*Group members: Marwane Bidamou (4), Eduard Koshkelyan (8), Roshan Maharjan (11)*

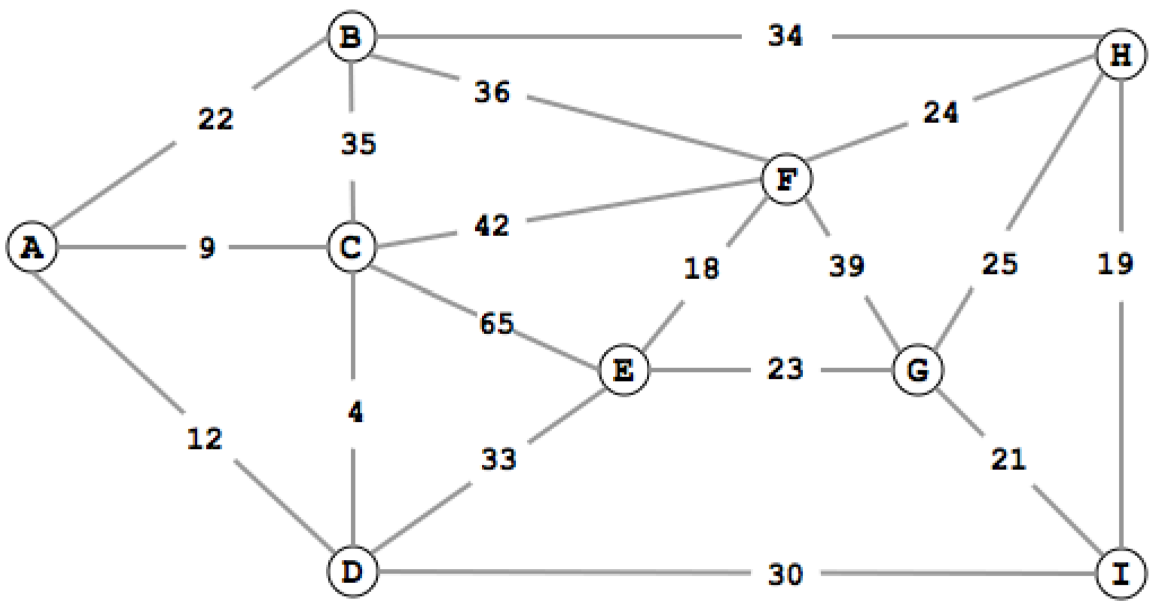


**Figure 1**

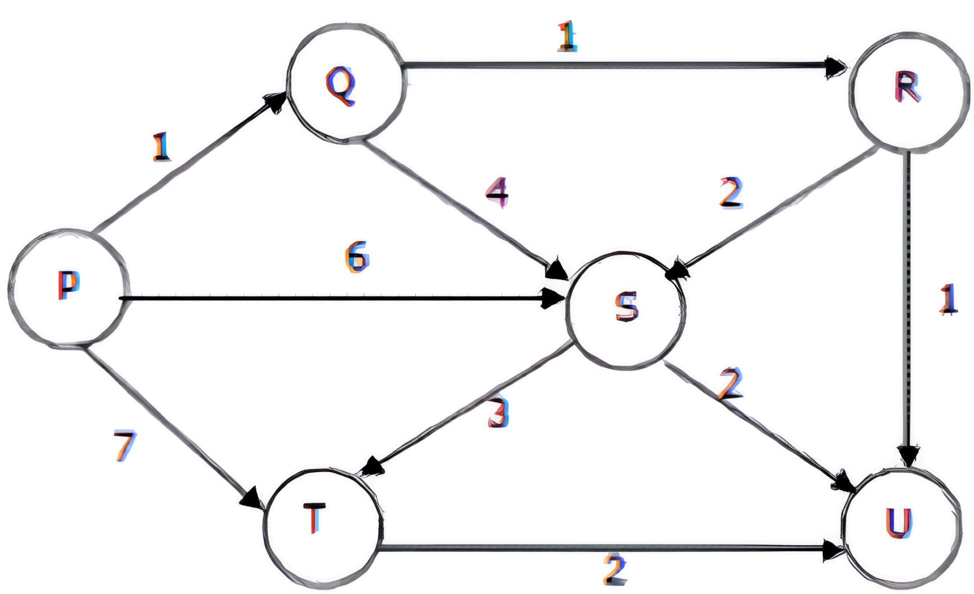


**Figure 2**

# Question 1

What is the adjacency matrix of the weighted graph G = (V,E) shown in Figure 1.

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| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| A | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| B | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| C | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| D | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| E | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| F | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| G | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| H | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| I | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |

# Question 2

Find the shortest path from A to all other vertices using Dijkstra's algorithm
(Slide 12). (Figure 1)
Start from A

**<u>Shortest distance from A to all vertices</u>**

**B[A] = {}**
**B[B] = B[A] ∪ { (A, B) } = {A, B }**
**B[C] = B[A] ∪ { (A, C) } = { (A, C) }**
**B[D] = B[A] ∪ { (A, D) } = { (A, D) }**
**B[E] = B[D] ∪ { (D, E) } = { (A, D), (D, E) }**
**B[F] = B[C] ∪ { (C, F) } = { (A, C), (C, F) }**
**B[G] = B[I] ∪ { (I, G) } = { (A, D), (D, I) (I, G) }**
**B[H] = B[B] ∪ { (B, H) } = { (A, B), (B, H) }**
**B[I] = B[D] ∪ { (D, I) } = { (A, D), (D, I) }**

