ADA

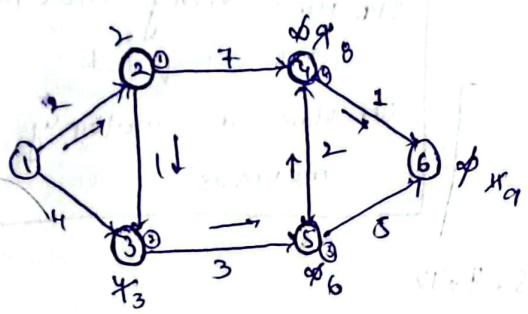
Friday -

15 - Dec - 2023

Lecture # 17

→ Greedy Algorithm

Dijkstra Algorithm ← Single Source Shortest Path.



(1) (2) → because of min weight. ∴ can update multiple times.

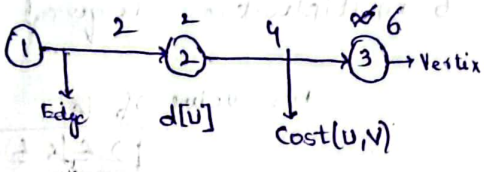
- Once we pick a vertex cannot again pick.
- Follow path with min weight for first vertex.
- we will pick that vertex which have min weight.

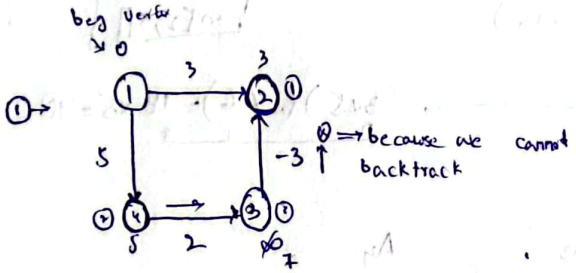
Updation (outflow)

if (d[u] + cost[u,v] < d[v])

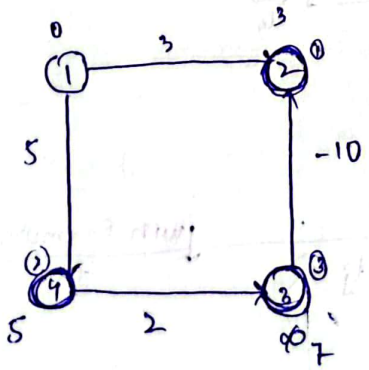
d[v] = d[u] + cost[u,v]

→ consider only outlink.





(2) →



→ Dynamic Programming

- For Optimisation Problem.
  (Either maximize or minimize)
- Exhaustive search in a controlled way.

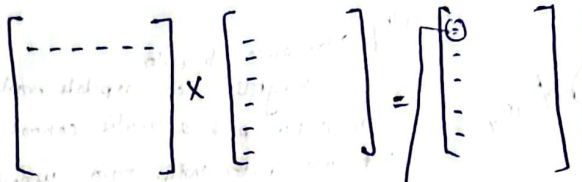
Matrix Chain Multiplication

A_1	A_2	A_3	A_4
3×5	5×3	3×8	8×5

QUESTIONS

Q1. How many multiplications are required.

Q2. How to multiply matrices.



5×6 · 6×8 · 5×8 = 40 · 5×8×6 = 240

For one element = 6 multiplications required

X	Y	Z
3×4	4×5	5×6

For value of k: $2 \le k \le 4$ → m[2,4]

(X·Y)Z = 60 + 90 = 150 (18×4 element multip)

X(YZ) = 120 + 72 = 192

(3×5)(5×6) = 18×5 = 90

Dimension

A_1	A_2	A_3	A_4
5 × 4	4 × 6	6 × 2	2 × 7
d_0 d_1	d_1 d_2	d_2 d_3	d_3 d_4

→ $[(A_1 \times A_2) \times (A_3 \times A_4)]$

→ $[((A_1 \times A_2) \times A_3) \times A_4]$ → Options

Ⓐ with Formula

$$m[i,j] = \min \{ m[i,k] + m[k+1,j] + d_{i-1} d_k d_j \}$$

$i \le k < j$

k = no. of possibilities

$$m[1,3] = \min \begin{cases} m[1,1] + m[2,3] + (5 \times 4 \times 2) & k=1 \\ m[1,2] + m[3,3] + (5 \times 6 \times 2) & k=2 \end{cases}$$

(d0 d1 d3)

	1	2	3	4
1	0	m1 120	88	158
2		0	m2 48	104
3			0	m3 84
4				0

- Two matrices

$m[1,2] = 5 \times 6 \times 4 = 120$

$m[2,3] = 4 \times 2 \times 6 = 48$

$m[3,4] = 6 \times 7 \times 2 = 84$

- 3 matrices

$m[1,3] = A_1 A_2 A_3 =$ → $(A_1 A_2) A_3$ or $A_1 (A_2 A_3)$

$m[2,4] = A_2 A_3 A_4 -$ → $(A_2 A_3) A_4$

- $m_1 = A_1 \times A_2$
- $m_2 = A_2 \times A_3$
- $m_3 = A_3 \times A_4$

→ Using Optimal Solution to solve another problem.

