

# Final Examination

## Important Notes – PLEASE READ FIRST

- Each submission must be a student's individual work. Collaborations between students are **not** allowed. We use MOSS and other software to detect plagiarism.
- If you adapt code or ideas from a book or website, acknowledge the book (title, edition/year, author(s), page numbers) or website (URL) in your programs or answers. You should not copy the code or text directly and then add some minor modifications, but to borrow ideas only.
- For Questions 4, 5 and 6, you can type the answers using a word processor or text editor and submit the document. You can also write the answers by hand and then scan the pages to submit.
- **Submission instruction:**
  - **websubmit:** <https://webapp.eecs.yorku.ca/submit/>
  - **submit command:** `submit 2011Z exam filename`
- Submit each answer/program after you finish it. Submit your work periodically and frequently before the deadline to avoid last-minute technical glitches.
- Do not modify the given class and method definitions. Do not add packages to the submitted Java files. Do not add I/O statements (e.g., scanner, print) to the submitted Java files. *Note:* These mistakes will mess up our automatic grading programs and produce incorrect outputs.

**The exam consists of 6 questions which have the same weight.**

### Question 1

Write a **recursive** Java method that rearranges an array of integers (duplicates allowed) so that all the negative numbers appear before all the non-negative numbers. Note that we do not want to sort the whole array, but only that all the negative numbers appear before all the non-negative numbers.

- Complete method `rearrangeArray()` in file [Rearrange.java](#). Submit only file [Rearrange.java](#).
- You can write your own main method to test your code, or use the main method in file [RearrangeMain.java](#) and input data manually from the standard input for testing.

*Hint:* Write an iterative algorithm first to help design the recursive version.

*Note:* Do not sort the array, which takes  $O(N \log N)$  given random inputs. Your method should run in  $O(N)$  time, where  $N$  is the number of elements in the array.

*Sample I/O:* Given the following array [10, 0, -2, 34, 45, -15, -23, 100, 70, -1, -2], one possible output is [-2, -1, -2, -23, -15, 45, 34, 100, 70, 0, 10]. Note that your output may be different depending on your algorithm, but all the negative numbers must appear before all the non-negative numbers in the final array.

## Question 2

Assume a singly linked list storing integers (duplicates allowed) and having a “head” pointer pointing to the first element of the list (see diagram below). Write a **recursive** method named `numOfTimes()` that returns the number of times an integer `k` appears in the linked list. The method returns 0 if the linked list does not contain integer `k` or is empty.

- Complete method `numOfTimes()` in file `LinkedList.java`. Submit only file `LinkedList.java`.
- You can write your own main method to test your code, or use the main method in file `LinkedListMain.java` and input data manually from the standard input for testing.

*Hint:* Write an iterative algorithm first to help design the recursive version.

*Note:* Your method should run in  $O(N)$  time, where  $N$  is the number of elements in the list.

*Sample I/O:* Given the following list (10, -50, 20, 8, -12, 20, 45, 20, -39, 68) and integer `k = 20`, method `numOfTimes()` returns 3. That is, number 20 appears 3 times on the list.



## Question 3

Given a binary tree that stores distinct integers (no duplicates), write a Java method that returns true if the tree contains number zero (0), and false otherwise. The method returns false if the tree is empty.

- Complete method `hasZero()` in file `BinaryTree.java`. Submit only file `BinaryTree.java`.
- You can write your own main method to test your code, or use the main method in file `BinaryTreeMain.java` and input data manually from the standard input for testing.

*Note:* These are general binary trees, **not binary search trees**. In the implementation of binary trees in file `BinaryTree.java`, the external nodes are not dummy nodes but contain actual data (integers).

*Sample I/O:* Given a tree contains the following integers (30, 50, -12, 0, 142, 8, -67, -109, 25), method `hasZero()` returns true.

## Question 4

This question refers to Problem 1 "Circular Arrays" of Assignment 3. Assume a method named `buildQ()` that is used to build a queue of size  $N$ . Each element is added to the queue one by one using method `insertLast()`. Every time the array is full, `buildQ()` extends the array by doubling its capacity and inserts the element into the new array as specified in the assignment. Obtain a **0**

running time bound for method buildQ() as a function of N, assuming the most efficient implementation and  $N = 2^k$  where k is a large positive integer. Show detailed calculations.  
*Note:* Name the submitted file q4.pdf, q4.doc, q4.docx, q4.txt, or q4.jpg, etc.

### Question 5

This question refers to Assignment 4, “AVL Trees”. Assume a method named buildAVL() that is used to build an AVL tree by inserting N keys (entries) into the tree one by one using method insert(). Note that the tree must be rebalanced after each insertion. Obtain a  $\theta$  running time bound for buildAVL() as a function of N. Show detailed calculations.

*Note:* Name the submitted file q5.pdf, q5.doc, q5.docx, q5.txt, or q5.jpg, etc.

### Question 6

Solve the following recurrence by obtaining a  $\theta$  bound for  $T(N)$  given that  $T(1) = \theta(1)$ . Show detailed calculations.

$$T(N) = T(N/2) + N - 1$$

*Note:* Name the submitted file q6.pdf, q6.doc, q6.docx, q6.txt, or q6.jpg, etc.