Homework #2

Section 1.1

28. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows tonight, then I will stay at home.
b) I go to the beach whenever it is a sunny summer day.
c) When I stay up late, it is necessary that I sleep until noon.

- $q \to p$ is the converse of $p \to q$ ~ Switch Order
- $\neg p \to \neg q$ is the inverse of $p \to q$ ~ negate
- $\neg q \to \neg p$ is the contrapositive of $p \to q$ ~ neg + switch

p q $p \to q$

a) If it snows tonight, then I will stay at home.

Converse: If, I stay at home, then it snowed tonight

Contrapositive: If I do not stay home, then it did not snow tonight.

Inverse: If it does not snow tonight, then I will not stay home

p q

b) I go to the beach whenever it is a sunny summer day.

Converse: If it is a sunny summer day, then I will go to the beach.

Contrapositive: If it's not a sunny summer day then I will not go to the beach

Inverse: If I do not go to the beach, then it was not a sunny summer day.

35. Construct a truth table for each of these compound propositions.

a) $p \to \neg q$ **b)** $\neg p \leftrightarrow q$
c) $(p \to q) \vee (\neg p \to q)$ **d)** $(p \to q) \wedge (\neg p \to q)$
e) $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$
f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

Biconditional $(\leftrightarrow, \equiv)$ iff → if and only if

p	q	p ↔ q
1	1	1
1	0	0
0	1	0
0	0	1

p = q then p ↔ q = 1

if they (p and q) are the same value then it's True.

a) $p \to \neg q$

p	q	¬q	p → ¬q
1	1	0	0
1	0	1	1
0	1	0	1
0	0	1	1

b) $\neg p \leftrightarrow q$

p	¬p	q	¬p ↔ q
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	0

c) $(p \to q) \vee (\neg p \to q)$

p	q	¬p	(p → q)	(¬p → q)	(p → q) ∨ (¬p → q)
1	1	0	1	1	1
1	0	0	0	1	1
0	1	1	1	1	1
0	0	1	1	0	1

d) $(p \to q) \wedge (\neg p \to q)$

p	q	¬p	(p → q)	(¬p → q)	(p → q) ∧ (¬p → q)
1	1	0	1	1	1
1	0	0	0	1	0
0	1	1	1	1	1
0	0	1	1	0	0

Conjunction $(\wedge, *, -)$

and

p	q	p ∧ q
1	1	1
1	0	0
0	1	0
0	0	0

e) $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$

p	¬p	q	¬p ↔ q	(p ↔ q)	(p ↔ q) ∨ (¬p ↔ q)
1	0	1	0	1	1
1	0	0	1	0	1
0	1	1	1	0	1
0	1	0	0	1	1

f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

(p ↔ q)	(¬p ↔ ¬q)	(¬p ↔ ¬q) ↔ (p ↔ q)
1		
0		
0		
1		

Disjunction $(\vee, +)$

p	q	p ∨ q
1	1	1
1	0	1
0	1	1
0	0	0

Conditional $(\to, \supset)$

p	q	p → q
1	1	1
1	0	0
0	1	1
0	0	1

out

Section 1.3

9. Show that each of these conditional statements is a tautology by using truth tables.

a) $(p \wedge q) \rightarrow p$ **b)** $p \rightarrow (p \vee q)$
c) $\neg p \rightarrow (p \rightarrow q)$ **d)** $(p \wedge q) \rightarrow (p \rightarrow q)$
e) $\neg(p \rightarrow q) \rightarrow p$ **f)** $\neg(p \rightarrow q) \rightarrow \neg q$

a) $(p \wedge q) \rightarrow p$

p	q	$(p \wedge q)$	$(p \wedge q) \rightarrow p$
1	1	1	1
1	0	0	1
0	1	0	1
0	0	0	1

Tautology

b) $p \rightarrow (p \vee q)$

p	q	$(p \vee q)$	$p \rightarrow (p \vee q)$
1	1	1	1
1	0	1	1
0	1	1	1
0	0	0	1

Tautology

c) $\neg p \rightarrow (p \rightarrow q)$

p	$\neg p$	q	$(p \rightarrow q)$	$\neg p \rightarrow (p \rightarrow q)$
1	0	1	1	1
1	0	0	0	1
0	1	1	1	1
0	1	0	1	1

Tautology

d) $(p \wedge q) \rightarrow (p \rightarrow q)$

p	q	$(p \wedge q)$	$(p \rightarrow q)$	$(p \wedge q) \rightarrow (p \rightarrow q)$
1	1	1	1	1
1	0	0	0	1
0	1	0	1	1
0	0	0	1	1

Tautology

e) $\neg(p \rightarrow q) \rightarrow p$

p	q	$(p \rightarrow q)$	$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \rightarrow p$
1	1	1	0	1
1	0	0	1	1
0	1	1	0	1
0	0	1	0	1

Tautology

f) $\neg(p \rightarrow q) \rightarrow \neg q$

p	q	$\neg q$	$(p \rightarrow q)$	$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \rightarrow \neg q$
1	1	0	1	0	1
1	0	1	0	1	1
0	1	0	1	0	1
0	0	1	1	0	1

Tautology

Associativity $(\vee, \wedge)$

$$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$$

~ can do "ors" as well

~ make sure the signs stay the same.

