

Homework #4 (section 2.1-2.3)

Section 2.1

4. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

- a) the set of people who speak English, the set of people who speak English with an Australian accent
- b) the set of fruits, the set of citrus fruits
- c) the set of students studying discrete mathematics, the set of students studying data structures
- d) the set of people who speak English, the set of people who speak English with an Australian accent

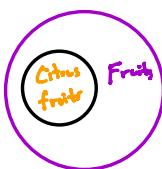
speak English with an Australian accent \subseteq of people who speak English.

is a subset



- b) the set of fruits, the set of citrus fruits

Citrus fruit \subseteq fruits



- c) the set of students studying discrete mathematics, the set of students studying data structures

neither

10. Determine whether these statements are true or false.

- a) $\emptyset \in \{\emptyset\}$
- b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$
- c) $\{\emptyset\} \in \{\emptyset\}$
- d) $\{\emptyset\} \in \{\{\emptyset\}\}$
- e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
- f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
- g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

a) $\emptyset \in \{\emptyset\}$

True

Because the empty set contains the
empty set.

b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$

True

Because the empty set is in the set

c) $\{\emptyset\} \in \{\emptyset\}$

False

Because the set contains the empty set,
not the set of the empty set

d) $\{\emptyset\} \in \{\{\emptyset\}\}$

True

Because the set contains the set of the empty set

e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$

Subset

True

Because the set of the empty set
is a subset of the set containing both
the Empty set and the set of the empty set

g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

False

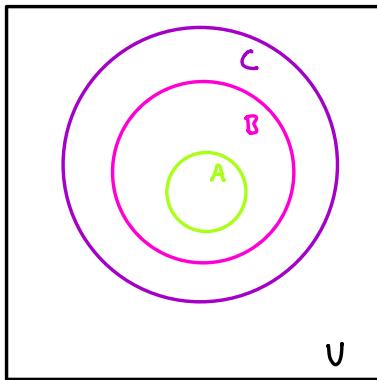
Because, the two sets are equal and to be a proper
subset, they can not be equal.

f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$

True

B/c the set as the empty set of the empty set is
contained within the second set, but the two
subsets are not equal.

14. Use a Venn diagram to illustrate the relationship $A \subseteq B$ and $B \subseteq C$.

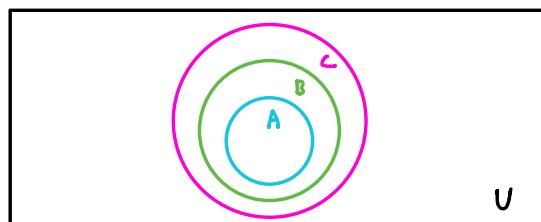


$U = \text{universe}$

Note:

$A \subseteq B$
C the " \subseteq " represents
that it can be equivalent

15. Use a Venn diagram to illustrate the relationships $A \subset B$ and $B \subset C$.



Note:

$A \subset B$

A must strictly be smaller
than B

19. What is the cardinality of each of these sets?

- a) $\{a\}$
- b) $\{\{a\}\}$
- c) $\{a, \{a\}\}$
- d) $\{a, \{a\}, \{a, \{a\}\}\}$

20. What is the cardinality of each of these sets?

- a) \emptyset
- b) $\{\emptyset\}$
- c) $\{\emptyset, \{\emptyset\}\}$
- d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

Note: Look at the commas!

a) $\{a\}$

1

a) \emptyset

0

b) $\{\{a\}\}$

1

b) $\{\emptyset\}$

1

c) $\{a, \{a\}\}$

2

c) $\{\emptyset, \{\emptyset\}\}$

2

d) $\{a, \{a\}, \{a, \{a\}\}\}$

3

d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

3

21. Find the **power set** of each of these sets, where a and b are distinct elements.

a) $\{a\}$ b) $\{a, b\}$ c) $\{\emptyset, \{\emptyset\}\}$

a) $\{a\}$

$$P(\{a\}) = \{\emptyset, \{a\}\}$$

Note:

