Introduction to Propositional Logic

Propositional Logic

A **statement** is a **declarative** sentence that can be **True (1)** or **False (0)**

Boolean logic ↗ in computer science ★

Examples of statements

- Milk is white T
- $|\emptyset| = 0$ ⇒ T
- Humans are just fish with legs ⇒ F

The point is not ~~whethere~~ whether or not these statements are True or False but rather that **they are statements**, which means in **propositional logic** we **can express them**.

- We can't express things like
    - Question, imperatives

Syntax

Propositions are denoted with **capital letters** P, Q, R, ...

P = 1 Cheated
Q = 1 Wrote an exam

Lowercase letters p, q, r... are used: for general propositions that have no meaning.

→ Used for general proofs

Connectives (some notations) · to change meanings

☆ Notations ☆

- p is a well-formed formula (wff)
- $\neg p$ is a wff — not p
- $p \wedge q$ is a wff — p and q ☆
- $p \vee q$ is a wff — p or q
- $p \rightarrow q$ is a wff — if p then q
- Implication → "If, then" $p \rightarrow q$

Example

Translate the following into English

P = I cheat R = I write an Exam

Q = I will get caught S = I will fail

- $(R \wedge P) \rightarrow (Q \wedge S)$

if I write an exam and I cheat

then I will get caught and I will fail

Example:

Translate into Propositional Logic:

if James does not die then Mary will not get any money and James' family will be happy.

$(\neg P) \rightarrow (\neg Q \wedge R)$

* not is a connective *

define our Keys

P = James dies

Q = Mary will get money

R = James' family will be happy

PR.3: IMPLICATIONS

Give the converse, inverse and contrapositive of the conditional statement:

Prof. B is happy when you get your homework done on time.

Converse $q \to p$

Inverse $\neg p \to \neg q$

Contrapositive $\neg q \to \neg p$

KEY

① First write in "If P, then q" form.
-So it's clear what is p and what is q

Prof. B is happy when you get your homework done on time.

Rewrite: If $\underbrace{\text{you get your homework done on time}}_{p}$, then $\underbrace{\text{Prof. B is happy}}_{q}$

Which property implies the other property.

★ "You have to get your HW done on time" ~ Happens 1st

p is called the hypothesis
q is called the conclusion

Converse $q \to p$ If Prof. B is happy, then you get your homework done on time.

Inverse $\neg p \to \neg q$

If you did not get your homework done on time, Prof. B is not happy

Contrapositive $\neg q \to \neg p$

If Prof. B is not happy, then you did not get your homework done on time.

More Notes

Implication $p \to q$ "If p, then q"

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

MORE ON IMPLICATIONS

In $p \to q$ there does not need to be any connection between the antecedent or the consequent. The "meaning" of $p \to q$ depends only on the truth values of p and q.

These implications are perfectly fine, but would not be used in ordinary English.

p

- "If the moon is made of green cheese, then I have more money than Bill Gates. "
- "If 1 + 1 = 3, then your grandma wears combat boots."

q

"Are related"

One way to view the logical conditional is to think of an obligation or contract.

- "If I am elected, then I will lower taxes."
- "If you get 100% on the final, then you will get an A."

If the politician is elected and does not lower taxes, then the voters can say that he or she has broken the campaign pledge. Something similar holds for the professor. This corresponds to the case where p is true and q is false.

Looking just at truth values.

IMPLICATIONS: CONVERSE, INVERSE, CONTRA-POSITIVE

From $p \to q$ we can form new conditional statements .

- $q \to p$ is the converse of $p \to q$ Switch Order
- $\neg p \to \neg q$ is the inverse of $p \to q$ negate
- $\neg q \to \neg p$ is the contrapositive of $p \to q$ neg + switch

Example: Find the converse, inverse, and contrapositive of "It raining is a sufficient condition for my not going to town."

If it is raining, then I will not go to town

p q

converse: If I do not go to town, then it is raining.
inverse: If it is not raining, then I will go to town. ✓
contrapositive: If I go to town, then it is not raining.

* Note that only the contrapositive has the same truth values as the original conditional statement. We call these equivalent. The converse and inverse are equivalent to one another, as well.

Applications of Propositional Logic

TRANSLATING ENGLISH SENTENCES

Steps to convert an English sentence to a statement in propositional logic using:

* "If I go to Harry's or to the country, I will not go shopping."

p I go q not r

Step 1: Identify atomic propositions and represent using propositional variables.

- p: I go to Harry's
- q: I go to the country.
- r: I will go shopping

Step 2: Determine appropriate logical connectives

- If p or q, then not r
- $(p \vee q) \rightarrow \neg r$

If p or q

PR.1: TRANSLATING ENGLISH SENTENCES

Convert each sentence into propositional logic.

a. You can access the Internet from campus only if you are a computer science major or you are not a freshman.

b. The automated reply cannot be sent when the file system is full.

a. You can access the Internet from campus only if you are a computer science major or you are not a freshman.

p q r

p: Campus internet access
q: computer science major
r: freshman

"if p, then q or not r

$\boxed{p \rightarrow (q \vee \neg r)}$

b. The automated reply cannot be sent when the file system is full.

p: automated reply can be sent
q: file system is full

$\boxed{q \rightarrow \neg p}$

"If q, then p"

CONSISTENT SYSTEM SPECIFICATIONS

Definition: A list of propositions is *consistent* if it is possible to assign truth values to the proposition variables so that each proposition is true.

Exercise: Are these specifications consistent?

1. • "The diagnostic message is stored in the buffer or it is retransmitted."
2. • "The diagnostic message is not stored in the buffer."
3. • "If the diagnostic message is stored in the buffer, then it is retransmitted."

p q 3

Solution: Let p denote "The diagnostic message is stored in the buffer." Let q denote "The diagnostic message is retransmitted" The specification can be written as:

$$(1)\ p \vee q,\ (2)\ \neg p,\ (3)\ p \to q$$

When p is false and q is true all three statements are true. So the specification is consistent.

F T

$p \vee q,$

look @ Truth Value

T

¬F = T

$\neg p,$

look @ Truth Value

T

F → T

$p \to q$

look @ Truth Value

T

PR.2: LOGIC PUZZLES (P.23 #18)

When planning a party, you want to know whom to invite. Among the people you would like to invite are three touchy friends. You know that if Jasmine attends, she will become unhappy if Samir is there, Samir will attend only if Kanti will be there, and Kanti will not attend unless Jasmine also does. Which combinations of these three friends can you invite so as not to make someone unhappy?

Yo

PR.3: LOGIC PUZZLES (P.23 #32A)

The police have three suspects for the murder of Mr. Cooper: Mr. Smith, Mr. Jones, and Mr. Williams. Smith, Jones, and Williams each declare that they did not kill Cooper. Smith also states that Cooper was a friend of Jones and that Williams disliked him. Jones also states that he did not know Cooper and that he was out of town the day Cooper was killed. Williams also states that he saw both Smith and Jones with Cooper the day of the killing and that either Smith or Jones must have killed him. Can you determine who the murderer was there is only murderer, and the two innocent men are telling the truth but the guilty man may be lying?

LOGIC PUZZLES

An island has two kinds of inhabitants, knights, who always tell the truth, and knaves, who always lie. You go to the island and meet A and B. A says "B is a knight.". B says "The two of us are of opposite types." What are A and B? Let p represent that A is a knight and q represent that B is a knight.

- Suppose A is a knight. Then p is true. Since a knight tells the truth, then q is also true. But this violates B's statement that A and B are different types. Since B would have to tell the truth, this scenario doesn't work.

~~$p \quad q$~~

~~$p \quad \neg q$~~ Not possible

$p \quad q$

$p \quad q$

- If A is a knave. He tells us that B is a knight, but is must be lying because he is a knave. Therefore B is also a knave. B tells us that he and A are of a different type, but must be lying since he is a knave. Therefore, both A and B are knaves. This is the only logical conclusion.

LOGIC PUZZLES

Can we represent this with a truth table? Yes, but it may be easier without using p and q. In order to find the truth, the values after each statement must match the possibilities on the left. Try this with me. (Note, I have filled in the A column for the first statement and B for the second statement, as those values wouldn't change.

Our Four Possibilities		A says "B is a knight"		B says "The two of us are of opposite type"	
A	B	A	B	A	B
Knight	Knight	Knight			Knight
Knight	Knave	Knight			Knave
Knave	Knight	Knave			Knight
Knave	Knave	Knave			Knave

Statement and Translation Examples

Translate the following sentences, given the following statements:

p: I finish writing my computer program before lunch
q: I shall play tennis in the afternoon
r: The sun is shining
s: The humidity is low

usually most difficult

Same as saying

p is necessary for q

$q \to p$

p is sufficient for q

$p \to q$

Problems:

1.) If the sun is shining, I shall play tennis this afternoon.

if statement

$\boxed{r \to q}$

- 2 statements

2.) Finishing the writing of my computer program before lunch is necessary for my playing tennis this afternoon.

p

connective

q

p is necessary for q

$q \to p$

Same as saying

$\boxed{q \to p}$

- 2 statements

3.) Low Humidity and sunshine are sufficient for me to play tennis this afternoon.

r

s

connectives

q

$\boxed{(s \wedge r) \to q}$

Sufficient for playing tennis this afternoon? (q)

P is sufficient for q

$p \to q$

Question 2

Determine which is a statement.

of the form

P

1) In 1999, Barack Obama released a K-pop album

Yes, it's a statement b/c it has a truth value, it is either True or False

2.) $17y + 20x$ is an integer $P(x, y)$

? of the form

No, not a statement b/c y and x are variables
it's not a statement if it has a variable in it.
~ tricky

3.) Tell me the time

No, this is a command

4.) I can't live without you. P

Yes, a statement
~ Can be True or False

Truth Tables

Logic

Recall: Each statements is True or False
1 0

~ Truth Tables will tell us all possible outcomes of statements and connectives !