LOGIL LAWS

11. Show that each conditional statement in Exercise 9 is a tautology without using truth tables.

a) $\underbrace{(p \wedge q)}_{p} \rightarrow \underbrace{p}_{q}$

DM LAW

$\neg(p \wedge q) \vee p$

$(\neg p \vee \neg q) \vee p$

$(\neg q \vee \neg p) \vee p$ ~ Commutativ

$\neg q (\neg p \vee p)$

Commutativity $(\vee, \wedge)$

$p \wedge q \Leftrightarrow q \wedge p$

$p \vee q \Leftrightarrow q \vee p$

~ We can flip the order

TABLE 7 Logical Equivalences Involving Conditional Statements.
$p \rightarrow q \equiv \neg p \vee q$
$p \rightarrow q \equiv \neg q \rightarrow \neg p$
$p \vee q \equiv \neg p \rightarrow q$
$p \wedge q \equiv \neg(p \rightarrow \neg q)$
$\neg(p \rightarrow q) \equiv p \wedge \neg q$
$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

22. Show that $(p \to q) \wedge (p \to r)$ and $p \to (q \wedge r)$ are logically equivalent.

All different Variations

Truth Table

p	q	r	$p \to q$	$p \to r$	$p \to q \wedge p \to r$	$(q \vee r)$	$p \to (q \wedge r)$
1	1	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1
0	0	1	1	1	1	0	1
0	1	0	1	1	1	0	1
1	0	1	0	1	0	0	0
0	0	0	1	1	1	0	1

logically Equivalent

32. Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.

All possible

p	q	r	$(p \wedge q)$	$(p \wedge q) \rightarrow r$	$(p \rightarrow r)$	$(q \rightarrow r)$	$(p \rightarrow r) \wedge (q \rightarrow r)$
1	1	1	1	1	1	1	1
1	1	0	1	0	0	0	1
1	0	0	0	1	0	1	1
0	1	1	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	0	1	1	0	0
1	0	1	0	1	1	1	1
0	0	0	0	1	1	1	1

Not Logically Equivalent

Conjunction $(\wedge, *, \cdot)$

p	q	$p \wedge q$ (and)
1	1	1
1	0	0
0	1	0
0	0	0

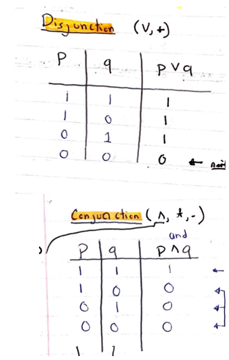
Conditional $(\rightarrow, \supset)$

p	q	$p \rightarrow q$
1	1	1
1	0	0
0	1	1
0	0	1

out

Disjunction $(\vee, +)$

p	q	$p \vee q$
1	1	1
1	0	1
0	1	1
0	0	0



Disjunction $(\vee, +)$

p	q	$p \vee q$
1	1	1
1	0	1
0	1	1
0	0	0

Conjunction $(\wedge, *, \cdot)$

and

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

61. Determine whether each of these compound propositions is satisfiable.

a) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

a) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

p	q	$\neg p$	$\neg q$	$(p \vee \neg q)$	$(\neg p \vee q)$	$(\neg p \vee \neg q)$	$(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$
1	1	0	0	1	1	0	1
1	0	0	1	1	0	1	1
0	1	1	0	0	1	1	1
0	0	1	1	1	1	1	1

Satisfiable

Section 1.4

8. Translate these statements into English, where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the domain consists of all animals.

a) $\forall x(R(x) \rightarrow H(x))$ **b)** $\forall x(R(x) \wedge H(x))$
c) $\exists x(R(x) \rightarrow H(x))$ **d)** $\exists x(R(x) \wedge H(x))$

R(x): " x is a rabbit"
H(x): "x hops"

∀x: for all
∃x: there exists

a) $\forall x(R(x) \rightarrow H(x))$

For all animals that is a rabbit, then that animal hops.

And

b) $\forall x(R(x) \wedge H(x))$

All

For all animals are rabbits and they hop.

c) $\exists x(R(x) \rightarrow H(x))$

There exists an Animal, that if is a rabbit, then it hops.

d) $\exists x(R(x) \wedge H(x))$

There exists an animal that is an Rabbit and that animal hops.

7. Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

a) $\forall x(C(x) \rightarrow F(x))$ **b)** $\forall x(C(x) \wedge F(x))$
c) $\exists x(C(x) \rightarrow F(x))$ **d)** $\exists x(C(x) \wedge F(x))$

a) $\forall x(C(x) \rightarrow F(x))$

For every x, if x is a comedian, then x is funny.

English: Every comedian is funny.

b) $\forall x(C(x) \wedge F(x))$

For every x, x is a comedian and x is funny.

NOTE:
"and" ~ combines them.

English: Everyone is a funny comedian.

c) $\exists x(C(x) \rightarrow F(x))$

There exists an x in the domain such that x is a comedian, then x is funny.

English: There exists a person that he/she is a comedian, then he/she is funny.

d) $\exists x(C(x) \wedge F(x))$

There exists an x in the domain, x is a comedian and x is funny.

English: There exists a funny comedian.

12. Let $Q(x)$ be the statement "$x + 1 > 2x$." If the domain consists of all integers, what are these truth values?

a) $Q(0)$ **b)** $Q(-1)$ **c)** $Q(1)$
d) $\exists x Q(x)$ **e)** $\forall x Q(x)$ **f)** $\exists x \neg Q(x)$
g) $\forall x \neg Q(x)$

a) $Q(0)$

$x + 1 > 2x$
$0 + 1 > 2(0)$
$1 > 0$
TRUE

b) $Q(-1)$

$x + 1 > 2x$
$-1 + 1 > 2(-1)$
$0 > -2$
TRUE

c) $Q(1)$

$x + 1 > 2x$
$1 + 1 > 2(1)$
$2 > 2$
FALSE

d) $\exists x Q(x)$

$x + 1 > 2x$
$0 + 1 > 2(0)$
$1 > 0$
TRUE

e) $\forall x Q(x)$

$x + 1 > 2x$
$1 + 1 > 2(1)$
$2 > 2$ FALSE
FALSE

f) $\exists x \neg Q(x)$

TRUE

g) $\forall x \neg Q(x)$

FALSE

13. Determine the truth value of each of these statements if the domain consists of all integers.

a) $\forall n(n+1>n)$ **b)** $\exists n(2n=3n)$

c) $\exists n(n=-n)$ **d)** $\forall n(3n \leq 4n)$

a) $\forall n(n+1>n)$

$n+1>n$

$0+1>0$

$1>0$ ✓

TRUE

b) $\exists n(2n=3n)$

$n=0$

$2(0)=3(0)$

$0=0$ ✓

TRUE

c) $\exists n(n=-n)$

$n=-n$

$n=0$

$0=-0$ ✓

TRUE

d) $\forall n(3n \leq 4n)$

~ True for all + integer

~ False for − integer

FALSE

28. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

a) Something is not in the correct place.

b) All tools are in the correct place and are in excellent condition.

c) Everything is in the correct place and in excellent condition.

d) Nothing is in the correct place and is in excellent condition.

e) One of your tools is not in the correct place, but it is in excellent condition.

$R(x)$: "x is in the correct place"

$E(x)$: "x is in excellent condition"

$T(x)$: "x is a [your] tool"

Domain: all things

a) Something is not in the correct place.

$$\exists x \neg R(x)$$

b) All tools are in the correct place $\wedge$ and are in excellent condition.

If something is a tool, then it is in the correct place place and in excellent condition:

$$\forall x\,(T(x) \rightarrow (R(x) \wedge E(x)))$$

c) Everything is in the correct place and in excellent condition.

$$\forall x\,(R(x) \wedge E(x))$$

d) Nothing is in the correct place and is in excellent condition.

$$\forall x \neg (R(x) \wedge E(x))$$

15. Determine the truth value of each of these statements if the domain for all variables consists of all integers.

a) $\forall n(n^2 \geq 0)$ **b)** $\exists n(n^2 = 2)$
c) $\forall n(n^2 \geq n)$ **d)** $\exists n(n^2 < 0)$

a) $\forall n(n^2 \geq 0)$ True

~ All integers ≥ 0

b) $\exists n(n^2 = 2)$ False

$n^2 = 2$
$n = \pm\sqrt{2}$
~ not an integer

c) $\forall n(n^2 \geq n)$ True

d) $\exists n(n^2 < 0)$ False

26. Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables.

a) Someone in your school has visited Uzbekistan.
b) Everyone in your class has studied calculus and C++.
c) No one in your school owns both a bicycle and a motorcycle.
d) There is a person in your school who is not happy.
e) Everyone in your school was born in the twentieth century.

Define the propositional function for the entire problems.

$Y(x)$: "x is in your school"
x is the domain

a) Someone in your school has visited Uzbekistan.

