Homework #4 (section 2.1-2.3)

Section 2.1

4. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

a) the set of people who speak English, the set of people who speak English with an Australian accent

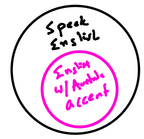
b) the set of fruits, the set of citrus fruits

c) the set of students studying discrete mathematics, the set of students studying data structures

a) the set of people who speak English, the set of people who speak English with an Australian accent

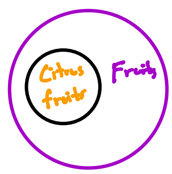
speak English with an Australian accent $\subseteq$ of people who speak English.

is a subset



b) the set of fruits, the set of citrus fruits

Citrus fruit $\subseteq$ fruits



c) the set of students studying discrete mathematics, the set of students studying data structures

neither

10. Determine whether these statements are true or false.

a) $\emptyset \in \{\emptyset\}$
b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$
c) $\{\emptyset\} \in \{\emptyset\}$
d) $\{\emptyset\} \in \{\{\emptyset\}\}$
e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

a) $\emptyset \in \{\emptyset\}$

True

~ Because the empty set contains the empty set.

b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$

True

Because the empty set is in the set

c) $\{\emptyset\} \in \{\emptyset\}$

False

Because the set contains the empty set, not the set of the empty set

d) $\{\emptyset\} \in \{\{\emptyset\}\}$

True

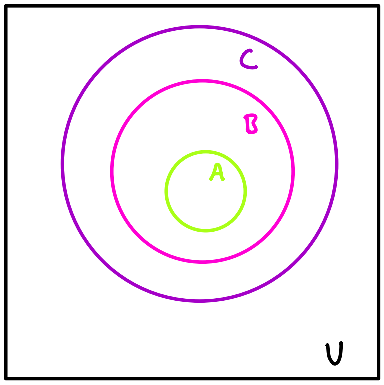
Because the set contains the set of the empty set

e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$

f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$

g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

14. Use a Venn diagram to illustrate the relationship $A \subseteq B$ and $B \subseteq C$.



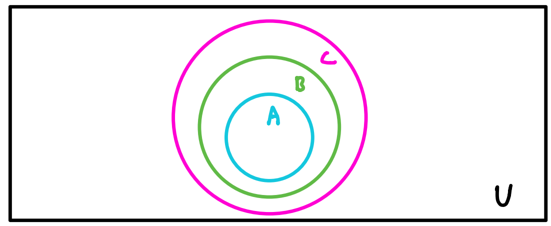
U = universe

Note:

$A \subseteq B$

the "_" represents that it can be equvilent

15. Use a Venn diagram to illustrate the relationships $A \subset B$ and $B \subset C$.



Note:

$A \subset B$

A must strictly be smaller than B

19. What is the cardinality of each of these sets?

a) $\{a\}$ **b)** $\{\{a\}\}$

c) $\{a, \{a\}\}$ **d)** $\{a, \{a\}, \{a, \{a\}\}\}$

20. What is the cardinality of each of these sets?

= size

a) $\emptyset$ **b)** $\{\emptyset\}$

c) $\{\emptyset, \{\emptyset\}\}$ **d)** $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

Note: look at the commas!

a) $\{a\}$

1

b) $\{\{a\}\}$

1

c) $\{a, \{a\}\}$

2

d) $\{a, \{a\}, \{a, \{a\}\}\}$

3

a) $\emptyset$

0

b) $\{\emptyset\}$

1

c) $\{\emptyset, \{\emptyset\}\}$

2

d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

3

21. Find the power set of each of these sets, where a and b are distinct elements.

a) $\{a\}$ **b)** $\{a, b\}$ **c)** $\{\emptyset, \{\emptyset\}\}$

Note:
power set is the set contains all possible subsets.

a) $\{a\}$

$P(\{a\}) = \{\emptyset, \{a\}\}$

b) $\{a, b\}$

$P(\{a,b\}) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

c) $\{\emptyset, \{\emptyset\}\}$

$P(\emptyset, \{\emptyset\}) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

27. Let $A = \{a, b, c, d\}$ and $B = \{y, z\}$. Find

a) $A \times B$. **b)** $B \times A$.

a) $A \times B$.

$A \times B = \{(a,y), (a,z), (b,y), (b,z), (c,y), (c,z), (d,y), (d,z)\}$

b) $B \times A$.

$B \times A = \{(y,a), (y,b), (y,c), (y,d), (z,a), (z,b), (z,c), (z,d)\}$

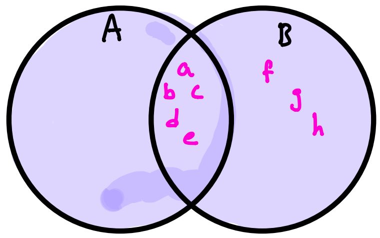
Section 2.2

4. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find

a) $A \cup B$. **b)** $A \cap B$.
c) $A - B$. **d)** $B - A$.

Union

a) $A \cup B$.

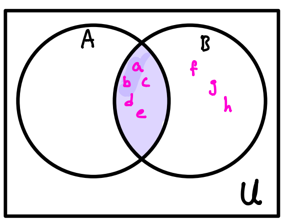


$A \cup B = \{a, b, c, d, e, f, g, h\}$

Let A and B be sets. The *union* of the sets A and B, denoted by $A \cup B$, is the set that contains those elements that are either in A or in B, or in both.

intersection

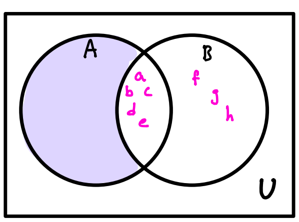
b) $A \cap B$.



