Tuesday. 16/04/2024.

LECTURE NO. 14

→ Regular Expressions:

Regular Language

Finite Automata | Regular Expression | Regular Grammar.

Context Free Languages.

Regular Language

Not a Regular Exp.
↳ Language = { $a^n b^n \mid n \geq 1$ }

For Equal No. a's & b's ← Memory Req. for This

Doesn't exist for it. ← Finite Automata can't be used. Push Down Automata

Regular Expression → ε-NFA → NFA → DFA
↓ Minimal DFA.

↓

→ a way of representing finite automata.
→ expression of strings and operators.

Operators:

i) * (Kleene's Closure)

$a^* \rightarrow$ 0 or more occurrences of a.

$a^0, a^1, a^2, a^3 \ldots$

$\epsilon, a, aa, aaa \ldots$

Note: If + operator is in power of expression than it's positive closure and if it's used in base of expression than it's union operator.

ii) + (positive closure)

$a^+ \rightarrow$ atleast 1 or more occurrences of 'a'.

iii) . (Concatenation).

$a \cdot b = \{ab\}$

Example:

$\rightarrow a^* b = \{b, ab, aab, aaab \ldots\}$

$a^0 b, a^1 b, a^2 b \ldots$

$\epsilon . b, ab, aab \ldots$

$\because \boxed{\begin{matrix} \epsilon . b = b \\ \phi . b = \phi \end{matrix}}$

$\rightarrow a^+ b = \{ab, aab, aaab \ldots\}$

$a^1 b, a^2 b, a^3 b$

$ab, aab, aaab$

iv) + (union)

$a+b = \{a, b\}$ Either a or b.

$(a+b)^0 = \epsilon$

$(a+b)^1 = \{a, b\}$

$(a+b)^2 = (a+b)(a+b)$

$= \{aa, ab, ba, bb\}$

→ A regular expression is said to be valid iff it can be derived from the primitive regular expression by a finite number of application of the rule.

r^+, r^*, $r_1 . r_2$, $r_1 + r_2$

primitive → Basic Building Blocks.

Primitive Regular Expressions.

$\Sigma = \{a, b\}$ $\quad$ $r_1 = \phi \Rightarrow$ Language $= \{\ \}$

→ ϕ, ϵ, a, b $\quad$ $r_1 = \epsilon \Rightarrow$ Language $= \{\epsilon\}$

$r_1 = a \Rightarrow$ Language $= \{a\}$

$r_1 = b \Rightarrow$ Language $= \{b\}$

→ Drive Language from a given regular expressions.

1	$r = \phi$	Language $(r) = \phi$
2	$r = \epsilon$	" $= \{\epsilon\}$
3	$r = a$	" $= \{a\}$
4	$r = a+b$	" $= \{a, b\}$
5	$r = a.b$	" $= \{ab\}$
6	$r = a+b+c$	" $= \{a, b, c\}$
7	$r = (ab+a).b$	" $= \{abb, ab\}$
8	$r = a^+$	" $= \{a, aa, aaa \ldots\}$
9	$r = a^*$	" $= \{\epsilon, a, aa, aaa \ldots\}$

(10) $r = (a + ba)(b + a)$

Language $(r) = \{ab, aa, bab, baa\}$

(11) $r = (a + \epsilon)(b + \phi)$

Language $(r) = \{ab, b\}$ $\cancel{\phi}$

(12) $r = (a+b)^2 = (a+b)(a+b)$

Language $(r) = \{aa, ab, ba, bb\}$

(13) $r = (a+b)^*$

Language $= \{\epsilon, a, b, aa, ab, ba, bb, \ldots\}$

aaa, bbb

abb

(14) $r = (a+b)^* (a+b) = (a+b)^+$

$\{\epsilon, a, b, aa, ab, ba, bb, aaa \ldots\} (a+b)$

$\epsilon . (a+b) = (a+b)$

$= \{a, b\}$.

$= \{a, b, aa, ab, ba, bb, aaa \ldots\}$

$= (a+b)^+$.

There can be multiple expressions for one Language but there can't be multiple Languages for one expression.

26/04/2024.
Friday

Lecture No. 16.

⇒ Regular Expression.

→ Starts and ends with the same symbol.

$$a(a+b)^*a + b(a+b)^*b + a + b.$$

→ starts and ends with different symbols.

$$a(a+b)^*b + b(a+b)^*a.$$

→ $|w| = 3$.

$$(a+b)^3 = (a+b)(a+b)(a+b).$$

→ $|w| \geq 3$

$$(a+b)^3 (a+b)^*$$
$$(a+b)^* (a+b)^3.$$

→ $|w| \leq 3$

$$\epsilon + (a+b) + (a+b)^2 + (a+b)^3$$
$$(a+b+\epsilon)^3$$

→ $|w|_a = 2$

$b^* a b^* a b^*$

→ $|w|_a \geq 2.$

$(a+b)^* a (a+b)^* a (a+b)^*$

Exp aababbab

$(a+b)^3 a (a+b)^2 a (a+b)^1$

→ $|w|_a \leq 2.$

$b^* (\epsilon + a) b^* (a+\epsilon) b^*.$

→ 3^{rd} Symbol is 'b'.

$(a+b)^2 b (a+b)^*$

$(a+b)(a+b) b (a+b)^*$

→ 3^{rd} Last Symbol is 'a'

$(a+b)^* a (a+b)^2$

→ $|w| \bmod 3 = 0$

$((a+b)^3)^*$

Exp: $\epsilon = ((a+b)^3)^0 = \epsilon$

$abb = ((abb)^3)^1$

$7/3$

$=2$

$5/3 = 2,$

$8/3 = 2.$ aabb

→ $|w| \bmod 3 = 2.$

$$(a+b)^2((a+b)^3)^*$$

Exp abbababb.

