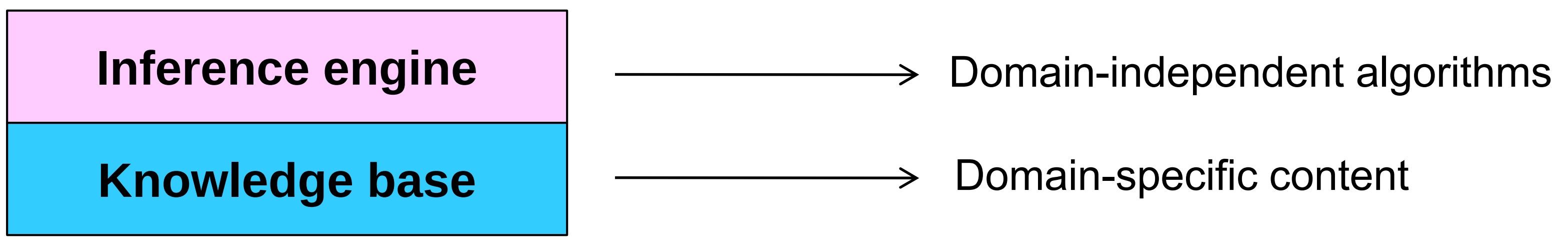
Logical Agents

- Knowledge-based agents
- Logic in general
- Propositional logic
- Inference rules and theorem proving
- First order logic

Knowledge-based agents



- Knowledge base (KB) = set of sentences in a formal language
- Declarative approach to building an agent (or other system):
  – Tell it what it needs to know
- Then it can ask itself what to do - answers should follow from the KB
- Distinction between data and program
- Fullest realization of this philosophy was in the field of expert systems or knowledge-based systems in the 1970s and 1980s

What is logic?

- **Logic** is a formal system for manipulating facts so that true conclusions may be drawn
  - "The tool for distinguishing between the true and the false" (Averroes)
- **Syntax:** rules for constructing valid sentences
  - E.g., $x + 2 \geq y$ is a valid arithmetic sentence, $\geq x2y +$ is not
- **Semantics:** "meaning" of sentences, or relationship between logical sentences and the real world
  - Specifically, semantics defines truth of sentences
  - E.g., $x + 2 \geq y$ is true in a world where $x = 5$ and $y = 7$

Propositional logic: Syntax

- **Atomic sentence:**
  - A *proposition symbol* representing a true or false statement
- **Negation:**
  - If **P** is a sentence, $\neg$**P** is a sentence
- **Conjunction:**
  - If **P** and **Q** are sentences, **P** $\wedge$ **Q** is a sentence
- **Disjunction:**
  - If **P** and **Q** are sentences, **P** $\vee$ **Q** is a sentence
- **Implication:**
  - If **P** and **Q** are sentences, **P** $\Rightarrow$ **Q** is a sentence
- **Biconditional:**
  - If **P** and **Q** are sentences, **P** $\Leftrightarrow$ **Q** is a sentence

- $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$ are called *logical connectives*

Propositional logic: Semantics

- A **model** specifies the true/false status of each proposition symbol in the knowledge base
  - E.g., **P** is true, **Q** is true, **R** is false
  - With three symbols, there are 8 possible models, and they can be enumerated exhaustively
- Rules for evaluating truth with respect to a model:

$\neg P$	is true	iff	P	is false			
$P \wedge Q$	is true	iff	P	is true	and	Q	is true
$P \vee Q$	is true	iff	P	is true	or	Q	is true
$P \Rightarrow Q$	is true	iff	P	is false	or	Q	is true
$P \Leftrightarrow Q$	is true	iff	$P \Rightarrow Q$	is true	and	$Q \Rightarrow P$	is true

Truth tables

- A **truth table** specifies the truth value of a composite sentence for each possible assignments of truth values to its atoms

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$true$	$true$
$false$	$true$	$true$	$false$	$true$	$true$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
$true$	$true$	$false$	$true$	$true$	$true$	$true$

- The truth value of a more complex sentence can be evaluated *recursively* or *compositionally*

Logical equivalence

- Two sentences are logically equivalent} iff true in same models: α ≡ ß iff α ╞ β and β ╞ α

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) \quad \text{de Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) \quad \text{de Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Entailment

- **Entailment** means that a sentence follows from the premises contained in the knowledge base:

$$KB \models \alpha$$

- Knowledge base *KB* entails sentence *α* if and only if *α* is true in all models where *KB* is true
  - E.g., $x + y = 4$ entails $4 = x + y$

Inference

- **Logical inference:** a procedure for generating sentences that follow from a knowledge base KB
- An inference procedure is **sound** if whenever it derives a sentence α, KB $\models \alpha$
  - A sound inference procedure can derive only true sentences
- An inference procedure is **complete** if whenever $KB \models \alpha$, α can be derived by the procedure
  - A complete inference procedure can derive every entailed sentence

Inference

- How can we check whether a sentence α is entailed by KB?
- How about we enumerate all possible models of the KB (truth assignments of all its symbols), and check that α is true in every model in which KB is true?
  - Is this sound?
  - Is this complete?
- Problem: if KB contains n symbols, the truth table will be of size 2^n
- Better idea: use *inference rules*, or sound procedures to generate new sentences or *conclusions* given the *premises* in the KB