- 32 is even T/1
or
- $A \subseteq B$ iff $x \in B$ implies $x \in A$ F/0

★ All ~~connection~~ connectives take a ~~limit~~ Truth value and output a truth value. ★

Taking a look at each Connective:

Negation ($\neg$, ~)

Truth Tables shows all possible combinations of truth conditions

Truth Table

P	¬P
1	0
0	1

shows all possible combination of truth conditions (P can either be True or False)

What does Negation do?

if P is True then the negation of P is going to be false (Vice-Verse)

$Neg\ P = 1 - P$

example: $P = 1 \quad \neg P = 1 - P$
$= 0$

nice way to find the value of the negation.

Mathematical way of looking at the problem

Conjunction ($\wedge$, $*$, $-$)

and

mostly use the carrot ($\wedge$) symbol

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

p and q

\# of rows = $2^{\#\text{ of statements}}$

All possible combinations

Conjunction: compound statement formed by joining two statements with the connector "and"

$p \wedge q = \min(p, q)$

The min of $p \wedge q$

← lowest value in row is 1

lowest value in row is 0

Mathematical Way of looking at this.

What does p and q do to the truth value?
~ only true when both p and q are true
~ in every other scenario it's false

Inclusive
or
Disjunction ~ "or"

Disjunction (V, +)

p	q	p V q
1	1	1
1	0	1
0	1	1
0	0	0

mathmatically ↓
$p \vee q = \max(p, q)$

p and q are true if at least one of p, q are true.

← neither p or q are true so p v q is false

Conditional ($\rightarrow$, $\supset$)

if p then q

p	q	p → q
1	1	1
1	0	0
0	1	1
0	0	1

it is only false if P is true and Q is false, the rest of the times it will be true

← Practice statement

if its sunny
I will wear
sunscreen

It's not sunny out

mathmatically
$p \rightarrow q = 1$
iff $p \leq q$
2^{nd} senerio : $1 \not\leq 0 = 0$

KEY
When am I lying to you?
I am lying to you if it is sunny out (which would be P is true) and I'm not wearing sunscreen.

This is like my promise to you I'm saying look it it's sunny out I promise I will wear sunscreen and if it is sunny and I'm not wearing sunscreen I lied to you therefore the conditional is false. But if it's not sunny outside so I am not lying to you am I.

It doesn't matter if I'm wearing sunscreen or not it's not sunny outside therefore I'm not violating any truth condition therefore it is true in the bottom two cases ☆

2 more connectives

Biconditional ($\leftrightarrow$, $\equiv$) iff $\rightarrow$ if and only if

p	q	p $\leftrightarrow$ q
1	1	1
1	0	0
0	1	0
0	0	1

$p = q$ then $p \leftrightarrow q = 1$

if they (p and q) are the same value then it's True.

Exclusive Or ($\oplus$, $\underline{\vee}$)

p	q	p $\oplus$ q
1	1	0
1	0	1
0	1	1
0	0	0

- opposite of the biconditional

$p \neq q$

then $p \oplus q = 1$

Proofs with Truth Tables

Formula: p and q are logically equivalent iff the truth conditions of p are the same as the truth conditions of q!

$p \Leftrightarrow q$ iff

p	q
x	x
y q	q y

the same, so p and q are logically equivalent.

Example:

Is $(p \wedge q) \Leftrightarrow \neg(p \vee q)$? (logically Equivalent)

NO! why?

~ The truth conditions for P and q ($p \wedge q$) and the truth conditions for $\neg(p \vee q)$ are not identical, there these are not logically equivalent.

"or"

p	q	$p \wedge q$	$p \vee q$	$\neg(p \vee q)$
1	1	1	1	0
1	0	0	1	0
0	1	0	1	0
0	0	0	0	1

NOT THE SAME

Steps to solve logically Equivalent Problems:

1st: Build truth table for p and q

- Then solve each column

For $\neg(p \vee q)$ we take the negation of the previous column $(p \vee q)$

2nd: Check statement

$\neg$: not
~ reverse value

example 2:

Is $\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$? Yes, they're logically equivalent

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p$	$\neg q$	$\neg p \vee \neg q$
1	1	1	0	0	0	0
1	0	0	1	0	1	1
0	1	0	1	1	0	1
0	0	0	1	1	1	1

∴ After doing the truth table, you can look at the truth conditions for the two formulas we want to compare

- By being logically equivalent if I ever have $\overbrace{\text{not P and q}}^{\neg(p \wedge q)}$ in a proof somewhere I can substitute that in with $\underbrace{\text{not P or not q}}_{(\neg p \vee \neg q)}$ b/c they're exactly the same thing.

Exercise:

Show that $(p \vee \neg p)$ is always true.
~ a tautology.

~ every single value in our truth table is going to be 1

p	$\neg p$	$p \vee \neg p$
1	0	1
0	1	1

→ Every single output is a 1

p or not p could be more elaborate

$\underbrace{(a \wedge b)}_{p} \vee \underbrace{\neg(a \wedge b)}_{\neg p}$] Combination of composition

Exercise:

Show that $(p \vee \neg p)$ is always False. (otherwise known as a contradiction)
~ a contradiction

p	$\neg p$	$p \wedge \neg p$
1	0	0
0	1	0

→ Every single output is a 0

Proofs using Truth Tables

Truth Table Examples

Logic

Example 1:

Draw the Truth Table for $\neg(p \vee \neg q) \to \neg p$

p	q	$\neg p$	$\neg q$	$p \vee \neg q$	$\neg(p \vee \neg q)$	$\neg(p \vee \neg q) \to \neg p$
1	1	0	0	1	0	1
1	0	0	1	1	0	1
0	1	1	0	0	1	1
0	0	1	1	1	0	1

for this column

Tautology ~ all outcomes in truth table is True (1).

$(\neg p \wedge q) \to \neg p$

Example 2:

$[p \wedge (p \to q)] \to q$

p	q	$p \to q$	$p \wedge (p \to q)$	$(p \wedge (p \to q)) \to q$
1	1	1	1	1
1	0	0	0	1
0	1	1	0	1
0	0	1	0	1

Tautology

Exclusive Or Example.

Example 1:

Write the Truth table for Exclusive or, ($p \oplus q$), then state whether $(p \oplus p) \oplus p$ is a tautology, contradiction or neither.

Xor

p	q	$p \oplus q$
1	1	0
1	0	1
0	1	1
0	0	0

p	$p \oplus p$	$(p \oplus p) \oplus p$
1	0	1
0	0	0

not different

- We can say $(p \oplus p) \oplus p = p$, logically equivalent, neither a tautology nor a contradiction.

Example 2:

Give the Truth table for $(p \oplus q) \vee (p \oplus \neg q)$

p	q	$\neg q$	$\overset{x}{p \oplus q}$	$\overset{y}{p \oplus \neg q}$	$x \vee y$
1	1	0	0	1	1
1	0	1	1	0	1
0	1	0	1	0	1
0	0	1	0	1	1

- Tautology

$$(p \oplus q) \vee (p \oplus \neg q) = 1$$

Sheffer Stroke Examples

~ Another logical operator

$p \uparrow q$ is to be read as "p nand q", and is equivalent to $\neg(\underbrace{p \wedge q}_{\neg p \vee \neg q})$.

not P or not q

Provide the truth table for $(p \uparrow q)$ and $(p \uparrow p)$.

p	q	$p \uparrow q$
1	1	0
1	0	1
0	1	1
0	0	1

at least one of them in the columns are false, so $p \uparrow q$ is True

equivalent

p	$p \uparrow p \Leftrightarrow \neg p$
1	0
0	1

$\neg \varphi \Leftrightarrow \varphi \uparrow \varphi$ ← Sheffer Stroke

Questions

1.) Using the sheffer stroke, provide the definition for $(p \wedge q)$

$$p \uparrow q \Leftrightarrow \neg(p \wedge q)$$

$$(p \wedge q) \Leftrightarrow \neg\neg(p \wedge q)$$

$$\Leftrightarrow \neg(p \uparrow q)$$

$$\Leftrightarrow (p \uparrow q) \uparrow (p \uparrow q)$$

$\neg \varphi \Leftrightarrow \varphi \uparrow \varphi$

$\varphi = p \uparrow q$

2.) find definition for $p \vee q$

$$(p \vee q) \quad \neg\neg p \vee \neg\neg q$$

$$\neg(\neg p \wedge \neg q)$$

Similar to $\neg(\underbrace{p \wedge q}_{\text{the sheffer stroke}})$

Something as need to define these in terms of the sheffler stroke

This "not", "and" is the sheffler stroke itself so our definition of $(p \vee q)$ is just

$$(p \uparrow p) \uparrow (q \uparrow q)$$

Logical Equivalent

Logic

Logic Laws

- We can use logical equivalences to reduce complex formulas into simpler ones

★ logical Equivalence is denoted ★ ↳ $\Leftrightarrow$ or $\equiv$

- Two new symbols

Truth table (always look like)

T	F
1	0
1	0
1	0
1	0

T : Tautology (always 1)

F : Contradiction (always 0)

first laws

Idenity

"and" ~ The Tautology is always true

$p \wedge T \Leftrightarrow p$

~ logically equivalent to P

note: solved proofs on next page.