$= (a+b)^2((a+b)^3)^2$

$= (a+b)(a+b)\ (a+b)^3(a+b)^3$

$=$ a b bab abb

→ $|w|_b \bmod 2 = 0$

$(b^2)^* a^* (b^2)^* a^* (b^2)^*$ ✗

$(a^* b\ a^* b\ a^*)^*$

① $\boxed{a^* + (a^* b\ a^* b a^*)^*}$ ~~b~~

② $\boxed{a^*(a^* b a^* b a^*)^*}$

Exp abbbb = ?

babbb = ?

$= a^0 + (a^* b a^* b a^*)^2$

$= \epsilon + (a^* b a^* b a^*)(a^* b a^* b a^*)$

$= \epsilon + (a^0 b a^1 b a^0)(a^0 b a^0 b a^0)$

bab bb

$=$

→ $|w|_a \bmod 3 = 1$

$(b^*(a)(b^*(b^*a\,b^*a\,b^*a\,b^*))^*)$ ✓

$b^*ab^*(b^*ab^*ab^*ab^*)^*$

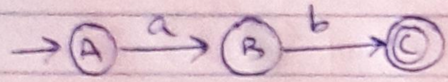
→ $|w|_b \bmod 3 = 2$.

$(a^*ba^*ba^*ba^*)^*$

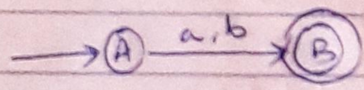
30/04/2024.

Lecture No. 17 — Tuesday

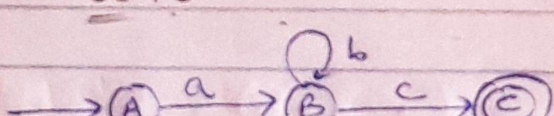
→ Finite Automata to Regular Expression.

1) 

$\underline{ab}$.

2) 

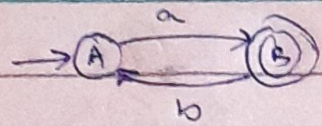
$\underline{a+b}$

3) 

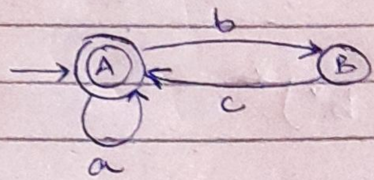
$\underline{ab^*c}$.

4) 

$(ab)^*$

(5) 

$a(ba)^*$ OR $(ab)^*a$.

(6) 

$a^* + (bc)^*$ ✗

$a^*(bc)^*$ ✗

Not generating abc bca

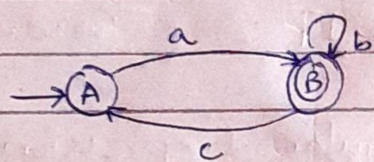
$(a^* + (bc)^*)^*$ ✓

OR

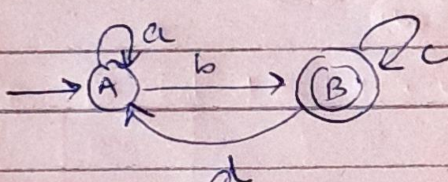
$(a + bc)^*$

Both equal.

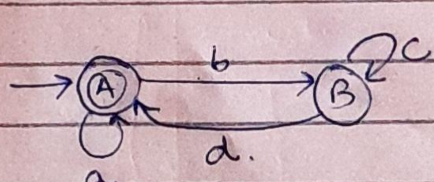
$\because (a^* + b^*)^* = (a+b)^*$.

(7) 

$a(b + ca)^*$.

(8) 

$a^*b(c + da^*b)^*$.

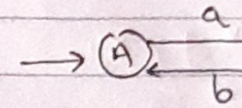
(9) 

Either a^* or bc^*d ka Kleene Closure.

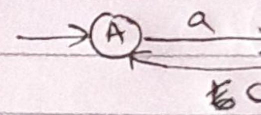
$\boxed{(a^* + (a^*bc^*da^*)^*)}$?

$(a + bc^*$

$(a^* + ($

(10) 

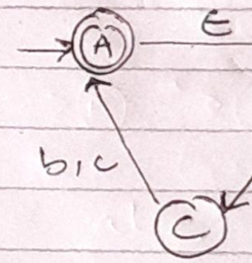
$a($

(11) 

$\cancel{a(b}$

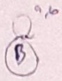
$= a($

$= a($

(12) 

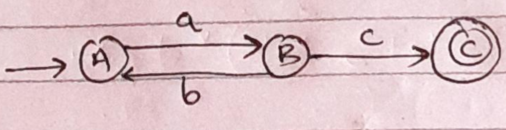
$(\epsilon . a(b$

$= (a(b+c$

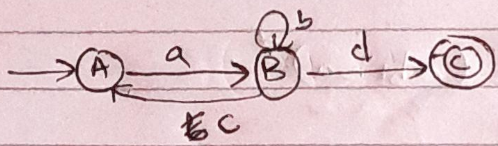


$(a+bc^*d)^*$

$(a^* + (bc^*d)^*)^* = (a+bc^*d)^*$

(10) 

$a(ba)^*c.$

(11) 

