1.3

7. Let $T(x, y)$ mean that student x likes cuisine y, where the domain for x consists of all students at your school and the domain for y consists of all cuisines. Express each of these statements by a simple English sentence.

a) $\neg T(\text{Abdallah Hussein, Japanese})$
b) $\exists x T(x, \text{Korean}) \wedge \forall x T(x, \text{Mexican})$
c) $\exists y(T(\text{Monique Arsenault}, y) \vee T(\text{Jay Johnson}, y))$
d) $\forall x \forall z \exists y((x \neq z) \rightarrow \neg(T(x, y) \wedge T(z, y)))$
e) $\exists x \exists z \forall y(T(x, y) \leftrightarrow T(z, y))$
f) $\forall x \forall z \exists y(T(x, y) \leftrightarrow T(z, y))$

10. Let $F(x, y)$ be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

a) Everybody can fool Fred.
b) Evelyn can fool everybody.
c) Everybody can fool somebody.
d) There is no one who can fool everybody.
e) Everyone can be fooled by somebody.
f) No one can fool both Fred and Jerry.
g) Nancy can fool exactly two people.
h) There is exactly one person whom everybody can fool.
i) No one can fool himself or herself.
j) There is someone who can fool exactly one person besides himself or herself.

31. Express the negations of each of these statements so that all negation symbols immediately precede predicates.

a) $\forall x \exists y \forall z T(x, y, z)$

b) $\forall x \exists y P(x, y) \vee \forall x \exists y Q(x, y)$

c) $\forall x \exists y (P(x, y) \wedge \exists z R(x, y, z))$

d) $\forall x \exists y (P(x, y) \rightarrow Q(x, y))$

1.6

18. What is wrong with this argument? Let $S(x, y)$ be "x is shorter than y." Given the premise $\exists s\, S(s, \text{Max})$, it follows that $S(\text{Max}, \text{Max})$. Then by existential generalization it follows that $\exists x\, S(x, x)$, so that someone is shorter than himself.

23. Identify the error or errors in this argument that supposedly shows that if $\exists x P(x) \wedge \exists x Q(x)$ is true then $\exists x (P(x) \wedge Q(x))$ is true.

1. $\exists x P(x) \vee \exists x Q(x)$	Premise
2. $\exists x P(x)$	Simplification from (1)
3. $P(c)$	Existential instantiation from (2)
4. $\exists x Q(x)$	Simplification from (1)
5. $Q(c)$	Existential instantiation from (4)
6. $P(c) \wedge Q(c)$	Conjunction from (3) and (5)
7. $\exists x (P(x) \wedge Q(x))$	Existential generalization

46. Determine the truth value of the statement $\exists x \forall y (x \leq y^2)$ if the domain for the variables consists of

a) the positive real numbers.

b) the integers.

c) the nonzero real numbers.

24. Identify the error or errors in this argument that supposedly shows that if $\forall x(P(x) \vee Q(x))$ is true then $\forall x P(x) \vee \forall x Q(x)$ is true.

1. $\forall x(P(x) \vee Q(x))$	Premise
2. $P(c) \vee Q(c)$	Universal instantiation from (1)
3. $P(c)$	Simplification from (2)
4. $\forall x P(x)$	Universal generalization from (3)
5. $Q(c)$	Simplification from (2)
6. $\forall x Q(x)$	Universal generalization from (5)
7. $\forall x(P(x) \vee \forall x Q(x))$	Conjunction from (4) and (6)

28. Use rules of inference to show that if $\forall x(P(x) \lor Q(x))$ and $\forall x((\neg P(x) \land Q(x)) \rightarrow R(x))$ are true, then $\forall x(\neg R(x) \rightarrow P(x))$ is also true, where the domains of all quantifiers are the same.

33. Use resolution to show that the compound proposition $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$ is not satisfiable.

1.7

6. Use a direct proof to show that the product of two odd numbers is odd.

18. Prove that if n is an integer and $3n + 2$ is even, then n is even using

a) a proof by contraposition.

b) a proof by contradiction.

20. Prove the proposition $P(1)$, where $P(n)$ is the proposition "If n is a positive integer, then $n^2 \geq n$." What kind of proof did you use?

24. Show that at least three of any 25 days chosen must fall in the same month of the year.

34. Is this reasoning for finding the solutions of the equation $\sqrt{2x^2 - 1} = x$ correct? *(1)* $\sqrt{2x^2 - 1} = x$ is given; *(2)* $2x^2 - 1 = x^2$, obtained by squaring both sides of (1); *(3)* $x^2 - 1 = 0$, obtained by subtracting x^2 from both sides of (2); *(4)* $(x - 1)(x + 1) = 0$, obtained by factoring the left-hand side of $x^2 - 1$; *(5)* $x = 1$ or $x = -1$, which follows because $ab = 0$ implies that $a = 0$ or $b = 0$.