Problem 1
Simulate the single pendulum shown in Figure 1. The pendulum is hinged to the ground via a revolute joint RJ.

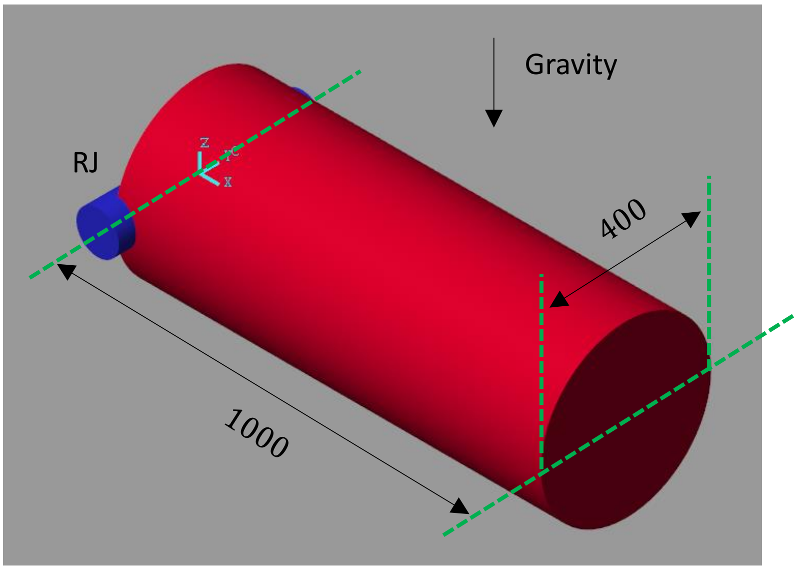
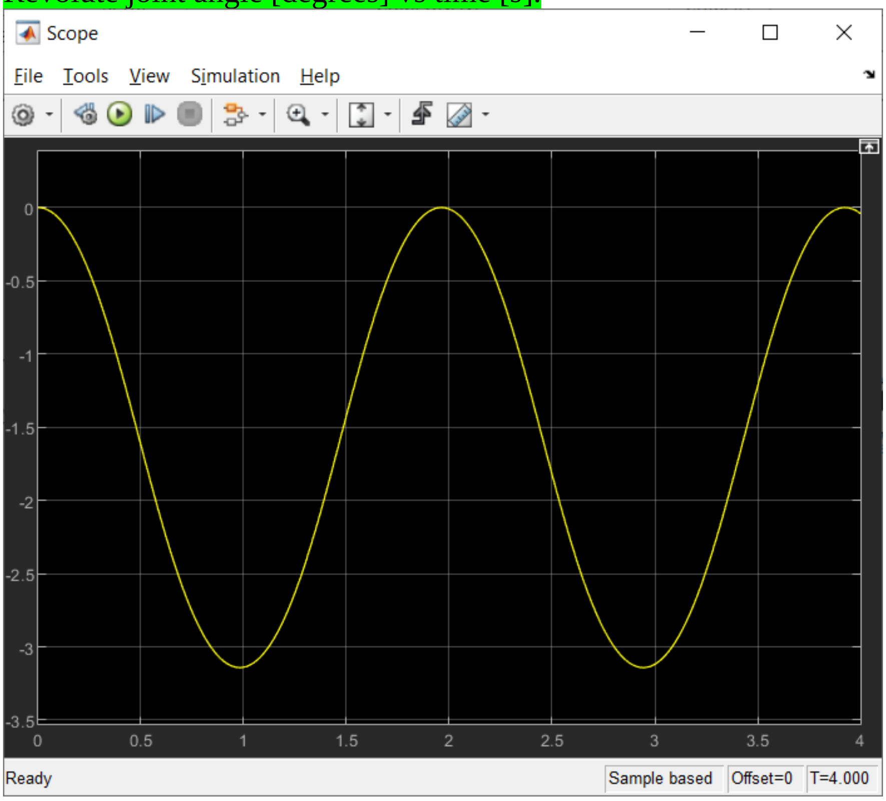


Figure 1. All measurements in [mm] - cylinder shown in horizontal position.

The pendulum is made of steel with density $\rho = 7800\,\frac{kg}{m^3}$ and has the shape of a solid cylinder.

Simulate the free motion of the pendulum for a total period of 4 seconds. Let the pendulum start from rest in horizontal position. Plot the rotation of the pendulum.

Revolute joint angle [degrees] vs time [s].



Problem 2

A double pendulum is to be modeled, see Figure 2. The pendulum consists of two bodies (Arm 1 and Arm 2). Arm 1 is connected to the frame with a revolute joint RJ. Arm2 is connected to Arm 1 with a universal joint UJ (universal joint = 2 revolute joints in series). Arm 1 has a quadratic cross section, *40 x 40 mm*, and has a mass of *8 kg*. Arm 2 has a quadratic cross section, *30 x 30 mm*, and has a mass of 12 kg. The pendulum is only subjected to gravitational forces (negative y-direction). There is no friction and no damping. The pendulum starts from rest (no initial velocities) in the position shown in Figure 2.

Make a model of the pendulum and plot the the x-, y- and z-coordinates of the point P from $t = 0$ s to $t = 3$ s.