$\cancel{a(ba+b)^*c}$ $\Rightarrow$ $\boxed{a(b^*(ca)^*)^*d}$ ✓

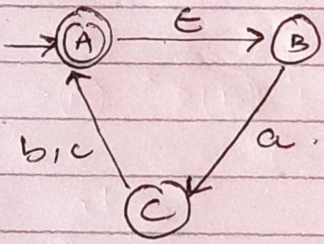
$= a(b+ca)^*d$

$= a(b^*(ca)^*)^*d$

$(b+ca)^* =$

$= (b^*(ca)^*)^*$

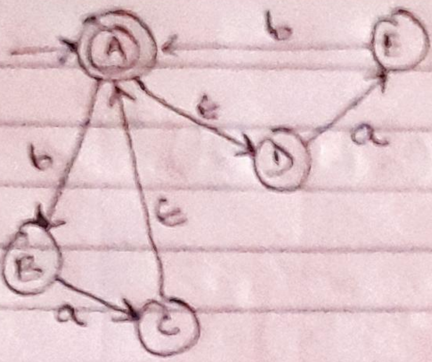
$= (b^*+ca^*)^*$

(12) 

$(\epsilon . a(b+c))^*$

$= (a(b+c))^* = (ab+ac)^*$

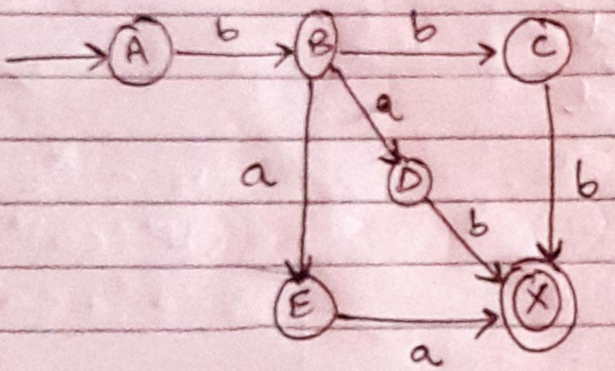
(13)



$$=\left((\epsilon.a.b)^* + (b.a.\epsilon)^*\right)^*$$

$$=\left((ab)^* + (ba)^*\right)^* = (ab+ba)^*$$

(14)

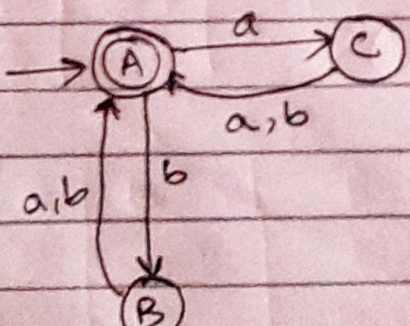


$b(bb+ab+aa)$

or

$bbb+bab+baa.$

(15)



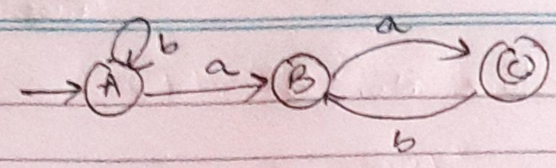
$$=\left(a(a+b)+b(a+b)\right)^*$$

$$=\left((a+b)(a+b)\right)^*$$

$$=\left((a+b)^2\right)^*$$

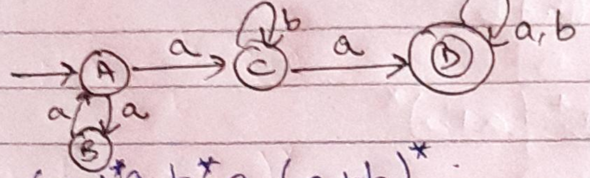
REGEX?

(16)



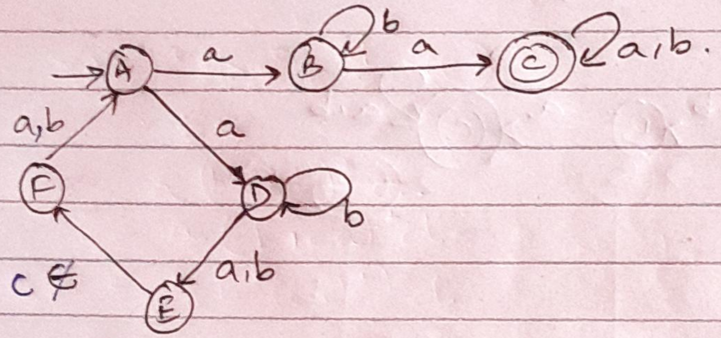
$b^* aa(ba)^*$

(17)



$(aa)^* a b^* a (a+b)^*$

(18)



$$\cancel{a b^* a (a+b)^* + a b^* (a+b) \cdot \epsilon \cdot (a+b)}$$

$$\cancel{= \left(ab^*\left(a(a+b)^* + (a+b)^2\right)\right)^*}$$

$(ab^*(a+b)c(a+b))^* ab^*a(a+b)^*$ ✓

07/05/2024
Tuesday

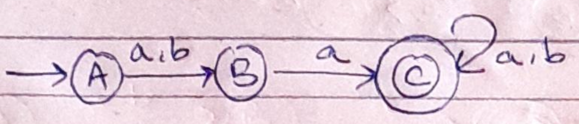
Lecture No. 18

→ Language = 2^{nd} Symbol is 'a'

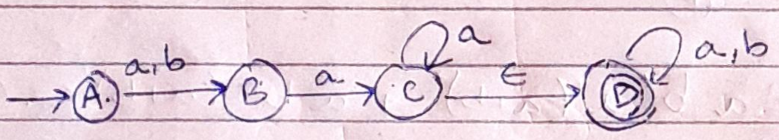
$r_1 = (a+b)\, a\, (a+b)^*$

$r_2 = (a+b)\, a^+ (a+b)^*$

$\underline{r_1}$.



