Section 1.1 Discrete Mathematics HW #2

X

**28.** State the converse, contrapositive, and inverse of each of these conditional statements.

**a)** If it snows tonight, then I will stay at home.
**b)** I go to the beach whenever it is a sunny summer day.
**c)** When I stay up late, it is necessary that I sleep until noon.

- $q \rightarrow p$ is the converse of $p \rightarrow q$ ~ Switch Order
- $\neg p \rightarrow \neg q$ is the inverse of $p \rightarrow q$ ~ negate
- $\neg q \rightarrow \neg p$ is the contrapositive of $p \rightarrow q$ ~ neg + switch

$p \rightarrow q$

**a)** If it snows tonight ($p$), then I will stay at home ($q$).

**Converse:** If, I stay at home, then it snowed tonight

**Contrapositive:** If I do not at stay home, then it did not snow tonight.

**Inverse:** If it does not snow tonight, then I will not stay home

**b)** I go to the beach ($p$) whenever it is a sunny summer day ($q$).

**Converse:** If it is a sunny summer day, then I will go to the beach.

**Contrapositive:** If it's not a sunny summer day then I will not go to the beach

**Inverse:** If I do not go to the beach, then it was not a sunny summer day.

**35.** Construct a truth table for each of these compound propositions.

**a)** $p \to \neg q$
**b)** $\neg p \leftrightarrow q$
**c)** $(p \to q) \vee (\neg p \to q)$
**d)** $(p \to q) \wedge (\neg p \to q)$
**e)** $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$
**f)** $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

**Biconditional** $(\leftrightarrow, \equiv)$ iff $\to$ if and only if

| p | q | $p \leftrightarrow q$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

$p = q$ then $p \leftrightarrow q = 1$

if they (p and q) are the same value then it's True.

**a)** $p \to \neg q$

| p | q | $\neg q$ | $p \to \neg q$ |
|---|---|---|---|
| 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 |

**b)** $\neg p \leftrightarrow q$

| p | $\neg p$ | q | $\neg p \leftrightarrow q$ |
|---|---|---|---|
| 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 |

**c)** $(p \to q) \vee (\neg p \to q)$

| p | q | $\neg p$ | $(p \to q)$ | $(\neg p \to q)$ | $(p \to q) \vee (\neg p \to q)$ |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 |

**d)** $(p \to q) \wedge (\neg p \to q)$

| p | q | $\neg p$ | $(p \to q)$ | $(\neg p \to q)$ | $(p \to q) \wedge (\neg p \to q)$ |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 |

**Conjunction** $(\wedge, *, -)$

and

| p | q | $p \wedge q$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

**e)** $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$

| p | $\neg p$ | q | $\neg p \leftrightarrow q$ | $(p \leftrightarrow q)$ | $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$ |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 |

**f)** $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

| $(p \leftrightarrow q)$ | $(\neg p \leftrightarrow \neg q)$ | $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$ |
|---|---|---|
| 1 | | |
| 0 | | |
| 0 | | |
| 1 | | |

**Disjunction** $(\vee, +)$

| p | q | $p \vee q$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |

**Conditional** $(\to, \supset)$

| p | q | $p \to q$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 1 |
| 0 | 0 | 1 |

out

**9.** Show that each of these conditional statements is a tautology by using truth tables.

**a)** $(p \wedge q) \to p$  **b)** $p \to (p \vee q)$
**c)** $\neg p \to (p \to q)$  **d)** $(p \wedge q) \to (p \to q)$
**e)** $\neg(p \to q) \to p$  **f)** $\neg(p \to q) \to \neg q$

**a)** $(p \wedge q) \to p$

| $p$ | $q$ | $(p \wedge q)$ | $(p \wedge q) \to p$ |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 0 | 0 | 1 |

Tautology

**b)** $p \to (p \vee q)$

| $p$ | $q$ | $(p \vee q)$ | $p \to (p \vee q)$ |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 |

Tautology

**c)** $\neg p \to (p \to q)$

| $p$ | $\neg p$ | $q$ | $(p \to q)$ | $\neg p \to (p \to q)$ |
|---|---|---|---|---|
| 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |

Tautology

**d)** $(p \wedge q) \to (p \to q)$

| $p$ | $q$ | $(p \wedge q)$ | $(p \to q)$ | $(p \wedge q) \to (p \to q)$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 |

Tautology

**e)** $\neg(p \to q) \to p$

| $p$ | $q$ | $(p \to q)$ | $\neg(p \to q)$ | $\neg(p \to q) \to p$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 |

Tautology

**f)** $\neg(p \to q) \to \neg q$

| $p$ | $q$ | $\neg q$ | $(p \to q)$ | $\neg(p \to q)$ | $\neg(p \to q) \to \neg q$ |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 |

Tautology

**Associativity** $(\vee, \wedge)$

$$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$$

~ can do "ors" as well

~ make sure the signs stay the same.

