

# Homework #1

## Section 1.1

1. Which of these sentences are propositions? What are the **truth values** of those that are propositions?
  - a) Boston is the capital of Massachusetts.
  - b) Miami is the capital of Florida.
  - c)  $2 + 3 = 5$ .
  - d)  $5 + 7 = 10$ .
  - e)  $x + 2 = 11$ .
  - f) Answer this question.

- a) Boston is the capital of Massachusetts.

**proposition ~ statement**  
**T**

- b) Miami is the capital of Florida.

**proposition ~ statement**  
**F**

- c)  $2 + 3 = 5$ .

**proposition**  
**T**

- d)  $5 + 7 = 10$ .

**proposition**  
**F**

- e)  $x + 2 = 11$ .

**not a proposition**  
**F**

- f) Answer this question.

**not a proposition**  
**F**

6. Suppose that **Smartphone A** has 256 MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP; **Smartphone B** has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and **Smartphone C** has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions. **True or False**

- a) Smartphone B has the most RAM of these three smartphones.
- b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.
- c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
- d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.
- e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

- a) Smartphone B has the most RAM of these three smartphones.

**True**

- b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.

**True ~ Higher resolution Camera**

- c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.

**False**

- d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.

**False**

- e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

**False**

### Smartphone A

**RAM: 256 MB**

**ROM: 32 GB**

**Camera: 8 MP**

### Smartphone B

**RAM: 288 MB**

**ROM: 64 GB**

**Camera: 4 MP**

### Smartphone C

**RAM: 128 MB**

**ROM: 32 GB**

**Camera: 5 MP**

9. Let  $p$  and  $q$  be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore," respectively. Express each of these compound propositions as an English sentence.

- a)  $\neg q$
- b)  $p \wedge q$
- c)  $\neg p \vee q$
- d)  $p \rightarrow \neg q$
- e)  $\neg q \rightarrow p$
- f)  $\neg p \rightarrow \neg q$
- g)  $p \leftrightarrow \neg q$
- h)  $\neg p \wedge (p \vee \neg q)$

$p = \text{Swimming at the New Jersey Shore is allowed}$

$q = \text{Sharks have been spotted near the shore}$

a)  $\sim \neg q$   
 $\sim$  ~~not operator~~

Sharks have **not** been spotted near the shore.

b)  $p \wedge q$

Connectives (some notation) → choose meaning	
• $p$ is a well-formed formula (wff)	
• $\neg p$ is a wff	$\neg p$
• $p \wedge q$ is a wff	$p \wedge q$ $\star$
• $p \vee q$ is a wff	$p \vee q$
• $\neg p \rightarrow q$ is a wff	$\neg p \rightarrow q$
• $p \leftrightarrow q$ is a wff	$p \leftrightarrow q$
• $p \text{ then } q$	$p \text{ then } q$

$\Leftrightarrow$  : if and only if

Swimming at the New Jersey Shore is allowed **and** Sharks have been spotted near the shore.

c)  $\neg p \vee q$

Swimming at the New Jersey Shore is **not** allowed, **or** Sharks have been spotted near the shore.

d)  $p \rightarrow \neg q$

Swimming at the New Jersey Shore is allowed, **then** Sharks have **not** been spotted near the shore.

e)  $\neg q \rightarrow p$

Sharks have **not** been spotted near the shore, **then** Swimming at the New Jersey Shore is allowed.

f)  $\neg p \rightarrow \neg q$

Swimming at the New Jersey Shore is **not** allowed, **then** Sharks have **not** been spotted near the shore.

g)  $p \leftrightarrow \neg q$

Swimming at the New Jersey Shore is allowed **if and only if** Sharks have **not** been spotted near the shore.

h)  $\neg p \wedge (p \vee \neg q)$

Swimming at the New Jersey Shore is **not** allowed, **and either** Swimming at the New Jersey Shore is allowed **or** Sharks have **not** been spotted near the shore.

10. Let  $p$  and  $q$  be the propositions "The election is decided" and "The votes have been counted," respectively. Express each of these compound propositions as an English sentence.

a)  $\neg p$   $\sim$  and

b)  $p \vee q$   
d)  $q \rightarrow p$

c)  $\neg p \wedge q$

e)  $\neg q \rightarrow \neg p$

f)  $\neg p \rightarrow \neg q$

g)  $p \leftrightarrow q$

h)  $\neg q \vee (\neg p \wedge q)$

$p$ : The election is decided

$q$ : The voter have been counted

a)  $\neg p$  The election is not decided

b)  $p \vee q$  The election is decided, or the voter have been counted.

c)  $\neg p \wedge q$  The election is not decided and the voter have been counted.

d)  $q \rightarrow p$  if the voter have been counted, then the election is decided.

e)  $\neg q \rightarrow \neg p$  if the voter have not been counted then the election is not decided.

f)  $\neg p \rightarrow \neg q$  If the election is not decided then the voter have not been counted.

g)  $p \leftrightarrow q$  The election is decided if and only if the voter have been counted.

h)  $\neg q \vee (\neg p \wedge q)$

12. Let  $p$ ,  $q$ , and  $r$  be the propositions

$p$  : You have the flu.