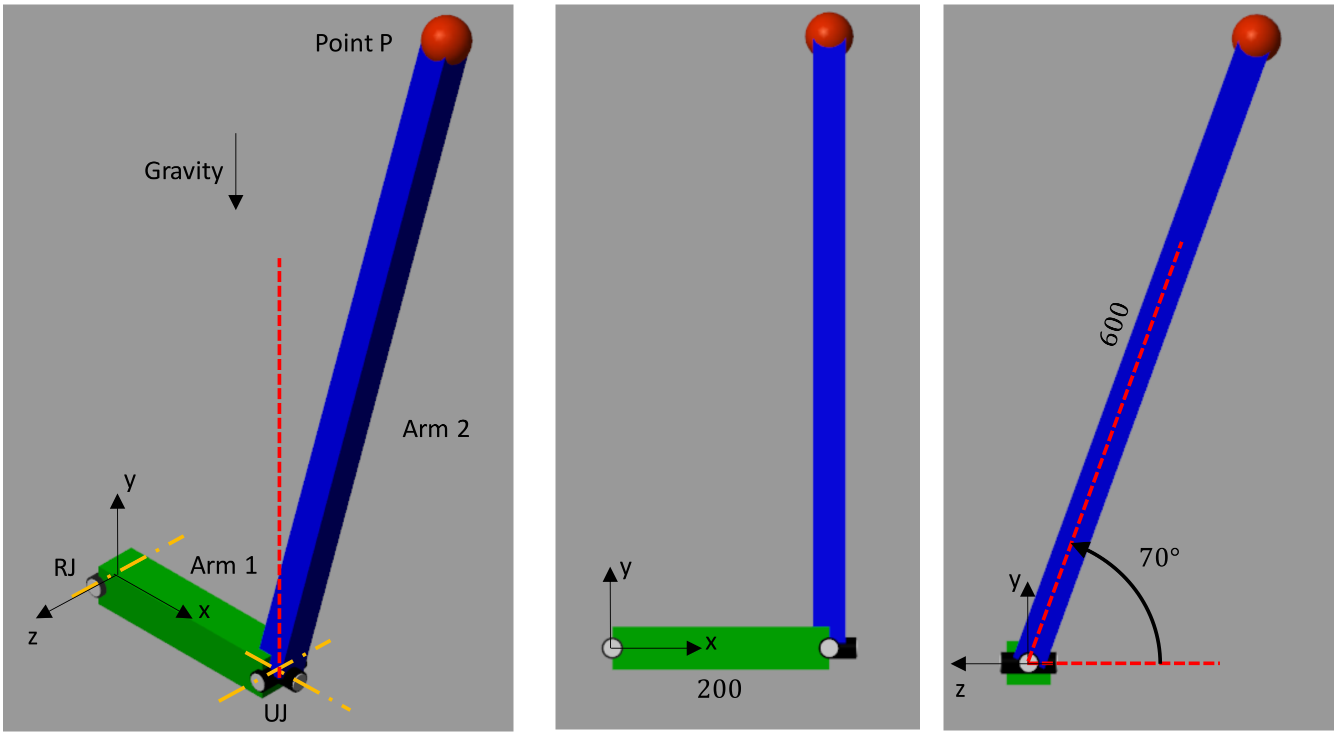
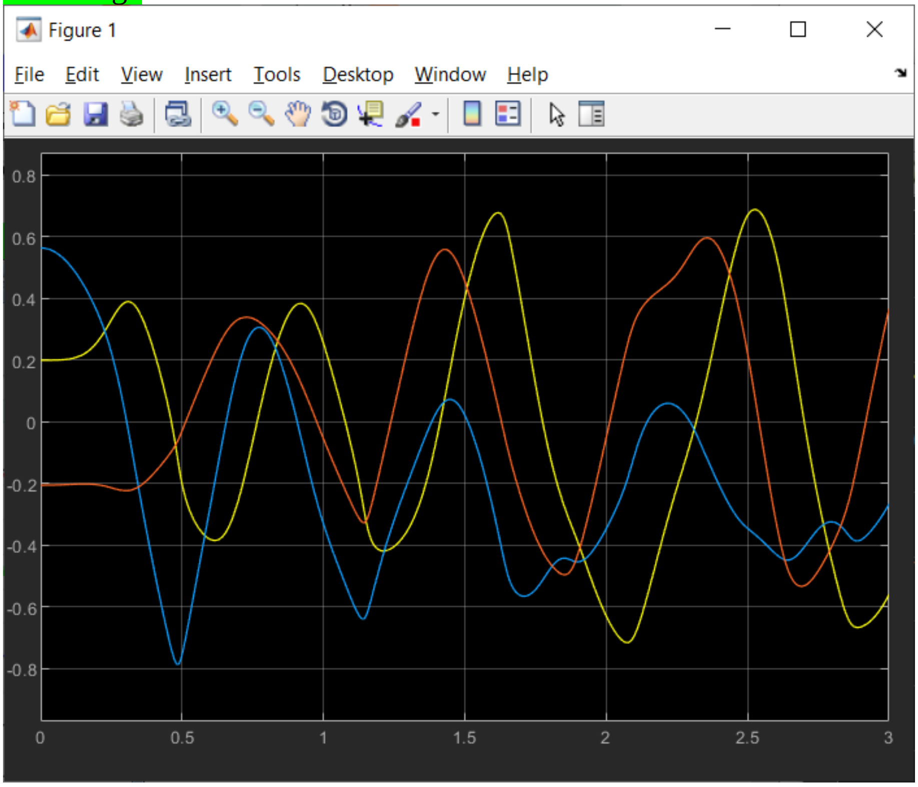


Figure 2 Double pendulum shown in initial position (isometric, xy-plane, yz-plane). All measurements are in mm or degrees.

Plot the x-y-z of coordinates point P in [m] vs time [s].
x is yellow
y is blue
z is orange



Expand the model of the double pendulum to the linkage mechanism shown in Figure 3 and Figure 4. One new body, Arm 3, is added. Arm 3 is connected to Arm 2 with a spherical joint, SJ. Arm 3 is also connected to ground with a revolute joint, RJ2.
Arm 3 has a quadratic cross section, *30 x 30* mm, and has a mass of *12 kg*. The angular velocity of Arm 1 around the global z-axis (RJ1) should increase linearly from *0 rad/s* to *6 rad/*s from *t = 0 s* to *t = 1.5* s and after that held constant at *6 rad/s*. The linkage mechanism starts from rest (no initial velocities) in the position shown in Figure 3 and Figure 4.

b) Make a model of the linkage mechanism and plot the reactive force in the revolute joint RJ2 from *t = 0 s* to *t = 3 s*.

