Roll No. _____

Pandit Deendayal Energy University

End Semester Examination:May, 2023

B.Tech. (Computer Engineering)

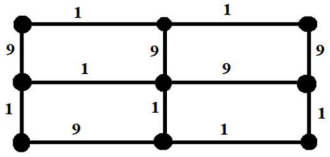
Semester - IV **Date:09/05/2023**

Course Name: Design and Analysis of Algorithms **Time: 3 hours**

Course Code: 20CP209T **Max. Marks: 100**

Instructions:

1. Do not write anything other than your roll number on question paper.
2. Read question paper properly and assume suitable data wherever essential by mentioning it clearly.

Que. No.	Description	Marks	CO
Q1	**Attempt following. (1 Mark for correct option, 2 Marks for valid justification.)**		
Q1 (A)	Let S is an NP-complete problem and Q and R be two other problems known to be in NP. Q is polynomial time reducible to S and S is polynomial-time reducible to R. Which of the following statements is/are correct? i. R is NP Complete ii. R is NP Hard iii. Q is NP complete iv. Q is NP Hard	3	CO-1
Q1 (B)	The solution to $T(n) = 5T(n/4) + 4n$ using Master's theorem is – i. $\theta(n^{\log_4 5})$ ii. $\theta(n^{\log_2 \sqrt{5}})$ iii. Both i and ii iv. Can't be solved using Master's theorem.	3	CO-2
Q1 (C)	Algorithm A multiplies two $68\,x\,68$ matrices using 132464 multiplications. Another algorithm B multiplies two $70\,x\,70$ matrices using 143640 multiplications and algorithm C multiplies two $72\,x\,72$ matrices using 155424 multiplications. Which algorithm yields the best asymptotic running time when used in D& C matrix multiplication algorithm- i. Algorithm A ii. Algorithm B iii. Algorithm C iv. All have same asymptotic running time	3	CO-3
Q1 (D)	Comparing the brute force method and branch and bound technique of solving travelling salesman problem, which is correct among the following- i. Brute force is more efficient ii. Branch & Bound is more efficient iii. Both techniques are same iv. Their complexities are incomparable	3	CO-4
Q1 (E)	In what manner is a state-space tree for a backtracking algorithm constructed? v. DFS vi. BFS vii. Both DFS & BFS viii. Nearest Neighbor First	3	CO-5
Q2	**Fill in the blanks with correct answers (1/3rd of total marks) and give proper reason to your answer (2/3rd of total marks) for each of the following.**		
Q2 (A)	Suppose problem P_1 can be reduced to problem P_2 in linear time (i.e.,$P_1 \leq_p P_2$). Then, if there exists a polynomial time algorithm for P_1, then there exists a polynomial time algorithm for P_2. This statement is ------ (True/ False)	3	CO-1
Q2 (B)	Consider the undirected, weighted graph as given. The number of minimum-weight spanning trees in the given graph is ----------. 	3	CO-3
Q2 (C)	Adding a number 'w' on the weight of every edge of a graph might change the shortest path between two vertices 'u' and 'v'. This statement is ------ (True/False)	3	CO-3
Q2 (D)	The optimal greedy algorithm and dynamic algorithm to solve the below 0/1 Knapsack problem gives ---------- and ---------- profit, respectively. $wt = \{1, 3, 2\}$, $profit = \{1, 12, 10\}$ and $W = 4$.	6	CO-4

<table>
<tr><td>Q3</td><td>Attempt Following</td><td></td><td></td></tr>
<tr><td>Q3 (A)</td><td>Obtain the amortized cost for performing n increment operations on a k-bit ($k \geq 4$) binary counter using potential method.
Hint: Use potential function as number of $1's$ in counter after i^{th} operation.</td><td>10</td><td>CO-1</td></tr>
<tr><td>Q3 (B)</td><td>Solve $a_n = 4a_{n-1} - 4a_{n-2} + n2^n + 4$ with $n \geq 2$, $a_0 = 0$ and $a_1 = 1$.</td><td>10</td><td>CO-2</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>Q4</td><td>Solve any two of the following:</td><td></td><td></td></tr>
<tr><td>Q4 (A)</td><td>Solve the following scheduling problem by giving step-by-step computations. Also, print the job sequence and maximum profit.
<table>
<tr><td>Deadline</td><td>7</td><td>2</td><td>5</td><td>3</td><td>4</td><td>5</td><td>2</td><td>7</td><td>3</td></tr>
<tr><td>Profit</td><td>15</td><td>20</td><td>30</td><td>18</td><td>18</td><td>10</td><td>23</td><td>16</td><td>25</td></tr>
</table></td><td>10</td><td>CO-3</td></tr>
<tr><td>Q4 (B)</td><td>Write the D & C algorithm for performing Large Integer Multiplication with complexity $O(n^2)$. Also, re-write the algorithm to achieve better complexity as to $O(n^2)$.</td><td>10</td><td>CO-3</td></tr>
<tr><td>Q4 (C)</td><td>Solve following Matrix Chain Multiplication using dynamic programming as per given order. The matrices have size 4 x 10, 10 x 3, 3 x 12, 12 x 7.</td><td>10</td><td>CO-3</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>Q5</td><td>Solve any one of the following:</td><td></td><td></td></tr>
<tr><td>Q5 (A)</td><td>Solve the travelling salesman problem consisting of 4 cities using branch and bound technique for the given distance matrix- $\begin{bmatrix} 0 & 10 & 15 & 20 \\ 5 & 0 & 9 & 10 \\ 6 & 13 & 0 & 12 \\ 8 & 8 & 9 & 0 \end{bmatrix}$</td><td>10</td><td>CO-5</td></tr>
<tr><td>Q5 (B)</td><td>Write algorithm for N-Queen using backtracking and find the complexity. Also, draw the state space tree for 4 Queen problem.</td><td>10</td><td>CO-5</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>Q6</td><td>Attempt Following</td><td></td><td></td></tr>
<tr><td>Q6 (A)</td><td>Given a set of n nuts of different sizes and n bolts of different sizes. There is a one-one mapping between nuts and bolts (i.e., each nut matches with exactly one bolt). There is a constraint that comparison of a nut to another nut or a bolt to another bolt is not allowed. It means nut can only be compared with bolt and bolt can only be compared with nut to see which one is bigger/smaller.
a) Give a D&C approach to match the nuts and the bolts efficiently. (5 marks)
b) Analyze the complexity of your approach by obtaining recurrence equation. (5 marks)</td><td>10</td><td>CO-6</td></tr>
<tr><td>Q6 (B)</td><td>You are traveling by a canoe down a river and there are n trading posts along the way. Before starting your journey, you are given for each $1 \leq i < j \leq n$, the fee $f_{i,j}$ for renting a canoe from post i to post j. These fees are arbitrary. For example, it is possible that $f_{1,3} = 10$ and $f_{1,4} = 5$. You begin at trading post 1 and must end at trading post n (using rented canoes). Your goal is to minimize the rental cost.
a) Write the recurrence for a DP that solves this problem. (3 marks)
b) Write its pseudo code. (3 marks)
c) Perform its complexity analysis. (4 marks)</td><td>10</td><td>CO-6</td></tr>
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==========Best Wishes !!! ==========