$q$  : You miss the final examination.

$r$  : You pass the course.

Express each of these propositions as an English sentence.

- a)  $p \rightarrow q$
- b)  $\neg q \leftrightarrow r$
- c)  $q \rightarrow \neg r$
- d)  $p \vee q \vee r$
- e)  $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$
- f)  $(p \wedge q) \vee (\neg q \wedge r)$

a)  $p \rightarrow q$

If you have the flu, then you miss the final examination.

b)  $\neg q \leftrightarrow r$

You don't miss the final exam if and only if you pass the course.

c)  $q \rightarrow \neg r$

If you miss the final examination, then you do not pass the course.

13. Let  $p$  and  $q$  be the propositions

$p$  : You drive over 65 miles per hour.

$q$  : You get a speeding ticket.

Write these propositions using  $p$  and  $q$  and logical connectives (including negations).

- You do not drive over 65 miles per hour.
- You drive over 65 miles per hour, but you do not get a speeding ticket.
- You will get a speeding ticket if you drive over 65 miles per hour.
- If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
- Driving over 65 miles per hour is sufficient for getting a speeding ticket.
- You get a speeding ticket, but you do not drive over 65 miles per hour.
- Whenever you get a speeding ticket, you are driving over 65 miles per hour.

- a) You do not drive over 65 miles per hour.

$$\neg p$$

$\sim$  and

- b) You drive over 65 miles per hour, but you do not get a speeding ticket.

$$p \wedge \neg q$$

"if then"  $\rightarrow$

- c) You will get a speeding ticket if you drive over 65 miles per hour.

$$p \rightarrow q$$

"if  $p$ , then  $q$ "  
 $\uparrow$  Hypothesis       $\downarrow$  Conclusion

- d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.

$$\neg p \rightarrow \neg q$$

"if  $p$ , then  $q$ "

"if  $p$ ,  $q$ "

" $p$  is sufficient for  $q$ "

" $q$  if  $p$ "

" $q$  when  $p$ "

"a necessary condition for  $p$  is  $q$ "

" $q$  unless  $\neg p$ "

For Conditionals  $\downarrow$

" $p$  implies  $q$ "

" $p$  only if  $q$ "

"a sufficient condition for  $q$  is  $p$ "

" $q$  whenever  $p$ "

" $q$  is necessary for  $p$ "

" $q$  follows from  $p$ "

- e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

$$p \rightarrow q$$

"and"

- f) You get a speeding ticket, but you do not drive over 65 miles per hour.

$$q \wedge \neg p$$

- g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

$$q \rightarrow p$$

14. Let  $p$ ,  $q$ , and  $r$  be the propositions

$p$  : You get an A on the final exam.  
 $q$  : You do every exercise in this book.  
 $r$  : You get an A in this class.

Write these propositions using  $p$ ,  $q$ , and  $r$  and logical connectives (including negations).

- a) You get an A in this class, but you do not do every exercise in this book.
- b) You get an A on the final, you do every exercise in this book, and you get an A in this class.
- c) To get an A in this class, it is necessary for you to get an A on the final.
- d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.
- e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.
- f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

- a) You get an A in this class, but you do not do every exercise in this book.

$$r \wedge \neg q$$

- b) You get an A on the final, you do every exercise in this book, and you get an A in this class.

$$p \wedge q \wedge r$$

- c) To get an A in this class, it is necessary for you to get an A on the final.

$$r \rightarrow p$$

- d) You get an A on the final, but you don't do every exercise in this book; nevertheless, you get an A in this class.

$$p \wedge \neg q \wedge r$$

- e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

$$p \wedge q \rightarrow r$$

- f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

$$r \leftrightarrow q \vee p$$

rown usually most difficult  
 some of  $p$  is necessary for  $q$ :  $q \rightarrow p$   
 $p$  is sufficient for  $q$ :  $p \rightarrow q$

15. Let  $p$ ,  $q$ , and  $r$  be the propositions

$p$  : Grizzly bears have been seen in the area.  
 $q$  : Hiking is safe on the trail.  
 $r$  : Berries are ripe along the trail.

Write these propositions using  $p$ ,  $q$ , and  $r$  and logical connectives (including negations).