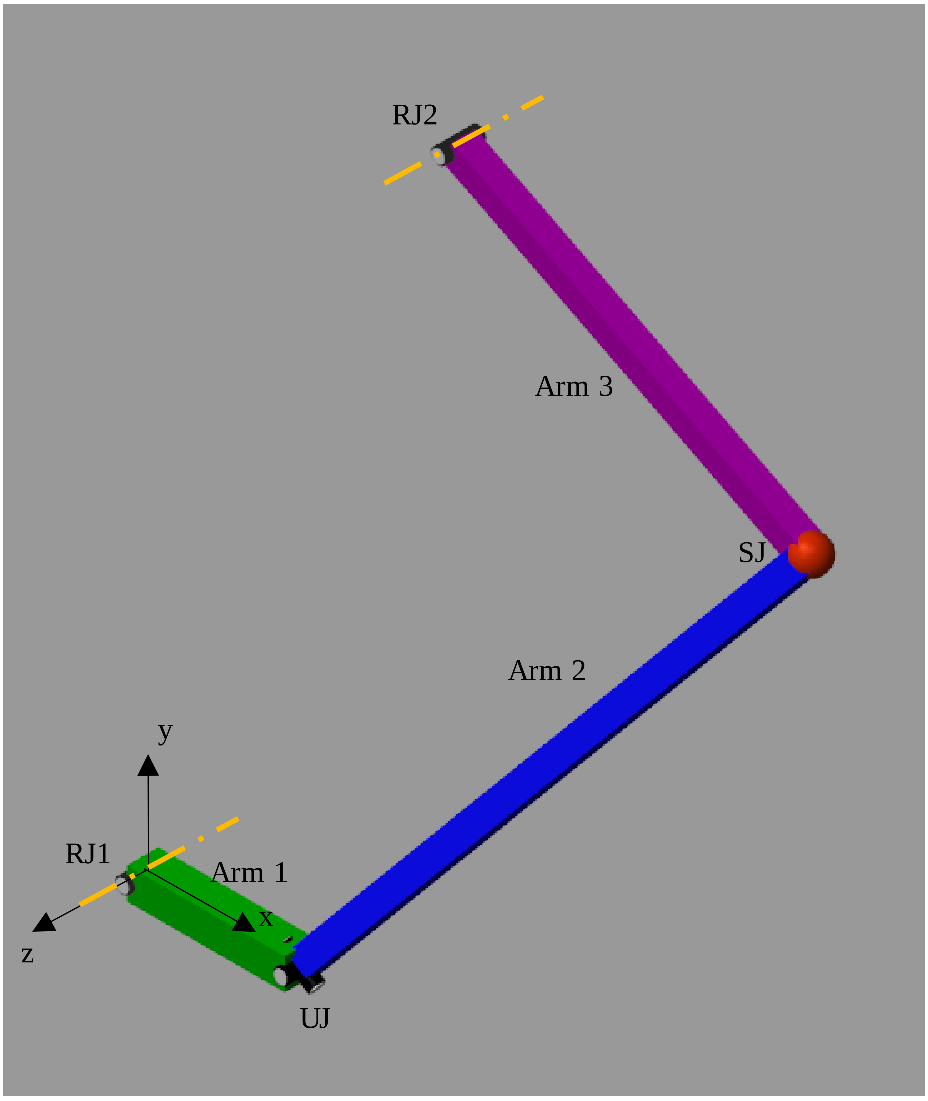


Figure 3 Linkage mechanism shown in initial position.

