Homework #7

Section 4.1

6. Show that if a, b, c, and d are integers, where $a \neq 0$, such that $a \mid c$ and $b \mid d$, then $ab \mid cd$.

10. What are the quotient and remainder when

a) 44 is divided by 8?

b) 777 is divided by 21?

c) -123 is divided by 19?

d) -1 is divided by 23?

e) -2002 is divided by 87?

f) 0 is divided by 17?

g) 1,234,567 is divided by 1001?

h) -100 is divided by 101?

14. Suppose that a and b are integers, $a \equiv 11 \pmod{19}$, and $b \equiv 3 \pmod{19}$. Find the integer c with $0 \leq c \leq 18$ such that

a) $c \equiv 13a \pmod{19}$.
b) $c \equiv 8b \pmod{19}$.
c) $c \equiv a - b \pmod{19}$.
d) $c \equiv 7a + 3b \pmod{19}$.
e) $c \equiv 2a^2 + 3b^2 \pmod{19}$.
f) $c \equiv a^3 + 4b^3 \pmod{19}$.

28. Decide whether each of these integers is congruent to 3 modulo 7.

a) 37 **b)** 66

c) -17 **d)** -67

Section 4.2

2. Convert the decimal expansion of each of these integers to a binary expansion.
a) 321 **b)** 1023 **c)** 100632

6. Convert the binary expansion of each of these integers to an octal expansion.

a) $(1111\ 0111)_2$

b) $(1010\ 1010\ 1010)_2$

c) $(111\ 0111\ 0111\ 0111)_2$

d) $(101\ 0101\ 0101\ 0101)_2$

7. Convert the hexadecimal expansion of each of these integers to a binary expansion.

a) $(80E)_{16}$ **b)** $(135AB)_{16}$

c) $(ABBA)_{16}$ **d)** $(DEFACED)_{16}$

26. Use Algorithm 5 to find 11^{644} **mod** 645.

Section 4.3

4. Find the prime factorization of each of these integers.

a) 39 **b)** 81 **c)** 101
d) 143 **e)** 289 **f)** 899

24. What are the greatest common divisors of these pairs of integers?

a) $2^2 \cdot 3^3 \cdot 5^5, 2^5 \cdot 3^3 \cdot 5^2$

b) $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13, 2^{11} \cdot 3^9 \cdot 11 \cdot 17^{14}$

c) $17, 17^{17}$

d) $2^2 \cdot 7, 5^3 \cdot 13$

e) $0, 5$

f) $2 \cdot 3 \cdot 5 \cdot 7, 2 \cdot 3 \cdot 5 \cdot 7$

26. What is the least common multiple of each pair in Exercise 24?