Let A and B be sets. The *intersection* of the sets A and B, denoted by $A \cap B$, is the set containing those elements in both A and B.

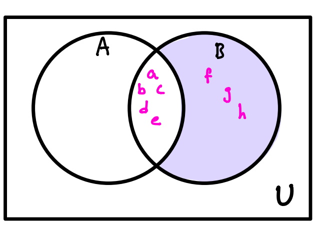
$A \cap B = \{a, b, c, d, e\}$

c) $A - B$.



$A - B = \emptyset$

d) $B - A$.

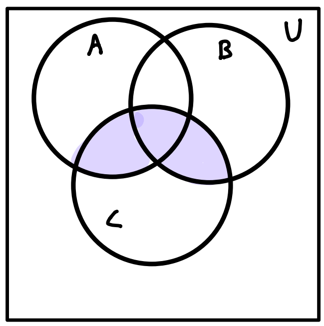


$B - A = \{f, g, h\}$

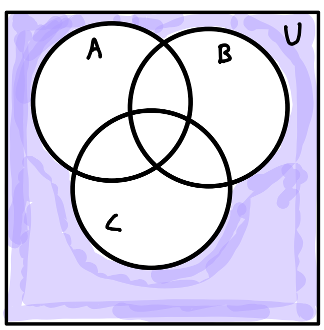
26. Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

a) $A \cap (B \cup C)$ **b)** $\overline{A} \cap \overline{B} \cap \overline{C}$
c) $(A - B) \cup (A - C) \cup (B - C)$

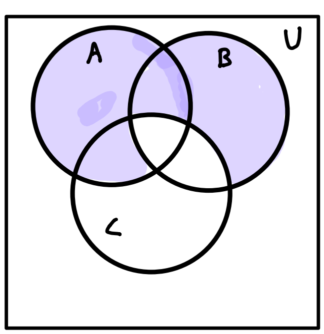
a) $A \cap (B \cup C)$



b) $\overline{A} \cap \overline{B} \cap \overline{C}$



c) $(A - B) \cup (A - C) \cup (B - C)$



30. Can you conclude that $A = B$ if A, B, and C are sets such that

a) $A \cup C = B \cup C$? **b)** $A \cap C = B \cap C$?

c) $A \cup C = B \cup C$ and $A \cap C = B \cap C$?

a) $A \cup C = B \cup C$?

No

Counter example: Given $A \cup C = B \cup C$

A and B might not be the same set

$A = \{1\}$

$B = \{0\}$

$C = \{0,1\}$

from Union

$A \cup C = \{0,1\}$

$A \cup B = \{0,1\}$

Which $A \cup C = B \cup C$ stands

While the two sets A and B are unequal.

$\therefore$ $A = B$ is not True.

b) $A \cap C = B \cap C$?

No

$A = \{0,1\}$

$B = \{0,2\}$

$C = \{0\}$

By Intersection:

$A \cap C = \{0\}$

$A \cap B = \{0\}$

c) $A \cup C = B \cup C$ and $A \cap C = B \cap C$?

47. Let $A_i = \{1, 2, 3, \ldots, i\}$ for $i = 1, 2, 3, \ldots$. Find

a) $\bigcup_{i=1}^{n} A_i$. **b)** $\bigcap_{i=1}^{n} A_i$.

a) $\bigcup_{i=1}^{n} A_i$.

b) $\bigcap_{i=1}^{n} A_i$.

50. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ if for every positive integer i,

a) $A_i = \{i, i+1, i+2, \ldots\}$.

b) $A_i = \{0, i\}$.

c) $A_i = (0, i)$, that is, the set of real numbers x with $0 < x < i$.

d) $A_i = (i, \infty)$, that is, the set of real numbers x with $x > i$.

a) $A_i = \{i, i+1, i+2, \ldots\}$.

b) $A_i = \{0, i\}$.

c) $A_i = (0, i)$, that is, the set of real numbers x with $0 < x < i$.

d) $A_i = (i, \infty)$, that is, the set of real numbers x with $x > i$.

Section 2.3 : functions

2. Determine whether f is a function from $\mathbf{Z}$ to $\mathbf{R}$ if

a) $f(n) = \pm n$.

b) $f(n) = \sqrt{n^2 + 1}$.

c) $f(n) = 1/(n^2 - 4)$.

6. Find the domain and range of these functions.

a) the function that assigns to each pair of positive integers the first integer of the pair

b) the function that assigns to each positive integer its largest decimal digit

c) the function that assigns to a bit string the number of ones minus the number of zeros in the string

d) the function that assigns to each positive integer the largest integer not exceeding the square root of the integer

e) the function that assigns to a bit string the longest string of ones in the string

8. Find these values.

a) $\lfloor 1.1 \rfloor$

b) $\lceil 1.1 \rceil$

c) $\lfloor -0.1 \rfloor$

d) $\lceil -0.1 \rceil$

e) $\lceil 2.99 \rceil$

f) $\lceil -2.99 \rceil$

g) $\lfloor \frac{1}{2} + \lceil \frac{1}{2} \rceil \rfloor$

h) $\lceil \lfloor \frac{1}{2} \rfloor + \lceil \frac{1}{2} \rceil + \frac{1}{2} \rceil$