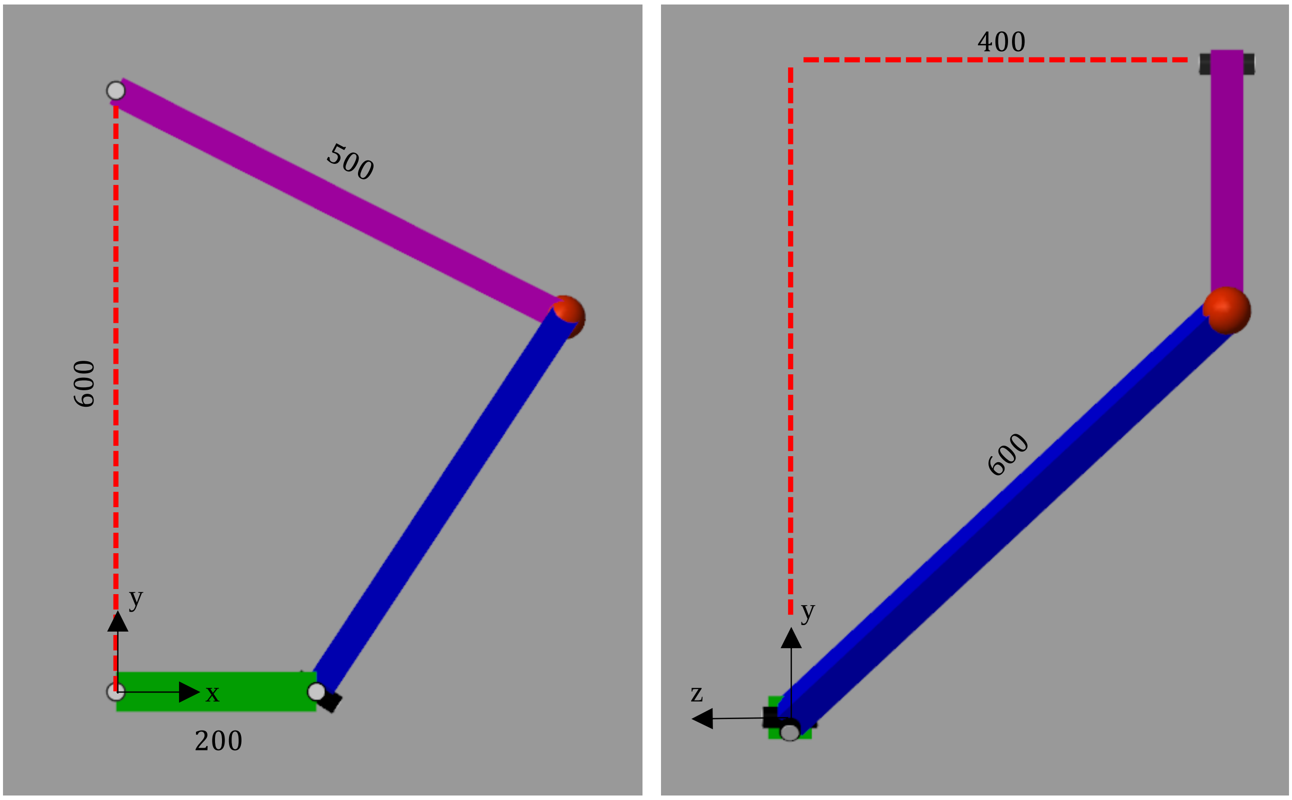
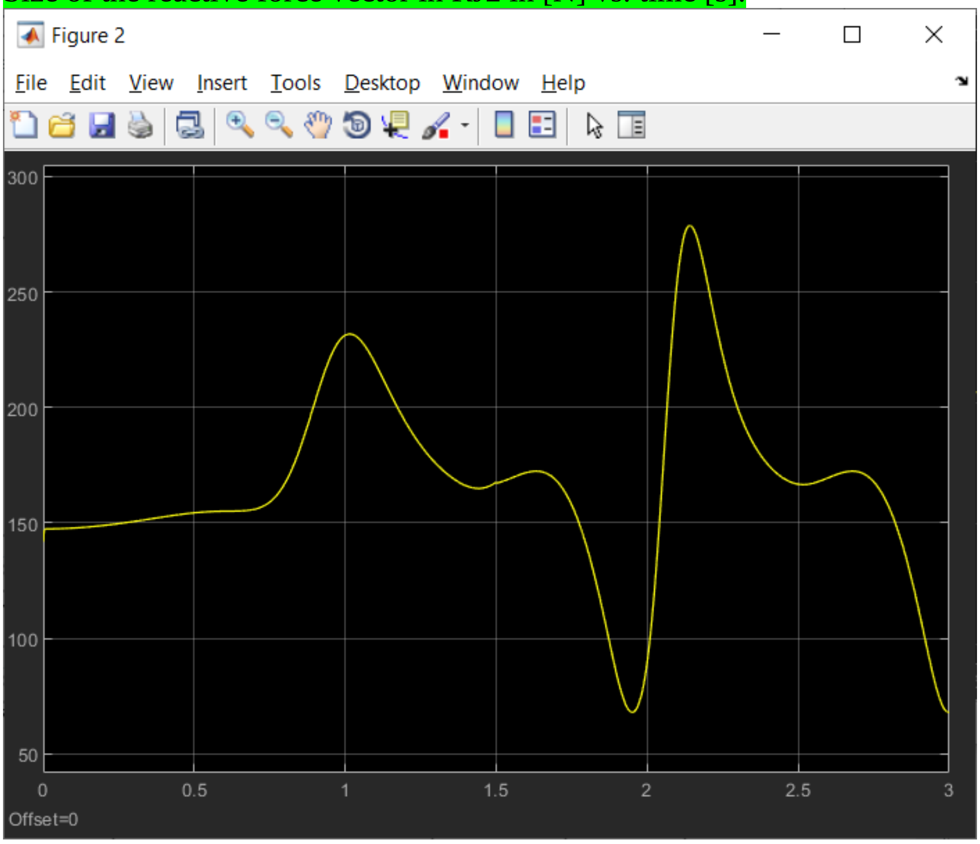


Figure 4 Linkage mechanism shown in initial position (xy-plane and yz-plane). All measurements are in [mm].

Size of the reactive force vector in RJ2 in [N] vs. time [s].



Problem 3
In Figure 5 is shown a mechanism. It consists of an arm (Arm) that is connected to the ground with a revolute joint (RJ). The Piston is connected to Arm via a spherical joint SJ2. The Piston is connected to the Cylinder via a prismatic joint PJ. Finally, the Cylinder is connected to the ground via a spherical joint SJ1. All the measurements are indicated in Figure 5.