Power set is the set containing all possible subsets.

b) $\{a, b\}$

$$P(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$

c) $\{\emptyset, \{\emptyset\}\}$

$$P(\{\emptyset, \{\emptyset\}\}) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$$

27. Let $A = \{a, b, c, d\}$ and $B = \{y, z\}$. Find

a) $A \times B$. b) $B \times A$.

a) $A \times B$.

$$A \times B = \{(a, y), (a, z), (b, y), (b, z), (c, y), (c, z), (d, y), (d, z)\}$$

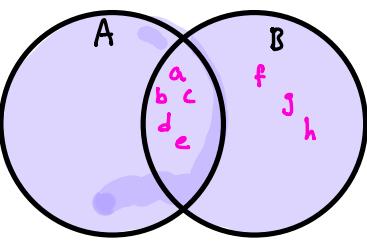
b) $B \times A$.

$$B \times A = \{(y, a), (y, b), (y, c), (y, d), (z, a), (z, b), (z, c), (z, d)\}$$

Section 2.2

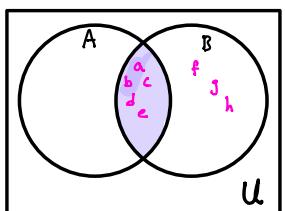
4. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$.
Find

- a) $A \cup B$.
- b) $A \cap B$.
- c) $A - B$.
- d) $B - A$.


a) $A \cup B$.

$$A \cup B = \{a, b, c, d, e, f, g, h\}$$

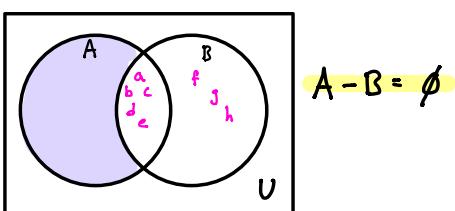
Let A and B be sets. The *union* of the sets A and B , denoted by $A \cup B$, is the set that contains those elements that are either in A or in B , or in both.


b) $A \cap B$.

$$A \cap B = \{a, b, c, d, e\}$$

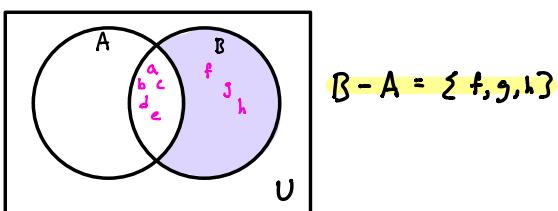
Let A and B be sets. The *intersection* of the sets A and B , denoted by $A \cap B$, is the set containing those elements in both A and B .

c) $A - B$.



$$A - B = \emptyset$$

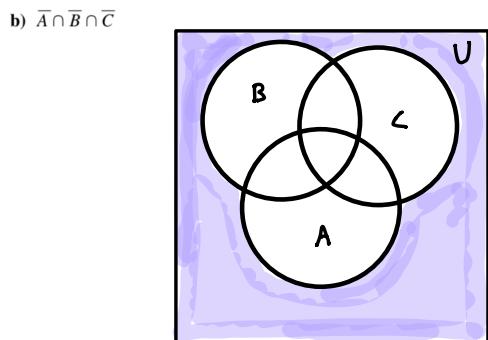
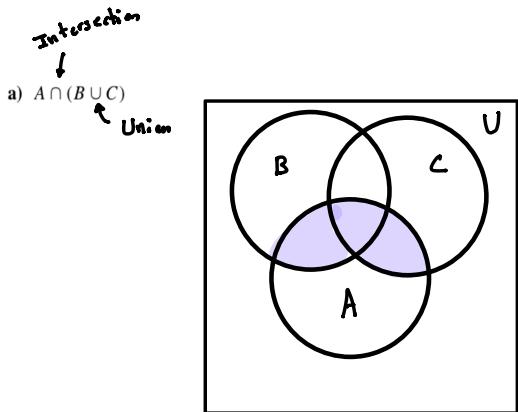
d) $B - A$.



