Homework #4 (section 2.1-2.3)

Section 2.1

4. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

a) the set of people who speak English, the set of people who speak English with an Australian accent

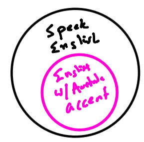
b) the set of fruits, the set of citrus fruits

c) the set of students studying discrete mathematics, the set of students studying data structures

a) the set of people who speak English, the set of people who speak English with an Australian accent

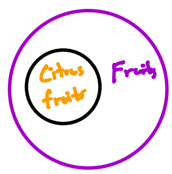
speak English with an Australian accent $\subseteq$ of people who speak English.

is a subset



b) the set of fruits, the set of citrus fruits

Citrus fruit $\subseteq$ fruits



c) the set of students studying discrete mathematics, the set of students studying data structures

neither

10. Determine whether these statements are true or false.

a) $\emptyset \in \{\emptyset\}$
b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$
c) $\{\emptyset\} \in \{\emptyset\}$
d) $\{\emptyset\} \in \{\{\emptyset\}\}$
e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

a) $\emptyset \in \{\emptyset\}$

True

~ Because the empty set contains the empty set.

b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$

True

Because the empty set is in the set

c) $\{\emptyset\} \in \{\emptyset\}$

False

Because the set contains the empty set, not the set of the empty set

d) $\{\emptyset\} \in \{\{\emptyset\}\}$

True

Because the set contains the set of the empty set

$\subset$ Subset

e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$

True

Because the set of the empty set is a subset of the set containing both the Empty set and the set of the Empty set

f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$

True

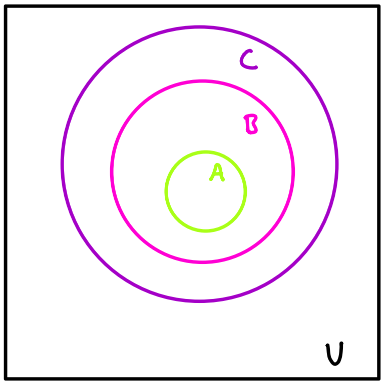
B/c the set of the empty set of the empty set is contained within the second subset, but the two subsets are not equal.

g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

False

Because, the two sets are equal and to be a proper subset, they can not be equal.

14. Use a Venn diagram to illustrate the relationship $A \subseteq B$ and $B \subseteq C$.



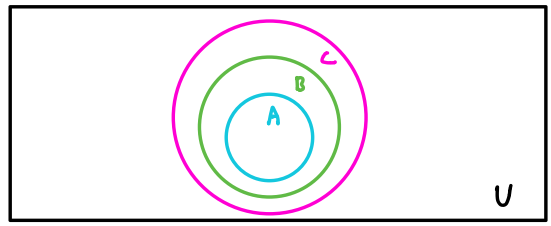
U = universe

Note:

$A \subseteq B$

the "_" represents that it can be equivilent

15. Use a Venn diagram to illustrate the relationships $A \subset B$ and $B \subset C$.



Note:

$A \subset B$

A must strictly be smaller than B

19. What is the cardinality of each of these sets?

a) $\{a\}$ **b)** $\{\{a\}\}$
c) $\{a, \{a\}\}$ **d)** $\{a, \{a\}, \{a, \{a\}\}\}$

20. What is the cardinality of each of these sets?

size

a) $\emptyset$ **b)** $\{\emptyset\}$
c) $\{\emptyset, \{\emptyset\}\}$ **d)** $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

Note: look at the commas!

a) $\{a\}$

1

b) $\{\{a\}\}$

1

c) $\{\underline{a}, \underline{\{a\}}\}$

2

d) $\{\underline{a}, \underline{\{a\}}, \underline{\{a, \{a\}\}}\}$

3

a) $\emptyset$

0

b) $\{\emptyset\}$

1

c) $\{\underline{\emptyset}, \underline{\{\emptyset\}}\}$

2

d) $\{\underline{\emptyset}, \underline{\{\emptyset\}}, \underline{\{\emptyset, \{\emptyset\}\}}\}$

3

21. Find the power set of each of these sets, where a and b are distinct elements.

a) $\{a\}$ **b)** $\{a, b\}$ **c)** $\{\emptyset, \{\emptyset\}\}$

Note:
power set is the set contains all possible subsets.

a) $\{a\}$

$P(\{a\}) = \{\emptyset, \{a\}\}$

b) $\{a, b\}$

$P(\{a,b\}) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

c) $\{\emptyset, \{\emptyset\}\}$

$P(\emptyset, \{\emptyset\}) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

27. Let $A = \{a, b, c, d\}$ and $B = \{y, z\}$. Find

a) $A \times B$. **b)** $B \times A$.

a) $A \times B$.

$A \times B = \{(a,y), (a,z), (b,y), (b,z), (c,y), (c,z), (d,y), (d,z)\}$

b) $B \times A$.

$B \times A = \{(y,a), (y,b), (y,c), (y,d), (z,a), (z,b), (z,c), (z,d)\}$

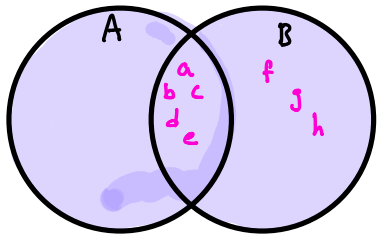
Section 2.2

4. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find

a) $A \cup B$. **b)** $A \cap B$.
c) $A - B$. **d)** $B - A$.

Union

a) $A \cup B$.