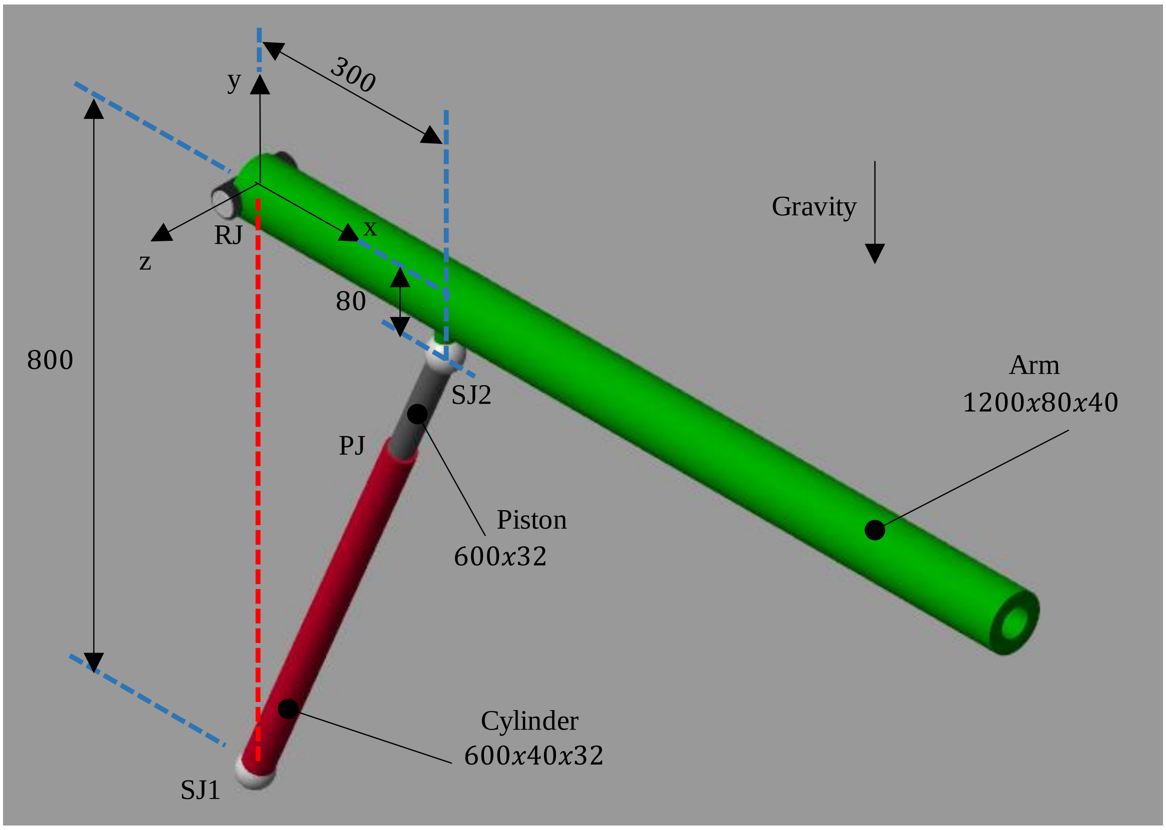


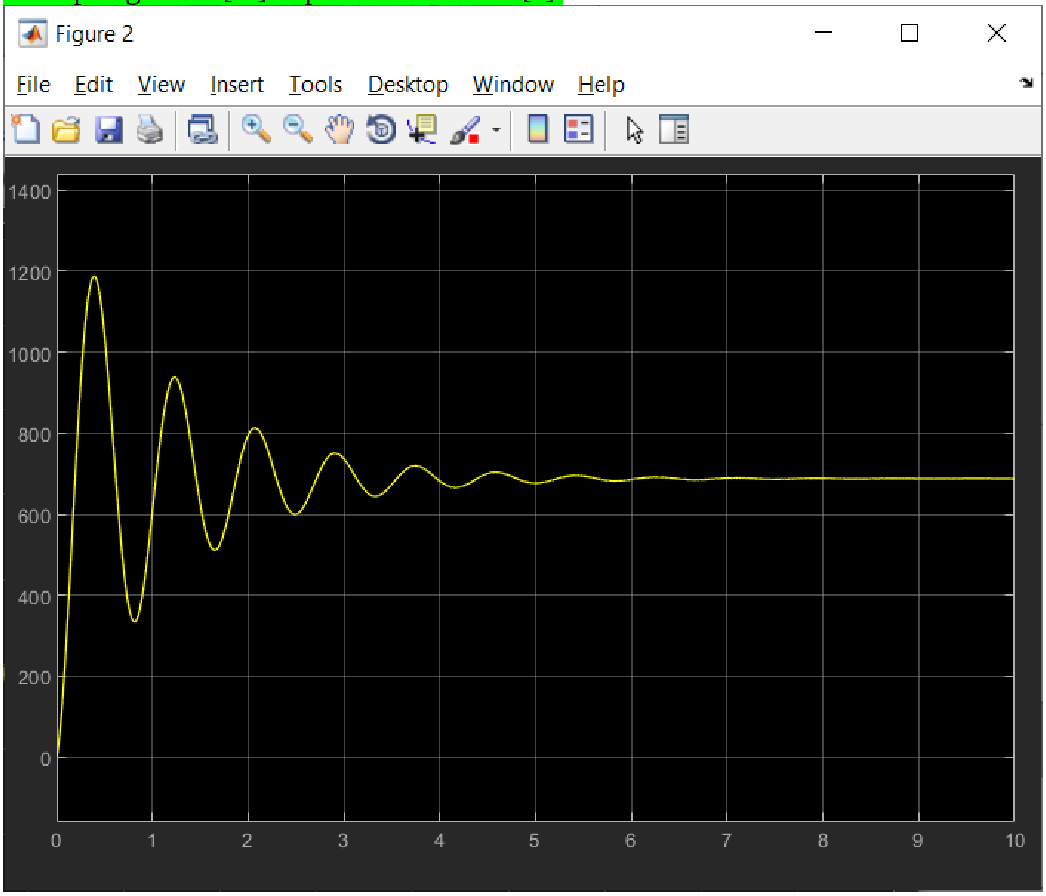
Figure 5. Mechanism with three bodies: Arm, Cylinder, and Piston. All measurements are in [mm] and the body measurements are: length x outer diameter x inner diameter.

All the components are made of steel with density $\rho = 7800\,\frac{kg}{m^3}$.

There is also a linear spring that is not shown in Figure 5. It is connected to the spherical joint SJ1 and to the mass center of the Piston. The spring has the following data; undeformed length, $L_0 = 480mm$, spring stiffness, $k = 10\,\frac{N}{mm}$, and linear damping, $b = 300\,\frac{N\cdot s}{m}$.

Simulate the mechanical system and let it start from rest with the Arm horizontal. Simulate for 10 seconds and plot the spring force.

The spring force [N] is plotted vs. Time [s].



Problem 4

In Figure 6 is shown a mechanism. It consists of three arms. Arm 1 is connected to the ground by means of a revolute joint RJ1 with joint axis along the global y-axis. Arm2 is connected to Arm 2 with a spherical joint SJ1. Arm 2 is connected to Arm 3 with another spherical joint SJ2. Finally, Arm 3 is connected to the ground with another revolute joint RJ2 with joint axis along the global z-axis. All the measurements are indicated in Figure 6.