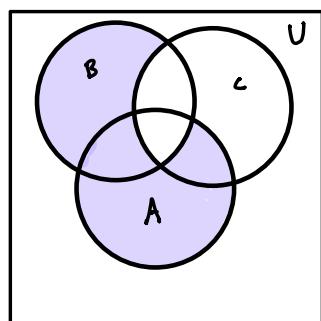
$$B - A = \{f, g, h\}$$

26. Draw the Venn diagrams for each of these combinations of the sets A , B , and C .

- a) $A \cap (B \cup C)$
- b) $\bar{A} \cap \bar{B} \cap \bar{C}$
- c) $(A - B) \cup (A - C) \cup (B - C)$



- c) $(A - B) \cup (A - C) \cup (B - C)$



30. Can you conclude that $A = B$ if A , B , and C are sets such that

- a) $A \cup C = B \cup C$?
- b) $A \cap C = B \cap C$?
- c) $A \cup C = B \cup C$ and $A \cap C = B \cap C$?

a) $A \cup C = B \cup C$?

No

Counter example: Given $A \cup C = B \cup C$

A and B might not be the same set

$$\begin{aligned} A &= \{1\} \\ B &= \{0\} \\ C &= \{0, 1\} \end{aligned}$$

from Union

$$\begin{aligned} A \cup C &= \{0, 1\} \\ A \cup B &= \{0, 1\} \end{aligned}$$

Which $A \cup C = B \cup C$ stands
While the two sets A and B are unequal.
 $\therefore A = B$ is not True.

b) $A \cap C = B \cap C$?

No

$$\begin{aligned} A &= \{0, 1\} \\ B &= \{0, 2\} \\ C &= \{0\} \end{aligned}$$

By Intersection:

$$\begin{aligned} A \cap C &= \{0\} \\ A \cap B &= \{0\} \end{aligned}$$

47. Let $A_i = \{1, 2, 3, \dots, i\}$ for $i = 1, 2, 3, \dots$. Find

a) $\bigcup_{i=1}^n A_i.$

b) $\bigcap_{i=1}^n A_i.$

$A_i = \{1, 2, 3, \dots, i\}$ for $i = 1, 2, 3, \dots$ the sets are increasing
 $A_1 \subseteq A_2 \subseteq \dots \subseteq A_n$
 $A_n = \{1, 2, 3, \dots, n\}$

b) $\bigcap_{i=1}^n A_i.$

$A_1 = \{1, 3\}$ looks to be the only chance of
being in the intersection

51. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ if for every positive integer i ,
- $A_i = \{-i, -i+1, \dots, -1, 0, 1, \dots, i-1, i\}$.
 - $A_i = \{-i, i\}$.
 - $A_i = [-i, i]$, that is, the set of real numbers x with $-i \leq x \leq i$.
 - $A_i = [i, \infty)$, that is, the set of real numbers x with $x \geq i$.

a) $A_i = \{-i, -i+1, \dots, -1, 0, 1, \dots, i-1, i\}$.

i is increasing and the set is getting larger $A_1 \subseteq A_2 \subseteq A_3$
 All the sets are subsets of the set of integers given, so every integer is included.

$$\bigcup_{i=1}^{\infty} A_i = \mathbb{Z}$$

And since A_i is a subset of each other

$$\bigcap_{i=1}^{\infty} A_i = A_1 = \{-1, 0, 1\}$$

b) $A_i = \{-i, i\}$.

All the sets are subsets of the set of integers and every nonzero integer is in exactly one of the sets.

$$\bigcup_{i=1}^{\infty} A_i = \mathbb{Z} - \{0\}$$

Each pair in these sets have no element in common, one will always be positive and the other negative. (e.g. $\{-1, 1\}$)

$$\bigcap_{i=1}^{\infty} A_i = \emptyset$$

