Homework #10 Sections 7.2, 9.1-9.3, 9.4-9.5

Section 7.2

3. Find the probability of each outcome when a biased die is rolled, if rolling a 2 or rolling a 4 is three times as likely as rolling each of the other four numbers on the die and it is equally likely to roll a 2 or a 4.

7. What is the probability of these events when we randomly select a permutation of $\{1, 2, 3, 4\}$?

a) 1 precedes 4.
b) 4 precedes 1.
c) 4 precedes 1 and 4 precedes 2.
d) 4 precedes 1, 4 precedes 2, and 4 precedes 3.
e) 4 precedes 3 and 2 precedes 1.

10. What is the probability of these events when we randomly select a permutation of the 26 lowercase letters of the English alphabet?

a) The first 13 letters of the permutation are in alphabetical order.

b) a is the first letter of the permutation and z is the last letter.

c) a and z are next to each other in the permutation.

d) a and b are not next to each other in the permutation.

e) a and z are separated by at least 23 letters in the permutation.

f) z precedes both a and b in the permutation.

Section 9.1

2. **a)** List all the ordered pairs in the relation $R = \{(a, b) \mid a \text{ divides } b\}$ on the set $\{1, 2, 3, 4, 5, 6\}$.

b) Display this relation graphically, as was done in Example 4.

c) Display this relation in tabular form, as was done in Example 4.

6. Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if

a) $x + y = 0$.
b) $x = \pm y$.
c) $x - y$ is a rational number.
d) $x = 2y$.
e) $xy \geq 0$.
f) $xy = 0$.
g) $x = 1$.
h) $x = 1$ or $y = 1$.

8. Show that the relation $R = \emptyset$ on a nonempty set S is symmetric and transitive, but not reflexive.

27. Let R be the relation $R = \{(a, b) \mid a \text{ divides } b\}$ on the set of positive integers. Find

a) R^{-1}.

b) $\overline{R}$.

43. How many of the 16 different relations on $\{0, 1\}$ contain the pair $(0, 1)$?

49. Find the error in the "proof" of the following "theorem."

"*Theorem*": Let R be a relation on a set A that is symmetric and transitive. Then R is reflexive.

"*Proof*": Let $a \in A$. Take an element $b \in A$ such that $(a, b) \in R$. Because R is symmetric, we also have $(b, a) \in R$. Now using the transitive property, we can conclude that $(a, a) \in R$ because $(a, b) \in R$ and $(b, a) \in R$.