$\underline{r_2}$



→ $\underline{r_1}$:

	a	b
→ A	{B}	{B}
B	{C}	ϕ
* C	{C}	{C}

	a	b
DFA → A	B	B
B	C	D
* C	C	C
D	D	D

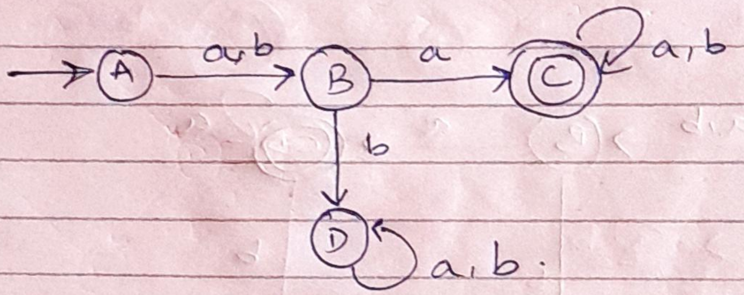
=

Minimal DFA

0 - Equivalence: {A, B, D} {C}

1 - Eq : {A, D} {B} {C}

2 - Eq : {A} {D} {B} {C}



→ $\underline{\underline{\gamma_2}}$:

ε - Closure →

NFA	a	b
→A	B	B
B	{C, D}	Φ
*C	{C, D}	{Φ}
*D	{D}	{D}.

A	{A}
B	{B}
C	{C, D}
D	{D}

$A \xrightarrow{\epsilon} A \xrightarrow{a} B \xrightarrow{\epsilon} B$. $B \xrightarrow{\epsilon} B \xrightarrow{a} C \xrightarrow{\epsilon} CD$

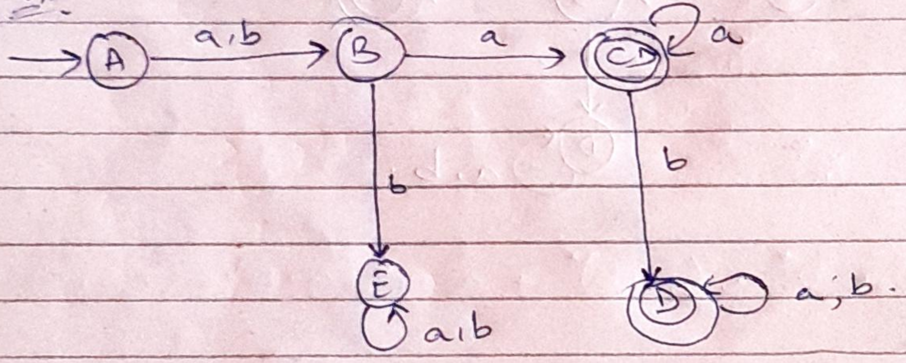
$C \xrightarrow{\epsilon} C \xrightarrow{a} C \xrightarrow{\epsilon} C$, $\rightarrow D$
$\rightarrow D \xrightarrow{a} D \rightarrow D$

$\rightarrow C \xrightarrow{\epsilon} C \xrightarrow{b} \Phi$
$\rightarrow D \xrightarrow{b} D \xrightarrow{\epsilon} D$.

DFA

	a	b
→ A	B	B
B	[CD]	[E]
* [CD]	[CD]	[D]
[E]	[E]	[E]
* [D]	[D]	[D]

DFA

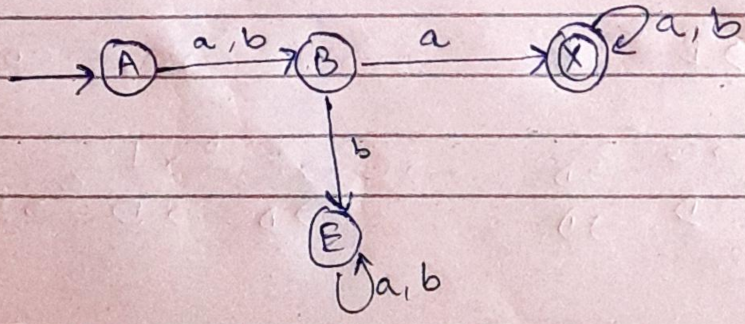


Min. DFA:

0- Equivalence: {A, B, E} {CD, D}

1- Equivalence: {A, E} {B} {CD, D}

2- Eq: {A} {E} {B} {CD, D}.

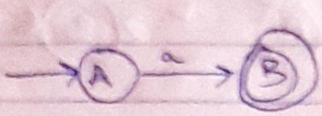


→ Regular Expression to Finite Automata

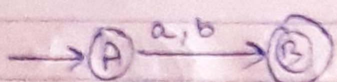
ϕ

ϵ

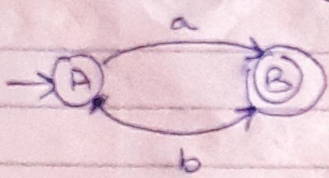
a



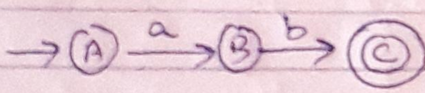
$a+b$



$a+b$

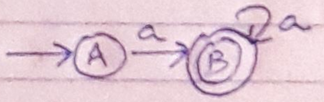


ab

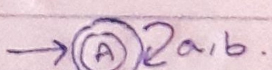


a^*

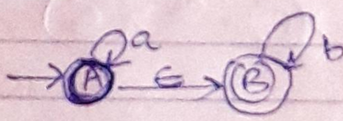
a^+



$(a+b)^*$

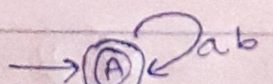


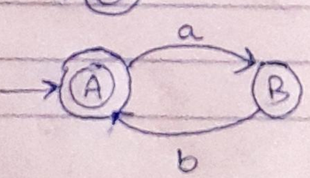
$a^* b^*$



ϵ is used to preserve the order.