Inference rules

- Modus Ponens

$$\frac{\alpha \Rightarrow \beta, \alpha}{\beta}$$

premises

conclusion

- And-elimination

$$\frac{\alpha \wedge \beta}{\alpha}$$

Inference rules

- And-introduction

$$\frac{\alpha,\ \beta}{\alpha \wedge \beta}$$

- Or-introduction

$$\frac{\alpha}{\alpha \vee \beta}$$

Inference rules

- Double negative elimination

$$\frac{\neg\neg\alpha}{\alpha}$$

- Unit resolution

$$\frac{\alpha \vee \beta, \neg\beta}{\alpha}$$

Resolution

$$\frac{\alpha \vee \beta, \neg\beta \vee \gamma}{\alpha \vee \gamma} \quad \text{or} \quad \frac{\alpha \vee \beta, \beta \Rightarrow \gamma}{\alpha \vee \gamma}$$

- Example:
  - α: “The weather is dry”
  - β: “The weather is rainy”
  - γ: “I carry an umbrella”

Resolution is complete

$$\frac{\alpha \vee \beta, \neg \beta \vee \gamma}{\alpha \vee \gamma}$$

- To prove KB $\models$ α, assume KB ∧ ¬ α and derive a contradiction
- Rewrite KB ∧ ¬ α as a conjunction of *clauses*, or disjunctions of *literals*
  - *Conjunctive normal form* (CNF)
- Keep applying resolution to clauses that contain complementary literals and adding resulting clauses to the list
  - If there are no new clauses to be added, then KB does not entail α
  - If two clauses resolve to form an empty clause, we have a contradiction and KB $\models$ α

Inference, validity, satisfiability

A sentence is valid if it is true in all models,

e.g., *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in some model

e.g., $A \vee B$, C

A sentence is unsatisfiable if it is true in no models

e.g., $A \wedge \neg A$

Satisfiability is connected to inference via the following:

$KB \models \alpha$ if and only if $(KB \wedge \neg \alpha)$ is unsatisfiable

Complexity of inference

- Propositional inference is *co-NP-complete*
  - *Complement* of the SAT problem: $\alpha \models \beta$ if and only if the sentence $\alpha \wedge \neg \beta$ is *unsatisfiable*
  - Every known inference algorithm has worst-case exponential running time
- Efficient inference possible for restricted cases

Definite clauses

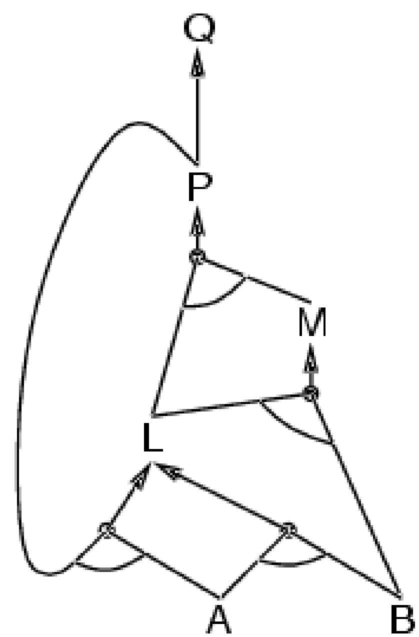
- A **definite clause** is a disjunction with exactly one positive literal
- Equivalent to $\underbrace{(\mathbf{P_1} \land \ldots \land \mathbf{P_n})}_{\textbf{premise} \text{ or } \textbf{body}} \Rightarrow \underbrace{\mathbf{Q}}_{\substack{\textbf{conclusion} \\ \text{or } \textbf{head}}}$

- Basis of logic programming (Prolog)
- Efficient (linear-time) complete inference through *forward chaining* and *backward chaining*

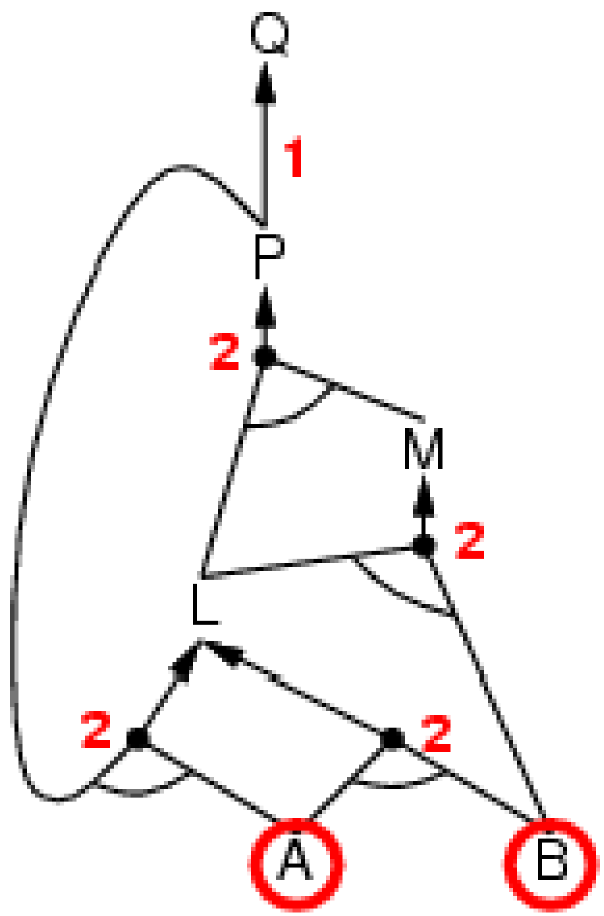
Forward chaining

- Idea: find any rule whose premises are satisfied in the *KB*, add its conclusion to the *KB*, and keep going until query is found