LOGIC LAWS

**11.** Show that each conditional statement in Exercise 9 is a tautology without using truth tables.

**a)** $(p \wedge q) \rightarrow p$

$\underbrace{p \wedge q}_{p} \rightarrow \underbrace{p}_{q}$

DM LAW

$\neg(p \wedge q) \vee p$

$(\neg p \vee \neg q) \vee p$

$(\neg q \vee \neg p) \vee p$ ~ Commutative

$\neg q (\neg p \vee p)$

Commutativity $(\vee, \wedge)$

$p \wedge q \Leftrightarrow q \wedge p$

$p \vee q \Leftrightarrow q \vee p$

~ We can flip the order

**TABLE 7** **Logical Equivalences Involving Conditional Statements.**

| |
|---|
| $p \rightarrow q \equiv \neg p \vee q$ |
| $p \rightarrow q \equiv \neg q \rightarrow \neg p$ |
| $p \vee q \equiv \neg p \rightarrow q$ |
| $p \wedge q \equiv \neg(p \rightarrow \neg q)$ |
| $\neg(p \rightarrow q) \equiv p \wedge \neg q$ |
| $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$ |
| $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$ |
| $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$ |
| $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$ |

22. Show that $(p \to q) \wedge (p \to r)$ and $p \to (q \wedge r)$ are logically equivalent.

All different Variations

| $p$ | $q$ | $r$ | $p \to q$ | $p \to r$ | $p \to q \wedge p \to r$ | $(q \vee r)$ | $p \to (q \wedge r)$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |

logically Equivalent

**32.** Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.

All possibilities

| $p$ | $q$ | $r$ | $(p \wedge q)$ | $(p \wedge q) \rightarrow r$ | $(p \rightarrow r)$ | $(q \rightarrow r)$ | $(p \rightarrow r) \wedge (q \rightarrow r)$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |

**Not Logically Equivalent**

Conjunction ($\wedge$, $*$, $\cdot$) and

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

Conditional ($\rightarrow$, $\supset$)

| $p$ | $q$ | $p \rightarrow q$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 1 |
| 0 | 0 | 1 |

Disjunction ($\vee$, $+$)

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |

X

## Section 1.4

**8.** Translate these statements into English, where $R(x)$ is "$x$ is a rabbit" and $H(x)$ is "$x$ hops" and the domain consists of all animals.

**a)** $\forall x(R(x) \to H(x))$ **b)** $\forall x(R(x) \wedge H(x))$
**c)** $\exists x(R(x) \to H(x))$ **d)** $\exists x(R(x) \wedge H(x))$

R(x): "x is a rabbit"
H(x): "x hops"

$\forall x$ : for all
$\exists x$: there exists

**a)** $\forall x(R(x) \to H(x))$

For all animals that is a rabbit, then that animal hops.

**b)** $\forall x(R(x) \overset{\text{And}}{\wedge} H(x))$

All For all animals are rabbits and they hop.

**c)** $\exists x(R(x) \to H(x))$

There exists an Animal, that if is a rabbit, then it hops.

**d)** $\exists x(R(x) \wedge H(x))$

There exists an animal that is an Rabbit and that animal hops.

**61.** Determine whether each of these compound propositions is satisfiable.

**a)** $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$
**b)** $(p \rightarrow q) \wedge (p \rightarrow \neg q) \wedge (\neg p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$
**c)** $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$

**a)** $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

| $p$ | $q$ | $\neg p$ | $\neg q$ | $(p \vee \neg q)$ | $(\neg p \vee q)$ | $(\neg p \vee \neg q)$ | $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |

Satisfiable

Disjunction $(\vee, +)$

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |

Conjunction $(\wedge, *, -)$ and

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

**12.** Let $Q(x)$ be the statement "$x + 1 > 2x$." If the domain consists of all integers, what are these truth values?

**a)** $Q(0)$ **b)** $Q(-1)$ **c)** $Q(1)$
**d)** $\exists x Q(x)$ **e)** $\forall x Q(x)$ **f)** $\exists x \neg Q(x)$
**g)** $\forall x \neg Q(x)$

