Discrete Mathematics

28. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows tonight, then I will stay at home.
b) I go to the beach whenever it is a sunny summer day.
c) When I stay up late, it is necessary that I sleep until noon.

- $q \to p$ is the converse of $p \to q$ ~ <u>Switch</u> <u>Order</u>
- $\neg p \to \neg q$ is the <u>inverse</u> of $p \to q$ ~ negate
- $\neg q \to \neg p$ is the contrapositive of $p \to q$ ~ neg + switch

$p \to q$

a) If it snows tonight (p), then I will stay at home (q).

Converse: If, I stay at home, then it snowed tonight

Contrapositive: If I do <u>not</u> at stay home, then it did <u>not</u> snow tonight.

Inverse: If it does <u>not</u> snow tonight, then I will <u>not</u> stay home

b) I go to the beach (p) whenever it is a sunny summer day (q).

Converse: <u>If</u> it is a sunny summer day, <u>then</u> I will go to the beach.

Contrapositive: If it's <u>not</u> a sunny summer day then I will not go to the beach

Inverse: If I do <u>not</u> go to the beach, <u>then</u> it was <u>not</u> a sunny summer day.

c) When I stay up late, it is necessary that I sleep until noon.

35. Construct a truth table for each of these compound propositions.

a) $p \to \neg q$ **b)** $\neg p \leftrightarrow q$
c) $(p \to q) \vee (\neg p \to q)$ **d)** $(p \to q) \wedge (\neg p \to q)$
e) $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$
f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

Biconditional ($\leftrightarrow$, $\equiv$) iff → if and only if

p	q	$p \leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

$p = q$ then $p \leftrightarrow q = 1$

if they (p and q) are the same value then it's True.

a) $p \to \neg q$

p	q	¬q	p → ¬q
1	1	0	0
1	0	1	1
0	1	0	1
0	0	1	1

b) $\neg p \leftrightarrow q$

p	¬p	q	¬p ↔ q
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	0

c) $(p \to q) \vee (\neg p \to q)$

p	q	¬p	(p → q)	(¬p → q)	(p → q) ∨ (¬p → q)
1	1	0	1	1	1
1	0	0	0	1	1
0	1	1	1	1	1
0	0	1	1	0	1

d) $(p \to q) \wedge (\neg p \to q)$

p	q	¬p	(p → q)	(¬p → q)	(p → q) ∧ (¬p → q)
1	1	0	1	1	1
1	0	0	0	1	0
0	1	1	1	1	1
0	0	1	1	0	0

Conjunction ($\wedge$, $*$, -)

and

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

e) $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$

p	¬p	q	¬p ↔ q	(p ↔ q)	(p ↔ q) ∨ (¬p ↔ q)
1	0	1	0	1	1
1	0	0	1	0	1
0	1	1	1	0	1
0	1	0	0	1	1

f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

(p ↔ q)		(¬p ↔ ¬q) ↔ (p ↔ q)
1		
0		
0		
1		

Disjunction ($\vee$, +)

p	q	$p \vee q$
1	1	1
1	0	1
0	1	1
0	0	0

Conditional ($\to$, $\supset$)

p	q	$p \to q$
1	1	1
1	0	0
0	1	1
0	0	1

out

9. Show that each of these conditional statements is a tautology by using truth tables.

a) $(p \land q) \rightarrow p$ **b)** $p \rightarrow (p \lor q)$
c) $\neg p \rightarrow (p \rightarrow q)$ **d)** $(p \land q) \rightarrow (p \rightarrow q)$
e) $\neg(p \rightarrow q) \rightarrow p$ **f)** $\neg(p \rightarrow q) \rightarrow \neg q$

a) $(p \land q) \rightarrow p$

p	q	$(p \land q)$	$(p \land q) \rightarrow p$
1	1	1	1
1	0	0	1
0	1	0	1
0	0	0	1

Tautology

b) $p \rightarrow (p \lor q)$

p	q	$(p \lor q)$	$p \rightarrow (p \lor q)$
1	1	1	1
1	0	1	1
0	1	1	1
0	0	0	1

Tautology

c) $\neg p \rightarrow (p \rightarrow q)$

p	$\neg p$	q	$(p \rightarrow q)$	$\neg p \rightarrow (p \rightarrow q)$
1	0	1	1	1
1	0	0	0	1
0	1	1	1	1
0	1	0	1	1

Tautology

d) $(p \land q) \rightarrow (p \rightarrow q)$

p	q	$(p \land q)$	$(p \rightarrow q)$	$(p \land q) \rightarrow (p \rightarrow q)$
1	1	1	1	1
1	0	0	0	1
0	1	0	1	1
0	0	0	1	1

Tautology

e) $\neg(p \rightarrow q) \rightarrow p$

p	q	$(p \rightarrow q)$	$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \rightarrow p$
1	1	1	0	1
1	0	0	1	1
0	1	1	0	1
0	0	1	0	1

Tautology

f) $\neg(p \rightarrow q) \rightarrow \neg q$

p	q	$\neg q$	$(p \rightarrow q)$	$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \rightarrow \neg q$
1	1	0	1	0	1
1	0	1	0	1	1
0	1	0	1	0	1
0	0	1	1	0	1

Tautology

11. Show that each conditional statement in Exercise 9 is a tautology without using truth tables.

a) $(p \wedge q) \rightarrow p$

22. Show that $(p \to q) \wedge (p \to r)$ and $p \to (q \wedge r)$ are logically equivalent.

All different Variations

p	q	r	$p \to q$	$p \to r$	$p \to q \wedge p \to r$	$(q \vee r)$	$p \to (q \wedge r)$
1	1	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1
0	0	1	1	1	1	0	1
0	1	0	1	1	1	0	1
1	0	1	0	1	0	0	0
0	0	0	1	1	1	0	1