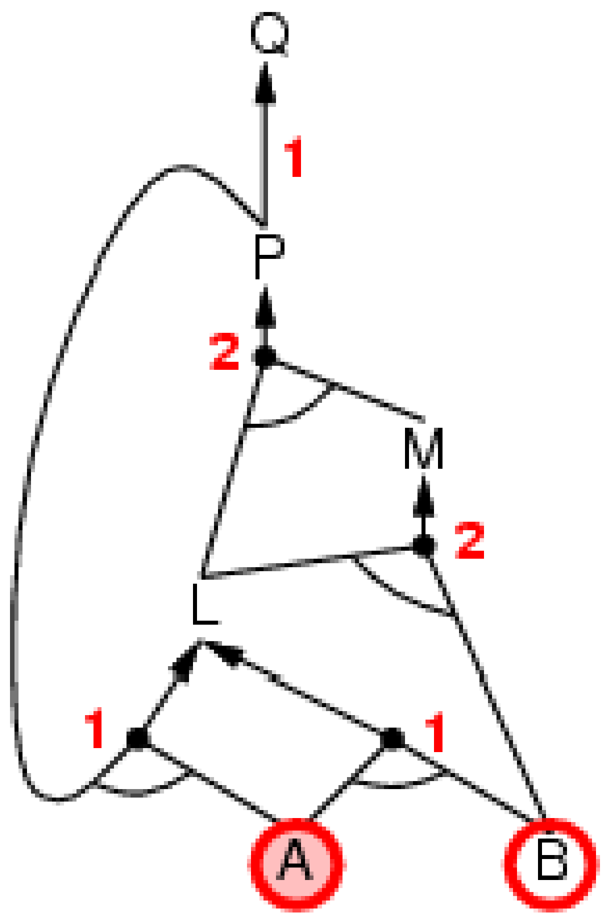
$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



