Homework #3

7. Let $T(x, y)$ mean that student x likes cuisine y, where the domain for x consists of all students at your school and the domain for y consists of all cuisines. Express each of these statements by a simple English sentence.

a) $\neg T(\text{Abdallah Hussein, Japanese})$
b) $\exists x T(x, \text{Korean}) \land \forall x T(x, \text{Mexican})$
c) $\exists y(T(\text{Monique Arsenault}, y) \lor T(\text{Jay Johnson}, y))$
d) $\forall x \forall z \exists y((x \neq z) \rightarrow \neg(T(x, y) \land T(z, y)))$
e) $\exists x \exists z \forall y(T(x, y) \leftrightarrow T(z, y))$
f) $\forall x \forall z \exists y(T(x, y) \leftrightarrow T(z, y))$

a) $\neg T(\text{Abdallah Hussein, Japanese})$

Abdallah Hussein does not like Japanese cuisine.

b) $\exists x T(x, \text{Korean}) \land \forall x T(x, \text{Mexican})$

In your school there exists a student who likes Korean and everyone likes Mexican cuisine

c) $\exists y(T(\text{Monique Arsenault}, y) \lor$ or
$T(\text{Jay Johnson}, y))$

There exists some cuisine that either Monique Arsenault or Jay Johnson likes.

d) $\forall x \forall z \exists y((x \neq z) \rightarrow \neg(T(x, y) \land T(z, y)))$

For every pair of distinct student at your school, there is some cuisine that at least one of them does not like.

e) $\exists x \exists z \forall y(T(x, y) \leftrightarrow T(z, y))$

There exists two students at your school, who like the same cusine.

10. Let $F(x, y)$ be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

a) Everybody can fool Fred.
b) Evelyn can fool everybody.
c) Everybody can fool somebody.
d) There is no one who can fool everybody.
e) Everyone can be fooled by somebody.
f) No one can fool both Fred and Jerry.
g) Nancy can fool exactly two people.
h) There is exactly one person whom everybody can fool.
i) No one can fool himself or herself.
j) There is someone who can fool exactly one person besides himself or herself.

a) Everybody can fool Fred.

$\forall x F(x, Fred)$ $\qquad$ $F(x,y)$ = "x" can fool "y"

b) Evelyn can fool everybody.

$\forall y F(Evelyn, y)$

c) Everybody can fool somebody.

"There exists a person in the World"

All people in the World

$\forall x \exists y F(x,y)$

d) There is no one who can fool everybody.

There does not exist a person in the World

All people in the World

$\neg \exists x \forall y F(x,y)$ → Can negate this using DM Law: $\forall x \exists y \neg F(x,y)$

e) Everyone can be fooled by somebody.

$\forall y \exists x F(x,y)$

f) No one can fool both Fred and Jerry.

There does not exist a person in the World

$\neg \exists x \left(F(x, Fred) \wedge F(x, Jerry)\right)$

g) Nancy can fool exactly two people.

$\exists x \exists y \left(F(Nancy, x) \wedge F(Nancy, y) \wedge (x \neq y) \wedge \forall z \left(F(Nancy, z) \rightarrow (z = x) \vee (z = y)\right)\right)$

h) There is exactly one person whom everybody can fool.

Exactly One

$\exists! \forall y F(x,y)$

i) No one can fool himself or herself.

$\forall x \neg F(x,x)$

j) There is someone who can fool exactly one person besides himself or herself.

$\exists! x \exists! y F(x,x) \wedge F(x,y)$

31. Express the negations of each of these statements so that all negation symbols immediately precede predicates.

a) $\forall x \exists y \forall z T(x, y, z)$
b) $\forall x \exists y P(x, y) \vee \forall x \exists y Q(x, y)$
c) $\forall x \exists y (P(x, y) \wedge \exists z R(x, y, z))$
d) $\forall x \exists y (P(x, y) \rightarrow Q(x, y))$

$\neg \exists x P(x) \equiv \forall x \neg P(x)$

$\neg \forall x P(x) \equiv \exists x \neg P(x)$

a) $\forall x \exists y \forall z T(x, y, z)$

$\neg \forall x \exists y \forall z T(x,y,z)$

$\exists x \neg \exists y \forall z T(x,y,z)$

$\exists x \forall y \neg \forall z T(x,y,z)$

$\exists x \forall y \exists z \neg T(x,y,z)$

b) $\forall x \exists y P(x, y) \vee \forall x \exists y Q(x, y)$

$\neg \forall x \exists y P(x,y) \wedge \neg \forall x \exists y Q(x,y)$

$\exists x \neg \exists y P(x,y) \wedge \exists x \neg \exists y Q(x,y)$

$\exists x \forall y \neg P(x,y) \wedge \exists x \forall y \neg Q(x,y)$

c) $\forall x \exists y (P(x, y) \wedge \exists z R(x, y, z))$

$\neg \forall x \exists y (P(x,y) \wedge \exists z R(x,y,z))$

$\exists x \neg \exists y (P(x,y) \wedge \exists z R(x,y,z))$

$\exists x \forall y \neg (P(x,y) \wedge \exists z R(x,y,z))$

$\exists x \forall y (\neg P(x,y) \vee \neg \exists z R(x,y,z))$

$\exists x \forall y (\neg P(x,y) \vee \forall z \neg R(x,y,z))$

d) $\forall x \exists y (P(x, y) \rightarrow Q(x, y))$

$\neg \forall x \exists y (P(x,y) \rightarrow Q(x,y))$

$\exists x \neg \exists y (P(x,y) \rightarrow Q(x,y))$

$\exists x \forall y \neg (P(x,y) \rightarrow Q(x,y))$

$\exists x \forall y (P(x,y) \rightarrow \neg Q(x,y))$

46. Determine the truth value of the statement $\exists x \forall y (x \leq y^2)$ if the domain for the variables consists of

a) the positive real numbers.
b) the integers.
c) the nonzero real numbers.

a) the positive real numbers.

there exists an x for all y that $x \leq y^2$

True

Counter example

b) the integers. ~ includes 0

True

c) the nonzero real numbers. – Could be −1, −2, .

