Homework #9 Sections 6.1 - 7.1

Section 6.1

1. There are 18 mathematics majors and 325 computer science majors at a college.

    a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?

    b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?

6. There are four major auto routes from Boston to Detroit and six from Detroit to Los Angeles. How many major auto routes are there from Boston to Los Angeles via Detroit?

7. How many different three-letter initials can people have?

Section 6.2

3. A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.

  a) How many socks must he take out to be sure that he has at least two socks of the same color?

  b) How many socks must he take out to be sure that he has at least two black socks?

6. Let d be a positive integer. Show that among any group of $d + 1$ (not necessarily consecutive) integers there are two with exactly the same remainder when they are divided by d.

12. How many ordered pairs of integers (a, b) are needed to guarantee that there are two ordered pairs (a_1, b_1) and (a_2, b_2) such that $a_1 \bmod 5 = a_2 \bmod 5$ and $b_1 \bmod 5 = b_2 \bmod 5$?

Section 6.3

11. How many bit strings of length 10 contain

    a) exactly four 1s?

    b) at most four 1s?

    c) at least four 1s?

    d) an equal number of 0s and 1s?

19. A coin is flipped 10 times where each flip comes up either
heads or tails. How many possible outcomes

    a) are there in total?
    b) contain exactly two heads?
    c) contain at most three tails?
    d) contain the same number of heads and tails?

21. How many permutations of the letters *ABCDEFG* contain

    a) the string *BCD*?

    b) the string *CFGA*?

    c) the strings *BA* and *GF*?

    d) the strings *ABC* and *DE*?

    e) the strings *ABC* and *CDE*?

    f) the strings *CBA* and *BED*?

33. Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have the same number of men and women?

Section 6.4

8. What is the coefficient of $x^8 y^9$ in the expansion of $(3x + 2y)^{17}$?

9. What is the coefficient of $x^{101}y^{99}$ in the expansion of $(2x - 3y)^{200}$?

12. The row of Pascal's triangle containing the binomial coefficients $\binom{10}{k}$, $0 \le k \le 10$, is:

1 10 45 120 210 252 210 120 45 10 1

Use Pascal's identity to produce the row immediately following this row in Pascal's triangle.

21. Prove that if n and k are integers with $1 \leq k \leq n$, then
$k \binom{n}{k} = n \binom{n-1}{k-1}$,

    a) using a combinatorial proof. [*Hint:* Show that the two sides of the identity count the number of ways to select a subset with k elements from a set with n elements and then an element of this subset.]

    b) using an algebraic proof based on the formula for $\binom{n}{r}$ given in Theorem 2 in Section 6.3.

∗32. Prove the binomial theorem using mathematical induction.

Section 6.5

5. How many ways are there to assign three jobs to five employees if each employee can be given more than one job?

8. How many different ways are there to choose a dozen donuts from the 21 varieties at a donut shop?

9. A bagel shop has onion bagels, poppy seed bagels, egg bagels, salty bagels, pumpernickel bagels, sesame seed bagels, raisin bagels, and plain bagels. How many ways are there to choose

   a) six bagels?

   b) a dozen bagels?

   c) two dozen bagels?

   d) a dozen bagels with at least one of each kind?

   e) a dozen bagels with at least three egg bagels and no more than two salty bagels?

15. How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 21,$$

where x_i, $i = 1, 2, 3, 4, 5$, is a nonnegative integer such that

a) $x_1 \geq 1$?
b) $x_i \geq 2$ for $i = 1, 2, 3, 4, 5$?
c) $0 \leq x_1 \leq 10$?
d) $0 \leq x_1 \leq 3$, $1 \leq x_2 < 4$, and $x_3 \geq 15$?

18. How many strings of 20-decimal digits are there that contain two 0s, four 1s, three 2s, one 3, two 4s, three 5s, two 7s, and three 9s?

27. There are 10 questions on a discrete mathematics final exam. How many ways are there to assign scores to the problems if the sum of the scores is 100 and each question is worth at least 5 points?

32. How many different strings can be made from the letters in *AARDVARK*, using all the letters, if all three *A*s must be consecutive?

6. What is the probability that a card selected at random from a standard deck of 52 cards is an ace or a heart?

14. What is the probability that a five-card poker hand contains cards of five different kinds?

18. What is the probability that a five-card poker hand contains a straight flush, that is, five cards of the same suit of consecutive kinds?

22. What is the probability that a positive integer not exceeding 100 selected at random is divisible by 3?