the numbers in any column by 1. Is it possible, using these operations, to obtain the table shown in Figure 1 from a table filled with zeros? If yes, construct a solution; if no, prove it is not possible. (5 points)

4	9	5
10	18	12
6	13	7

Figure 1

4. **HW** An *arrangement* of lines in the plane $\mathbf{R}^2$ is a set of lines satisfying the property that no point in the plane $x \in \mathbf{R}^2$ is the intersection of three or more lines.
   - Draw a set of lines that are a legal arrangement. (1 point)
   - Draw a set of lines that are not a legal arrangement. (1 point)

   An arrangement divides the plane up into *cells* or polyhedral regions that have segments of the arrangement's lines as borders. Two cells are called *neighbors* if they share a border segment (shared border points do not count). A *2-coloring* of the cells of an arrangement is an assignment of "black" or "white" to each cell such that no neighboring cells share the same color.
   Use mathematical induction to prove that an arrangement's cells can always be 2-colored.
   - Write mathematically the exact statement that you are going to prove. (2 points)
   - Prove the above statement using mathematical induction. For credit, you must prove the basis step, state the inductive hypothesis, and give a correct proof of the inductive step, specifying where you use the inductive hypothesis and justifying intermediate steps of your proof. (4 points)

5. **HW** The Fibonacci sequence is defined $F_1 = 0, F_2 = 1$, and then for all $k \geq 2$, we define $F_k = F_{k-1} + F_{k-2}$. The first few terms of Fibonacci sequence are:

   $0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \dots$

   While not all positive integers are Fibonacci numbers (e.g., 4), surprisingly we can express any positive integer as the sum of distinct terms in the Fibonacci sequence.
   Use strong induction to prove the following theorem:

   Theorem. *Every positive integer n can be expressed as the sum of distinct terms in the Fibonacci sequence.*

   - Write mathematically the exact statement that you are going to prove. (2 points)
   - Prove the above statement using strong induction. For credit, you must prove the basis step(s), state the inductive hypothesis, and give a correct proof of the inductive step, specifying where you use the inductive hypothesis and justifying intermediate steps of your proof. (4 points)

6. **HW** Alice and Bob play the following game: starting with a pile of n stones, Alice and Bob take turns, each removing 1, 2, 3, or 4 stones from the pile. The player who removes the last stone loses the game. Alice goes first. For which values of n is this game a forced win by Bob? By Alice? Use strong induction to prove your answer.
   - Write mathematically the exact statement that you are going to prove. (2 points)

- Prove the above statement using strong induction. For credit, you must prove the basis step(s), state the inductive hypothesis, and give a correct proof of the inductive step, specifying where you use the inductive hypothesis and justifying intermediate steps of your proof. (4 points)

Gerry Brady
Tuesday September 24 21:38:09 CDT 2019