

Direct Proofs

Terminology

Definition: Something given (no have to Proof)

Theorem: Something to be proved

Results corollary

example

$$\text{Let } m = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

~ never have to proof
b/c we gave it a def.

Direct Proof.

If φ , then ψ .

Assume φ

Show that ψ .

Prove: If x is odd, x^2 is odd

Assume x is odd

$$x = 2n + 1$$

$$x^2 = (2n + 1)^2$$

$$= 4n^2 + 4n + 1$$

$$= 2n(2n + 2) + 1 \sim \text{Proven that } x^2 \text{ is also odd}$$

Note: We know that this will
compute a odd number

Assume antecedent True

Prove conclusion/Consequence

Prove: If x, y are odd, then xy is odd.

$$x = 2k + 1$$

$$y = 2j + 1$$

Case 3 ✓

$$xy = (2k + 1)(2j + 1)$$

$$= 4kj + 2k + 2j + 1$$

Odd

← The important thing is
that we have a + 1
at the end.
So we can say it's
odd.

If we said $X \text{ or } Y$

Case 1 $x = 2k + 1$ ✓
 $y = 2j + 1$

Case 2 $x = 2k$ ✓
 $y = 2j + 1$

- Would have to prove
all 3 cases.

Direct Proof Examples

Prove: If $5|2a$ for $a \in \mathbb{Z}$, then $5|a$

Prove: Every odd integer is a difference of two squares. ($13 = 7^2 - 6^2$)

Certesian Product Proof Examples

Prove that $A \times (B - C) = (A \times B) - (A \times C)$

Prove that $A \times B \subseteq C \times D$ iff $A \subseteq C$ and $B \subseteq D$

Proof by Case

Prove: $\forall \psi \rightarrow X$

Assume ψ
Show X .

Assume ψ
Show X

Prove: If $n \in \mathbb{Z}$, $n^2 + 3n + 4$ is even

Proof by Cases Examples

Prove: If $m+n$ and $n+p$ are even, where $m, n, p \in \mathbb{Z}$, then $m+p$ is even.

Prove: If $x, y \in \mathbb{R}$, then $\max(x, y) + \min(x, y) = x + y$

Proof by Contraposition

Prove: $\varphi \rightarrow \psi$

Assume $\neg \psi$
Show $\neg \varphi$

Prove: If $7x+9$ is even, then x is odd.

Proof by Contradiction

We want to prove φ

- ① Assume $\neg \varphi$
- ② Find some contradiction $\varphi \wedge \neg \varphi$
- ③ Claim $\neg \neg \varphi$
 $= \varphi$

Show that $\sqrt{2}$ is irrational

Prove that $(A-B) \cap (B-A) = \emptyset$

Mathematical Induction