$U(x)$: "x has visited Uzbekistan"

$\exists x\, U(x)$ ~ Domain is just your school

$\exists x\, (Y(x) \land U(x))$ ~ Domain is all people

$V(x,y)$: "person x has visited country y"

$\exists x\, (Y(x) \land V(x, \text{Uzbekistan}))$

b) Everyone in your class has studied calculus and C++.

~ $\forall x$

Propositional Function
$C(x)$: "x has studied Calculus"
$P(x)$: "x has studied C++"

$\forall x\, (C(x) \land P(x))$ ~ Domain is just your schoolmate

$\forall x\, (Y(x) \rightarrow (C(x) \land P(x)))$ ~ Domain is all people.

$S(x,y)$: person x has studied subject y

$\forall x\, (Y(x) \rightarrow (S(x, \text{Calculus}) \land S(x, C{+}{+})))$

c) No one in your school owns both a bicycle and a motorcycle.

$B(x)$: "x owns a bike"
$M(x)$: "x owns a motorcycle"

$\forall x (\neg (B(x) \lor M(x)))$

27. Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables.

a) A student in your school has lived in Vietnam.

b) There is a student in your school who cannot speak Hindi.

c) A student in your school knows Java, Prolog, and C++.

d) Everyone in your class enjoys Thai food.

e) Someone in your class does not play hockey.

a) A student in your school has lived in Vietnam.

1st Domain: People in the school
2nd Domain: People in the World

A(x) Represents "x is in your school"
V(x) Represents "x has lived in Vietnam"
V(x,y) Represents "x has lived in y"

Varying the Domain: $\exists x\, V(x)$ ~ There exists a student who lived in Vietnam.

Predicate with one variables: $\exists x\, (V(x) \land A(x))$

Predicate with two variables: $\exists x\, (A(x) \land V(x, \text{Vietnam}))$

b) There is a student in your school who cannot speak Hindi.

30. Suppose the domain of the propositional function $P(x, y)$ consists of pairs x and y, where x is 1, 2, or 3 and y is 1, 2, or 3. Write out these propositions using disjunctions and conjunctions.

a) $\exists x\, P(x, 3)$ **b)** $\forall y\, P(1, y)$

c) $\exists y \neg P(2, y)$ **d)** $\forall x \neg P(x, 2)$

$\wedge$: and for $\forall x$

$\vee$: or for $\exists x$

a) $\exists x\, P(x, 3)$

$P(1,3) \vee P(2,3) \vee P(3,3)$

b) $\forall y\, P(1, y)$

$P(1,1) \wedge P(1,2) \wedge P(1,3)$

c) $\exists y \neg P(2, y)$ $\neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)$

d) $\forall x \neg P(x, 2)$ $\neg P(1,2) \wedge \neg P(2,2) \wedge \neg P(3,2)$

31. Suppose that the domain of $Q(x, y, z)$ consists of triples x, y, z, where $x = 0, 1$, or 2, $y = 0$ or 1, and $z = 0$ or 1. Write out these propositions using disjunctions and conjunctions.

a) $\forall y Q(0, y, 0)$ **b)** $\exists x Q(x, 1, 1)$

c) $\exists z \neg Q(0, 0, z)$ **d)** $\exists x \neg Q(x, 0, 1)$

a) $\forall y Q(0, y, 0)$

$Q(0,0,0) \wedge Q(0,1,0)$

b) $\exists x Q(x, 1, 1)$

$Q(0,1,1) \vee Q(1,1,1) \vee Q(2,1,1)$

c) $\exists z \neg Q(0, 0, z)$

$\neg Q(0,0,0) \vee \neg Q(0,0,1)$

d) $\exists x \neg Q(x, 0, 1)$

$\neg Q(0,0,1) \vee \neg Q(1,0,1) \vee \neg Q(2,0,1)$

39. Translate these specifications into English where $F(p)$ is "Printer p is out of service," $B(p)$ is "Printer p is busy," $L(j)$ is "Print job j is lost," and $Q(j)$ is "Print job j is queued."

a) $\exists p(F(p) \wedge B(p)) \to \exists j L(j)$

b) $\forall p B(p) \to \exists j Q(j)$

c) $\exists j(Q(j) \wedge L(j)) \to \exists p F(p)$

d) $(\forall p B(p) \wedge \forall j Q(j)) \to \exists j L(j)$

a) $\exists p(F(p) \wedge B(p)) \to \exists j L(j)$

If there is a printer that is both out of service and busy then the print job is lost.

b) $\forall p B(p) \to \exists j Q(j)$

If all printers are busy, then there is a print job in queue.

c) $\exists j(Q(j) \wedge L(j)) \to \exists p F(p)$

If there is a job that is in queue and lost then that printer is out of service.