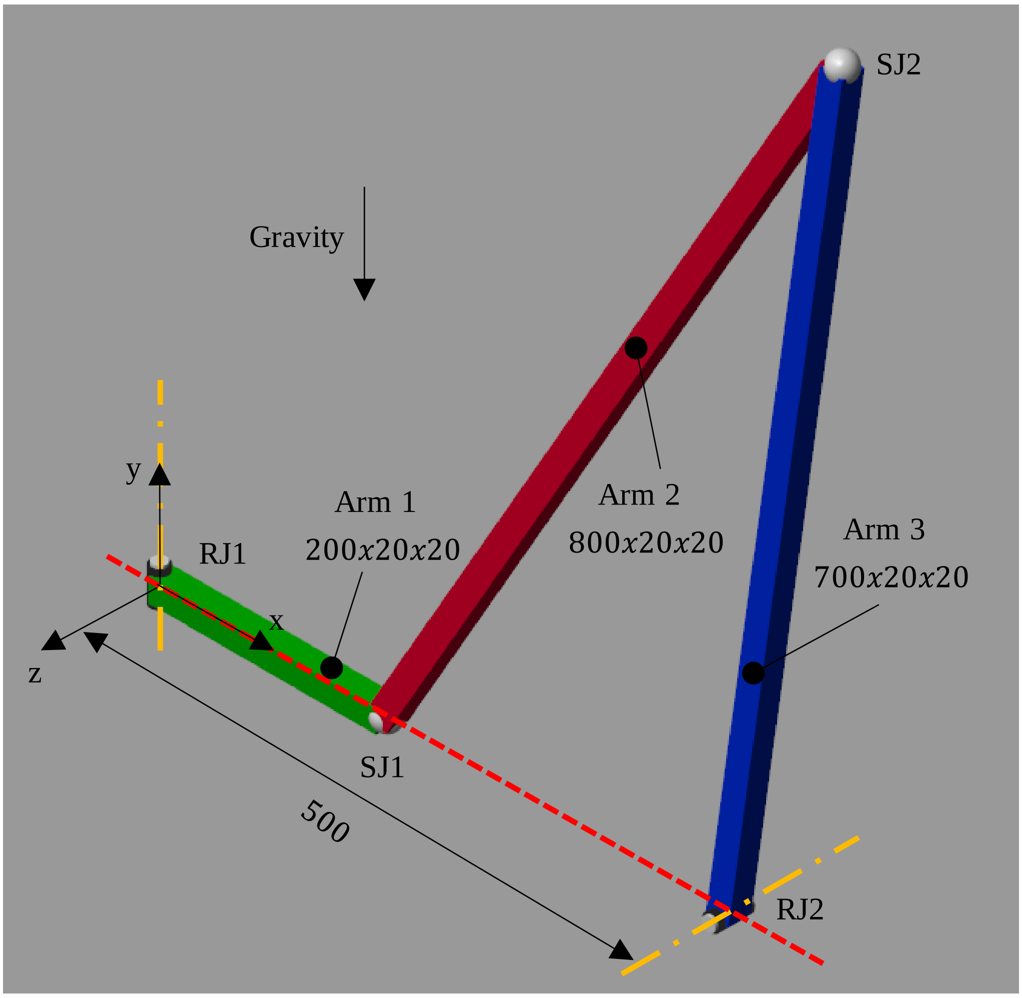


Figure 6. Mechanism with three movable links, Arm1, Arm2, and Arm 3. All measurements are in [mm] and the body measurements are: length x height x width. The rotational axes of RJ1 and RJ2 are indicated.

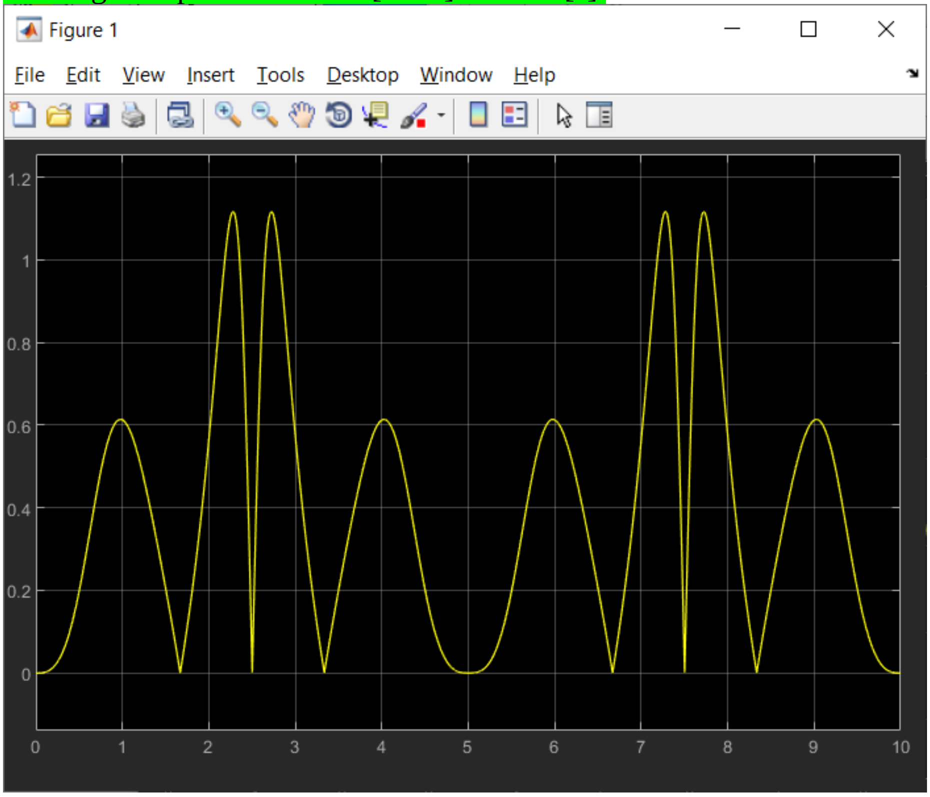
All the components are made of steel with density $\rho = 7800\,\frac{kg}{m^3}$.

