**MPCS 50103 Discrete Mathematics—Autumn 2019**

**Homework 7: revised. This problem set is due Monday November 18 at 11:59 pm.**

*Reading*: Rosen 7e, chapter 10, sections 10.1–10.4.

**Homework assignment**:

- "DO" exercises are strongly recommended to check your understanding of the concepts. **Solve them but do not submit them**.
- **Problems labeled "HW" are homework problems that you are required to submit**.
- You are responsible for the material covered in **both** "DO" exercises and HW problems.

**"Do" Exercises (solve but do *not* submit):**

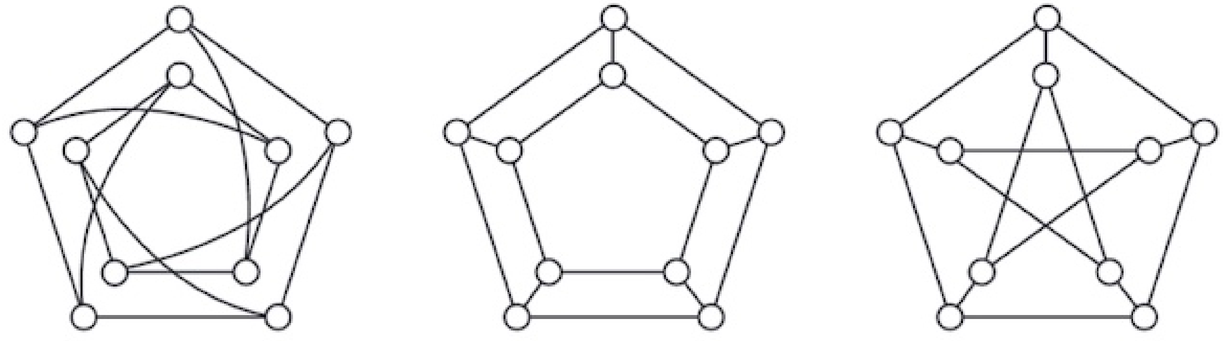
1. **"DO"** Rosen 7e, section 10.2, exercises 23, 33, 39, 41, 43a, 45, 46, and 49, on pages 665–667.
2. **"DO"** Rosen 7e, section 10.3, exercises 35, 39, 41, 43, and 55, on pages 676–677.
3. **"DO"** Rosen 7e, section 10.4, exercises 21 and 23, on pages 690–691.

**Homework Problems: DUE Monday November 18 at 11:59 pm**

- **Collaboration policy**: There is no penalty for acknowledged collaboration. To acknowledge collaboration, give the names of students with whom you worked at the beginning of your homework submission and mark any solution that relies on your collaborators' ideas. Note: you must work out and write up each homework solution by yourself without assistance.
  **DO NOT COPY or rephrase someone else's solution.**
  The same requirement applies to books and other written sources: you should acknowledge all sources that contributed to your solution of a homework problem. Acknowledge specific ideas you learned from the source.
  **DO NOT COPY or rephrase solutions from written sources.**
- **Internet policy**: Looking for solutions to homework problems on the internet, even when acknowledged, is STRONGLY DISCOURAGED. If you find a solution to a homework problem on the internet, do not not copy it. Close the website, work out and write up your solution by yourself, and cite the url of website in your writeup. Acknowledge specific ideas you learned from the website.
  **Copied solutions obtained from a written source, from the internet, or from another person will receive ZERO credit and will be flagged to the attention of the instructor.**
- Write out your work for every problem. If you just write your answer without showing your work, you will not receive credit.

