3.2 The Growth of Functions

Big-O Notation

Let f and g be functions from the set of integers or the set of real numbers to the set of real numbers. We say that f (x) is O(g(x)) if there are constants C and k such that.

$|f(x)| \leq C|g(x)|$

whenever x > k. [This is read as "f (x) is big-oh of g(x)."]

To show that $f(x)$ is $O(g(x))$ we need to find only one pair of constants C and K. The witness such that $|f(x)| < C|g(x)|$ whenever $x > K$

Example 1

Show that $f(x) = x^2 + 2x + 1$ is $O(x^2)$.

We can take $C = 4$ and $K = 1$ as witnesses to show that $f(x)$ is $O(x^2)$

$f(x) = x^2 + 2x + 1 < 4x^2$ when $x > 1$

- Could use other values of witnesses to prove

Example 2: Show that $7x^2$ is $O(x^3)$

When $x > 7$, $7x^2 < x^3$

We can take $C = 1$ and $K = 7$ as witnesses

$7x^2 < x^3$ for $x > 7$

Example 3

Show that n^2 is not $O(n)$

- We must show that no pairs of witnesses C and K exist such that $n^2 \leq Cn$ whenever $n > K$.

Big-O Estimates for Some Important Functions

Thereom 1:

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, where $a_0, a_1, \dots, a_{n-1}, a_n$ are real numbers. Then $f(x)$ is $O(x^n)$.