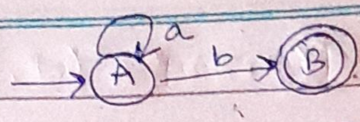
$(ab)^*$



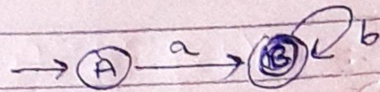


OFF topic nahi haa krein.

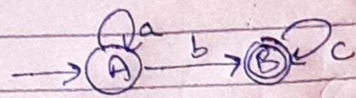
a^*b



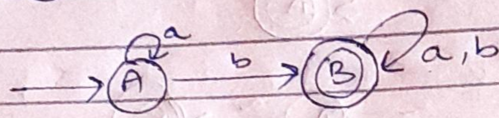
ab^*



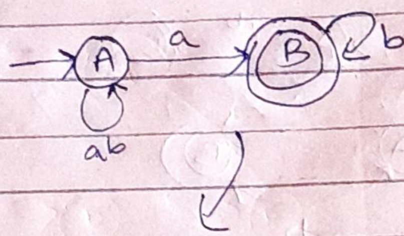
a^*bc^*

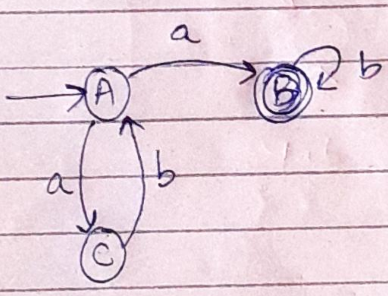


$(a^*b(a+b)^*)$

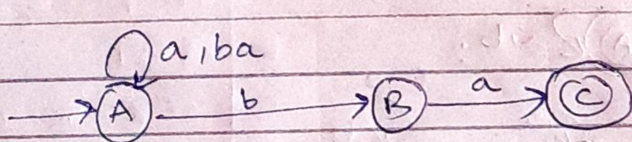


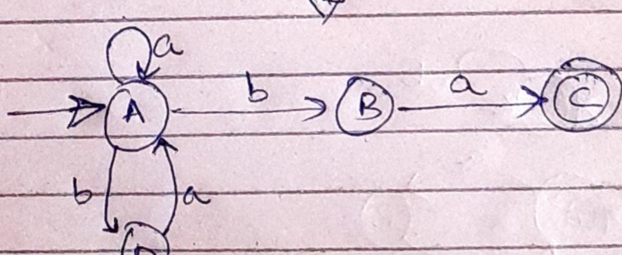
$(ab)^*ab^*$



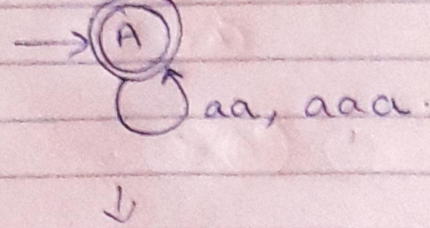


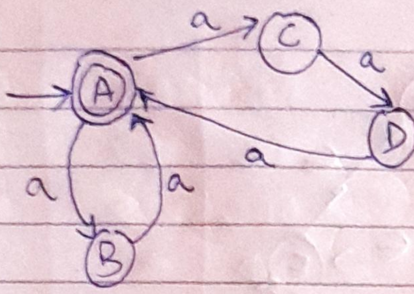
$(a+ba)^*ba$



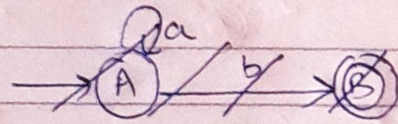


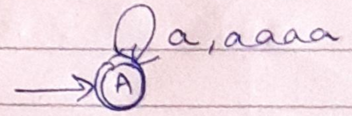
• $(aa + aaa)^*$

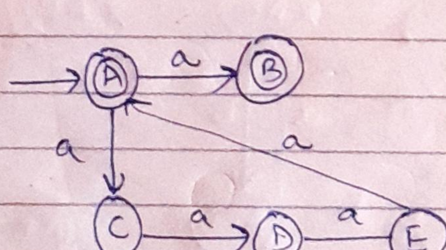