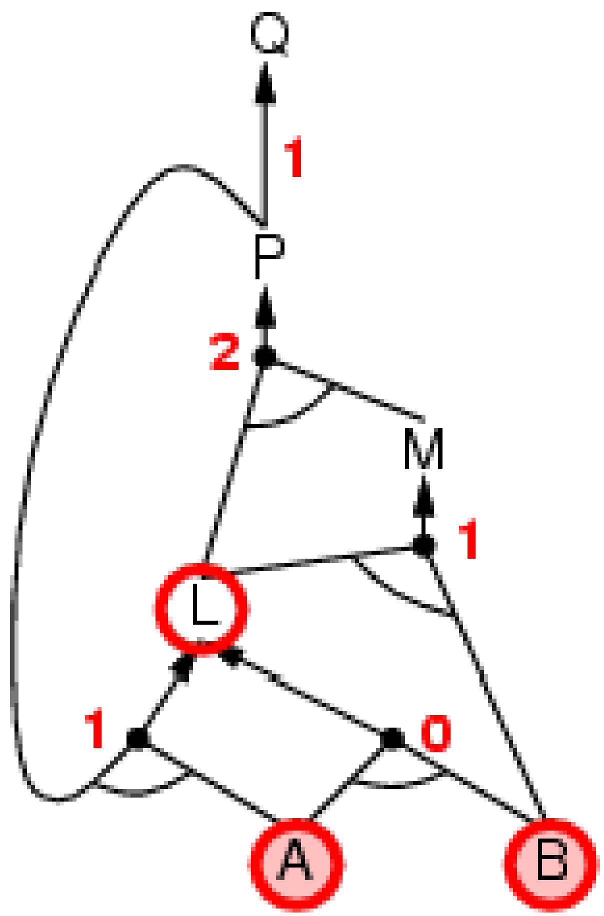
Forward chaining example



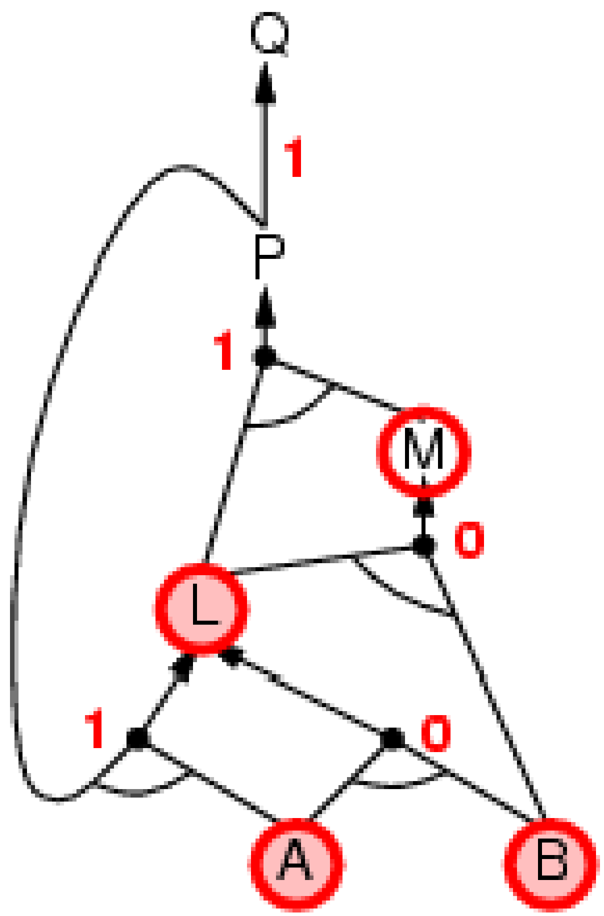
Forward chaining example



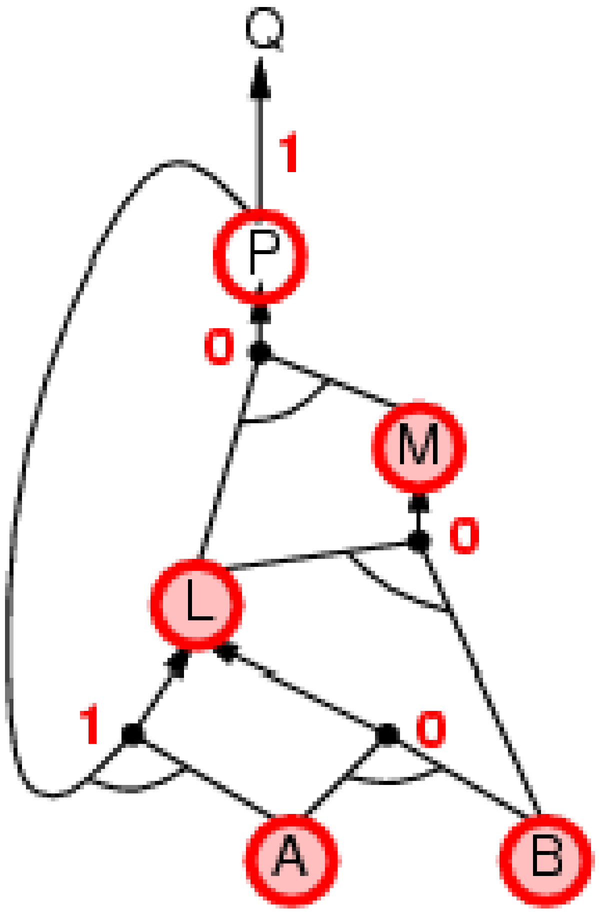
Forward chaining example



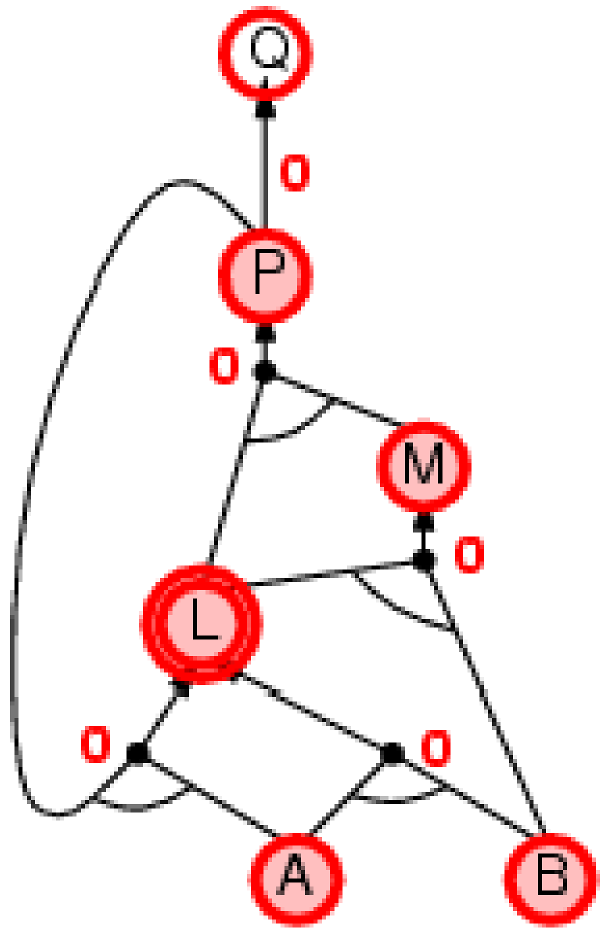
Forward chaining example