(logically Equivalent

32. Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.

p	q	r	$(p \wedge q)$	$(p \wedge q) \rightarrow r$	$(p \rightarrow r)$	$(q \rightarrow r)$	$(p \rightarrow r) \wedge (q \rightarrow r)$
1	1	1					
1	1	0					
1	0	0					
0	1	1					
0	0	1					
0	1	0					
1	0	1					
0	0	0					

8. Translate these statements into English, where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the domain consists of all animals.

a) $\forall x(R(x) \rightarrow H(x))$ **b)** $\forall x(R(x) \wedge H(x))$
c) $\exists x(R(x) \rightarrow H(x))$ **d)** $\exists x(R(x) \wedge H(x))$

R(x): "x is a rabbit"
H(x): "x hops"

$\forall x$: for all
$\exists x$: there exists

a) $\forall x(R(x) \rightarrow H(x))$

For all animals that is a rabbit, then that animal hops.

And

b) $\forall x(R(x) \wedge H(x))$

All

For all animals are rabbits and they hop.

c) $\exists x(R(x) \rightarrow H(x))$

There exists an Animal, that if is a rabbit, then it hops.

d) $\exists x(R(x) \wedge H(x))$

There exists an animal that is an Rabbit and that animal hops.

61. Determine whether each of these compound propositions is satisfiable.

a) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

b) $(p \rightarrow q) \wedge (p \rightarrow \neg q) \wedge (\neg p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$

c) $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$

a) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

1.4

12. Let $Q(x)$ be the statement "$x + 1 > 2x$." If the domain consists of all integers, what are these truth values?

a) $Q(0)$ **b)** $Q(-1)$ **c)** $Q(1)$
d) $\exists x Q(x)$ **e)** $\forall x Q(x)$ **f)** $\exists x \neg Q(x)$
g) $\forall x \neg Q(x)$

a) $Q(0)$

$x + 1 > 2x$
$0 + 1 > 2(0)$
$1 > 0$

TRUE

b) $Q(-1)$

$x + 1 > 2x$
$-1 + 1 > 2(-1)$
$0 > -2$

TRUE

c) $Q(1)$

$x + 1 > 2x$
$1 + 1 > 2(1)$
$2 > 2$

FALSE

d) $\exists x Q(x)$

$x + 1 > 2x$
$0 + 1 > 2(0)$ →
$1 > 0$

TRUE

e) $\forall x Q(x)$

$x + 1 > 2x$
$1 + 1 > 2(1)$
$2 > 2$ FALSE

FALSE

f) $\exists x \neg Q(x)$

TRUE

g) $\forall x \neg Q(x)$

FALSE

13. Determine the truth value of each of these statements if the domain consists of all integers.

a) $\forall n(n+1>n)$ **b)** $\exists n(2n=3n)$

c) $\exists n(n=-n)$ **d)** $\forall n(3n \le 4n)$

a) $\forall n(n+1>n)$

$n+1>n$

$0+1>0$

$1>0$ ✓

TRUE

b) $\exists n(2n=3n)$

$2n=3n$

$n=\frac{3}{2}n$

$\frac{2}{3}=\frac{3}{2}\cdot\frac{2}{3}$ ~ 3/2 isnt an integer.

FALSE

c) $\exists n(n=-n)$

$n=-n$

FALSE

d) $\forall n(3n \le 4n)$

$3n \le 4n$

TRUE

27. Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables.

a) A student in your school has lived in Vietnam.

b) There is a student in your school who cannot speak Hindi.

c) A student in your school knows Java, Prolog, and C++.

d) Everyone in your class enjoys Thai food.

e) Someone in your class does not play hockey.

a) A student in your school has lived in Vietnam.

1^{st} Domain: People in the school
2^{nd} Domain: People in the World

A(x) Represents "x is in your school"
V(x) Represents "x has lived in Vietnam"
V(x,y) Represents "x has lived in y"

Varying the Domain: $\exists x\, V(x)$ ~ There exists a student who lived in Vietnam.

Predicate with one Variables: $\exists x\,(V(x) \wedge A(x))$

Predicate with two Variables: $\exists x\,(A(x) \wedge V(x, \text{Vietnam}))$

b) There is a student in your school who cannot speak Hindi.

30. Suppose the domain of the propositional function $P(x, y)$ consists of pairs x and y, where x is 1, 2, or 3 and y is 1, 2, or 3. Write out these propositions using disjunctions and conjunctions.

a) $\exists x\, P(x, 3)$

b) $\forall y\, P(1, y)$

c) $\exists y \neg P(2, y)$

d) $\forall x\, \neg P(x, 2)$

39. Translate these specifications into English where $F(p)$ is "Printer p is out of service," $B(p)$ is "Printer p is busy," $L(j)$ is "Print job j is lost," and $Q(j)$ is "Print job j is queued."

a) $\exists p(F(p) \wedge B(p)) \rightarrow \exists j L(j)$

b) $\forall p B(p) \rightarrow \exists j Q(j)$

c) $\exists j(Q(j) \wedge L(j)) \rightarrow \exists p F(p)$

d) $(\forall p B(p) \wedge \forall j Q(j)) \rightarrow \exists j L(j)$

a) $\exists p(F(p) \wedge B(p)) \rightarrow \exists j L(j)$

50. Show that $\forall x\, P(x) \vee \forall x\, Q(x)$ and $\forall x\, (P(x) \vee Q(x))$ are not logically equivalent.