ADA

Wednesday

20 - Dec - 2023

Lecture #18

→ Dynamic Programing (To find global Optimum Solution

① - Identify and solve subproblems.

② - Use Optimal substructure to solve Larger problems.

→ Optimal Solution to subproblem.

→ 0/1 Knapsack Problem

m = 8 (bag capacity)

n = 4

1 → is because we can fulfill weight in the capacity.

Capacity →

P_i	W_i	0	1	2	3	4	5	6	7	8	
1	2	0	0	1	1	1	1	1	1	1	assuming we have one obj.
2	3	0	0	1	2	2	3	3	3	3	
5	4	0	0	1	2	5	5	6	7	7	
6	5	0	0	1	2	5	6	6	7	8	
		y_1	y_2	y_3	y_4						
		0	1	0	1						

Optimal Sol.

8 - 6 = 2

2 - 2 = 0 (Exit)

Index

FORMULA :-

$$X[i,w] = \max\{X[i-1,w], X[i-1, w-w[i]] + P[i]\}$$

Adding profit in this particular index

Example :-

$$X[4,5] = \max\{X[3,5], X[3,5-5]+6\}$$

$X[5,-4]$

w[i] → corresponding weight

$$= \max\{X[3,5], X[3,0]+6\}$$
$$= \max\{5, 6\}$$
$$= 6$$

$$X[4,6] = \max\{X[3,6], X[3,6-5]+6\}$$
$$= \max\{X[3,6], X[3,1]+6\}$$
$$= \max\{6, 6\}$$
$$= 6$$

$$X[4,7] = \max\{X[3,7], X[3,7-5]+6\}$$
$$= \max\{7, 1+6\}$$
$$= \max\{7, 7\} \rightarrow (1,6) \text{ or } (2,5)$$
$$= 7$$

$$X[4,8] = \max\{X[3,8], X[3,8-5]+6\}$$
$$= \max\{7, 2+6\}$$
$$= \max\{7, 8\}$$
$$= 8$$

24-8=21

Backtracking

y_1 y_2 y_3 y_4 y_5

Weights

Items = {2, 4, 3, 1, 5}

Profits = {10, 5, 6, 8, 3}

Example #2 m=8 n=5

P_i	W_i	0	1	2	3	4	5	6	7	8
10	2	0	0	10	10	10	10	10	10	10
5	4	0	0	10	10	10	10	15	15	15
6	3	0	0	10	10	10	16	16	16	16
8	1	0	8	10	18	18	18	24	24	24
3	5	0	8	10	18	18	24	24	24	24

y_1 $y_2$0 $y_3$1 $y_4$1 $y_5$0 → [0 0 1 1 0]

$$X[i, W] = \max\{X[i-1, W], X[i-1, W-W[i]] + P[i]\}$$

$$X[2,5] = \max[X[1,5], X[1,5-4] + 5]$$
$$= \max[10, X[1,1] + 5]$$
$$= \max[10, 0+5]$$
$$= \max[10]$$

$$X[2,6] = \max\{X[1,6], X[1,6-4] + 5\}$$
$$= \max\{X[1,6], X[1,2] + 5\}$$
$$= \max\{10, 10+5\}$$
$$= 15$$

$$x[2,7] = \max\{x[1,7], x[1,7-4]+5\}$$
$$= \max\{16, 15\}$$
$$= 15$$

→ Backtracking

y_1	y_2	y_3	y_4	y_5
1	0	1	1	0