• $\cancel{a^*b(a+b)^*}$ $(a+aaaa)^*$





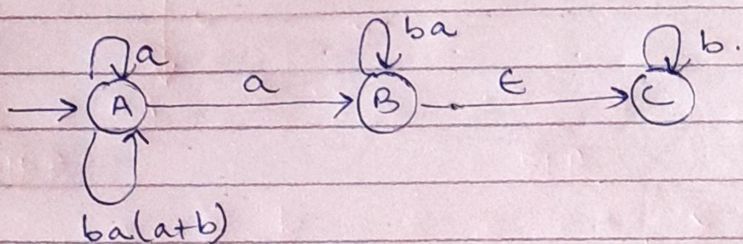


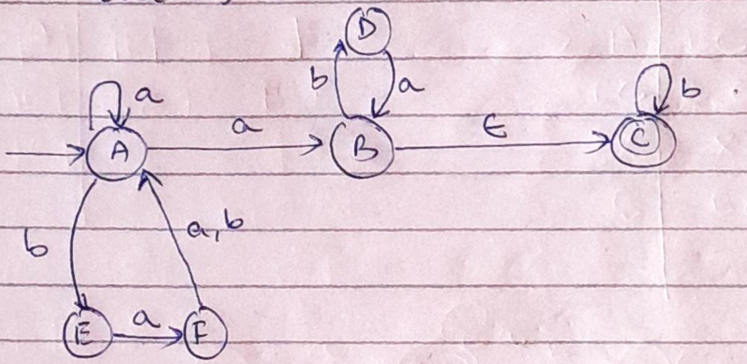
14/05/2024.
Tuesday

Lecture # 20

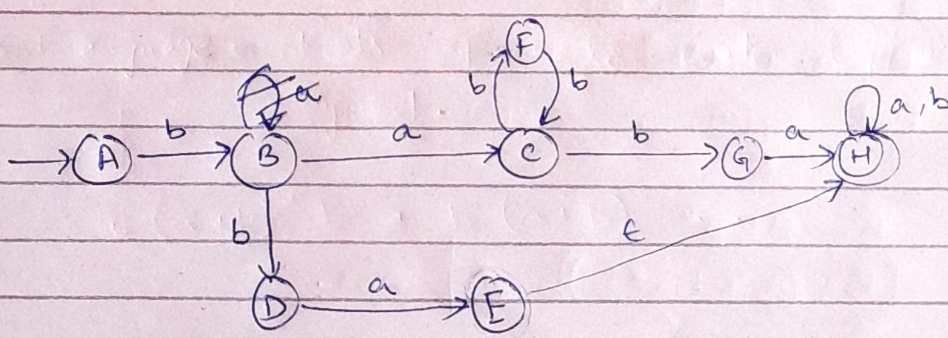
→ Regular Expression to Finite Automata.

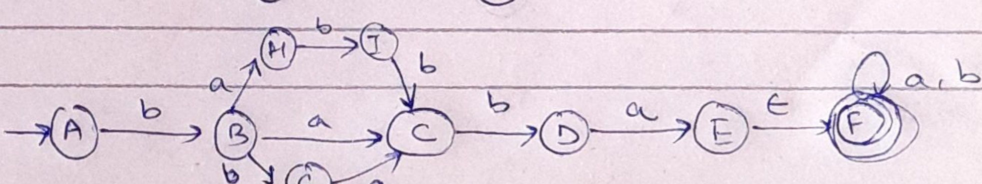
1) $[a+ba(a+b)]^* a(ba)^* b^*$



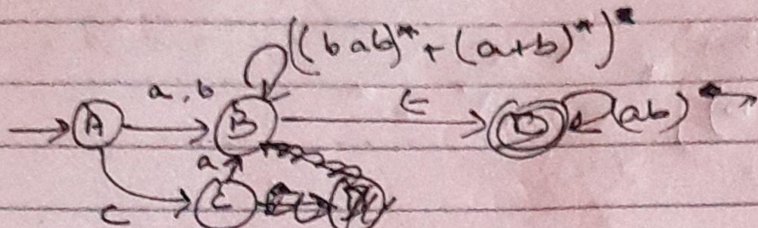


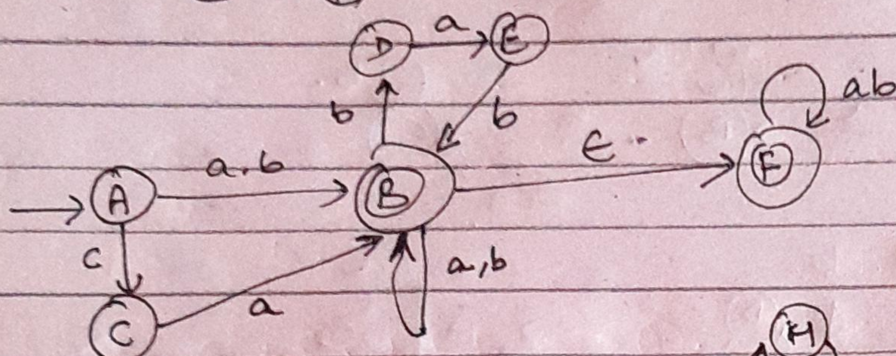
2) $b(a+ba+abb)(ba(a+b)^*)$

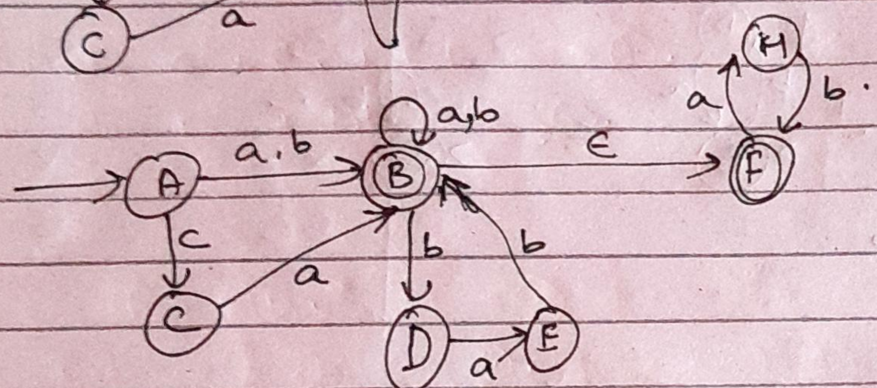




3) $(a+b+ca)((bab)^* + (a+b)^*)^*(ab)^*$







$r_0 = ((a+b)(a+\epsilon)^*(b) + \epsilon)$

$r_1 = ((a+b)\{a^*(b) + \epsilon)$

17/05/2024.
Friday.

Lecture # 21

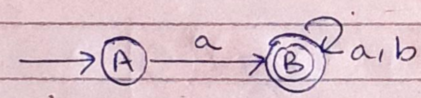
→ ARDEN'S THEOREM:

FA → RE.