The rotation of Arm 1 around the global y-axis (RJ1) is prescribed as a function of time:

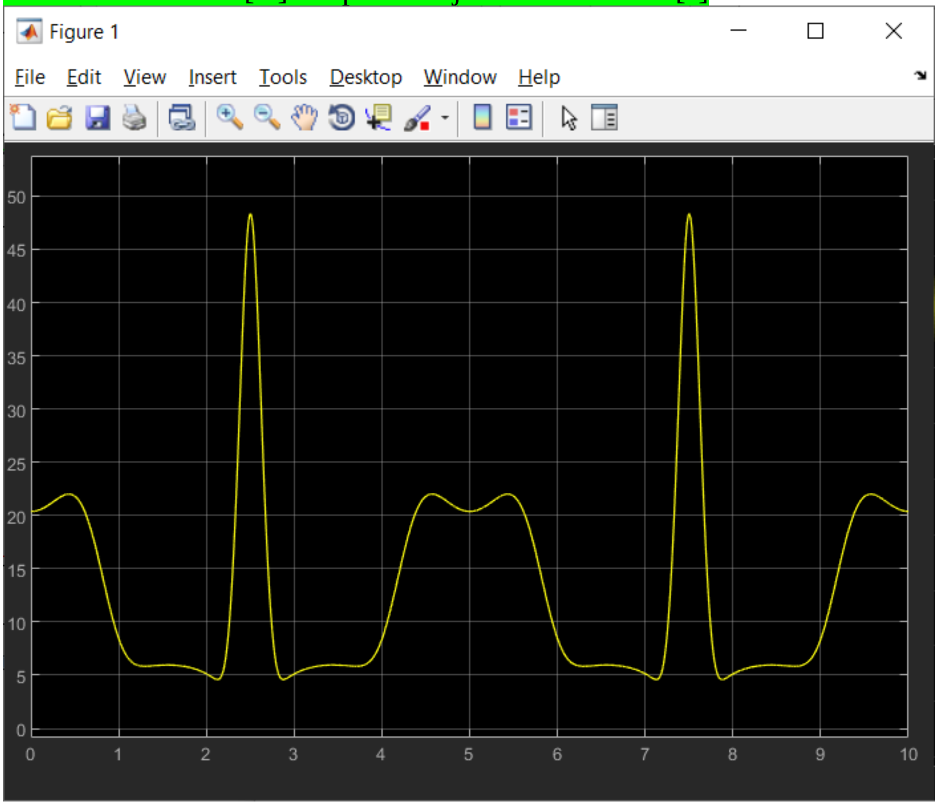
$$\theta = 2 \cdot \pi \cdot \left\{1 + sin\left[\frac{\pi}{5} \cdot (t - 2.5)\right]\right\}$$

With θ in [rad] and t in [sec]. In Figure 6 the mechanism is shown in the initial position, $\theta = 0$. Simulate the system for 10 seconds and plot the size of the angular speed vector of Arm 3 and the size of the reactive force vector in SJ2.

The angular speed of Arm 3 [rad/s] vs time [s].



The reactive force [N] in spherical joint SJ2 vs time [s].



Problem 5 (see Appendix)

Simulate the two stage gearbox in Figure 7.

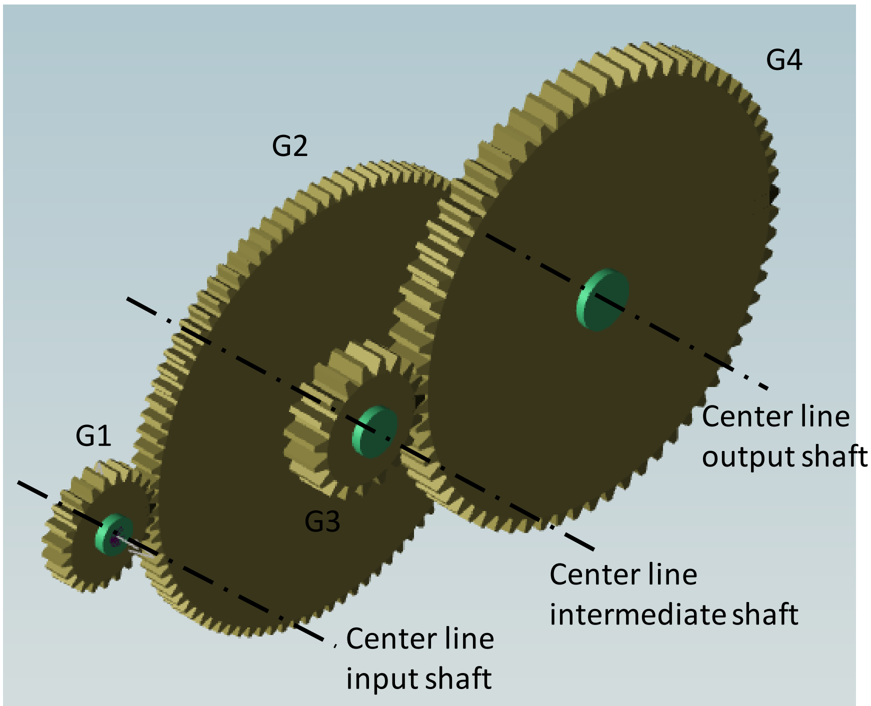


Figure 7. Two stage gearbox.

The input shaft is ramped up to a speed of 1 rad/s in 0.5 s. The output shaft is subjected to an external torque around its center axes of 100 Nm.

All the gears are spur gears (no helical angle) with normal pressure angle of 20°. In Table 1 other main data of the four gears of the gearbox are listed

Table 1

Gear	Number of teeth	Module [mm]	Width [mm]	Shaft
G1	23	5	30	Input
G2	93	5	30	Intermediate
G3	19	7	45	Intermediate
G4	67	7	45	Output

Modeling: Use revolute joint to represent the shaft bearings.

Problem 6 (see Appendix)
Simulate the gear connection in Figure 8.

