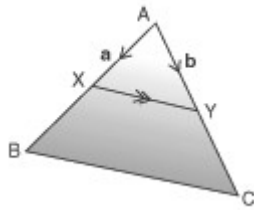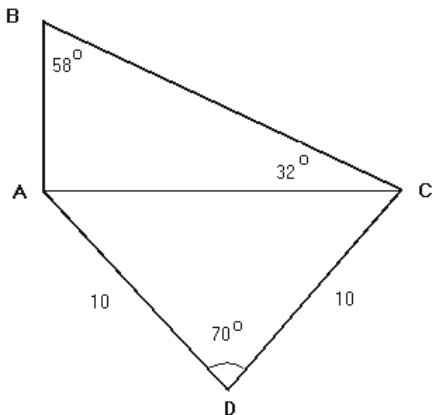# Maths Questions

---

Q: In the triangle ABC the points x and y are the mid points of AB and AC. Show the line XY is parallel to BC and half its length.
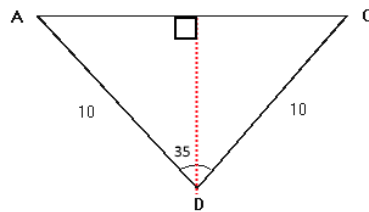
$XY = \frac{1}{2} AC - \frac{1}{2} AB$

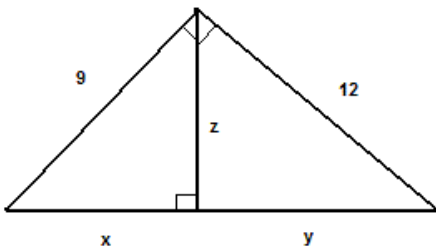$XY = \frac{1}{2}(AC - AB)$ ← Sub BC = AC - AB

$XY = \frac{1}{2} BC$

Q: Find the length of side AB in the figure to the left

$\sin(\Theta) = O/C$

$\sin(35°) = \frac{1}{2} AC / AD$

$AC = 2\sin(35°)(10)$

$AC = 11.42$

$\tan(\Theta) = O/A$

$\tan(32°) = AB / AC$

$AB = \tan(32°)(11.42)$

$AB = 7.169$

Q: Find the lengths x, y and z

3 equations using pythagoras:

$(x + y)^2 = 9^2 + 12^2$

$12^2 = z^2 + y^2$

$9^2 = z^2 + x^2$

Rearrange 1:	*Substitute 3 from 2:*	*Solve for z:*
$(x + y)^2 = 9^2 + 12^2$	$12^2 - 9^2 = z^2 + y^2 - (z^2 + x^2)$	$z^2 = 12^2 - y^2$
$x = 15 - y$	$63 = y^2 - x^2$	$z = \sqrt{(12^2 - 9.6^2)}$
	$63 = y^2 - (15 - y)^2$	$z = 7.2$
	$63 = y^2 - (225 - 15y - 15y - y^2)$	
	$y = 9.6$	
	$x = 15 - 9.6 = 5.4$	

Q: Find a, b and c so that the graph of the quadratic function f(x) = ax² + bx + c has a vertex at (-2, 1) and passes through the point (0,-3)

Vertex Form:

$f(x) = a(x - h)^2 + k$

$f(x) = a(x + 2)^2 + 1$ ← *sub in (0, -3)*

$-3 = a(0 + 2)^2 + 1$

$a = -1$
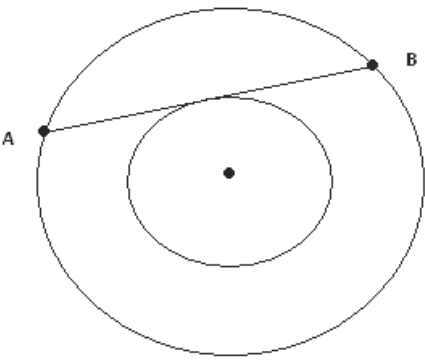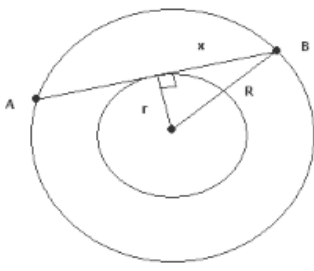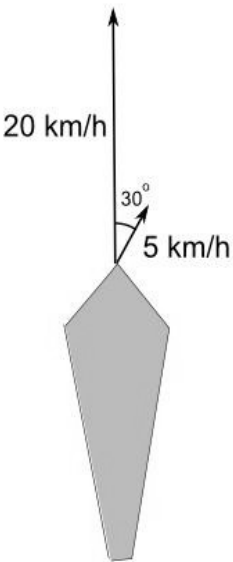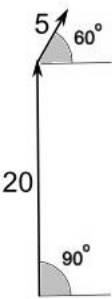
$f(x) = a(x + 2)^2 + 1$

