MPCS 50103 Discrete Mathematics—Autumn 2019

Homework 4: Revised. This problem set is due Monday October 28 at 11:59 pm.

Reading: Rosen 7e, chapter 6, sections 6.2, 6.4–6.5; chapter 7, sections 7.1–7.2.

Written assignment:

- Solve the following "DO" exercises and homework problems "HW".
- "DO" exercises are strongly recommended to check your understanding of the concepts. **Do not submit them**.
- **Problems labeled "HW" must be submitted**.
- You are responsible for the material covered in **both** "DO" exercises and HW problems.

"Do" Exercises (*not* to be submitted):

1. **"DO"** Rosen 7e, section 6.2, exercises 13, 23, 27, 37, and 41, on pages 405–406.
2. **"DO"** Rosen 7e, section 6.4, exercises 9, 21a, and 27a on pages 421–422.
3. **"DO"** Rosen 7e, section 6.5, exercises 1, 9, 15, 23, 35, and 45, on pages 432–434.
4. **"DO"** Rosen 7e, section 7.1, exercises 27a and 36, on pages 452–453.
5. **"DO"** Rosen 7e, section 7.2, exercises 2 and 28, on pages 466–468.

Assigned problems (to be submitted Monday October 28 at 11:59 pm):

- **Collaboration policy: If you work with others, indicate their names as part of your submission. You must answer each question by yourself without assistance. It is a violation of this policy to submit a solution that you cannot explain orally to the instructor/TAs.**
- **Looking for solutions to problems on the internet, even when acknowledged, is STRONGLY DISCOURAGED.**
- **Write out your work for every problem. If you just write your answer without showing your work, you will not receive credit.**
- **In this problem set, do not simplify exponents, factorials, or binomial coefficients. For example, write 2^8, not 256; 5!, not 120; "6 choose 3" in symbolic form, not 20.**

1. **HW** Answer each of the following counting questions and explain your answer in each case.
   - How many solutions are there to the equation:
     $$x_1 + x_2 + x_3 + x_4 + x_5 = 21,$$
     where all x_i, $i = 1, 2, 3, 4, 5$, are nonnegative integers and moreover $0 \le x_1 < 3$ and $x_3 \ge 11$?
   - In how many ways can 21 indistinguishable candies be distributed among 5 (distinguishable) children so that no child gets more than 10 candies?

   For each question, show your work and explain your answer. (4 points each)

2. **HW** Three UChicago students are ordering shakes at Five Guys on 53rd Street. Five Guys allows a customer to choose any combination of all their flavors for a milk shake (so, for instance, you can order a Banana/Chocolate/Peanut Butter shake). The flavors available are:
   Banana, Cherries, Chocolate, Coffee, Malted Milk, Oreo, Peanut Butter, Salted Caramel, Strawberries
   Other than flavors, a customer can also add Whipped Cream and Bacon. The three students want to try every single flavor, but they only want to buy one shake each. Their idea is to order three shakes, each with a combination of flavors that does not have any flavor in common with the other two shakes, but that as a whole covers all the 9 flavors.

- How many different ways are there for them to order if none of them want to add bacon or whipped cream?
- How many different ways are there if they want exactly one shake to have bacon, but any number of shakes can have whipped cream?

For each question, show your work and explain your answer. (4 points each)

3. **HW** Let Odd_n denote the **number** of subsets of an n-element set that have an odd number of elements and let Even_n denote the **number** of subsets that have an even number of elements. For $n \geq 1$, prove that $\text{Odd}_n = \text{Even}_n$. Either use the Binomial Theorem or give a combinatorial proof. (4 points)

4. **HW** Give careful and complete proofs for the following two problems.
   - Suppose S is a set of $n + 1$ integers taken from the set $\{1, 2, \ldots, 2n\}$. Prove, using the Pigeonhole Principle, that some pair of integers in S is relatively prime.
   - Prove, using the Pigeonhole Principle, that if 5 points are picked in the interior of a square with a side length of 2, then at least 2 of these points are no farther than $\sqrt{2}$ apart.

   (5 points each)

5. **HW** A box contains 10 balls, 6 of which are green and 4 of which are blue.
   - Three balls are removed from the box; the balls are wrapped in paper so that their color is hidden from view. Find the probability that a fourth ball removed from the box is blue. Assume that the 10 balls are equally likely to be drawn from the box. Show your work and explain your answer. (2 points)
   - Find the probability that all 3 of the balls removed will be green if it is known that at least one of the removed balls is green. Again, assume that each of the 10 balls is equally likely to be drawn from the box. Show your work and explain your answer. (4 points)

6. **HW** MPCS 50103 students Alice and Bob announce their RSA public keys as $(N, 13)$ and $(N, 17)$, respectively; because they are good friends, they use the same modulus N. After learning that Cletis employed RSA to send the same message to Alice and Bob, Eve succeeds at retrieving the encrypted texts that Cletis sent to the two recipients. How can Eve recover Cletis's plain text from the two encrypted texts? Show all your work and explain your answer. (6 points)

Gerry Brady
Wednesday October 23 22:39:11 CDT 2019