If P and Q are two regular expression over Σ and if P does not contain ϵ Then the following equation in R given by $R = Q + RP$ has a unique solution $R = QP^*$.

$$R = Q + RP \longrightarrow R = QP^*$$

$$R = Q + RP$$
$$R = Q + (Q + RP)P$$
$$R = Q + QP + RPP$$
$$R = Q + QP + RP^2$$
$$R = Q + QP + (Q + RP)P^2$$
$$R = Q + QP + QP^2 + RP^3$$
$$R = Q(P^0 + P^1 + P^2 + P^3 + - - -)$$
$$R = QP^*$$

1) 

1) For initial state use Epsilon must and if there are any incoming edges Then also mention them.

$A = \epsilon \rightarrow ①$

$B = Aa + Ba + Bb \longrightarrow ②$.

2) for other states only incoming edges

3) Solve Equations for Accepting state

$$\underbrace{B}_{R} = \underbrace{Aa}_{Q} + \underbrace{B}_{R}\underbrace{(a+b)}_{P}$$

$R = QP$

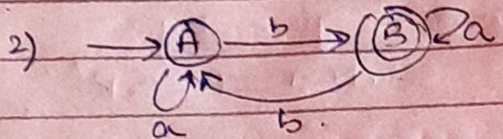
$B = Aa(a+b)^* \rightarrow (3)$ $\therefore R = QP^*$

Put eq(1) in eq(3).

$B = \epsilon . a(a+b)^*$

$\boxed{B = a(a+b)^*}$

2)



$A = \epsilon + Aa + Bb \longrightarrow (1)$.

$\underbrace{B}_{R} = \underbrace{B}_{R}\underbrace{a}_{P} + \underbrace{Ab}_{Q} \longrightarrow (2)$.

$B = Aba^* \longrightarrow (3)$

Put eq(3) in eq(1).

$A = \epsilon + Aa + Aba^*b$

$\underbrace{A}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{A}_{R}\underbrace{(a + ba^*b)}_{P}$

$A = \epsilon . (a + ba^*b)^*$

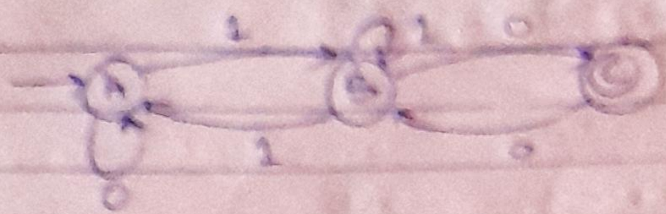
$\boxed{A = (a + ba^*b)^*} \longrightarrow (4)$.

Put eq(4) in eq(3).

$B = Aba^*$

$\boxed{B = (a + ba^*b)^*ba^*}$ Ans.

4)



$A = \epsilon + A0 + B1 \rightarrow (1)$

$B = A1 + B1 + C0 \rightarrow (2)$

$C = B0 \rightarrow (3)$

$B = A1 + B1 + B00$

$B = A1 + B(1 + 0.0)$

$B = A1(\cancel{+}1 + 0^2)^* \rightarrow (4)$

$A = \epsilon + A0 + A1(1+0^2)^*1$

$A = \epsilon + A(0 + 1(1+0^2)^*1)$

$A = \epsilon.(0 + 1(1+0^2)^*1)^* \rightarrow (5)$

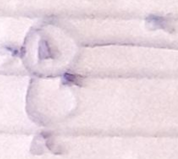
Put eq (5) in eq (4).

$B = \cancel{\alpha}\,\epsilon(0 + 1(1+0^2)^*1)^*1(1+0^2)^* \rightarrow (6)$

Put eq (6) in eq (3).

$\boxed{C = (0 + 1(1+0^2)^*1)^*1(1+0^2)^*0}$ Ans.

5)



$A = \epsilon + Aa$

$B = Aa + Bb$

$C = Ba \rightarrow$

$B = Aa + B$

$B = Aa +$

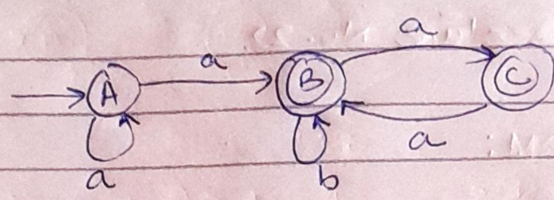
$B = Aa(b$

$B = a^*a($

$C = a^*a$

$A = a^*$

5)



$A = \epsilon + Aa \rightarrow (1)$

$B = Aa + Bb + Ca \rightarrow (2)$

$C = Ba \rightarrow (3)$

$B = Aa + Bb + Baa$

$B = Aa + B(b+aa)$

$B = Aa(b+aa)^* \rightarrow (4)$

$A = \epsilon + Aa$

$A = \epsilon . a^* \rightarrow (5)$

$\boxed{A = a^*}$

$\boxed{B = a^* a(b+aa)^*}$

$\boxed{C = a^* a(b+aa)^* a}$

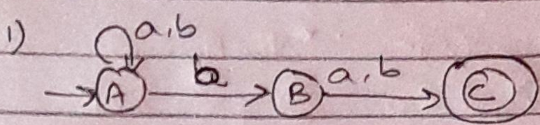
$R = a^* a(b+aa)^* + a^* a(b+aa)^* a.$

21/05/2024.
Tuesday.

Lecture No. 22

→ ARDEN'S THEOREM:

1)



$A = \epsilon + Aa + Ab \rightarrow (1)$

$B = Ab \rightarrow (2)$

$C = Ba + Bb \rightarrow (3).$

$C = B(a+b)$

$\Rightarrow (1)$ $\quad A = \epsilon + A(a+b)$

$\quad R = Q \quad R \quad P$

$\boxed{A = \epsilon \cdot (a+b)^*} \rightarrow (4).$

Put eq (4) in eq (2).

$B = (a+b)^* b \rightarrow (5).$

Put eq (3).

