Homework #5

Chapter 2, Section 3

12. Determine whether each of these functions from **Z** to **Z** is one-to-one.

a) $f(n) = n - 1$ **b)** $f(n) = n^2 + 1$
c) $f(n) = n^3$ **d)** $f(n) = \lceil n/2 \rceil$

Injective (one-to-one) Functions

- Each Value in the range corresponding to exactly one element in the domain.

a) $f(n) = n - 1$

if $f(a) = f(b)$ then $a = b$ ← To prove one-to-one problems

$f(a) = 3$ $f(b) = 3$ ✓

$a = 4$ $b = 4$ ✓

Yes one-to-one

b) $f(n) = n^2 + 1$

if $f(a) = f(b)$ then $a = b$

$f(a) = 5$ $f(b) = 5$

$a = 2$ $b = -2$ ✗

$f(a) = f(b)$ but $a \neq b$

Not one-to-one

c) $f(n) = n^3$

if $f(a) = f(b)$ then $a = b$

One-to-one

$f(a) = 8$ $f(b) = 8$ ✓

$a = 2$ $b = 2$ ✓

d) $f(n) = \lceil n/2 \rceil$

if $f(a) = f(b)$ then $a = b$

13. Which functions in Exercise 12 are onto?

Surjective (onto) Functions

Every element in the codomain maps to at least one element in the domain

a) $f(n) = n - 1$

onto

- because every integer is 1 less than some integer

b) $f(n) = n^2 + 1$

Not onto

- Because everything is positive (n^2), so the range cannot include any negative integer

c) $f(n) = n^3$

d) $f(n) = \lceil n/2 \rceil$

22. Determine whether each of these functions is a bijection from **R** to **R**.

a) $f(x) = -3x + 4$
b) $f(x) = -3x^2 + 7$
c) $f(x) = (x + 1)/(x + 2)$
d) $f(x) = x^5 + 1$

a) $f(x) = -3x + 4$

b) $f(x) = -3x^2 + 7$

c) $f(x) = (x + 1)/(x + 2)$

d) $f(x) = x^5 + 1$

26. a) Prove that a strictly increasing function from **R** to itself is one-to-one.

b) Give an example of an increasing function from **R** to itself that is not one-to-one.

30. Let $S = \{-1, 0, 2, 4, 7\}$. Find $f(S)$ if

a) $f(x) = 1$.

b) $f(x) = 2x + 1$.

c) $f(x) = \lceil x/5 \rceil$.

d) $f(x) = \lfloor (x^2 + 1)/3 \rfloor$.

33. Suppose that g is a function from A to B and f is a function from B to C.

a) Show that if both f and g are one-to-one functions, then $f \circ g$ is also one-to-one.

b) Show that if both f and g are onto functions, then $f \circ g$ is also onto.

***34.** If f and $f \circ g$ are one-to-one, does it follow that g is one-to-one? Justify your answer.

***35.** If f and $f \circ g$ are onto, does it follow that g is onto? Justify your answer.

43. Let $g(x) = \lfloor x \rfloor$. Find

a) $g^{-1}(\{0\})$.

b) $g^{-1}(\{-1, 0, 1\})$.

c) $g^{-1}(\{x \mid 0 < x < 1\})$.

Chapter 2 Section 4

4. What are the terms a_0, a_1, a_2, and a_3 of the sequence $\{a_n\}$, where a_n equals

a) $(-2)^n$?
b) 3?
c) $7 + 4^n$?
d) $2^n + (-2)^n$?

9. Find the first five terms of the sequence defined by each of these recurrence relations and initial conditions.

a) $a_n = 6a_{n-1}$, $a_0 = 2$
b) $a_n = a_{n-1}^2$, $a_1 = 2$
c) $a_n = a_{n-1} + 3a_{n-2}$, $a_0 = 1$, $a_1 = 2$
d) $a_n = na_{n-1} + n^2 a_{n-2}$, $a_0 = 1$, $a_1 = 1$
e) $a_n = a_{n-1} + a_{n-3}$, $a_0 = 1$, $a_1 = 2$, $a_2 = 0$

12. Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if

a) $a_n = 0$. **b)** $a_n = 1$.
c) $a_n = (-4)^n$. **d)** $a_n = 2(-4)^n + 3$.

19. Suppose that the number of bacteria in a colony triples every hour.

a) Set up a recurrence relation for the number of bacteria after n hours have elapsed.

b) If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours?

29. What are the values of these sums?

a) $\sum_{k=1}^{5} (k+1)$

b) $\sum_{j=0}^{4} (-2)^j$

c) $\sum_{i=1}^{10} 3$

d) $\sum_{j=0}^{8} (2^{j+1} - 2^j)$