$p \vee F \Leftrightarrow p$

"or"

Let's reduce this Problem:

$(p \vee F) \wedge (q \vee T)$ — Rule we did

$p \wedge (q \vee T)$ — Idenity Law

$p \wedge T$ — Domination Law

p — Idenity Law

Domination

$p \vee T \Leftrightarrow T$

~ True is always True

$p \wedge F \Leftrightarrow F$

~ False is always False

~ in domination P, doesnt matter

~ We can see how the Truth and the False are dominating over the formula

Double Negation

$\neg\neg p \Leftrightarrow p$

~ crosses out★

★ **DeMorgan's Law** ~ Used everywhere !

flip

distributed

$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

Example:

$\neg(\neg p \wedge \neg q)$

$\neg\neg p \vee \neg\neg q$ — DeMorgans Law

$p \vee q$ — Double Negation

$3 \times (1+2)$ — Another way to look

$(3 \times 1) + (3 \times 2)$

Distributive law

$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

Absorption Law

$p \wedge (p \vee q) \Leftrightarrow p$

$p \vee (p \wedge q) \Leftrightarrow p$

example:

$\neg\neg p \vee ((p \vee F) \wedge \neg\neg q)$

$p \vee ((p \vee F) \wedge q)$ Double Negation ×2

$p \vee (p \wedge q)$ Idenity

p Absorption

Commutativity ($\vee, \wedge$)

$p \wedge q \Leftrightarrow q \wedge p$

$p \vee q \Leftrightarrow q \vee p$

~ We can flip the order

Inverse Laws

$p \wedge \neg p \Leftrightarrow F$

$p \vee \neg p \Leftrightarrow T$

Associativity ($\vee, \wedge$)

$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$

~ can do "ors" as well

~ make sure the signs stay the same.

Conditional Law

$p \rightarrow q \Leftrightarrow \neg p \vee q$

☆ Truth Table ☆

p	q	p→q	¬p	¬p ∨ q
1	1	1	0	1
1	0	0	0	0
0	1	1	1	1
0	0	1	1	1

∴ logically Equivilent

Exercise:

Show $(\neg(p \wedge q) \wedge q)$ is logically equivalent to $(\neg p \wedge q)$

$\neg(p \wedge q) \wedge q$ — Whenever I see a ¬ (not) outside of brackets I want to DeMorgans them together

$(\neg p \vee \neg q) \wedge q$ — DeMorgan's Law

see these ∨ (or's) and ∧ (ands) are different so use

$(q \wedge \neg p) \vee (q \wedge \neg q)$ — Distributivity

$(q \wedge \neg p) \vee F$ — Inverse

What happens if I have anything or False? ~ that's left with anything we had before

$q \wedge \neg p$ — Identity Law: $(p \vee F \Leftrightarrow p)$

$\neg p \wedge q$ — Commutativity Law

~ can flip anything around

Proofs ☆

Identity Laws

example: $p \wedge T \Leftrightarrow p$

Truth Table

p	T	$p \wedge T$
T	T	T
F	T	F

We see that p and $p \wedge T$ are logically Equivalent

Example: $p \vee F \Leftrightarrow p$

Truth Table

p	F	$p \vee F$
T	F	T
F	F	F

logically Equivalent

absorption Law

example: $p \wedge (p \vee q) \Leftrightarrow p$

Truth Table:

p	q	$p \vee q$	$p \wedge (p \vee q)$
1	1	1	1
1	0	1	1
0	1	1	0
0	0	0	0

p and $p \wedge (p \vee q)$ are logically equivalent.

Domination law proofs

$p \vee T \iff T$

Truth Table

p	T	p v T
1	1	1
0	1	1

$p \wedge F \iff F$

p	F	p ^ F
1	0	0
0	0	0

Inverse Law Proofs

$p \wedge \neg p \iff F$

p	¬p	p ^ ¬p	F
1	0	0	0
0	1	0	0

$p \vee \neg p \iff T$

p	¬p	p v ¬p	T
1	0	1	1
0	1	1	1

DeMorgan Law Proofs

$\neg(p \wedge q) \iff \neg p \vee \neg q$

p	q	(p ^ q)	¬(p ^ q)	¬p	¬q	¬p v ¬q
1	1	1	0	0	0	0
1	0	0	1	0	1	1
0	1	0	1	1	0	1
0	0	0	1	1	1	1

$\neg(p \vee q) \iff \neg p \wedge \neg q$

p	q	(p v q)	¬(p v q)	¬p	¬q	¬p ^ ¬q
1	1	1	0	0	0	0
1	0	1	0	0	1	0
0	1	1	0	1	0	0
0	0	0	1	1	1	1

Logic Laws Example 1.

1.) Use Laws to reduce: $\neg[(p \wedge q) \rightarrow r]$ $\qquad p \rightarrow q \Leftrightarrow \neg p \vee q$

1.) $\neg(\neg(p \wedge q) \vee r)$ ← Conditional Law ~ perform this law 1st.

2.) $\neg\neg(p \wedge q) \wedge \neg r$ DeMorgan's Law

3.) $p \wedge q \wedge \neg r$ Double Negation

2.) Prove that $(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \Leftrightarrow (p \wedge q)$

$\rightarrow (p \wedge (p \wedge q)) \rightarrow (p \wedge p \wedge q)$

$r \vee s \quad \wedge$

1.) $(\neg p \vee q) \wedge (p \wedge q)$ ~p identity law

2.) $((p \wedge q) \wedge q) \vee ((p \wedge q) \wedge \neg p)$ Distribution law $p \wedge (r \vee s) \Leftrightarrow (p \wedge r) \vee (p \wedge s)$

reduction

3.) $(p \wedge q) \vee ((p \wedge q) \wedge \neg p)$ idenity law

~ all conjunctions

4.) $(p \wedge q) \vee F$ Negation

5) $p \wedge q$

~ if we have something that could be True or something that's always False it's always going to take the upper value.

Wnt this to reduce to True

3.) Show that $(p \vee q) \rightarrow (q \rightarrow q)$ is a tautology $\Leftrightarrow T$

1) $\neg(p \vee q) \vee (q \rightarrow q)$ definition of $\rightarrow$

2.) $\neg(p \vee q) \vee \underbrace{(\neg q \vee q)}$ do the $\rightarrow$ again

3.) $\neg(p \vee q) \ \underline{\vee} \ T$ - b/c this is the law of the excluded middle

4.) T - if we have something ~ always be True or True, its always going to take the upper value with the or, and True is always True

If we have this statement: $p \rightarrow q$ ①

1 this is always True, then that means the conditional will always be a 1.

$\varphi \rightarrow (q \rightarrow q)$ - We know its a 1 b/c if we have $q \rightarrow q$ this is always True

1 1

- always a Tautology !

Logic Laws Examples 2

Show that $p \to q$ and $\neg q \to \neg p$ are logically equivalent without using truth tables or a "contrapositive" law.