- a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.
- b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.
- c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.
- d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.
- e) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.
- f) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

$\wedge \sim$  and

$\vee \sim$  or

- a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.

$$r \wedge \neg p$$

- b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

$$\neg p \wedge q \wedge r$$

- c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

$$\text{if } r^{\rightarrow} \quad r \rightarrow (q \leftrightarrow \neg p)$$

- d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.

$$\neg q \wedge \neg p \wedge r$$

- e) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.

$$q \rightarrow (\neg r \wedge \neg p)$$

- f) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

$$(p \wedge r) \rightarrow \neg q$$

"if  $p$ , then  $q$ "

"if  $p$ ,  $q$ "

" $p$  is sufficient for  $q$ "

" $q$  if  $p$ "

" $q$  when  $p$ "

"a necessary condition for  $p$  is  $q$ "

" $q$  unless  $\neg p$ "

" $p$  implies  $q$ "

" $p$  only if  $q$ "

"a sufficient condition for  $q$  is  $p$ "

" $q$  whenever  $p$ "

" $q$  is necessary for  $p$ "

" $q$  follows from  $p$ "

16. Determine whether these biconditionals are true or false.
- $2 + 2 = 4$  if and only if  $1 + 1 = 2$ .
  - $1 + 1 = 2$  if and only if  $2 + 3 = 4$ .
  - $1 + 1 = 3$  if and only if monkeys can fly.
  - $0 > 1$  if and only if  $2 > 1$ .

a)  $2 + 2 = 4$  if and only if  $1 + 1 = 2$ .

True

### Biconditional Truth Table

b)  $1 + 1 = 2$  if and only if  $2 + 3 = 4$ .

False

c)  $1 + 1 = 3$  if and only if monkeys can fly.

True

d)  $0 > 1$  if and only if  $2 > 1$ .

False

P	q	$P \leftrightarrow q$
1	1	1
1	0	0
0	1	0
0	0	1

if p and q  
are the same  
Value, then it's  
TRUE

22. Write each of these statements in the form "if  $p$ , then  $q$ " in English. [Hint: Refer to the list of common ways to express conditional statements provided in this section.]
- a) It is necessary to wash the boss's car to get promoted.
  - b) Winds from the south imply a spring thaw.
  - c) A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.
  - d) Willy gets caught whenever he cheats.
  - e) You can access the website only if you pay a subscription fee.
  - f) Getting elected follows from knowing the right people.
  - g) Carol gets seasick whenever she is on a boat.

Analyze the sentence  
- find the cause and effect

cause ~ if  
effect ~ then

- a) It is necessary to wash the boss's car to get promoted.

1) Rewrite in "if  $p$ , then  $q$ " form.

If you wash your boss's car, then you will get promoted.

- b) Winds from the south imply a spring thaw.

cause: Wind from the south  
effect: Spring thaw

- c) A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.

cause: bought less than a year ago  
effect: Warranty is good

If you bought the computer less than a year ago, then the warranty is good.

- d) Willy gets caught whenever he cheats.

cause: Cheats  
effect: Caught

If Willy cheats, then he will get caught.

- e) You can access the website only if you pay a subscription fee.

cause: Pay for Subscription  
effect: Access to Website

- f) Getting elected follows from knowing the right people.

cause: Knowing the right people  
effect: Gets elected

- g) Carol gets seasick whenever she is on a boat.

cause: On a boat  
effect: Gets seasick

18. Determine whether each of these conditional statements is true or false.

- a) If  $1 + 1 = 3$ , then unicorns exist.
- b) If  $1 + 1 = 3$ , then dogs can fly.
- c) If  $1 + 1 = 2$ , then dogs can fly.
- d) If  $2 + 2 = 4$ , then  $1 + 2 = 3$ .

- a) If  $1 + 1 = 3$ , then unicorns exist.

$$F \rightarrow F$$

**True**

- b) If  $1 + 1 = 3$ , then dogs can fly.

$$F \rightarrow F$$

**True**

- c) If  $1 + 1 = 2$ , then dogs can fly.

$$T \rightarrow F$$

**False**

- d) If  $2 + 2 = 4$ , then  $1 + 2 = 3$ .

$$T \rightarrow T$$

**True**

**Conditional**

P	Q	$P \rightarrow Q$
1	1	1
1	0	0
0	1	1
0	0	1

**if  $P$  then  $Q$**

it is only false if P is true  
and Q is false, the rest of the  
times it will be true

20. For each of these sentences, determine whether an **inclusive** or, or an **exclusive** or, is intended. Explain your answer.

- Experience with C++ or Java is required.
- Lunch includes soup or salad.
- To enter the country you need a passport or a voter registration card.
- Publish or perish.

- a) Experience with C++ or Java is required.

experience with C++ ✓  
experience with Java ✓

You need to know at least one of  
the languages!