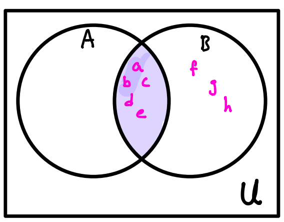


$A \cup B = \{a, b, c, d, e, f, g, h\}$

Let A and B be sets. The *union* of the sets A and B, denoted by $A \cup B$, is the set that contains those elements that are either in A or in B, or in both.

intersection

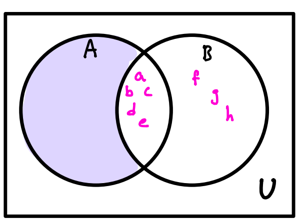
b) $A \cap B$.



Let A and B be sets. The *intersection* of the sets A and B, denoted by $A \cap B$, is the set containing those elements in both A and B.

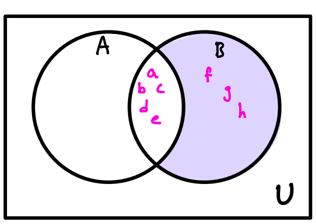
$A \cap B = \{a, b, c, d, e\}$

c) $A - B$.



$A - B = \emptyset$

d) $B - A$.



$B - A = \{f, g, h\}$

26. Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

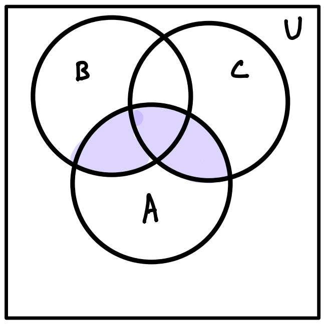
a) $A \cap (B \cup C)$ **b)** $\overline{A} \cap \overline{B} \cap \overline{C}$

c) $(A - B) \cup (A - C) \cup (B - C)$

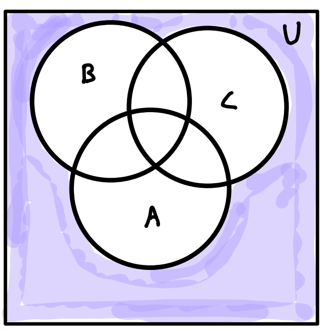
Intersection

a) $A \cap (B \cup C)$

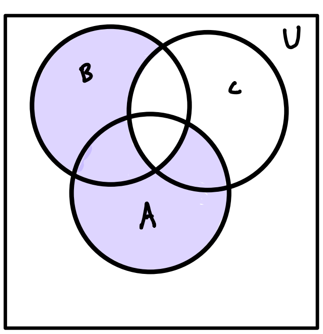
Union



b) $\overline{A} \cap \overline{B} \cap \overline{C}$



c) $(A - B) \cup (A - C) \cup (B - C)$



30. Can you conclude that $A = B$ if A, B, and C are sets such that

a) $A \cup C = B \cup C$?   **b)** $A \cap C = B \cap C$?

c) $A \cup C = B \cup C$ and $A \cap C = B \cap C$?

a) $A \cup C = B \cup C$?

No

Counter example: Given $A \cup C = B \cup C$

A and B might not be the same set

$A = \{1\}$
$B = \{0\}$
$C = \{0,1\}$

from Union

$A \cup C = \{0,1\}$
$A \cup B = \{0,1\}$

Which $A \cup C = B \cup C$ stands
While the two sets A and B are unequal.
$\therefore$ $A = B$ is <u>not True</u>.

b) $A \cap C = B \cap C$?

No

$A = \{0,1\}$
$B = \{0,2\}$
$C = \{0\}$

By Intersection:

$A \cap C = \{0\}$
$A \cap B = \{0\}$

47. Let $A_i = \{1, 2, 3, \ldots, i\}$ for $i = 1, 2, 3, \ldots$. Find

a) $\bigcup_{i=1}^{n} A_i$.   **b)** $\bigcap_{i=1}^{n} A_i$.

a) $\bigcup_{i=1}^{n} A_i$.

$A_i = \{1,2,3,\ldots i\}$ for $i = 1,2,3\ldots$ the sets are increasing

$A_1 \subseteq A_2 \subseteq \ldots \subseteq A_n$

$A_n = \{1,2,3,\ldots n\}$

b) $\bigcap_{i=1}^{n} A_i$.

$A_1 = \{1\}$

$A_1 = \{1\}$ looks to be the only chance of being in the intersection.

51. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ if for every positive integer i,

a) $A_i = \{-i, -i+1, \ldots, -1, 0, 1, \ldots, i-1, i\}$.

b) $A_i = \{-i, i\}$.

c) $A_i = [-i, i]$, that is, the set of real numbers x with $-i \le x \le i$.

d) $A_i = [i, \infty)$, that is, the set of real numbers x with $x \ge i$.

a) $A_i = \{-i, -i+1, \ldots, -1, 0, 1, \ldots, i-1, i\}$.

i is increasing and the set is getting larger $A_1 \subseteq A_2 \subseteq A_3$

All the sets are subsets of the set of integers given, so every integer is included.

$\bigcup_{i=1}^{\infty} A_i = \mathbb{Z}$

And since A_1 is a subset of each other

$\bigcap_{i=1}^{\infty} A_i = A_1 = \{-1, 0, 1\}$

b) $A_i = \{-i, i\}$.

All the sets are subsets of the set of integers and every nonzero integer is in exactly one of the sets.

$\bigcup_{i=1}^{\infty} A_i = \mathbb{Z} - \{0\}$

Each pair in these sets have no element in common, one will always be positive and the other negative. (e.g. $\{-1, 1\}$)

$\bigcap_{i=1}^{\infty} A_i = \emptyset$

c) $A_i = [-i, i]$, that is, the set of real numbers x with $-i \le x \le i$.

d) $A_i = [i, \infty)$, that is, the set of real numbers x with $x \ge i$.