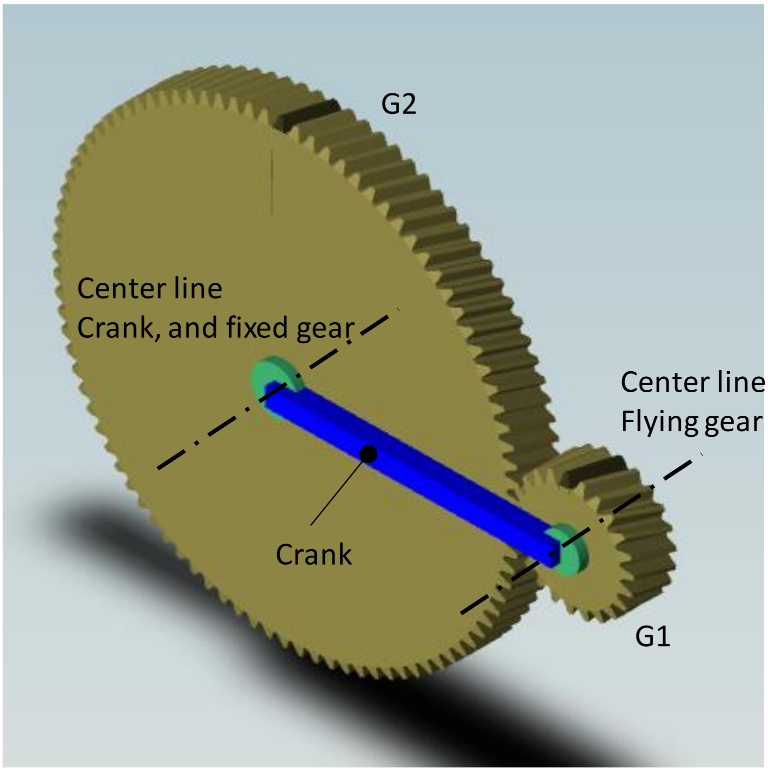


Figure 8. Gear connection with flying gear.

The crank is connected to the ground via a revolute joint. At the end of the crank the gear G1 (flying gear) is connected via another revolute joint (axis parallel with first revolute joint. G1 is in contact with the gear G2 that is fixed to the ground along an axis parallel to the revolute joints. When the crank rotates around its center line, G1 is forced to roll on G2.

The crank speed is ramped up to 1 rad/s in 0.5 s and the fixed gear is held fixed (no speed).

Both gears are helical gears with normal pressure angle of 20° and helix angle of 15°. In Table 2 other main data of the two gears of the mechanism are listed

Table 2

Gear	Number of teeth	Module [mm]	Width [mm]	Revolute joint connection
G1	23	5	50	Crank
G2	93	5	50	Ground

Appendix - Basic gear and gear mesh geometry
A simple spur gear mesh is shown in Figure A1. The number of teeth is referred to as z, hence, z_1 and z_2 for the two gears, respectively. The two gears roll of each other along each their roll circle. The diameter of the roll circle for a spur gear is $d = m \cdot z$, where m is the module, normally measured in [mm]. Two gears in contact always have the same module. The center distance is the average roll diameter: $c = 0.5 \cdot m \cdot (z_1 + z_2)$.

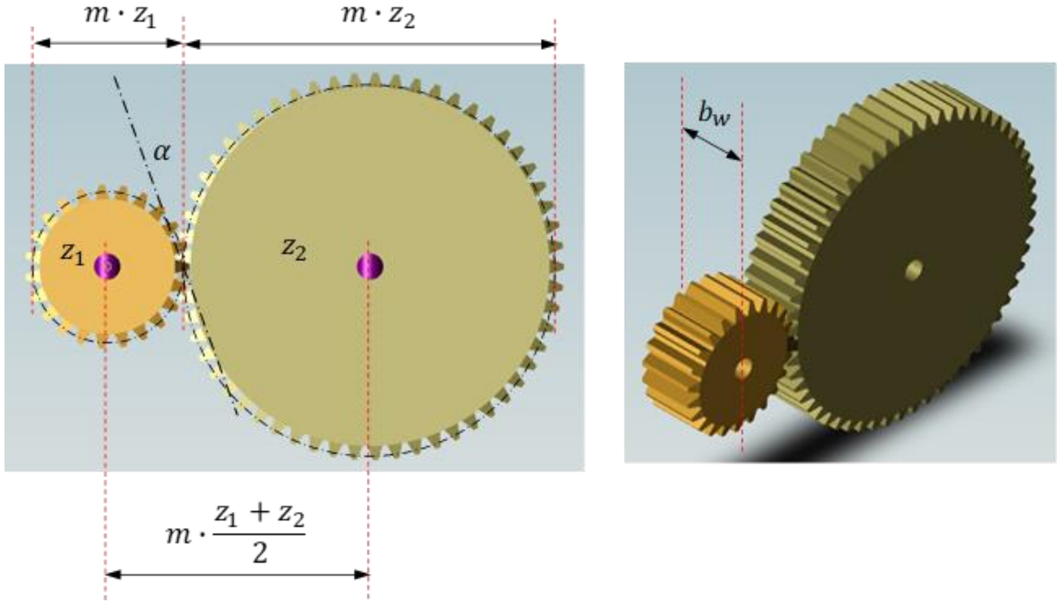


Figure A1 Basic geometrical data for a spur gear mesh.

The line of contact is not tangential to the roll circles. It has an angle, referred to as the normal pressure angle, that normally always is $\alpha = 20°$. Another important parameter is the face width, b_w.

The teeth flank of a spur gear is parallel to the axis of rotation whereas the teeth flank of a helical gear has an angle, β = the helix angle, relative to the axis of rotation, see Figure A2. Also, the center distance is affected: $c = 0.5 \cdot m \cdot (z_1 + z_2)/cos\beta$.

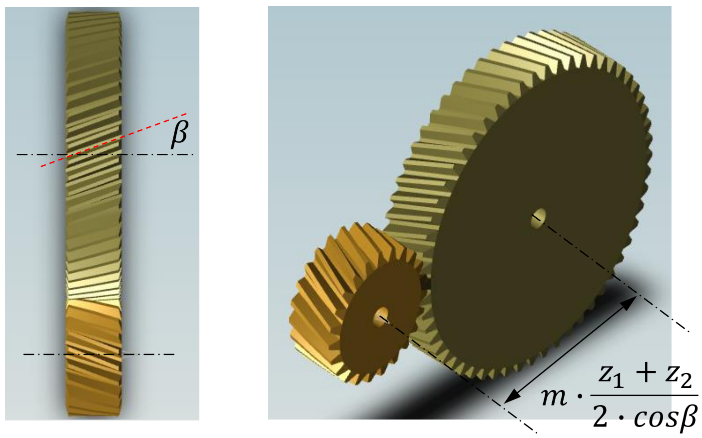


Figure A2 Basic geometrical data for a helical gear mesh.