Mathematical Induction Chapter 5.1-5.4

Proof Using Mathematical Induction - Summation Formulas

Mathematical Induction

To prove $P(x)$ is true for $x \in \mathbb{Z}^+$, where $P(x)$ is a propositional function, we complete two steps:

1.) Basis Step - Verify $P(1)$ is true

2.) Inductive step - Verify if $P(K)$ is true, then $P(K+1)$ is true $\forall K \in \mathbb{Z}^+$

Inductive Hypothesis: $P(K)$ is true

Must Show: $P(K) \rightarrow P(K+1)$

Conclusion: $P(x)$ is true $\forall K \in \mathbb{Z}^+$

Proving a Summation Formula

$\sum_{i=1}^{i}$

Show $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

Use mathematical induction to show that for all non-negative integer n, $1+2+2^2+\ldots+2^n=2^{n+1}-1$

Conjective and prove a summation formula for the sum of the first n positive odd integers.

Prove $1 + 3 + 5 + \dots + (2n - 1) = n^2$

Proof Using Mathematical Induction - Inequalities

Proving an Inequality:

Prove $n < 2^n$ $\forall n \in \mathbb{Z}^+$ using mathematical induction.

Let $P(n)$: $n < 2^n$ $\forall n \in \mathbb{Z}^+$

1. Basis: $P(1)$

$P(1)$: $1 < 2^1$

$1 < 2^1$

2. Inductive $P(k) \rightarrow P(k+1)$

Prove $2^n < n!$ $\forall n \in \mathbb{Z}^+$ and $n \geq 4$.

1. Basis $P(4)$

2. Inductive $P(k) \rightarrow P(k+1)$

Proof Using Mathematical Induction - Divisibilty

Use mathematical induction to prove $7^{n+2} + 8^{2n+1}$ is divisible by 57 for all non-negetive integers n.

Let $P(n)$: $7^{n+2} + 8^{2n+1}$ is divisible by 57

① Basis: $P(0)$

$P(0)$: $7^{0+2} + 8^{2(0)+1} = 7^2 + 8 =$

② Inductive:

IH: $7^{k+2} + 8^{2k+1}$

The Well-Ordering Principle and Strong Induction

The Well-Ordering Principle

Every non-empty set of non-negative integers has a least element

$\mathbb{Z}^+$ $\qquad$ $n = 1$

$n \in \mathbb{Z}^+$ $\quad$ $n \geq 4$ $\qquad$ $n = 4$

non-negative int $\qquad$ $n = 0$

Prove that every amount of postage of \$ 0.12 or more can be formed using \$ 0.04 and \$ 0.05 stamps.

$P(n)$ is the statement that postage of n-cents can be formed using 4-cent and 5-cent stamps if $n \geq 12$.

Basis: show postage of 12-cents can be made

Inductive: show if $P(k)$ is true, then $P(k+1)$ is true for $k \geq 12$

Revisiting Recursive Definitions

Fibonacci Numbers

Recall the set of Fibonacci numbers:

0, 1, 1, 2, 3, 5, 8, 13, 21, ...

How each number found?

add 2 previous terms

① Initial conditions

$f_0 = 0$

$f_1 = 1$

② Function

$f_n = f_{n-1} + f_{n-2}, \quad n \geq 2$

Recursive Defined Functions

Basis Step: Specifies the value of the function for the first term(s)

Recursive Step: Gives a rule for finding subsequent values using a previous value(s) beginning at those defined in the basis step.

If f is defined recursively by $f(0) = 2$ and $f(n+1) = 3f(n) - 1$, find $f(1)$, $f(2)$, $f(3)$, and $f(4)$.

Give a recursive definition for a^n for $a \in \mathbb{R}$ and non-negative and $n \in \mathbb{Z}^+$

Give a recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3 \ldots$ if $a_n = 2n + 1$

Recursively Defined Sets and structures

Basis step: Specifies an inital collection of elements

Recursive step: Gives rules for forming new elements from those already in the set.

Recursively Defined Set

Let S be a subset of the integers defined

recursively by:

Basis step:

Recursive Step:

List the elements of S produced by the first 5 application of the recursive definition.

Recursively Defined structure

A set of rooted trees, where a rooted tree consists of a set of vertices

Example of Rooted Trees Defn:

Basis Step:

Recursive Step 1:

Step 2:

Structurel Induction

Recursive Algorithms