S = Different Orders

S →

	1	2	3	4
1			1	3
2				3
3				
4				

value of k through (A)

$m[1,3] \rightarrow (A_1 A_2) A_3$ → shows multiple of $(A_1 A_2) A_3$

$= 120 + 0 + 60$

$(5 \times 6 \times 4)$

$= 180$

$m[1,3] \rightarrow A_1 (A_2 A_3)$

$= 0 + 48 + 40 = 88$

$5 \times 4 \quad 4 \times 2$

$10 \times 4 = 40$

m

(A) → min

$m[1,3] = \min \begin{cases} 0 + 48 + 40 = 88 \checkmark \\ 120 + 0 + 60 = 180 \end{cases}$

Extra ①
op#2)

m[2,4] → k=2
→ k=3

i = row = 2
j = columns = 4

$$m[2,4] = \min \begin{cases} m[2,2] + m[3,4] + (4 \times 6 \times 7) \\ m[2,3] + m[4,4] + (4 \times 2 \times 7) \end{cases}$$

~~min~~ { 0 + ~~48~~ +
48
~~120~~ + [6×7×2]

$$= \min \begin{cases} 0 + 84 + 168 = 252 & k=2 \\ 48 + 0 + 56 = 104 ✓ & k=3 \end{cases}$$

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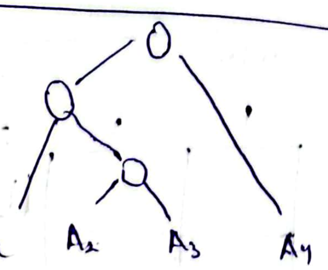
m[1,4]

$1 \le k \le 4$

i = 1
j = 4

k = 1, 2, 3

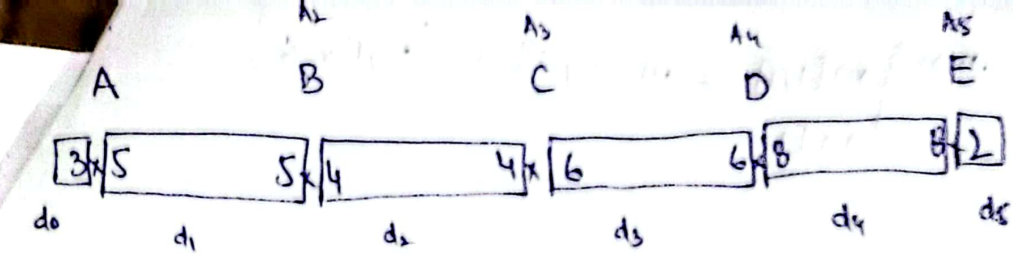


$$m[1,4] = \min \begin{cases} m[1,1] + m[2,4] + (5 \times 4 \times 7) \\ m[1,2] + m[3,4] + (5 \times 6 \times 7) \\ m[1,3] + m[4,4] + (5 \times 2 \times 7) \end{cases}$$

$$= \min \begin{cases} 0 + 104 + \cancel{56}\ 140 = 244 & k=1 \\ 120 + 84 + \cancel{84}\ 210 = 414 & k=2 \\ 88 + 0 + 70 = 158 ✓ & k=3 \end{cases}$$

①=k ③=k

→ [[(A1) (A2 A3)][(A4)]] → See S table



$$m[i,j] = \min \{ m[i,k] + m[k+1,j] + d_{i-1} d_k d_j \}$$

$i \le k < j$ $\quad$ i = rows $\quad$ j = colums .

m →

	1	2	3	4	5
1	0	60			
2		0	120		
3			0	192	
4				0	96
5					0

s →

	1	2	3	4	5
1					
2					
3					
4					
5					

$$m[1,2] = \min \{ m[1,1] + m[2,2] + (3 \times 5 \times 4) \quad \cancel{m[1,2] + m[3,2] + (5 \times 4 \times 4)} \}$$

$1 \le k < 2$

$k = 1$

~~$k \neq 1$~~

$i = 1$

$j = 2$

$$= \min \{ 0 + 0 + 60 = 60 \}$$

.

$m[1,3] = \min \begin{cases} m[1,1] + m[2,3] + (3 \times 5 \times 6) \\ m[1 \end{cases}$

$i = 1$

$j = 3$

$1 \leq k < 3$

$k = 1$

$k = 2$