**a)** $Q(0)$

$x + 1 > 2x$
$0 + 1 > 2(0)$
$1 > 0$
TRUE

**b)** $Q(-1)$

$x + 1 > 2x$
$-1 + 1 > 2(-1)$
$0 > -2$
TRUE

**c)** $Q(1)$

$x + 1 > 2x$
$1 + 1 > 2(1)$
$2 > 2$
FALSE

**d)** $\exists x Q(x)$

$x + 1 > 2x$
$0 + 1 > 2(0)$
$1 > 0$
TRUE

**e)** $\forall x Q(x)$

$x + 1 > 2x$
$1 + 1 > 2(1)$
$2 > 2$ FALSE
FALSE

**f)** $\exists x \neg Q(x)$

TRUE

**g)** $\forall x \neg Q(x)$

FALSE

**13.** Determine the truth value of each of these statements if the domain consists of all integers.

**a)** $\forall n(n + 1 > n)$ **b)** $\exists n(2n = 3n)$

**c)** $\exists n(n = -n)$ **d)** $\forall n(3n \leq 4n)$

**a)** $\forall n(n + 1 > n)$

$n + 1 > n$

$0 + 1 > 0$

$1 > 0$ ✓

TRUE

**b)** $\exists n(2n = 3n)$

$n = 0$

$2(0) = 3(0)$

$0 = 0$ ✓

TRUE

**c)** $\exists n(n = -n)$

$n = -n$

$n = 0$

$0 = -0$ ✓

TRUE

**d)** $\forall n(3n \leq 4n)$

~ True for all + integer

~ False for − integer

FALSE

**27.** Translate each of these statements into logical expressions in three different ways by varying the domain and by using predicates with one and with two variables.

**a)** A student in your school has lived in Vietnam.

**b)** There is a student in your school who cannot speak Hindi.

**c)** A student in your school knows Java, Prolog, and C++.

**d)** Everyone in your class enjoys Thai food.

**e)** Someone in your class does not play hockey.

**a)** A student in your school has lived in Vietnam.

$1^{st}$ Domain: People in the school

$2^{nd}$ Domain: People in the World

$A(x)$ Represents " x is in your school"

$V(x)$ Represents " x has lived in Vietnam"

$V(x,y)$ Represents " x has lived in y"

Varying the Domain: $\exists x\, V(x)$ ~ There exists a student who lived in Vietnam.

Predicate with one Variables: $\exists x\,(V(x) \land A(x))$

Predicate with two Variables: $\exists x\,(A(x) \land V(x, \text{Vietnam}))$

**b)** There is a student in your school who cannot speak Hindi.

**30.** Suppose the domain of the propositional function $P(x, y)$ consists of pairs $x$ and $y$, where $x$ is 1, 2, or 3 and $y$ is 1, 2, or 3. Write out these propositions using disjunctions and conjunctions.

**a)** $\exists x\, P(x, 3)$ **b)** $\forall y\, P(1, y)$
**c)** $\exists y \neg P(2, y)$ **d)** $\forall x\, \neg P(x, 2)$

$\wedge$ : and for $\forall x$
$\vee$ : or for $\exists x$

**a)** $\exists x\, P(x, 3)$

$P(1,3) \vee P(2,3) \vee P(3,3)$

**b)** $\forall y\, P(1, y)$

$P(1,1) \wedge P(1,2) \wedge P(1,3)$

**c)** $\exists y \neg P(2, y)$ $\neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)$

**d)** $\forall x\, \neg P(x, 2)$ $\neg P(1,2) \wedge \neg P(2,2) \wedge \neg P(3,2)$

**39.** Translate these specifications into English where $F(p)$ is "Printer $p$ is out of service," $B(p)$ is "Printer $p$ is busy," $L(j)$ is "Print job $j$ is lost," and $Q(j)$ is "Print job $j$ is queued."

**a)** $\exists p(F(p) \wedge B(p)) \rightarrow \exists j L(j)$

**b)** $\forall p B(p) \rightarrow \exists j Q(j)$

**c)** $\exists j(Q(j) \wedge L(j)) \rightarrow \exists p F(p)$

**d)** $(\forall p B(p) \wedge \forall j Q(j)) \rightarrow \exists j L(j)$

**a)** $\exists p(F(p) \wedge B(p)) \rightarrow \exists j L(j)$

If there is a printer that is both out of service and busy then the print job is lost.

**b)** $\forall p B(p) \rightarrow \exists j Q(j)$

If all printers are busy, then there is a print job in queue.

**c)** $\exists j(Q(j) \wedge L(j)) \rightarrow \exists p F(p)$

If there is a job that is in Queue and lost then that printer is out of service.

**50.** Show that $\forall x\, P(x) \vee \forall x\, Q(x)$ and $\forall x\, (P(x) \vee Q(x))$ are not logically equivalent.