$\boxed{C = (a+b)^* b (a+b)}$ Ans.

2) 

$A = \epsilon + Aa + Ab \rightarrow (1)$

$B = Aa \rightarrow (2)$

$C = Ba \rightarrow (3)$

$X = Cb + Xa + Xb \rightarrow (4)$

$\Rightarrow (1)$ $A = \epsilon + A(a+b)$

$A = \epsilon (a+b)^* \rightarrow (5)$

Put in eq (2).

$B = (a+b)^* a \rightarrow (6)$

Put eq (6) in eq (3).

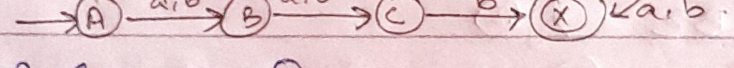
$C = (a+b)^* aa \rightarrow (7)$

$\rightarrow (4)$ $X = Cb + X(a+b)$

$X = Cb(a+b)^*$

Put value of C from eq (7).

$\boxed{X = (a+b)^* aab (a+b)^*}$ Ans.

3) 

$A = \epsilon \rightarrow (1)$

$B = Aa + Ab \rightarrow (2)$ $\Rightarrow$ $B = A(a+b) \Rightarrow B = \epsilon(a+b)$

$C = Ba + Bb \rightarrow (3)$ $\boxed{B = (a+b)}$ Put in eq (3).

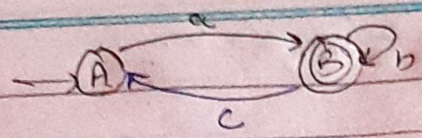
$X = Cb + Xa + Xb \rightarrow (4)$ $\Rightarrow (3)$ $C = B(a+b)$

$X = Cb + X(a+b)$ $C = (a+b)(a+b)$

$X = Cb(a+b)^* \rightarrow (5)$

$\boxed{X = (a+b)^2 b (a+b)^*}$ Ans.

4)



$\cancel{A = \epsilon}$

$A = \epsilon + Bc \rightarrow ①$

$\underset{R}{B} = \underset{Q}{Aa} + \underset{RP}{Bb} \rightarrow ②$

$B = Aab^* \rightarrow ③$

Put eq ③ in eq ①

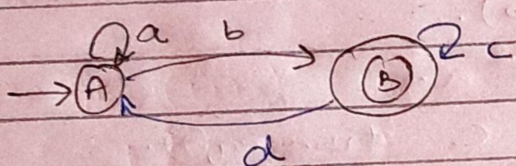
$A = \epsilon + Aab^*c$

$A = \epsilon(ab^*c)^* \rightarrow ④$

Put eq ④ in eq ③

$\boxed{B = \epsilon(ab^*c)^*ab^*}$ Ans.

5)



$\underset{}{\overset{Q}{(\epsilon + Bd)}} + \overset{RP}{Aa}$

$A = \epsilon + Aa + Bd \rightarrow ①$

$B = Ab + Bc \rightarrow ②$

$B = Abc^* \rightarrow ③$

Put in eq ①

$A = \epsilon + Aa + Abc^*d$

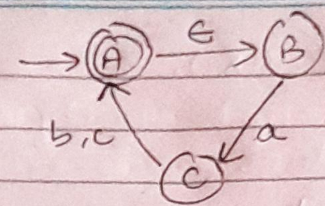
$A = \epsilon + A(a + bc^*d)$

$A = \epsilon(a + bc^*d)^* \rightarrow ④$

Put eq ④ in eq ③

$\boxed{B = (a + bc^*d)^*bc^*}$ Ans.

6)



$A = \epsilon + Cb + Cc \rightarrow (1)$

$B = A\epsilon \rightarrow (2)$

$C = Ba \rightarrow (3)$.

$\boxed{C = A\epsilon . a} \rightarrow (4)$.

Put eq (4) in eq (1).

$A = \epsilon + C(b+c)$

$A = \epsilon + A\epsilon . a(b+c)$.

$A = \cancel{\epsilon(1 + a(b+c))}$

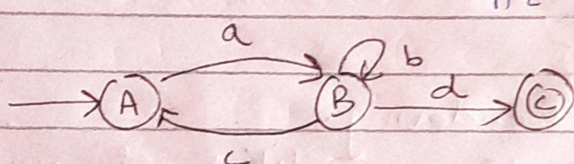
$\boxed{\cancel{A = \epsilon + \epsilon a(b+c)}}$

$A = \epsilon . (\epsilon . a(b+c))^*$.

$\boxed{A = (\epsilon . a(b+c))^*}$ Ans.

$A = (a(b+c))^*$ Ans

(7)



$A = \epsilon + Bc \rightarrow (1)$

$B = Aa + Bb \rightarrow (2)$

$C = Bd \rightarrow (3)$

eq (2) $\Rightarrow$ $B = Aab^* \rightarrow (4)$

Put in eq (1).

$A = \epsilon + Aab^*c$

$A = \epsilon(ab^*c)^* \rightarrow (5)$

Put eq (5) in eq (4).

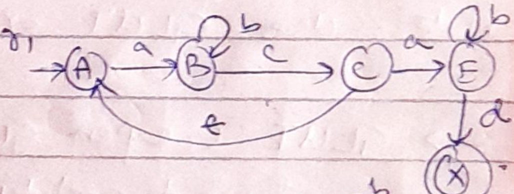
$B = (ab^*c)^* ab^* \rightarrow (6)$.

Put eq (6) in eq (3).

r_1 $\boxed{C = (ab^*c)^* ab^*d}$ Ans.

r_2 $\boxed{C = a(ca+b)^* d}$?

r_1



$r_2 =$