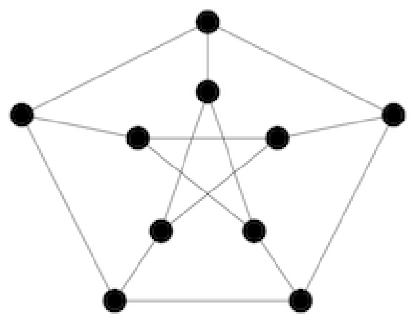
1. **HW** Recall: A sequence of nonnegative integers $d = (d_1, d_2, \ldots, d_n)$, $n > 1$, is ***graphic*** if there exists a simple graph with degree sequence $d$.
   Determine whether or not the following sequences are graphic. If the sequence is graphic, draw the corresponding simple graph. If not, provide a proof of impossibility. (2 points each)
   - $(1, 1, 2, 2, 3, 3)$
   - $(1, 1, 2, 2, 3, 3, 4, 4)$
   - $(2, 2, 2, 2, 2, 3, 4, 5)$
   - $(2, 2, 3, 3, 3, 3, 3, 4, 5)$

2. **HW** Among the following graphs, which pairs (if any) are isomorphic? If a pair of these graphs is isomorphic, describe an isomorphism between them, making sure to clearly label the vertices in both graphs. If a pair is not isomorphic, use a graph invariant to prove they are not. Recall that the graph invariants we have learned so far are number of vertices, number of edges, degree sequence, subgraph/circuit of a given length, and connectedness. (2 points for each pair)



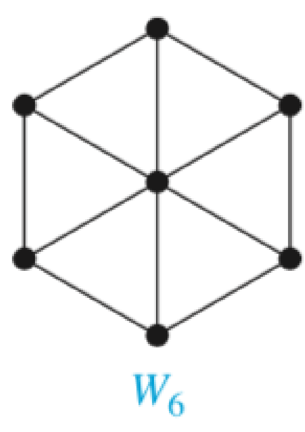
$G_1$ $G_2$ $G_3$

3. **HW Definition**. An undirected simple graph $G$ is called ***regular*** of degree $k$ (or ***k-regular***) if all the vertices of $G$ have degree exactly $k$. For example, if $\deg(v) = 3$ for all vertices $v$ in $V(G)$, then we call $G$ *3-regular*. The following graph is an example of a 3-regular graph:



    - Suppose that $G$ is 3-regular and has $n$ vertices. How many edges does $G$ have? Prove your answer. (2 points)
    - Draw two nonisomorphic connected simple 3-regular graphs on 6 vertices. (3 points)
    - Prove that your graphs are not isomorphic (use a graph invariant). (2 points)
    - Let $G$ be a $k$-regular graph with $k$ odd. Prove that the number of vertices in $G$ must be even. (3 points)

4. **HW** Recall: The ***complement*** $G^c$ of the simple graph $G = (V, E)$ is a simple graph with the same vertices as $G$, and $(u, v)$ is an edge of $G^c$ if and only if it is not an edge of $G$.
    - Draw the complement of the wheel $W_6$. (1 point)



$W_6$

    - Give an expression for the number of edges in $G^c$, given $G = (V, E)$. (2 points)
    - For a simple graph $G$ on $n$ vertices, with degree sequence $(d_1, d_2, \ldots, d_n)$, give an expression for the degree sequence of $G^c$. (2 points)
    - For simple graphs $G_1$ and $G_2$, prove that $G_1$ and $G_2$ are isomorphic if and only if ${G_1}^c$ and ${G_2}^c$ are isomorphic. (3 points)

- Prove that if $G$ is not connected, then $G^c$ is connected. Is the converse true? (3 points)

5. **HW** Draw all nonisomorphic simple graphs on a fixed set of four vertices. (Make sure not to miss any, and do not repeat, i.e., no pair of your drawings should represent isomorphic graphs.) (6 points; you lose 2 points for every mistake up to 3 mistakes)

6. **HW** Prove by induction: Every simple graph with $n$ vertices and $n - k$ edges, where $k$ is an integer and $k < n$, has at least $k$ connected components.
For credit, you must prove the basis step, state the inductive hypothesis, and give a correct proof of the inductive step, specifying where you use the inductive hypothesis and justifying intermediate steps of your proof. (4 points)

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*Gerry Brady*
*Thursday November 14 15:03:40 CST 2019*