1. $p \to q$
2. $\neg p \vee q$ definition of $\to$ (conditional law)
3. $\neg\neg q \vee \neg p$ community law
4. $\neg q \to \neg p$ def of $\to$

Show that $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent

1. $(p \to r) \wedge (q \to r)$
2. $(\neg p \vee r) \wedge (\neg q \vee r)$ $\to$ def
3. $r \vee (\neg q \wedge \neg p)$ Distributed law
4. $(\neg q \wedge \neg p) \vee r$ communitive law
5. $\neg(\neg q \wedge \neg p) \to r$ $\to$ def
6. $(\neg\neg q \vee \neg\neg p) \to r$ DeMorgans'
7. $(q \vee p) \to r$ Double Negation

Conditionals

~ Challenging ~

Conditionals ★ The Conditional is equated logically to the Converse — ~~and~~ contrapositive ★ nothing else ★

$p \rightarrow q$

$\boxed{\neg p \vee q}$ logically equivilent

Converse

$q \rightarrow p$

~ flipping the order

$\boxed{\neg q \vee p}$ logically Equivalent

~ logically Equivalent

~ logically Equivelent

Inverse

$\neg p \rightarrow \neg q$

Just adding the negations to p and q

$\neg\neg p \vee \neg q$ Double Negation

$\boxed{p \vee \neg q}$ logically Equivant

Contrapositive

$\neg q \rightarrow \neg p$

$\neg\neg q \vee \neg p$ using double negation

$\boxed{q \vee \neg p}$ logically equivilent

Biconditional

- conjunction of two conditionals

not a standard operator

$\underset{\text{if and only if}}{p \leftrightarrow q} \iff \overset{p \text{ only if } q}{(p \rightarrow q)} \wedge \overset{p \text{ if } q}{(q \rightarrow p)}$

Truth Table Proof →

p	q	$p \leftrightarrow q$	$(p \rightarrow q)$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
1	1	1	1	1	1
1	0	0	0	1	0
0	1	0	1	0	0
0	0	1	1	1	1

logically equivelent

"if p, then q"
"if p, q"
"p is sufficient for q"
"q if p"
"q when p"
"a necessary condition for p is q"
"q unless $\neg p$"
"p implies q"
"p only if q"
"a sufficient condition for q is p"
"q whenever p"
"q is necessary for p"
"q follows from p"

Extra Notes on Conditionals

If $\underbrace{\text{I study hard,}}_{p}$ then $\underbrace{\text{I will pass.}}_{q}$

If p then q

$p \to q$ ("p implies q")

Either I don't study hard, or I pass

"or"

$\neg p \vee q$

Vascously True statements

When the hypothesis is false, the statement is vacuously true.

Ex: if Trefor is a unicorn, then everyone get's an A.

if p then q

$p \to q$

Translation Examples

"I will pass if I get a good exam mark." — if p then q

$G \to P$

"I will pass only if I get a good exam mark."

$P \to G$

"I will pass if and only if I get a good exam score."

$P \leftrightarrow G$

~ closely related in terms of the biconditional.

This means if you get a good exam mark and you don't pass then it's false, because it's a lie.

If I pass the course $P \to G$ if P is a 1 then I better be getting a good exam mark, so I will pass only if I get a good exam mark.

$P \to G$, ← FALSE

- if I pass and don't get a good exam mark then this is going to be false, I will only pass if I get a good exam mark. This says I passed, even though I didn't get a good exam mark, which is a lie.

~ Think of it when someone is lying.

Examples

Mark is writing an exam on propositional logic. During the Exam, Dr. Cheatmaster notices that Mark is acting ~~up~~ rather suspicious. Suspecting Mark of cheating, Dr. Cheatmaster walks up behind Mark and noticinges a cheat sheet. Dr. Cheatmaster says

"If you don't give me your cheat sheet, then you will fail the course."

Because Mark doesn't want to fail, he gives Dr. Cheatmaster his cheat sheet. After reviewing the cheatsheet, Dr. Cheatmaster fails Mark.

NO

Did Dr. Cheatmister lie to Mark? Explain your answer using the truth conditions of the conditional and its logical equivalencies.

Answer:

$$\neg C \rightarrow F \Longleftrightarrow \neg F \rightarrow C$$

But Mark gave Dr. Cheatmaster the cheat sheet

C

I doesn't matter if he gave him his cheatsheet, he was probably going to fail

Conditional Examples

Determine whether the conditionals are true or False.

(i) If $\underbrace{2 \times 4 = 6}_{\text{FALSE}}$, then $\underbrace{3 + 6 = 7}_{\text{FALSE}}$

$0 \rightarrow 0$ True!

(ii) If $\underbrace{1 + 1 = 2}_{\text{TRUE}}$, then $\underbrace{3 \times 6 = 18}_{\text{TRUE}}$

$1 \rightarrow 1$ True!

(iii) $\underbrace{2 \times 7 = 14}_{\text{TRUE}}$ $\boxed{\text{only if}}$ $\underbrace{6 \times 2 = 18}_{\text{FALSE}}$

$1 \rightarrow 0$ False!

When we have "only if"

if P then Q } Equivalent
P only if Q

Question #2:

"If it snows I will Cry"

$P \to Q$

Converse

$Q \to P$

~reverse

If I will cry then it snows

Inverse

$\neg P \to \neg Q$

if it doesnt snow, then I will not cry

Contra positive

$\neg Q \to \neg P$

If I won't cry, it doesnt snow

^ This is logically equivalent to the 1st conditional.

Section 1.4

Rules of Inference

• Deducing logical outcomes

if it rains, I will get wet.
It's raining
$\therefore$ I will get wet.

if R then W

$R \to W$ Premise
R R is happening
$\therefore W$

Conclusion

→ Logical Argument ←

★ We take one or two premises and we deduce a conclusion from them

$((R \to W) \land R) \to W$

Some thing

with a Truth Table this would be a Tautology

Rules

• A set of premises $P_1, P_2, \ldots P_n$ prove some conclusion q in an argument.

$$(P_1 \land P_2 \land \ldots \land P_n) \to q$$

• An argument is valid if the premises logically entail the conclusion

~ if all our rules of inference takes us from the premises to the conclusion then it's valid. If there are improper steps or steps that aren't logically equivalent in there, then it's invalid.

Laws ★ Rules of Inferences ★

① Modus Ponens MPP

→ sometimes referred to as affirming the antecedent

$$\begin{array}{l} p \to q \\ \underline{p \qquad} \\ \therefore q \end{array}$$

like sticking the thing into the arrow and getting an output

② Modus Tollens MTT

$$\begin{array}{l} p \to q \\ \underline{\neg q \qquad} \\ \therefore \neg p \end{array}$$

~opposite way

$\neg p \to \neg p$ logically Equivalent

③ Hypothetical Syllogism HS

like transitivity

$$\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$$

Take short cut

$p \to q \to r$

④ Disjunctive Syllogism DS

$$\begin{array}{l} p \vee q \\ \underline{\neg p \qquad} \\ \therefore q \end{array}$$

~ q must be True.

⑤ Addition $\vee I$

$$\begin{array}{l} \underline{p \qquad} \\ \therefore p \vee q \end{array}$$

~if p is True then p or q is True.

⑦ Conjunction $\wedge I$

$$\begin{array}{l} p \\ \underline{q \qquad} \\ \therefore p \wedge q \end{array}$$

if p and q are both True, the $p \wedge q$ are going to be true

⑥ Simplification $\wedge E$

← and elimination b/c you're removing the and from it.

$$\begin{array}{l} \underline{p \wedge q} \\ \therefore p \\ \therefore q \end{array}$$

① Exercise

Task:
- Prove that (1) - (3) entail $\neg J$.
- Justify each step

1. R Premise
2. $R \to D$ Premise
3. $D \to \neg J$ Premise

4. D - using line 1, 2, MPP
   - stuck R in and got D

5. $\neg J$ - using 3, 4, MPP

Another way to do things (Hypothetical Syllogism)

4. $R \to \neg J$ - using 2, 3, HS
5. $\neg J$ - 1, 4 MPP

② Exercise

Task:
Show that (1) - (3) entail R

1. $(\neg R \lor \neg F) \to (S \land L)$ Premise
2. $S \to T$ Premise
3. $\neg T$ Premise

4. $\neg S$ 2, 3, MTT ✓
5. $\neg(S \land L) \to \neg(\neg R \lor \neg F)$ 1, Contrapositive
6. $(\neg S \lor \neg L) \to (\neg\neg R \land \neg\neg F)$ 5, De Morgans
7. $(\neg S \lor \neg L) \to (R \land F)$ 6, Double Negation
8. $\neg S \lor \neg L$ 4, Addition

9. $R \land F$ 7, 8 MPP

10. R 9, simplication

Predicate Logic and Negating Quantifiers

Predicate Logic and Quantifiers
↳ Can have terms?
- We want to talk about variables

- $E(x) = x$ is even

two-place predicate

open
- $\boxed{G(x, y) = x \text{ is greater than } y}$ NOT A STATEMENT!

Can stick in constants in

closed
$G(2,1) = 2$ is greater than 1 True! ~ has a truth Value
$G(3,6) = 3$ is greater than 6 False!

What's the difference b/w closed and open formula, well closed formulas have truth values and open do not have truth values. ∴ $G(x,y)$ is not a statement. Which means we can't even have it in propositional logic at all, we need predicate logic in order to say something like $G(x,y)$ is greater than y. and assigned truth values to constants plugged into there variables and that's one thing predicate logic lets us do. But the more important thing it does is gives us quantifiers

We want to introduce quantifiers

Universal quantifier
$\forall x\, P(x)$: "For all x, x is P"
(for all x)

Existential quantifier
$\exists x\, P(x)$: "For some x, x is P"
There exists x

Examples

We use this notation everywhere in mathematics

① For every real number n, there is a real number m such that $m^2 = n$.

$\forall n \in \mathbb{R} \quad \exists m \in \mathbb{R} : m^2 = n$ or $P(m,n) = m^2 = n$

For all real numbers n — Exist an m in the reals

② Given two rationals, x and y, $\sqrt{ab}$ will ~~ab~~. also be rational.