**A[A] = 0**

A[B] = A[A] + wt(B) = 0 + 22 = 22
**A[C] = A[A] + wt(C) = 0 + 9 = 9**
**A[D] = A[A] + wt(D) = 0 + 12 = 12**

A[B] = A[C] + wt(B) = 9 + 35 = 44
A[D] = A[C] + wt(D) = 9 + 4 = 13
**A[F] = A[C] + wt(F) = 9 + 42 = 51**
A[E] = A[C] + wt(E) = 9 + 65 = 74

**A[E] = A[D] + wt(E) = 12 + 33 = 45**
**A[I] = A[D] + wt(I) = 12 + 30 = 42**

**A[H] = A[B] + wt(H) = 22 + 34 = 56**
A[F] = A[B] + wt(F) = 22 + 36 = 58

A[G] = A[I] + wt(G) = 42 + 21 = 63
A[H] = A[I] + wt(H) = 42 + 19 = 61

A[G] = A[E] + wt(G) = 45 + 23 = 68
A[F] = A[E] + wt(F) = 45 + 18 = 63

A[G] = A[F] + wt(G) = 51 + 39 = 90
A[H] = A[F] + wt(H) = 51 + 24 = 75

A[G] = A[H] + wt(G) = 56 + 25 = 81

# Question 3

What is the time complexity?
=> O(m log n)

# Question 4

Find a minimum spanning tree using Kruskal's Algorithm (Figure 1)
Sorting edge by weight

| Edge | Weight |
|---|---|
| C, D | 4 |
| A, C | 9 |
| A, D | 12 |
| E, F | 18 |
| H, I | 19 |

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| | |
|---|---|
| G, I | 21 |
| A, B | 22 |
| E, G | 23 |
| F, H | 24 |
| G, H | 25 |
| D, I | 30 |
| D, E | 33 |
| B, H | 34 |
| B, C | 35 |
| B, F | 36 |
| F, G | 39 |
| C, F | 42 |
| C, E | 65 |

A B C D E F G H I

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A | B | C | D | E | F | G | H | I |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A | B | C, D | E | F | G | H | I |

| | | | | | | |
|---|---|---|---|---|---|---|
| A, C, D | B | E | F | G | H | I |

| | | | | | |
|---|---|---|---|---|---|
| A, C, D | B | E, F | G | H | I |

| | | | | |
|---|---|---|---|---|
| A, C, D | B | E, F | G | H, I |

| | | | |
|---|---|---|---|
| A, C, D | B | E, F | G, H, I |

| | | |
|---|---|---|
| A, B, C, D | E, F | G, H, I |

| | |
|---|---|
| A, B, C, D | E, F, G, H, I |

| |
|---|
| A, B, C, D, E, F, G, H, I |

# Question 5

What is the time complexity?
=> O(mlog n)

# Question 6

What is the adjacency matrix of the weighted directed Acyclic graph G = (V,E) shown in Figure 2.

| | P | Q | R | S | T | U |
|---|---|---|---|---|---|---|
| P | 0 | 1 | 0 | 1 | 1 | 0 |
| Q | 0 | 0 | 1 | 1 | 0 | 0 |
| R | 0 | 0 | 0 | 1 | 0 | 1 |
| S | 0 | 0 | 0 | 0 | 1 | 1 |
| T | 0 | 0 | 0 | 0 | 0 | 1 |
| U | 0 | 0 | 0 | 0 | 0 | 0 |

# Question 7

Find the shortest path from P to U. (Figure 2). (Use the algorithm starting at slide 33).
=>
**Topological Sort:**
One possible topological order: P, Q, S, T, R, U.

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**Initialize Distances:**

P: 0
Q: ∞
S: ∞
T: ∞
R: ∞
U: ∞

**Relaxation:**
*From P:*

P -> Q: Q = 0 + 1 = 1
P -> S: S = 0 + 6 = 6
P -> T: T = 0 + 7 = 7

*From Q:*
Q -> R: R = 1 + 1 = 2
Q -> S: S = min(6, 1 + 4) = 5

*From S:*
S -> T: T = min(7, 5 + 3) = 7
S -> U: U = 5 + 2 = 7

*From T:*
T -> U: U = min(7, 7 + 2) = 7

*From R:*
R -> U: U = min(7, 2 + 1) = 3

# Question 8

What is the time complexity?
=> O(n+m)

# Question 9

Can you use Dijkstra's algorithm (Slide 12) to find the shortest path from P to U? (Figure 2).
=> Yes

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# Question 10

If "Yes", find the shortest path from P to U using Dijkstra's algorithm (Slide 12) (Figure 2).

=>

Start from P

**B[P] = {}**

**B[Q] = B[P] ∪ { (P, Q) } = { (P, Q) }**

**B[R] = B[Q] ∪ { (Q, R) } = { (P, Q), (Q, R) }**

**B[U] = B[R] ∪ { (R, U) } = { (P, Q), (Q, R), (R, U) }**

**A[P] = 0**

**A[Q] = A[P] + wt(Q) = 0 + 1 = 1**

A[S] = A[P] + wt(S) = 0 + 6 = 6

A[T] = A[P] + wt(T) = 0 + 7 = 7

**A[R] = A[Q] + wt(R) = 1 + 1 = 2**

A[S] = A[R] + wt(S) = 2 + 2 = 4

**A[U] = A[R] + wt(U) = 2 + 1 = 3**

**Therefore, the shortest path from P to U is P -> Q -> R -> U.**