A Reference for Myself in Analysis and Topology

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1 Mathematical Logic

2 Set Theory

2.1 Zermelo-Fraenkel-Choice Axioms

2.1.1 (Sets are objects). If A is a set, then A is also an object. In particular, given two sets A and B, it is meaningful to ask whether A is also an element of B.

2.1.2 (Equality of sets). Two sets A and B are equal, $A = B$ *iff* every element of A if an element of B and vice versa. To put it another way, $A = B$ *if and only if* every element x of A belongs also to B, and every element y of B belongs also to A.

2.1.3 (Empty set). There exists a set $\emptyset$, known as the empty set, which contains no elements, i.e., for every object x we have $x \notin \emptyset$.

2.1.4 (Singleton sets and pair sets). If a is an object, then there exists a set $\{a\}$ whose only element is a, i.e., for every object y, we have $y \in A$ *if and only if* $y = a$; we refer to $\{a\}$ as the *singleton set* whose element is a. Furthermore, if a and b are objects, then there exists a set $\{a, b\}$ whose only elements are a and b; i.e., for every object y, we have $y \in \{a, b\}$ *if and only if* $y = a$ or $y = b$; we refer to this set as the *pair set* formed by a and b.

2.1.5 (Pairwise union). Given any two sets

3 The Natural Numbers

3.1 Peano Axioms

3.1.1 0 is a natural number.

3.1.2 If n is a natural number, then $n{+}{+}$ is also a natural number.

3.1.3 0 is not the successor of any natural number; i.e., we have $n{+}{+} \neq 0$ for every natural number n.

3.1.4 Different natural numbers must have different successors; i.e., if $n,\ m$ are natural numbers, and $n \neq m$, then $n{+}{+} \neq m{+}{+}$. Equivalently, if $n{+}{+} = m{+}{+}$, then we must have $n = m$.

3.1.5 (Principle of Mathematical Induction). Let $P(n)$ be any property pertaining to a natural number n. Suppose that $P(0)$ is true, and suppose that whenever $P(n)$ is true, $P(n{+}{+})$ is also true. Then $P(n)$ is true for every natural number n.

4 Formal Construction of Z, Q, R

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