Both the Same

$\forall x, y \in \mathbb{Q}, \sqrt{xy} \in \mathbb{Q}$

or

$\forall x \in \mathbb{Q} \ \forall y \in \mathbb{Q}$

- Translating some mathematical statements into predicate logic

Negating Quantifiers ~ Tricky

- Define $\forall x, \exists x$ for a universe with elements $\{1, 2, \dots, n\}$

$\forall x P(x) \iff P(1) \wedge P(2) \wedge \dots \wedge P(n)$

This is True if every x in the universe is P

$\exists x P(x) \iff P(1) \vee P(2) \vee \dots \vee \dots P(n)$

Definition of Existent X

Show that $\neg\forall x\,[P(x)] \iff \exists x\,[\neg P(x)]$

$$\boxed{\neg\forall x\, P(x)} = \neg(P(1) \wedge P(2) \wedge \ldots \wedge P(n))$$

DeMorgan's Law $= \neg p(1) \vee \neg p(2) \vee \ldots \vee P(n)$

$$\boxed{= \exists x\,[\neg P(x)]}$$

$\exists x\, P(x) = P(1) \vee P(2) \vee \ldots \vee P(n)$

All Equivalencies

$$\forall x\, P(x) \iff \neg\exists x\,[\neg P(x)]$$

$$\exists x\, P(x) \iff \neg\forall x\,[\neg P(x)]$$

$$\neg\forall x\, P(x) \iff \exists x\,[\neg P(x)]$$

$$\neg\exists x\, P(x) \iff \forall x\,[\neg P(x)]$$

Negation written as a minus

$\neg\forall x\, P(x)$

$\| \quad \| \sim$ reduced

$- \forall + P$

No negation wrote as a plus

$+\exists - P$

$\exists x \neg P(x)$

How do we get the logical equivalence?

~ We just reverse the negations and pluses and change the quantifiers

~ then you translate back

Another Example:

$\exists x\, P(x)$

$+\exists + P$

$-\forall - P$

$\neg\forall x\,[\neg P(x)]$

logically Equivalent

Negate the following:

$$\neg \left[\forall x \left[\exists y \left[P(x,y) \wedge Q(y) \right] \right] \right]$$

$$\exists x \neg \left[\exists y \left[P(x,y) \wedge Q(y) \right] \right]$$

$$\exists x \forall y \neg \left[P(x,y) \wedge Q(y) \right]$$

DeMorgans LAW

$$\exists x \forall y \left[\neg P(x,y) \vee \neg Q(y) \right]$$

1. 4 Predicate Logic

When Propositional Logic Fails

If I say,

- All candy made with chocolate is delicious
- M&M's are made with chocolate

Does it follow that M&M's are delicious?

We can't model this relationship with propositions.

This is where we need predicate logic which include:

- **Variables**: x, y, z, these are the subjects of the statement(s)
- **Predicates**: a property the variable can have. (ex. "is greater than 3")
- **Quantifiers**: covered in the next video

Predicates

- Statements involving variables, such as "$x < 2$" and "$x + y = z$" are often found in mathematical assertions, in computer programs and in system specifications. The statements are neither true nor false when the values of the variables aren't specified.

- The statement "x is less than 2" has two parts. First, the variable x is the subject of the statement. The second part, the predicate, "is less than 2" refers to the property that the subject of our statement can have. The predicate, "is less than 2" can be denoted by $P(x)$, where P denotes the predicate and x the variable.

$X < 2$

X is the subject of the statement

The predicate is " less than 2"

$P(x)$

Propositional Functions

<u>Propositional</u> functions become <u>propositions</u> (and have <u>truth values</u>) when their variables are each replaced by a value from the <u>*domain*</u> (or *bound* by a quantifier, as we will see later).

The statement $P(x)$ is said to be the value of the propositional function P at x.

For example, let $P(x)$ denote $\boxed{\text{“}x > 0\text{”}}$ and the <u>domain be the integers.</u> Then:

$P(-3)$ *is false.* $-3 > 0$ False

$P(0)$ *is false.* $0 > 0$ False

$P(3)$ *is true.* $3 > 0$ True

Universe

Often the domain is denoted by U. So in this example U <u>is the integers</u>.

Examples of Propositional Functions

Variables Domain

Let “$x + y = z$” be denoted by $R(x, y, z)$ and U (for all three variables) be the integers. Find these truth values:

Right now this is considered a <u>propositional function.</u>

Now that we've Given x,y,z Values its now a <u>Proposition</u> (Has Truth Value of True or False)

$R(2, -1, 5)$ $2 + -1 = 5$ False

$R(3, 4, 7)$ $3 + 4 = 7$ True

$R(x, 3, z)$ Not a <u>Proposition</u>

2 Variables not yet defined

Propositional Functions

Exemples

S(x,y) = x is in state y
Domain: All systems x and states y

1. $\exists x S(x, \text{open})$
2. $\forall x (S(x, \text{malfunctioning}) \vee S(x, \text{diagnostic}))$
3. $\exists x S(x, \text{open}) \vee \exists x S(x, \text{diagnostic})$
4. $\exists x \neg S(x, \text{available})$
5. $\forall x \neg S(x, \text{working})$

1. There exists a System that is open.
2. Every system is either malfunctioning or diagnostic.
3. Some systems are open, or some systems are diagnostic
4. There exist a system that is unavailable
5. None of these systems are working.

Example 1:

$\forall x$

Every user has access to an electronic mailbox

Propositional Function

$M(x)$ = x can access a mailbox

need to Quantify it.

$\forall x\, M(x)$

$M(x,y)$ = x can access system y

$\forall x\, M(x, \text{mailbox})$

Example 2:

then

The system mailbox can be accessed by everyone in the group if the file system is locked.

Define predicate

$L(\text{file system}) \rightarrow \forall x\, M(x)$

$L(y)$ = system y is Locked

$M(x)$ = x can access a mailbox

Example 3

The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.

$D(y)$ = system y diagnostic

$D(\text{proxy}) \rightarrow D(\text{firewall})$

Example 4

At least one router is functioning normally if the throughput is between 100 Kbps and 500 Kbps and the proxy server is not in diagnostic mode.

$D(y)$

y = routers

Define Domain

$N(y)$ = y is functioning normally

$T(a,b)$ = throughput is between a & b

$(T(100, 500) \wedge \neg D(\text{proxy})) \rightarrow \exists y\, N(y)$

Compound Expressions

Connectives from propositional logic carry over to predicate logic.

If $P(x)$ denotes "$x > 0$," find these truth values:

$P(3) \vee P(-1)$	$T \vee F$	*Solution*: T
$P(3) \wedge P(-1)$	$T \wedge F$	*Solution*: F
$P(3) \rightarrow P(-1)$	$T \rightarrow F$	*Solution*: F
$P(3) \rightarrow \neg P(-1)$	$T \rightarrow T$	*Solution*: T

Expressions with variables are not propositions and therefore do not have truth values. For example,

$$P(3) \wedge P(y)$$
$$P(x) \rightarrow P(y)$$

When used with quantifiers (to be introduced next), these expressions (propositional functions) become propositions.

If $P(x) \rightarrow x > 0$

"or"

$P(3) \vee P(-1)$ $\quad$ $T \vee F$ $\quad$ Solution: T

$3 > 0$ $\quad$ $-1 > 0$

TRUE $\quad$ FALSE

Quantifiers

QUANTIFIERS

AS WE KNOW, A PROPOSITIONAL FUNCTION $P(x)$ IS NOT A PROPOSITION UNTIL IT HAS A TRUTH VALUE. UP TO THIS POINT, WE COULD ONLY DO THIS BY ASSIGNING A VALUE TO OUR VARIABLE.

EX. IF $P(x)$ REPRESENTS " $x > 0$ ", FIND THE TRUTH VALUE FOR $P(4)$.

QUANTIFIERS

NOW WE WILL TURN A PROPOSITIONAL FUNCTION INTO A PROPOSITION USING A QUANTIFIER.

LET'S FOCUS ON THE TWO MOST WIDELY USED QUANTIFIERS:

"FOR ALL"	$\forall$	UNIVERSAL QUANIFIER
"THERE EXISTS"	$\exists$	EXISTENTIAL QUANTIFIER

THE UNIVERSAL QUANTIFIER "$\forall$"

THE STATEMENT, $\boxed{\forall x\, P(x)}$ TELLS US THAT THE PROPOSITION P(x) MUST BE TRUE FOR ALL VALUES OF X IN THE DOMAIN OF DISCOURSE / UNIVERSE.

EX. LET P(x) REPRESENT "$x > 0$". FIND EACH TRUTH VALUE FOR $\boxed{\forall x\, P(x)}$ "for every x in the domain"

a. U is $\mathbb{Z}$ ~ Integers $-3 \in \mathbb{Z}$ $-3 > 0$ False $\rightarrow$ "Counter Example"

b. U is $\mathbb{Z}^+$ ~ Positive Integers   TRUE

THE EXISTENTIAL QUANTIFIER "$\exists$"

THE STATEMENT, $\boxed{\exists x\, P(x)}$, TELLS US THAT THE PROPOSITION P(x) IS TRUE FOR SOME VALUE(S) OF X IN THE DOMAIN OF DISCOURSE / UNIVERSE.