**Inclusive**

- b) Lunch includes soup or salad.

**Exclusive**

-you want get both

- c) To enter the country you need a passport or a voter registration card.

**Inclusive**

- d) Publish or perish.

**Inclusive**

**Exclusive**

Or ( $\oplus, \vee$ )

P	q	$P \oplus q$
1	1	0
1	0	1
0	1	1
0	0	0

opposite of biconditional

$P \neq q$  then  $P \oplus q = 1$

**Inclusive**  
Disjunction ( $\vee, +$ )

Disjunction ~ "or"



P	q	$P \vee q$
1	1	1
1	0	1
0	1	1
0	0	0

Mathematically ↗

$$P \vee q = \max(P, q)$$

P and q are true if at least  
one of P, q are true.  
← neither P or q are true so  $P \vee q$  is false.

23. Write each of these statements in the form "if  $p$ , then  $q$ " in English. [Hint: Refer to the list of common ways to express conditional statements.]

- a) It snows whenever the wind blows from the northeast.
- b) The apple trees will bloom if it stays warm for a week.
- c) That the Pistons win the championship implies that they beat the Lakers.
- d) It is necessary to walk 8 miles to get to the top of Long's Peak.
- e) To get tenure as a professor, it is sufficient to be world-famous.
- f) If you drive more than 400 miles, you will need to buy gasoline.
- g) Your guarantee is good only if you bought your CD player less than 90 days ago.
- h) Jan will go swimming unless the water is too cold.

Let  $p$  and  $q$  be propositions. The conditional statement  $p \rightarrow q$  is the proposition "if  $p$ , then  $q$ ." The conditional statement  $p \rightarrow q$  is false when  $p$  is true and  $q$  is false, and true otherwise. In the conditional statement  $p \rightarrow q$ ,  $p$  is called the *hypothesis* (or *antecedent* or *premise*) and  $q$  is called the *conclusion* (or *consequence*).

$$P \rightarrow q$$

"if $p$ , then $q$ "	" $p$ implies $q$ "
"if $p, q$ "	" $p$ only if $q$ "
" $p$ is sufficient for $q$ "	"a sufficient condition for $q$ is $p$ "
" $q$ if $p$ "	" $q$ whenever $p$ "
" $q$ when $p$ "	" $q$ is necessary for $p$ "
"a necessary condition for $p$ is $q$ "	" $q$ follows from $p$ "
" $q$ unless $\neg p$ "	

- a) It snows whenever the wind blows from the northeast.

If the wind blows from the Northeast, then it snows.

- b) The apple trees will bloom if it stays warm for a week.

If it stays warm for a week, then the apple trees will bloom.

- c) That the Pistons win the championship implies that they beat the Lakers.

If the Pistons win the championship, then they beat the Lakers.

- d) It is necessary to walk 8 miles to get to the top of Long's Peak.

If you get to the top of Long's Peak, then you must have walked 8 miles.

- e) To get tenure as a professor, it is sufficient to be world-famous.

If you're world famous, then you will get tenure as a professor.

- f) If you drive more than 400 miles, you will need to buy gasoline.

If you drive more than 400 miles, then you will need to buy gasoline.

- g) Your guarantee is good only if you bought your CD player less than 90 days ago.

If your guarantee is good, then you must have bought your CD player 90 days ago.

- h) Jan will go swimming unless the water is too cold.

If the water is not too cold, then Jan will go swimming.

2004 CHAMPS!  
Mr. Big Shot!

27. State the converse, contrapositive, and inverse of each of these conditional statements.

- a) If it snows today, I will ski tomorrow.
- b) I come to class whenever there is going to be a quiz.
- c) A positive integer is a prime only if it has no divisors other than 1 and itself.

$\text{P}$        $\text{q}$        $\text{P} \rightarrow \text{q}$

- a) If it snows today, I will ski tomorrow.

$\text{q} \rightarrow \text{P}$  Converse: If I ski tomorrow, then it snowed today.

$\neg \text{q} \rightarrow \neg \text{P}$  Contrapositive: If I do not ski tomorrow, then it did not snow today.

$\neg \text{P} \rightarrow \neg \text{q}$  Inverse: If it does not snow today, then I will not ski tomorrow.

- b) I come to class whenever there is going to be a quiz. If there is going to be a quiz, then I will come to class.

$\text{q} \rightarrow \text{P}$  Converse: If I come to class, then there is going to be a quiz.

$\neg \text{q} \rightarrow \neg \text{P}$  Contrapositive: If I don't come to class, then there is not going to be a quiz.

$\neg \text{P} \rightarrow \neg \text{q}$  Inverse: If there is not going to be a quiz, then I will not go to class.