# Homework 6 2.6-3.2

## Section 2.6:

**5.** Find a matrix **A** such that

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \mathbf{A} = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}.$$

[*Hint:* Finding **A** requires that you solve systems of linear equations.]

**13.** In this exercise we show that matrix multiplication is associative. Suppose that $\mathbf{A}$ is an $m \times p$ matrix, $\mathbf{B}$ is a $p \times k$ matrix, and $\mathbf{C}$ is a $k \times n$ matrix. Show that $\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}$.

If $\mathbf{A}$ and $\mathbf{B}$ are $n \times n$ matrices with $\mathbf{AB} = \mathbf{BA} = \mathbf{I}_n$, then $\mathbf{B}$ is called the **inverse** of $\mathbf{A}$ (this terminology is appropriate because such a matrix $\mathbf{B}$ is unique) and $\mathbf{A}$ is said to be **invertible**. The notation $\mathbf{B} = \mathbf{A}^{-1}$ denotes that $\mathbf{B}$ is the inverse of $\mathbf{A}$.

**18.** Show that

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 3 \end{bmatrix}$$

is the inverse of

$$\begin{bmatrix} 7 & -8 & 5 \\ -4 & 5 & -3 \\ 1 & -1 & 1 \end{bmatrix}.$$

**19.** Let $\mathbf{A}$ be the $2 \times 2$ matrix

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Show that if $ad - bc \neq 0$, then

$$\mathbf{A}^{-1} = \begin{bmatrix} \frac{d}{ad - bc} & \frac{-b}{ad - bc} \\ \frac{-c}{ad - bc} & \frac{a}{ad - bc} \end{bmatrix}.$$

**27.** Let

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

Find

**a)** $\mathbf{A} \vee \mathbf{B}$. **b)** $\mathbf{A} \wedge \mathbf{B}$. **c)** $\mathbf{A} \odot \mathbf{B}$.

**28.** Find the Boolean product of $\mathbf{A}$ and $\mathbf{B}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}.$$

# Section 3.2: Growth of functions

**8.** Find the least integer $n$ such that $f(x)$ is $O(x^n)$ for each of these functions.

**a)** $f(x) = 2x^2 + x^3 \log x$

**b)** $f(x) = 3x^5 + (\log x)^4$

**c)** $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$

**d)** $f(x) = (x^3 + 5 \log x)/(x^4 + 1)$

**14.** Determine whether $x^3$ is $O(g(x))$ for each of these functions $g(x)$.

**a)** $g(x) = x^2$

**b)** $g(x) = x^3$

**c)** $g(x) = x^2 + x^3$

**d)** $g(x) = x^2 + x^4$

**e)** $g(x) = 3^x$

**f)** $g(x) = x^3/2$

**22.** Arrange the function $(1.5)^n$, $n^{100}$, $(\log n)^3$, $\sqrt{n}\log n$, $10^n$, $(n!)^2$, and $n^{99}+n^{98}$ in a list so that each function is big-$O$ of the next function.

**30.** Show that each of these pairs of functions are of the same order.

**a)** $3x + 7$, $x$

**b)** $2x^2 + x - 7$, $x^2$

**c)** $\lfloor x + 1/2 \rfloor$, $x$

**d)** $\log(x^2 + 1)$, $\log_2 x$

**e)** $\log_{10} x$, $\log_2 x$

**32.** Show that if $f(x)$ and $g(x)$ are functions from the set of real numbers to the set of real numbers, then $f(x)$ is $O(g(x))$ if and only if $g(x)$ is $\Omega(f(x))$.