EX. LET P(x) REPRESENT "$x > 0$". FIND EACH TRUTH VALUE FOR $\exists x\, P(x)$

a. U is $\mathbb{Z}$ $7 \in \mathbb{Z}$ $7 > 0$ TRUE

b. U is $\mathbb{Z}^-$ FALSE
~ All negative integers are < 0, so false

THE QUANTIFIERS

UNIVERSAL	EXISTENTIAL
$\forall$	$\exists$
"FOR ALL"	"THERE EXISTS"
WHEN TRUE? WHEN $P(x)$ IS TRUE FOR EVERY x IN THE DOMAIN	WHEN TRUE? THERE IS AN x IN THE DOMAIN FOR WHICH $P(x)$ IS TRUE
WHEN FALSE? THERE IS AN x IN THE DOMAIN FOR WHICH $P(x)$ IS FALSE	WHEN FALSE? WHEN $P(x)$ IS FALSE FOR EVERY x IN THE DOMAIN
$\forall x P(x) \equiv P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n)$	$\exists x P(x) \equiv P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n)$

PRACTICE

LET $P(x)$ BE "$x^2 > 0$" IF THE DOMAIN CONSISTS OF ALL INTEGERS. FIND THE TRUTH VALUES OF:

$\forall x\, P(x)$ $\quad 0 \in \mathbb{Z} \quad 0^2 > 0$ FALSE $\quad \therefore \forall x\, P(x)$ is FALSE

$\exists x\, P(x)$ $\quad 1 \in \mathbb{Z} \quad 1^2 > 0$ TRUE $\quad \therefore \exists x\, P(x)$ is TRUE

LET $P(x)$ BE "$x^2 < 0$". IF THE DOMAIN CONSISTS OF ALL INTEGERS, FIND THE TRUTH VALUES OF:

$\forall x\, P(x)$ FALSE $\quad (-1)^2 < 0 \quad 1 < 0$, False $\quad \therefore \forall x\, P(x)$ is False

$\exists x\, P(x)$ FALSE $\quad \therefore \exists x\, P(x)$ is False

PRACTICE

LET $P(x)$ BE "$x+1=2x$" IF THE DOMAIN CONSISTS OF ALL INTEGERS. FIND THE TRUTH VALUES OF:

$\forall x\, P(x)$ $\quad 0 \in \mathbb{Z},\quad 0+1 = 2(0)$: FALSE $\quad \therefore \forall x\, P(x)$ is False

$\exists x\, P(x)$ $\quad 1 \in \mathbb{Z},\quad 1+1 = 2(1)$: TRUE $\quad \therefore \exists x\, P(x)$ is True

LET $P(x)$ BE "$x^2 < 16$". IF THE DOMAIN CONSISTS OF ALL INTEGERS, FIND THE TRUTH VALUES OF:

$\forall x\, P(x)$ FALSE

$\exists x\, P(x)$ TRUE

THE UNIQUENESS QUANTIFIER $\boxed{\exists !}$

THE STATEMENT, $\exists! x\, P(x)$, TELLS US THAT THE PROPOSITION $P(x)$ IS TRUE FOR EXACTLY ONE VALUE OF x IN THE DOMAIN OF DISCOURSE / UNIVERSE.

GIVE THE TRUTH VALUE FOR $\exists! x\, P(x)$ FOR EACH PROPOSITION IN THE DOMAIN OF ALL INTEGERS.

a. $P(x)$ REPRESENTS "$2x = 4$" $\quad 2x = 4$, $2(2) = 4$ $\quad$ TRUE

b. $P(x)$ REPRESENTS "$2x > 4$" $\quad 2x > 4$, $x > 2$ $\quad$ FALSE ~ There is more than one integer that $x > 2$. (e.g 3,4)

c. $P(x)$ REPRESENTS "$2x = 3$" $\quad 2x = 3$, $2(1.5) = 3$, $x = 1.5$ $\quad$ FALSE ~ 1.5 isnt an integer

1.4.3 Negating and Translating Quantifiers

QUANTIFIERS WE'VE LEARNED

LET P(x) BE THE STATEMENT "X HAS TAKEN A COURSE IN PROGRAMMING" FOR THE DOMAIN OF STUDENTS IN YOUR CLASS.

What does this mean?

$\forall x P(x)$ Every student in my class has taken a course in programming.

$\exists x P(x)$ There is a student in my class who has taken a course in programming.

NEGATING QUANTIFIERS

LET P(x) BE THE STATEMENT "X HAS TAKEN A COURSE IN PROGRAMMING" FOR THE DOMAIN OF STUDENTS IN YOUR CLASS.

$\neg \forall x P(x)$ There is a student in my class who hasn't taken a course in programming.

$\neg \forall x P(x) \xleftrightarrow{\text{Equivalent}} \exists x \neg P(x)$

$\neg \exists x P(x)$ All students in my class have not taken a course in programming.

$\neg \exists x P(x) \xleftrightarrow{\text{Equivalent}} \forall x \neg P(x)$

DeMorgan's Laws for Quantifiers

$\neg \exists x P(x) \equiv \forall x \neg P(x)$

- True when $P(x)$ is false for every x
- False when there is an x for which $P(x)$ is true

$\neg \forall x P(x) \equiv \exists x \neg P(x)$

- True when there is an x for which $P(x)$ is false
- False when $P(x)$ is true for every x

Translating and Negating

Negate the statements

"There is an honest politician"

"All Americans eat cheeseburgers"

$H(x)$ Represents "x is Honest" For domain of all Politicians

$\exists x\, H(x)$

"Not" operator

$\neg \exists x\, H(x) \equiv \boxed{\forall x \neg H(x)}$ <u>Every politician is dishonest</u>

$C(x)$ Represents "x Eats Cheeseburgers" For domain of all Americans.

$\forall x\, C(x)$

$\neg \forall x\, C(x) \equiv \exists x \neg C(x)$ Not every American eats cheeseburgers.

TRANSLATING

SOME STUDENT IN THIS CLASS HAS VISITED MEXICO.

DOMAIN: STUDENTS IN CLASS

m(x) Rep: "X has visited Mexico"

There exist a student

$\boxed{\exists x\, M(x)}$

DOMAIN: ALL PEOPLE

Two Propositional Functions

m(x) Rep: "X has visited Mexico"

C(x) Rep: "X is a student in this class"

$\boxed{\exists x\,(M(x) \wedge C(x))}$

PRACTICE

EVERY STUDENT IN THIS CLASS HAS VISITED CANADA OR MEXICO.

DOMAIN: STUDENTS IN THIS CLASS

m(x) Rep: " X has visited Mexico"

C(x) Rep: " X has visited Canada"

$\forall x\,(C(x) \vee m(x))$ One Way to write this.

DOMAIN: ALL PEOPLE

S(x) Rep: " X is a student in this class"

$\forall x\,(\underline{S(x)} \rightarrow (\underline{C(x) \vee m(x)}))$

~ using "if, then"

Negating Quantifiers and Translation Examples

Translate the following:

- Every student is majoring in math or computer science

$$\forall x\,[S(x) \rightarrow (M(x) \vee C(x))]$$

- For all ~~ABC~~ real x, y, $x > y$ if $x^2 > y^2$

$$\forall x \in \mathbb{R}, \forall y \in \mathbb{R}\ [(x^2 > y^2) \rightarrow (x > y)]$$

- For all $\varepsilon > 0$, there is an $\delta > 0$ such that $|f(x) - L| < \varepsilon$ when $|x - a| < \delta$.

$$\forall \varepsilon > 0, \exists \delta > 0\ [(|x - a| < \delta) \rightarrow (|f(x) - L| < \varepsilon)]$$

Negate:

$$\exists x\,(Px \wedge Qx)$$

$$\neg\left(\exists x\,(Px \wedge Qx)\right)$$

$$(\neg\exists x)(Px \wedge Qx)$$

$$\forall x\ \neg(Px \wedge Qx)$$

$$\forall x\,[\neg Px \vee \neg Qx]$$

Negate:

$$\forall x\,(Px \rightarrow Qx)$$

$$\neg\forall x\,(Px \rightarrow Qx)$$

$$\exists x\,[\neg(Px \rightarrow Qx)]$$

$$\exists x\,[\neg(\neg Px \vee Qx)]$$

$$\exists x\,[Px \wedge \neg Qx]$$

Unique Quantifier Examples

Determine the Truth Values.

(i) $n \in \mathbb{Z}$ $\quad$ $\exists n\, (n^2 = 2)$

we know $n = \pm\sqrt{2}$

FALSE

(ii) $x \in \mathbb{R}$ $\quad$ $\forall x\, (x^2 + 2 \geq 1)$

$x^2 \geq 0$ $\quad$ $0 + 2 \geq 1$

TRUE

(iii) $x \in \mathbb{Z}$ $\quad$ $\forall x\, ((-x)^2 = x^2)$

$0^2 = 0^2$

$(-3)^2 = 3^2$

$3^2 = (-3)^2$

TRUE

If $\exists!\, x\, P(x)$ means "there exists a unique x such that $P(x)$",

means unique

determine the truth values of the following:

(i) $\exists!\, x\, (x + 1 = 2x)$ for $x \in \mathbb{Z}$

(ii) $\exists!\, x\, (x > 1)$ for $x \in \{0, 1, 2, 3\}$

Which of the following are always True?

