4. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

a) the set of people who speak English, the set of people who speak English with an Australian accent

b) the set of fruits, the set of citrus fruits

c) the set of students studying discrete mathematics, the set of students studying data structures

10. Determine whether these statements are true or false.

a) $\emptyset \in \{\emptyset\}$ **b)** $\emptyset \in \{\emptyset, \{\emptyset\}\}$

c) $\{\emptyset\} \in \{\emptyset\}$ **d)** $\{\emptyset\} \in \{\{\emptyset\}\}$

e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$ **f)** $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$

g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

14. Use a Venn diagram to illustrate the relationship $A \subseteq B$ and $B \subseteq C$.

20. What is the cardinality of each of these sets?

a) $\emptyset$ **b)** $\{\emptyset\}$

c) $\{\emptyset, \{\emptyset\}\}$ **d)** $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

27. Let $A = \{a, b, c, d\}$ and $B = \{y, z\}$. Find

a) $A \times B$. **b)** $B \times A$.

2.2.4

4. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find

a) $A \cup B$. **b)** $A \cap B$.

c) $A - B$. **d)** $B - A$.

26. Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

a) $A \cap (B \cup C)$ **b)** $\overline{A} \cap \overline{B} \cap \overline{C}$

c) $(A - B) \cup (A - C) \cup (B - C)$

30. Can you conclude that $A = B$ if A, B, and C are sets such that

a) $A \cup C = B \cup C$? **b)** $A \cap C = B \cap C$?

c) $A \cup C = B \cup C$ and $A \cap C = B \cap C$?

47. Let $A_i = \{1, 2, 3, \ldots, i\}$ for $i = 1, 2, 3, \ldots$. Find

a) $\bigcup_{i=1}^{n} A_i$.

b) $\bigcap_{i=1}^{n} A_i$.

50. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ if for every positive integer i,

a) $A_i = \{i, i+1, i+2, \ldots\}$.

b) $A_i = \{0, i\}$.

c) $A_i = (0, i)$, that is, the set of real numbers x with $0 < x < i$.

d) $A_i = (i, \infty)$, that is, the set of real numbers x with $x > i$.

2.3

2. Determine whether f is a function from **Z** to **R** if

a) $f(n) = \pm n$.

b) $f(n) = \sqrt{n^2 + 1}$.

c) $f(n) = 1/(n^2 - 4)$.

6. Find the domain and range of these functions.

a) the function that assigns to each pair of positive integers the first integer of the pair

b) the function that assigns to each positive integer its largest decimal digit

c) the function that assigns to a bit string the number of ones minus the number of zeros in the string

d) the function that assigns to each positive integer the largest integer not exceeding the square root of the integer

e) the function that assigns to a bit string the longest string of ones in the string

8. Find these values.

a) $\lfloor 1.1 \rfloor$
b) $\lceil 1.1 \rceil$
c) $\lfloor -0.1 \rfloor$
d) $\lceil -0.1 \rceil$
e) $\lceil 2.99 \rceil$
f) $\lceil -2.99 \rceil$
g) $\lfloor \frac{1}{2} + \lceil \frac{1}{2} \rceil \rfloor$
h) $\lceil \lfloor \frac{1}{2} \rfloor + \lceil \frac{1}{2} \rceil + \frac{1}{2} \rceil$