$f(x) = -(x + 2)^2 + 1$ ← *sub in 'a'*

$f(x) = -(x + 2)(x + 2) + 1$

$f(x) = -x^2 - 4x - 3$ ← of the form ax² + bx + c

$b = -4, c = -2$

Q: The two circles below are concentric (have same center). The radius of the large circle is 10 and that of the small circle is 6. What is the length of the chord AB?

r = radius of small circle = 6
R = radius of large circle = 10
x = $^1/_2$ AB

$r^2 + x^2 = R^2$
$6^2 + x^2 = 10^2$
x = 8
AB = 16

$A = \begin{bmatrix} 2 & 1 \\ 5 & -3 \end{bmatrix}$ $B = \begin{bmatrix} -2 & 4 \\ 3 & -2 \end{bmatrix}$

Q: What is 2A + B^T?

2A = | 4   2 | B^T = | -2   3 | 2A + B^T = | 2   5 |
     | 10 -6 | | 4   -2 | | 14 -8 |

$A = \begin{bmatrix} 3 & -1 & 2 \\ -2 & 4 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 0 \\ -1 & 4 \\ -3 & 2 \end{bmatrix}$

Q: What is AB?

AB = | (3*2) + (-1*-1) + (2*-3)   (3*0) + (-1*4) + (2*2) | = | 1   0 |
     | (-2*2) + (-1*4) + (0*-3)   (-2*0) + (4*4) + (0*2) | | -8 16 |

$A = \begin{bmatrix} 1 & x \\ 2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

Q: If AB = BA then what is the value of x?

AB = | 1+x   1+2x | BA = | 3   x+3 | 1+x = 3 *as AB = BA*
     | 5   8 | | 5   x+6 | x = 2

Q: If a = (5, -12), what is the magnitude and direction of a?

‖a‖ = √(5² + (-12) ²)   Θ = tan⁻¹(y/x) from polar form
‖a‖ = 13   Θ = tan⁻¹(-12, 5) = 292.6°

$\|a\| = \sqrt{5^2 + (-12)^2}$ $\Theta = \tan^{-1}(y/x)$ from polar form
$\|a\| = 13$ $\Theta = \tan^{-1}(-12, 5) = 292.6°$

---

20 km/h
30°
5 km/h

Q: A ship is heading due north at 20 km/h but is blown off course by the wind which is blowing from 30° west of south at 5 km/h. What is the speed of the ship, and in which direction is it traveling?

Convert from polar to Cartesian:
(x, y) = (‖r‖cosΘ, ‖r‖sinΘ)
ship (x, y) = (20cos(90°), 20sin(90°)) = (0, 20)
wind (x, y) = (5cos(60°), 5sin(60°)) = (2.5, 4.33)

Add the two vectors:
Final = (0, 20) + (2.5, 4.33) = (2.5, 24.330)

Convert back to polar coordinates:
r = √(x² + y²)   Θ = tan⁻¹(y/x)
r = 24.46 km/h   Θ = 84.1° from +x axis (east)

Q: A circle of center (-3 , -2) passes through the points (0 , -6) and (a , 0). Find a.

Distances from center to any point on the circle are equal
$\sqrt{((-6 + 2)^2 + (0 + 3)^2)} = \sqrt{((a + 3)^2 + (0 + 2)^2)}$
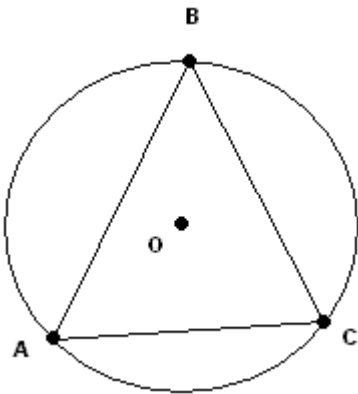$16 + 9 = (a + 3)(a + 3) + 4$
$a^2 + 6a + 9 = 21$
$a^2 + 6a - 12 = 0$ ← *use the quadratic formula*
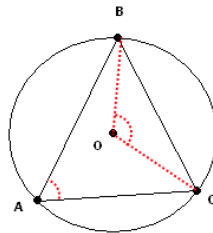$a = (-6 \pm \sqrt{(6^2 - 4(-12))}) / 2$
$a = -3 \pm (\sqrt{84} / 2)$
$a = -3 \pm (\sqrt{84} / \sqrt{4})$
$a = -3 \pm \sqrt{21}$

Q: In the triangle ABC sides AB and CB have equal lengths and the measure of angle ABC is equal to 36 degrees. What is the measure of angle BOC where O is the center of the circle?

Angles BAC and BCA the same
$36° + 2BAC = 180°$
$BAC = 72°$

Angle BOC (central angle) twice the size of BAC and BCA (inscribed angle)
$BOC = 144°$