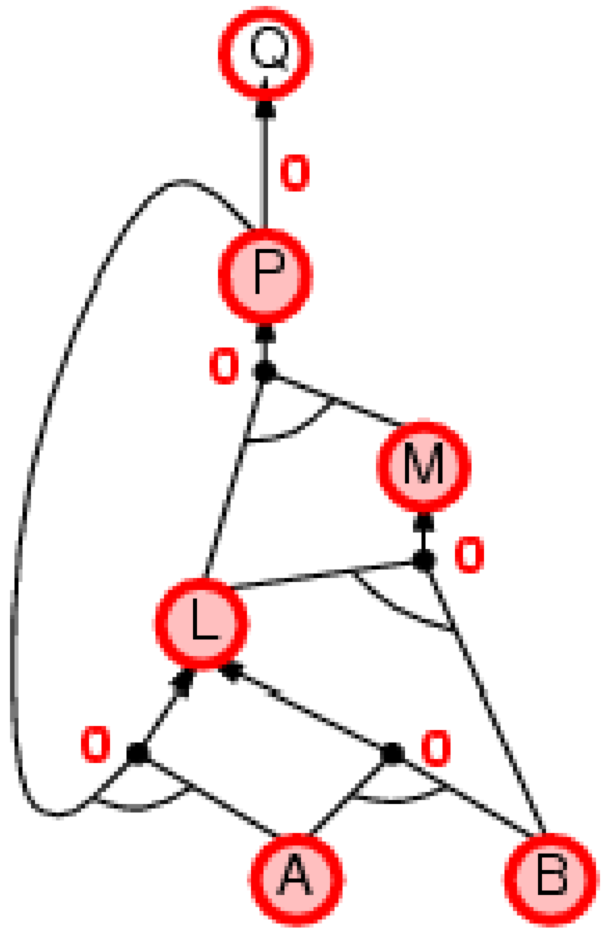
Forward chaining example



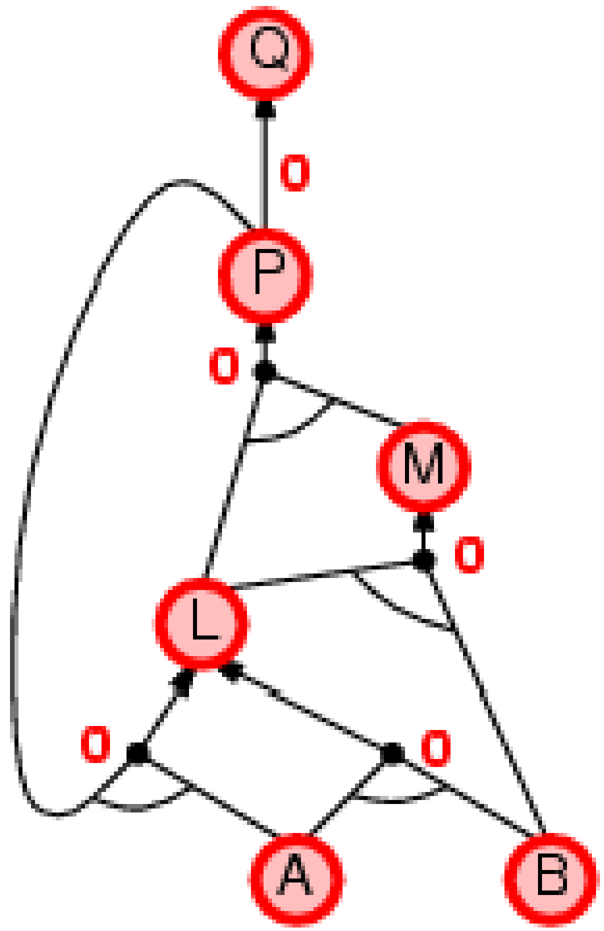
Forward chaining example



Forward chaining example



Forward chaining example



Backward chaining

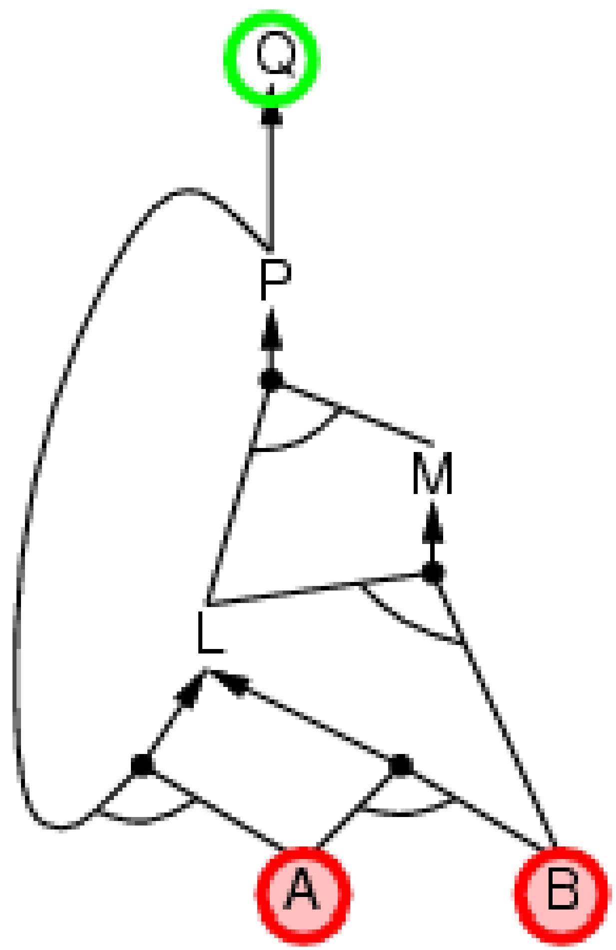
Idea: work backwards from the query q:

to prove q by BC,

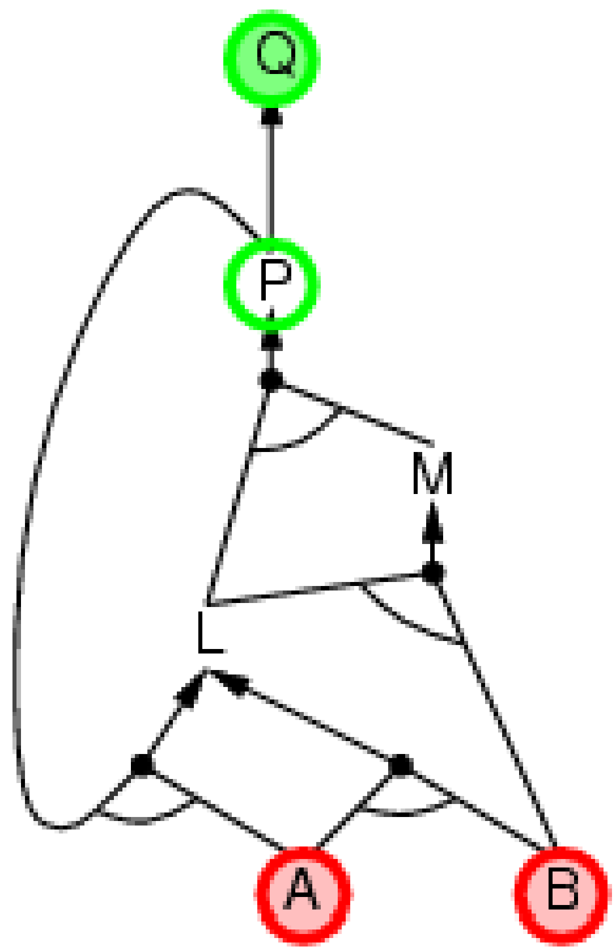
check if q is known already, or

prove by BC all premises of some rule concluding q

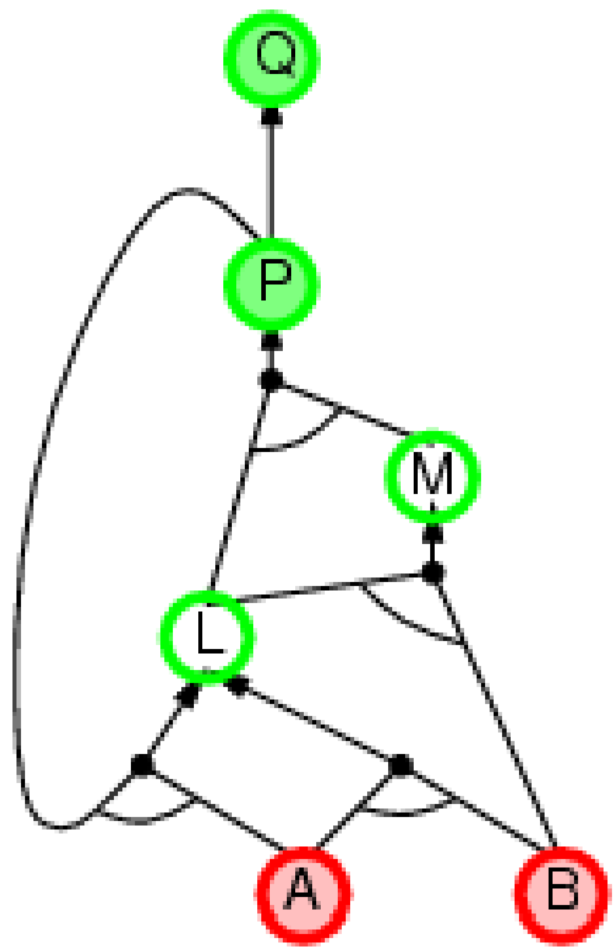
Backward chaining example



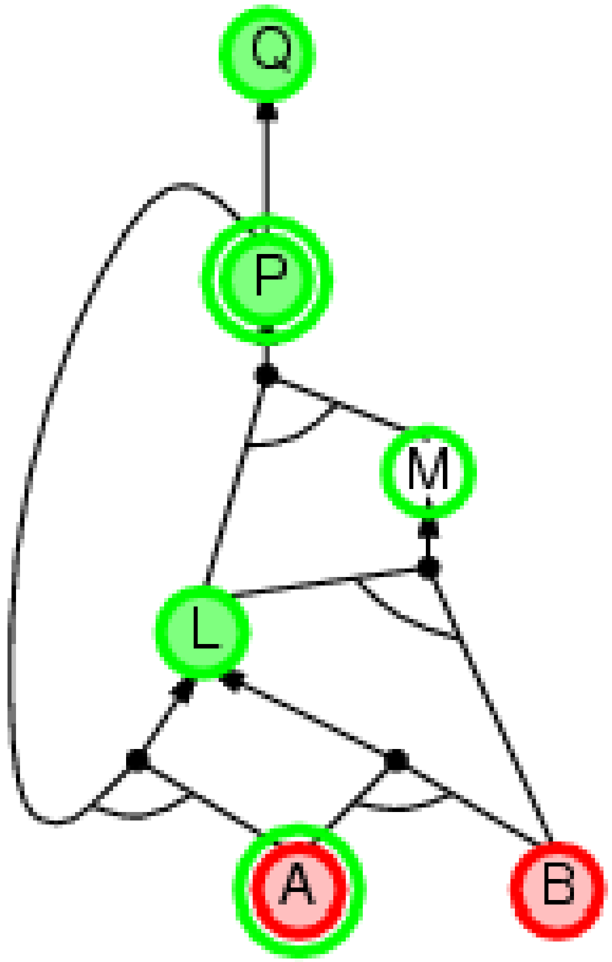
Backward chaining example



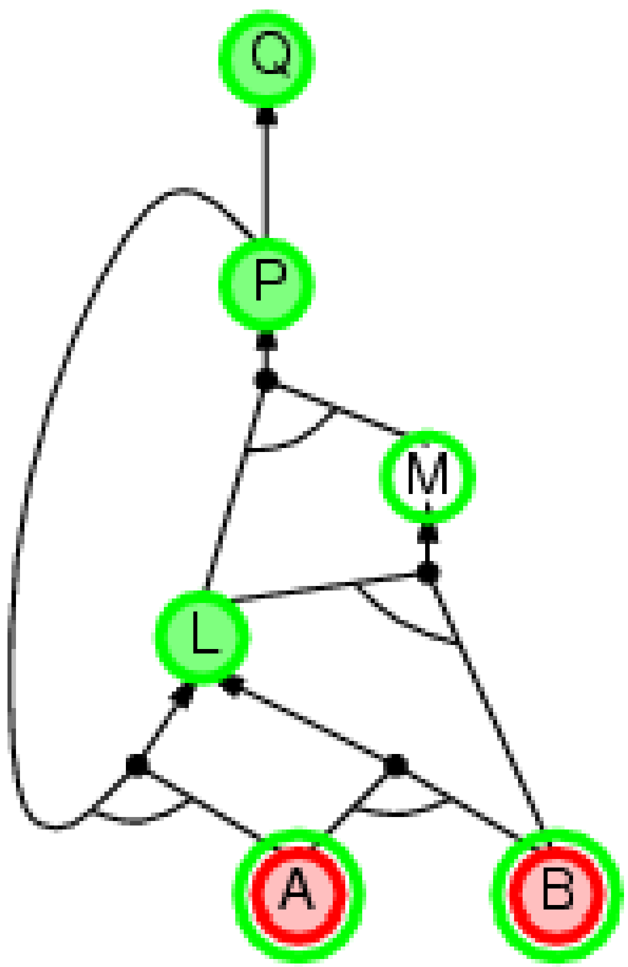
Backward chaining example



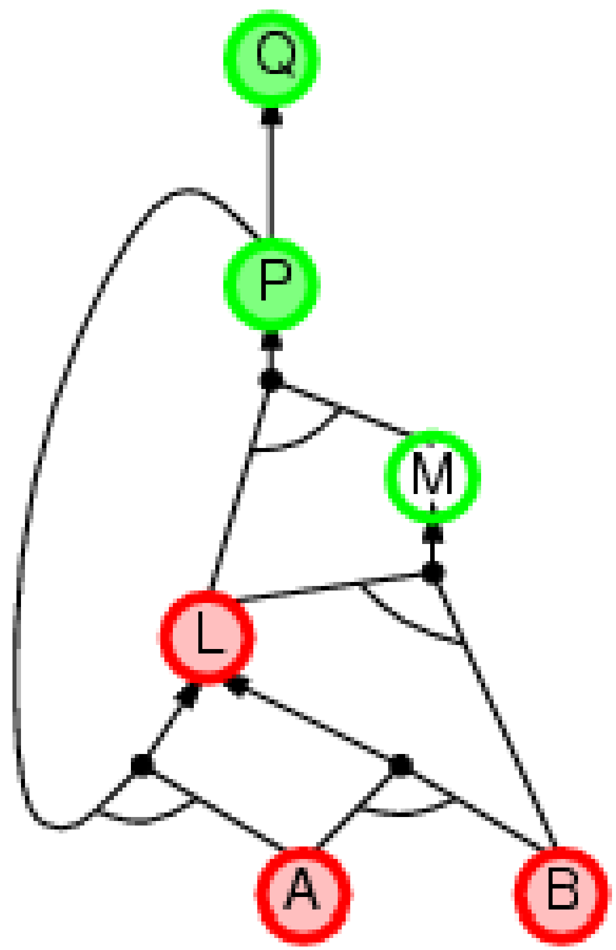
Backward chaining example



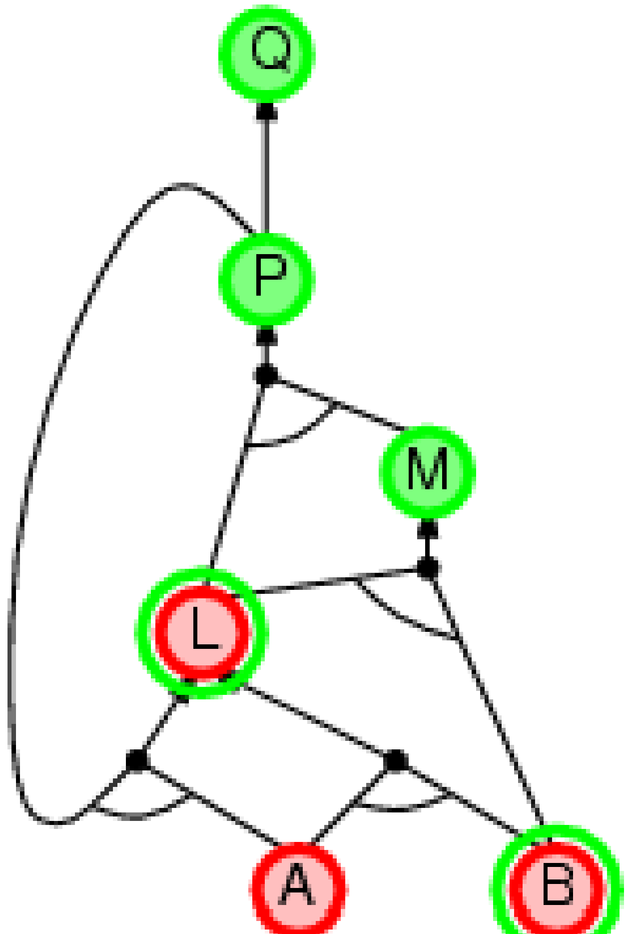
Backward chaining example



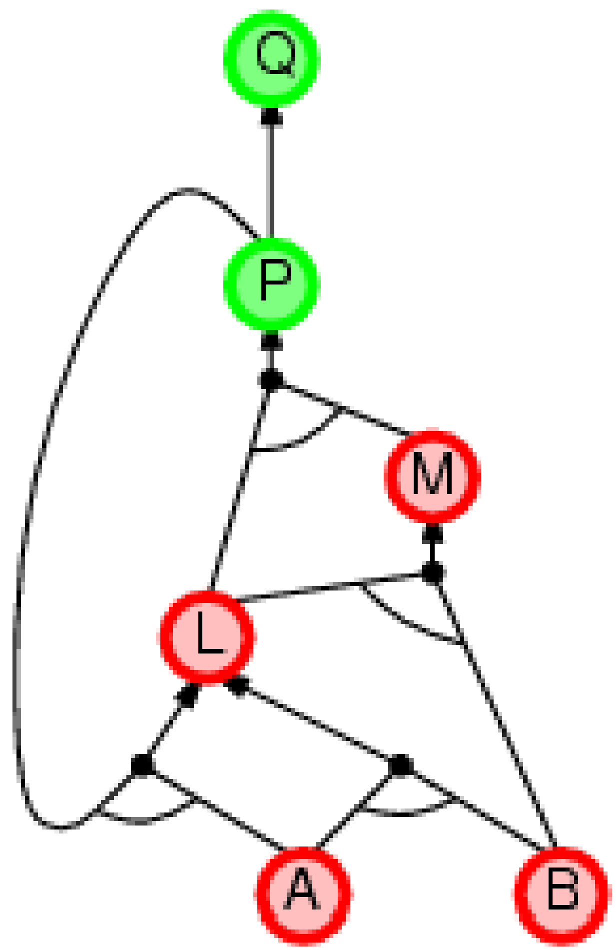