True

Section 1.6

18. What is wrong with this argument? Let $S(x, y)$ be "x is shorter than y." Given the premise $\exists s S(s, \text{Max})$, it follows that $S(\text{Max}, \text{Max})$. Then by existential generalization it follows that $\exists x S(x, x)$, so that someone is shorter than himself.

S(Max, Max) does not follow from the previous premise S(s, Max)

23. Identify the error or errors in this argument that supposedly shows that if $\exists x P(x) \wedge \exists x Q(x)$ is true then $\exists x(P(x) \wedge Q(x))$ is true.

1. $\exists x P(x) \vee \exists x Q(x)$	Premise
2. $\exists x P(x)$	Simplification from (1)
3. $P(c)$	Existential instantiation from (2)
4. $\exists x Q(x)$	Simplification from (1)
5. $Q(c)$	Existential instantiation from (4)
6. $P(c) \wedge Q(c)$	Conjunction from (3) and (5)
7. $\exists x(P(x) \wedge Q(x))$	Existential generalization

Simplification

$$\frac{p \wedge q}{\therefore q}$$

Existential Instantiation (EI)

$$\frac{\exists x\, P(x)}{\therefore P(c) \text{ for some } c}$$

1. $\exists x\, P(x) \vee \exists x\, Q(x)$	Premise
2. $\exists x\, P(x)$	Simp 1
3. $P(c)$	EI 2
4. $\exists x\, Q(x)$	Simp 1
Error → 5. $Q(c)$	EI 4

- We can't assume that c that makes P true is the same as the c that makes Q true

24. Identify the error or errors in this argument that supposedly shows that if $\forall x(P(x) \vee Q(x))$ is true then $\forall x P(x) \vee \forall x Q(x)$ is true.

1.	$\forall x(P(x) \vee Q(x))$	Premise
2.	$P(c) \vee Q(c)$	Universal instantiation from (1)
3.	$P(c)$	Simplification from (2)
4.	$\forall x P(x)$	Universal generalization from (3)
5.	$Q(c)$	Simplification from (2)
6.	$\forall x Q(x)$	Universal generalization from (5)
7.	$\forall x(P(x) \vee \forall x Q(x))$	Conjunction from (4) and (6)

Universal Instantiation (UI)

$$\frac{\forall x\, P(x)}{\therefore\ P(c)}$$

1. $\forall x\,(P(x) \vee Q(x))$ Premise

2. $P(c) \vee Q(c)$ UI ①

ERROR → 3. $P(c)$ Simplify ②

Simplification

$$\frac{p \wedge q}{\therefore\ q}$$

- We can't simplify here at step 3 because Simplification requires a $\wedge$ not a $\vee$.

- Same Error Occurs at step 5.

27. Use rules of inference to show that if $\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.

$\forall x(P(x) \rightarrow (Q(x) \wedge S(x))) \wedge \forall x(P(x) \wedge R(x)) \equiv \underline{\forall x(R(x) \wedge S(x))}$

1.	$\forall x(P(x) \wedge R(x))$	Premise
2.	$P(a) \wedge R(a)$	UI ①
3.	$P(a)$	Simplify ②
4.	$\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$	Premise
5.	$Q(a) \vee S(a)$	UMP ③ and ④
6.	$S(a)$	Simplify ⑤
7.	$R(a)$	Simplify (2)
8.	$R(a) \wedge S(a)$	Conjunction ⑦ and ⑥
9.	$\forall x(R(x) \wedge S(x))$	UG ⑤

Universal Instantiation (UI)

$$\frac{\forall x\, P(x)}{\therefore\ P(c)}$$

Universal Modus Ponens

$\forall x\,(P(x) \rightarrow Q(x))$

$P(a)$ where a is a particular element in the domain

$\therefore\ Q(a)$

Conjunction

$$\frac{\begin{matrix} p \\ q \end{matrix}}{\therefore\ p \wedge q}$$

Universal Generalization (UG)

$$\frac{P(c) \text{ for an arbitrary } c}{\therefore\ \forall x\, P(x)}$$

1.7 Proofs

- Going thru Video Series

6. Use a direct proof to show that the product of two odd numbers is odd.

18. Prove that if n is an integer and $3n + 2$ is even, then n is even using

a) a proof by contraposition.

b) a proof by contradiction.

20. Prove the proposition $P(1)$, where $P(n)$ is the proposition "If n is a positive integer, then $n^2 \geq n$." What kind of proof did you use?

24. Show that at least three of any 25 days chosen must fall in the same month of the year.

34. Is this reasoning for finding the solutions of the equation $\sqrt{2x^2 - 1} = x$ correct? *(1)* $\sqrt{2x^2 - 1} = x$ is given; (2) $2x^2 - 1 = x^2$, obtained by squaring both sides of (1); (3) $x^2 - 1 = 0$, obtained by subtracting x^2 from both sides of (2); (4) $(x - 1)(x + 1) = 0$, obtained by factoring the left-hand side of $x^2 - 1$; (5) $x = 1$ or $x = -1$, which follows because $ab = 0$ implies that $a = 0$ or $b = 0$.