$\exists! x\, P(x) \rightarrow \exists x\, P(x)$

$\exists! x\, \neg P(x) \rightarrow \neg \forall x\, P(x)$

$\exists x\, P(x) \rightarrow \exists! x\, P(x)$

Conditionals ☆ Extra Notes ☆

extra logic

Conditional $P \rightarrow Q$

if P, "the hypothesis" then Q ~ "the conclusion"

Converse $Q \rightarrow P$

Inverse $\neg P \rightarrow \neg Q$

Contrapositive $\neg Q \rightarrow \neg P$

Bio conditional $P \Leftrightarrow Q$

Examples

① If you study then you will get a good grade

Whatever comes after the "If", will be the hypothesis

Whatever comes after the "then", that's your conclusion.

What would the Converse be?

- in reverse

If you get a good grade then you studied.

~ But that's not necessary true, you could've gotten a good grade b/c the teacher made the test really easy or you're just naturally good @ math.

What about the Inverse?

- Add word not in there (negated)

if you do not study, then you will not get a good grade.

What about the Contrapositive?

~ combination of the converse and inverse and negating them

if you do not get a good grade, then you did not study

2 Angles

- form line, they are supplementary, add up to 180° ✓

extra logic

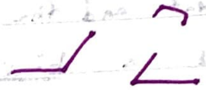
② If 2 Angles form a linear pair then they are supplementary

What would be the converse of the statement?

"if 2 Angles are supplementary then they form a linear pair"

Is this True?

Thats not necessary true → We could have two angles here



Example of the converse error. Just b/c you say something in reverse doesnt mean its True you know forward or a conditional statement.

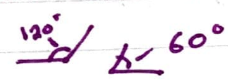
- Notice they're not adjacent to each other, they're not sharing a vertex or and a array.
- Not a linear pair

What about the Inverse of the statement?

"If 2 Angles do not form a linear pair then they are not supplementary."

True or False?

FALSE, b/c just b/c they don't form a linear pair doent mean they're not supplemenery., they could still add up to 180°.



What about the Contrapositive?

- switch the hypothesis and conclusion and we're going to negate them both.

" If 2 angles are not supplementary then they do not form a linear pair."

True or False?

TRUE, b/c if they're not supplementary, that mean they don't add up to 180° theres no way that they could be forming a straight line, a linear pair

~ Plus we know if the Conditional is True, then the contrapositive will automatically be true, they have the same truth value.

(3.) If 2 lines form right angles then they are Perpendicular.

~ That makes sense, it's True

What about as a converse statement?

"if two lines are perpendicular then they form right angles

~ The conditional statement and the converse statement are both true, we can write this as a bi conditional statement.

Bi meaning two and conditional statement meaning ~~as~~ its a conditional Statement forward and reverse. $P \Longleftrightarrow Q$

How do we write this as a Biconditional?

~ replace with "If and Only if"

2 lines form right angles if and only if they are perpindicular

Conditional statements

$\boxed{\begin{array}{l}\text{Def: } p \to q \text{ means:} \\ \text{"If } p \text{ is TRUE then } q \text{ is TRUE"}\end{array}}$

Practice Problems

Section 1.1

8. Let p and q be the propositions

p : I bought a lottery ticket this week.
q : I won the million dollar jackpot.

Express each of these propositions as an English sentence.

a) $\neg p$ **b)** $p \vee q$ **c)** $p \rightarrow q$
d) $p \wedge q$ **e)** $p \leftrightarrow q$ **f)** $\neg p \rightarrow \neg q$
g) $\neg p \wedge \neg q$ **h)** $\neg p \vee (p \wedge q)$

a) $\neg p$ I did not buy a lottery ticket this week

b) $p \vee q$ I bought a lottery ticket this week. or I won the million dollar jackpot.

c) $p \rightarrow q$ If I bought a lottery ticket this week. then I won the million dollar jackpot.

d) $p \wedge q$ I bought a lottery ticket this week. and I won the million dollar jackpot.

e) $p \leftrightarrow q$ I bought a lottery ticket this week. if end Only if I won the million dollar jackpot.

f) $\neg p \rightarrow \neg q$ If I did not buy a lottery ticket this week then I did not win the millionaire jackpot

g) $\neg p \wedge \neg q$ I did not buy a lottery ticket this week end I did not win the millionaire jackpot

h) $\neg p \vee (p \wedge q)$ I did not buy a lottery ticket this week or

10. Let p and q be the propositions "The election is decided" and "The votes have been counted," respectively. Express each of these compound propositions as an English sentence.

a) $\neg p$ ~ and
b) $p \vee q$ ~ or
c) $\neg p \wedge q$
d) $q \rightarrow p$
e) $\neg q \rightarrow \neg p$
f) $\neg p \rightarrow \neg q$
g) $p \leftrightarrow q$
h) $\neg q \vee (\neg p \wedge q)$

p: The election is decided

q: The Voter have been counted

a) $\neg p$ The election is not decided

b) $p \vee q$ The election is decided, or the voter have been counted.

c) $\neg p \wedge q$ The election is not decided and the voter have been counted.

d) $q \rightarrow p$ if the voter have been counted, then the election is decided.

e) $\neg q \rightarrow \neg p$ if the votes have not been counted then the election is not decided.

f) $\neg p \rightarrow \neg q$ If the election is not decided then the votes have not been counted.

g) $p \leftrightarrow q$ The election is decided if and Only if the voter have been counted.

h) $\neg q \vee (\neg p \wedge q)$

15. Let p, q, and r be the propositions

p : Grizzly bears have been seen in the area.
q : Hiking is safe on the trail.
r : Berries are ripe along the trail.

Write these propositions using p, q, and r and logical connectives (including negations).

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.
b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.
c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.
d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.
e) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.
f) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

$\wedge$ ~ and
$\vee$ ~ or

and

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.

$r \wedge \neg p$

b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

$\neg p \wedge q \wedge r$

c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

if $r \rightarrow$

$r \rightarrow (q \leftrightarrow \neg p)$

p : Grizzly bears have been seen in the area.
q : Hiking is safe on the trail.
r : Berries are ripe along the trail.

d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.

$\neg q \wedge \neg p \wedge r$

e) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.

$q \rightarrow (\neg r \wedge \neg p)$

"if p, then q"
"if p, q"
"p is sufficient for q"
"q if p"
"q when p"
"a necessary condition for p is q"
"q unless $\neg p$"
"p implies q"
"p only if q"
"a sufficient condition for q is p"
"q whenever p"
"q is necessary for p"
"q follows from p"

~ "if"

f) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

$(p \wedge r) \rightarrow \neg q$

11. Let p and q be the propositions

p : It is below freezing.
q : It is snowing.

Write these propositions using p and q and logical connectives (including negations).

a) It is below freezing and snowing.
b) It is below freezing but not snowing.
c) It is not below freezing and it is not snowing.
d) It is either snowing or below freezing (or both).
e) If it is below freezing, it is also snowing.
f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
g) That it is below freezing is necessary and sufficient for it to be snowing.

$\wedge$ ~ and
$\vee$ ~ or

"if p, then q"	"p implies q"
"if p, q"	"p only if q"
"p is sufficient for q"	"a sufficient condition for q is p"
"q if p"	"q whenever p"
"q when p"	"q is necessary for p"
"a necessary condition for p is q"	"q follows from p"
"q unless $\neg p$"	

a) It is below freezing and snowing.

$p \wedge q$

~"and"

b) It is below freezing but not snowing.

p: It is below freezing
q: It is snowing

$p \wedge \neg q$

c) It is not below freezing and it is not snowing.

$\neg p \wedge \neg q$

d) It is either snowing or below freezing (or both).

$q \vee p$

e) If it is below freezing, it is also snowing.

$p \rightarrow q$

f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.

$(p \vee q) \wedge (p \rightarrow \neg q)$

g) That it is below freezing is necessary and sufficient for it to be snowing.

$p \leftrightarrow q$

12. Let p, q, and r be the propositions

p : You have the flu.
q : You miss the final examination.
r : You pass the course.

Express each of these propositions as an English sentence.

a) $p \to q$ **b)** $\neg q \leftrightarrow r$
c) $q \to \neg r$ **d)** $p \vee q \vee r$
e) $(p \to \neg r) \vee (q \to \neg r)$
f) $(p \wedge q) \vee (\neg q \wedge r)$

a) $p \to q$

If you have the flu, then you miss the final examination

b) $\neg q \leftrightarrow r$

You don't miss the final exam if and only if you pass the course

c) $q \to \neg r$

If you miss the final examination, then you do not pass the course.

d) $p \vee q \vee r$

e) $(p \to \neg r) \vee (q \to \neg r)$

f) $(p \wedge q) \vee (\neg q \wedge r)$

14. Let p, q, and r be the propositions

p : You get an A on the final exam.
q : You do every exercise in this book.
r : You get an A in this class.