Backward chaining example



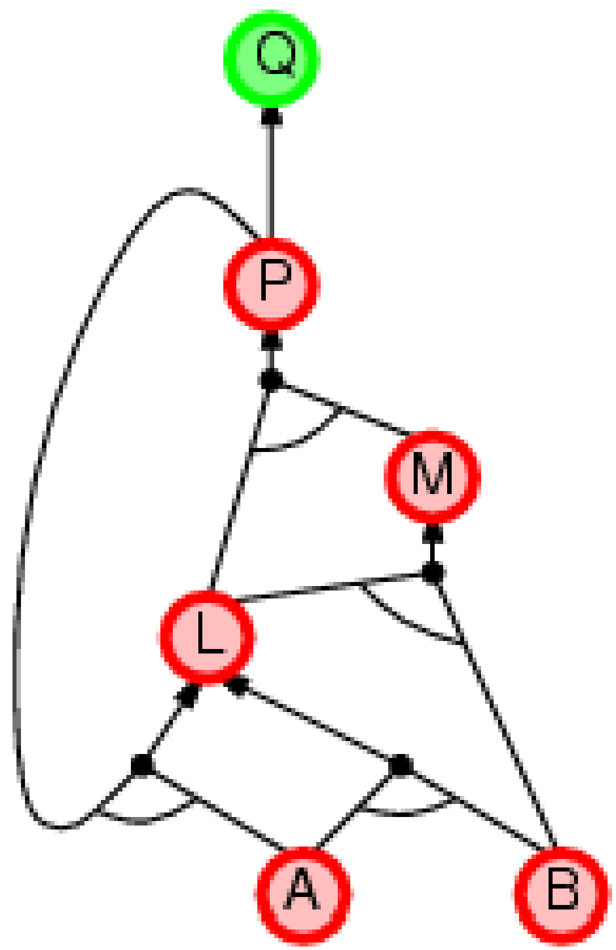
Backward chaining example



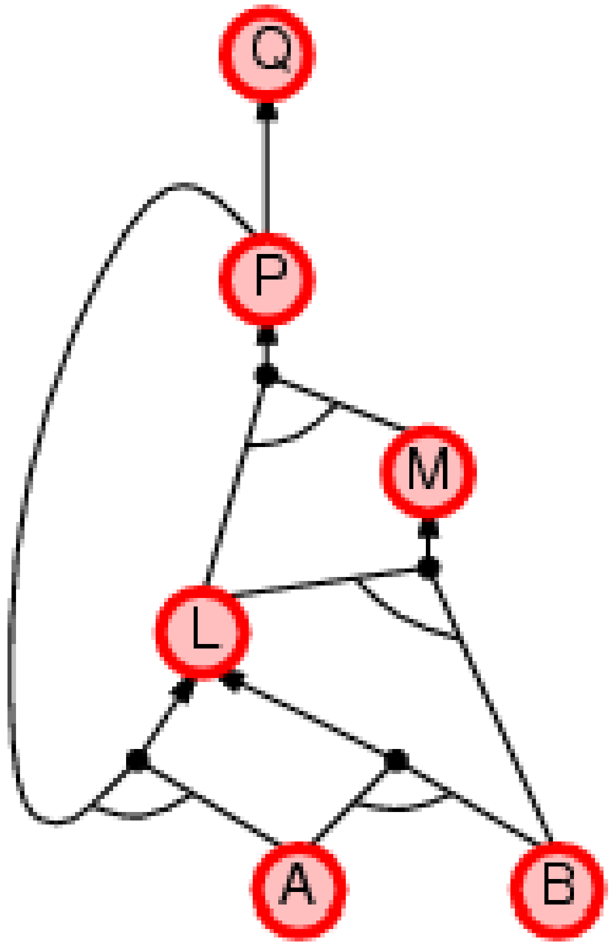
Backward chaining example



Backward chaining example



Backward chaining example



Forward vs. backward chaining

- Forward chaining is data-driven, automatic processing
  - May do lots of work that is irrelevant to the goal

- Backward chaining is goal-driven, appropriate for problem-solving
  - Complexity can be much less than linear in size of KB

Summary

- Logical agents apply inference to a knowledge base to derive new information and make decisions
- Basic concepts of logic:
  - syntax: formal structure of sentences
  - semantics: truth of sentences wrt models
  - entailment: necessary truth of one sentence given another
  - inference: deriving sentences from other sentences
  - soundness: derivations produce only entailed sentences
  - completeness: derivations can produce all entailed sentences
- Resolution is complete for propositional logic
- Forward, backward chaining are linear-time, complete for definite clauses