MPCS 50103 Discrete Mathematics—Autumn 2019

Homework 8. This problem set is due Monday November 25 at 11:59 pm.

Reading: Rosen 7e, chapter 10, sections 10.4–10.5; chapter 11, section 11.1, pages 785–786 of section 11.4.

Written assignment:

- "DO" exercises are strongly recommended to check your understanding of the concepts. **Solve them but do not submit them**.
- **Problems labeled "HW" are homework problems that you are required to submit**.
- You are responsible for the material covered in **both** "DO" exercises and HW problems.

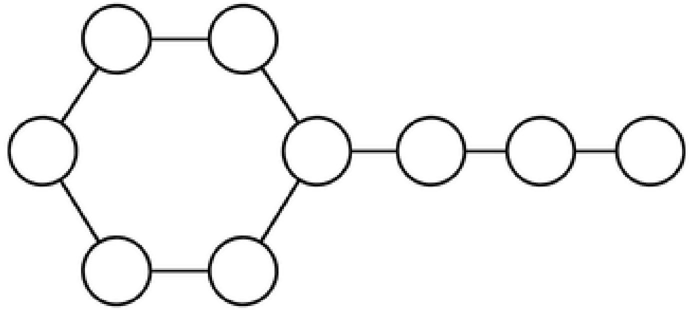
"Do" Exercises (solve but do *not* submit):

1. **"DO"** Rosen 7e, section 10.4, exercises 31, 33, 43, and 47, on pages 691–692.
2. **"DO"** Rosen 7e, section 10.5, exercises 1, 6, 15, 27, 31, 33, 35, 41, 46, 47, and 48, on pages 704–706.
3. **"DO"** Rosen 7e, section 11.1, exercise 47, on page 757.
4. **"DO"** Rosen 7e, section 11.4, exercises 9, 11, and 12, on page 795.

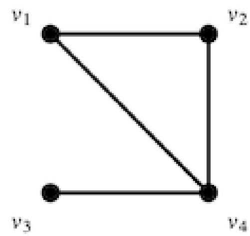
Homework Problems: DUE Monday November 25 at 11:59 pm

- **Collaboration policy**: There is no penalty for acknowledged collaboration. To acknowledge collaboration, give the names of students with whom you worked at the beginning of your homework submission and mark any solution that relies on your collaborators' ideas. Note: you must work out and write up each homework solution by yourself without assistance.
  DO NOT COPY or rephrase someone else's solution.
  The same requirement applies to books and other written sources: you should acknowledge all sources that contributed to your solution of a homework problem. Acknowledge specific ideas you learned from the source.
  DO NOT COPY or rephrase solutions from written sources.
- **Internet policy**: Looking for solutions to homework problems on the internet, even when acknowledged, is STRONGLY DISCOURAGED. If you find a solution to a homework problem on the internet, do not not copy it. Close the website, work out and write up your solution by yourself, and cite the url of website in your writeup. Acknowledge specific ideas you learned from the website.
  Copied solutions obtained from a written source, from the internet, or from another person will receive ZERO credit and will be flagged to the attention of the instructor.
- Write out your work for every problem. If you just write your answer without showing your work, you will not receive credit.

1. **HW**
   - Prove: Any connected subgraph of a tree is a tree. (4 points)
   - Prove: Every tree with at least two vertices has at least two leaves. (4 points)
2. **HW** The *lollipop graph* L_n is the simple graph consisting of a cycle on n vertices, plus a "stick" of floor(n/2) vertices hanging off one of the vertices of the cycle. Below is a picture of L_6. How many spanning trees does L_n have? Explain your answer. (4 points)



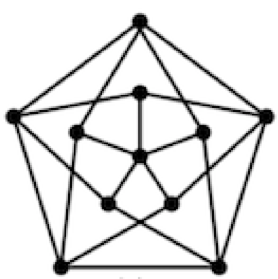
3. **HW Definition**. In a simple connected graph $G = (V, E)$, a vertex v is called a ***cut vertex*** if $G - v$ has at least two nonempty connected components. An edge e in E is called a ***cut edge*** if $G - e$ has at least two nonempty connected components.
   - Identify all the cut vertices and cut edges in the following graph. (1 point)

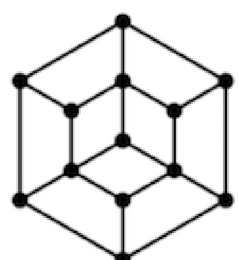


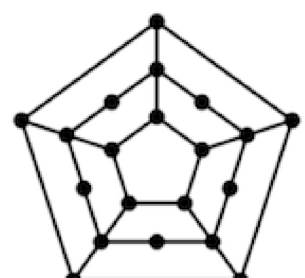
   - Suppose that G is a simple connected graph, and that every vertex of G has even degree. Prove that G has no cut edge. (4 points)
   - Suppose that G is a simple connected graph. Prove that G has a vertex that is not a cut vertex. (4 points)

4. **HW Definition**. A graph G is ***Eulerian*** if it has an *Euler circuit*, i.e., a simple circuit containing every edge of G. (Note: simple circuits can repeat vertices, but not edges.)
   - Prove or disprove: Every Eulerian simple bipartite graph has an even number of edges. (4 points)

5. **HW** Determine whether each of the following graphs has a Hamilton circuit. If it does, identify the Hamilton circuit in the graph, making sure to label the vertices clearly. If it does not, give an argument to show why no Hamilton circuit exists. (2 points each)


(a)


(b)


(c)

6. **HW** Suppose we want to prove that for any simple undirected graph $G = (V, E)$, if G is connected and $|E| = |V| - 1$, then G is a tree. What is wrong with the following proof? **Explain clearly where the logical error(s) in the proof lie and what it is (they are).**

   We already assume that the considered graph is connected, so all we need to prove is that it has no simple circuit. We proceed by induction on the number of vertices. For $|V| = 1$, we have a single vertex and no edge, so the statement holds. Assume the implication holds for any graph $G = (V, E)$ on n vertices. We want to prove it for a graph $G' = (V', E')$ arising from G by adding a new vertex. In order that the assumption $|E'| = |V'| - 1$ holds for G', we must also add one new edge, and because we assume G' is connected, this new edge must connect the new vertex to some vertex in V. Hence the new vertex has degree 1 and so it cannot be contained in a simple circuit. And because G' has no simple circuit (by the inductive hypothesis), we get that neither does G' have a simple circuit, which finishes the induction step.

(4 points)

Gerry Brady
Wednesday November 20 14:18:02 CST 2019