Write these propositions using p, q, and r and logical connectives (including negations).

a) You get an A in this class, but you do not do every exercise in this book.
b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
c) To get an A in this class, it is necessary for you to get an A on the final.
d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.
e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

...n own

Usually most difficult

Same as saying

p is necessary for q

$q \to p$

p is sufficient for q

$p \to q$

this afternoon.

and

a) You get an A in this class, but you do not do every exercise in this book.

$r \wedge \neg q$

b) You get an A on the final, you do every exercise in this book, and you get an A in this class.

$p \wedge q \wedge r$

c) To get an A in this class, it is necessary for you to get an A on the final.

$r \to p$

d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

$p \wedge \neg q \wedge r$

e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

$p \wedge q \to r$

f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

$r \leftrightarrow q \vee p$

22. Write each of these statements in the form "if p, then q" in English. [*Hint:* Refer to the list of common ways to express conditional statements provided in this section.]

a) It is necessary to wash the boss's car to get promoted.
b) Winds from the south imply a spring thaw.
c) A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.
d) Willy gets caught whenever he cheats.
e) You can access the website only if you pay a subscription fee.
f) Getting elected follows from knowing the right people.
g) Carol gets seasick whenever she is on a boat.

Analyze the sentence
- find the cause and effect

Cause ~ if
effect ~ then

a) It is necessary to wash the boss's car to get promoted.

1.) Re write in "if p, then q" form.

If you wash your boss's car, then you will get promoted

b) Winds from the south imply a spring thaw.

Cause: Wind from the south
effect: Spring thaw

c) A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.

Cause: bought less than a year ago
effect: Warranty is good

If you bought the computer less than a year ago, the the warranty is good.

d) Willy gets caught whenever he cheats.

Cause: Cheats
effect: Caught

If Will cheats, than he will get caught.

e) You can access the website only if you pay a subscription fee.

Cause: Pay for Subscription
effect: Access to Website

f) Getting elected follows from knowing the right people.

Cause: Knowing the right people
effect: Gets elected

g) Carol gets seasick whenever she is on a boat.

Cause: On a boat
effect: Gets seasick

More Practice Problems

1.1

27. State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows today, I will ski tomorrow.
b) I come to class whenever there is going to be a quiz.
c) A positive integer is a prime only if it has no divisors other than 1 and itself.

p q $p \to q$

a) If it snows today, I will ski tomorrow.

$q \to p$ Converse: If I ski tomorrow, then it snowed today.
$\neg q \to \neg p$ Contrapositive: If I do not ski tomorrow, then it did not snow today.
$\neg p \to \neg q$ Inverse: If it does not snow today, then I will not ski tommorrow.

p q

b) I come to class whenever there is going to be a quiz. If there is going to be a quiz, then I will come to class.

$q \to p$ Converse: If I come to class, then there is going to be a quiz.
$\neg q \to \neg p$ Contrapositive: If I don't come to class, then there is not going to be a quiz
$\neg p \to \neg q$ Inverse: If there is not going to be a quiz, then I will not go to class

1.4

7. Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.

a) $\forall x(C(x) \to F(x))$ **b)** $\forall x(C(x) \wedge F(x))$
c) $\exists x(C(x) \to F(x))$ **d)** $\exists x(C(x) \wedge F(x))$

a) $\forall x(C(x) \to F(x))$

For every x, if x is a comedian, then x is funny.

English: Every Comedian is funny.

b) $\forall x(C(x) \wedge F(x))$

For every x, x is a comedian and x is funny.

NOTE: "and" ~ Combines them.

English: Everyone is a funny Comedian.

c) $\exists x(C(x) \to F(x))$

There exists an x in the domain such that x is a comedian, then x is funny.

English: There exists a person that he/she is a comedian, then he/she is funny.

d) $\exists x(C(x) \wedge F(x))$

There exists an x in the domain, x is a comedian and x is funny.

English: There exists a funny comedian

1.4

15. Determine the truth value of each of these statements if the domain for all variables consists of all integers.

a) $\forall n(n^2 \geq 0)$ **b)** $\exists n(n^2 = 2)$
c) $\forall n(n^2 \geq n)$ **d)** $\exists n(n^2 < 0)$

a) $\forall n(n^2 \geq 0)$ True

~ All integers ≥ 0

b) $\exists n(n^2 = 2)$ False

$n^2 = 2$
$n = \pm\sqrt{2}$
~ not an integer

c) $\forall n(n^2 \geq n)$ True

d) $\exists n(n^2 < 0)$ False

26. Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables.

a) Someone in your school has visited Uzbekistan.
b) Everyone in your class has studied calculus and C++.
c) No one in your school owns both a bicycle and a motorcycle.
d) There is a person in your school who is not happy.
e) Everyone in your school was born in the twentieth century.

Define the propositional function for the entire problems.

$Y(x)$: "x is in your school"

x is the domain

a) Someone in your school has visited Uzbekistan.

$U(x)$: "x has visited Uzbekistan"

$\exists x\, U(x)$ ~ Domain is just your school

$\exists x\, (Y(x) \land U(x))$ ~ Domain is all people

$V(x,y)$: "person x has visited country y"

$\exists x\, (Y(x) \land V(x, \text{Uzbekistan}))$

b) Everyone in your class has studied calculus and C++.

~ $\forall x$

Propositional Function
$C(x)$: "x has studied Calculus"
$P(x)$: "x has studied C++"

$\forall x\, (C(x) \land P(x))$ ~ Is Domain is just your schoolmate

$\forall x\, (Y(x) \rightarrow (C(x) \land P(x)))$ ~ Domain is all people.

$S(x,y)$: person x has studied subject y

$\forall x\, (Y(x) \rightarrow (S(x, \text{Calculus}) \land S(x, \text{C++})))$

c) No one in your school owns both a bicycle and a motorcycle.

$B(x)$: "x owns a bike"
$M(x)$: "x owns a motorcycle"

$\forall x (\neg (B(x) \lor M(x)))$

28. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

a) Something is not in the correct place.

b) All tools are in the correct place and are in excellent condition.

c) Everything is in the correct place and in excellent condition.

d) Nothing is in the correct place and is in excellent condition.

e) One of your tools is not in the correct place, but it is in excellent condition.

R(x): "x is in the correct place"

E(x): "x is in excellent condition"

T(x): "x is a [your] tool"

Domain: all things

a) Something is not in the correct place.

$\exists x \neg R(x)$

b) All tools are in the correct place $\wedge$ and are in excellent condition.

If something is a tool, then it is in the correct place place and in excellent condition:

$\forall x (T(x) \rightarrow (R(x) \wedge E(x)))$

c) Everything is in the correct place and in excellent condition.

$\forall x (R(x) \wedge E(x))$

d) Nothing is in the correct place and is in excellent condition.

$\forall x \neg (R(x) \wedge E(x))$

31. In each case we just have to list all the possibilities, joining them with $\vee$ if the quantifier is $\exists$, and joining them with $\wedge$ if the quantifier is $\forall$.

a) $Q(0,0,0) \wedge Q(0,1,0)$ **b)** $Q(0,1,1) \vee Q(1,1,1) \vee Q(2,1,1)$

c) $\neg Q(0,0,0) \vee \neg Q(0,0,1)$ **d)** $\neg Q(0,0,1) \vee \neg Q(1,0,1) \vee \neg Q(2,0,1)$

31. Suppose that the domain of $Q(x, y, z)$ consists of triples x, y, z, where $x = 0, 1,$ or 2, $y = 0$ or 1, and $z = 0$ or 1. Write out these propositions using disjunctions and conjunctions.

a) $\forall y Q(0, y, 0)$ **b)** $\exists x Q(x, 1, 1)$
c) $\exists z \neg Q(0, 0, z)$ **d)** $\exists x \neg Q(x, 0, 1)$

a) $\forall y Q(0, y, 0)$

$Q(0,0,0) \wedge Q(0,1,0)$

b) $\exists x Q(x, 1, 1)$

$Q(0,1,1) \vee Q(1,1,1) \vee Q(2,1,1)$

c) $\exists z \neg Q(0, 0, z)$

$\neg Q(0,0,0) \vee \neg Q(0,0,1)$

d) $\exists x \neg Q(x, 0, 1)$

$\neg Q(0,0,1) \vee \neg Q(1,0,1) \vee \neg Q(2,0,1)$

38. Translate these system specifications into English where the predicate $S(x, y)$ is "x is in state y" and where the domain for x and y consists of all systems and all possible states, respectively.

a) $\exists x S(x, \text{open})$

b) $\forall x (S(x, \text{malfunctioning}) \vee S(x, \text{diagnostic}))$

c) $\exists x S(x, \text{open}) \vee \exists x S(x, \text{diagnostic})$

d) $\exists x \neg S(x, \text{available})$

e) $\forall x \neg S(x, \text{working})$

a) $\exists x S(x, \text{open})$

There exists a system that is open

51. Show that $\exists x\, P(x) \land \exists x\, Q(x)$ and $\exists x\, (P(x) \land Q(x))$ are not logically equivalent.