19/01/2024 Friday.

Lec No. 25

→ BRANCH & BOUND:

- NP-Hard and NP-Complete → Research Area.

* Polynomial-Time $O(n^k)$
  - Linear Search
  - Binary Search
  - Insertion / Bubble / Merge Sort.
  - Matrix Multiplication

* Exponential Time.
  - 0/1 knapsack → 2^n
  - TSP-Travelling Salesman Problem → 2^n
  - Graph Coloring → 2^n
  - Hamiltonian Cycle → 2^n

→ Associations / Similarities b/w exponential time algorithms.

→ Unable to write deterministic Polynomial Time Algorithm, write non-deterministic Polynomial Time Algorithm.

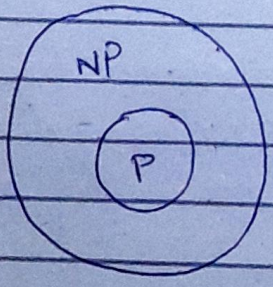
- Deterministic → Each and Every Statement is clear and known.
- Non-Deterministic → Some of the Statements are not known.

↳ Non-Deterministic Algo.

```
Algorithm NSearch (A, n, key)
{
    j = choice()
    if (key = A[j])
    {
        Print (j)
        Success()}
    Print (0)
    failure()
}
```

P → Set of those deterministic Polynomial time algorithms.

NP → Set of those non-deterministic Polynomial time algorithms.



$P \subseteq NP$.

CNF - SAT

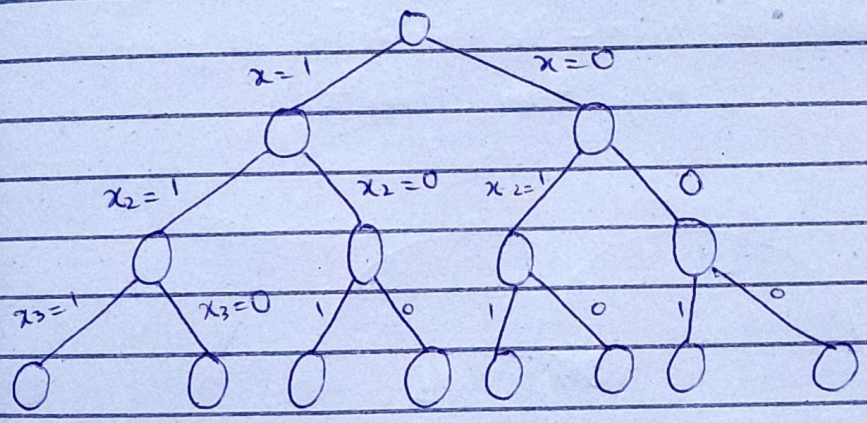
- Satisfiability Problem

$$x_i = \{x_1, x_2, x_3\} \rightarrow 2^3 \text{ Possibilities}$$
$$2^n \text{ for } n \text{ values.}$$

$$CNF = (\bar{x}_1 \vee x_2 \vee \bar{x}_3) \wedge (x_1 \wedge \bar{x}_2 \vee \bar{x}_3)$$

↓

Conjunctive Normal Form.

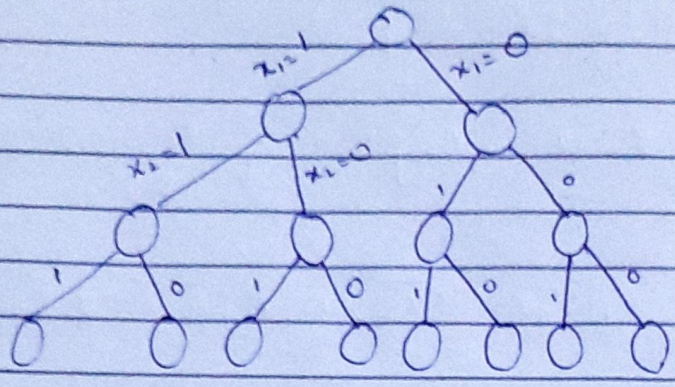


- 0/1 Knapsack Problem.

$P = \{10, 15, 20\}$

$W = \{4, 8, 3\}$

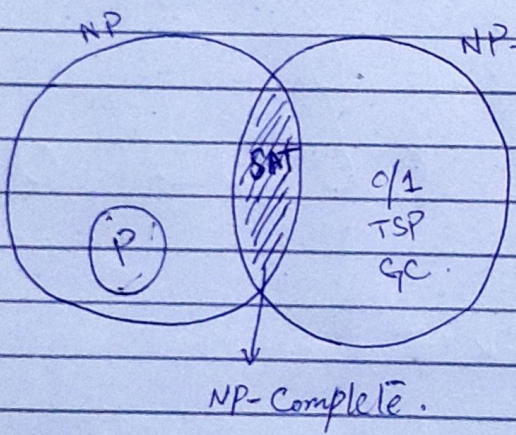
$x_i = \{0/1, 0/1, 0/1\}$



→ Only Exponential

→ NP-Hard : reduce into a base problem

→ NP-Complete : Non-Deterministic Polynomial Time Algorithm exists.



(P=NP)

π. Reduction? → A mathematical process to find Associations.

SAT ∝ 0/1 knapsack Problem.

↓

NP-Hard