CSE310 HW01, Tuesday, 01/26/2021, Due: Tuesday, 02/02/2021

Please read the instructions carefully. **You have to use the companion answer sheet (which is a fillable PDF file) to type/select your answers to the questions described here**. Adobe Acrobat Reader can be found at `https://get.adobe.com/reader/`. Hand-written assignment (or photo of it) will not be graded. **Submit the filled PDF file of the answer sheet on Gradescope, following the link on Canvas**. You should name your file using the format **CSE310-HW01-LastName-FirstName.pdf**.

Q1 (12 points) In the following table, there are 12 entries in the form nij, where $i = 1, 2, 3$ and $j = 1, 2, 3, 4$. Each of these entries denotes the largest integer n such that $f(n)$ milliseconds does not exceed t, where $f(n)$ is the function corresponding to the row of the entry and t is the time corresponding to the column of the entry. For example, for entry $n23$, we have $f(n) = 3^n$ and $t = 1$ hour. Hence $n23$ should be the largest integer n such that 3^n milliseconds is no more than 1 hour.

On the answer sheet, enter the values for nij, $i = 1, 2, 3$, $j = 1, 2, 3, 4$.

	1 second	1 minute	1 hour	1 day
$10n^2 + 100$	$n11$	$n12$	$n13$	$n14$
3^n	$n21$	$n22$	$n23$	$n24$
$n!$	$n31$	$n32$	$n33$	$n34$

Q2 (6 points) For each of the following pairs of functions $f(n)$ and $g(n)$, decide whether we have $f(n) \in O(g(n))$, $f(n) \in \Omega(g(n))$, or $f(n) \in \Theta(g(n))$. On the answer sheet, check each of the boxes that is true.

(a) $f(n) = \sum_{i=1}^{n} i$, $g(n) = n \times (\log n)^3$.

(b) $f(n) = n^{1000}$, $g(n) = 1.00001^n$.

(c) $f(n) = n^3 + \sum_{i=1}^{n} i$, $g(n) = \sum_{i=1}^{n} (2i)^2$.

(d) $f(n) = (2 \times n)!$, $g(n) = 2 \times n!$.

Q3 (12 points) This question tests your understanding of the Insertion sort algorithm as stated in the textbook and the lecture slides. Assume that we use Insertion sort to sort the array A with 5 elements where the initial values of the array elements (from $A[1]$ to $A[5]$) are A :

13	14	11	15	12

. During the execution of the algorithm, we may have to write into one of the 5 memory locations of the array elements, i.e., write into $A[i]$ for some $i = 1, 2, 3, 4, 5$.

Every time we write into one of these locations, we say that A is overwritten. Check the corresponding box on the answer sheet to answer each of the following questions.

(a) What is the array content immediately after A is overwritten the 1st time?

(b) What is the array content immediately after A is overwritten the 3rd time?

(c) What is the array content immediately after A is overwritten the 5th time?

(d) What is the array content immediately after A is overwritten the 7th time?

Q4 (10 pts) This question tests your understanding of proofs for asymptotic notations.

(a) Let $f(n) = 5n^2 + 10000$. In order to prove that $f(n) \in O(n^2)$, we need to find a positive constant $c > 0$ and an integer $N \geq 1$ such that

$$f(n) \leq c \times n^2, \text{for every } n \geq N. \quad (1)$$

Answer the following questions on the answer sheet.

(a1) Will $c = 6, N = 100$ make the proof correct?

(a2) Will $c = 6, N = 200$ make the proof correct?

(a3) Will $c = 7, N = 100$ make the proof correct?

(a4) Will $c = 6, N = 50$ make the proof correct?

(a5) Will $c = 5, N = 100$ make the proof correct?

(b) Let $g(n) = 5n^2 - 10000$. In order to prove that $g(n) \in \Omega(n^2)$, we need to find a positive constant $c > 0$ and an integer $N \geq 1$ such that

$$g(n) \geq c \times n^2, \text{for every } n \geq N. \quad (2)$$

Answer the following questions on the answer sheet.

(b1) Will $c = 4, N = 100$ make the proof correct?

(b2) Will $c = 4, N = 200$ make the proof correct?

(b3) Will $c = 3, N = 100$ make the proof correct?

(b4) Will $c = 4, N = 50$ make the proof correct?

(b5) Will $c = 5, N = 100$ make the proof correct?

Q5 (10 pts) Select **True** or **False** to each of the following statements on the answer sheet.

(a) If $f(n) \in O(n)$ and $g(n) \in O(n)$, then $f(n) + g(n) \in O(n)$.

(b) If $f(n) \in O(n)$ and $g(n) \in O(n^2)$, then $f(n) + g(n) \in O(n)$.

(c) If $f(n) \in O(n)$, then $n^2 \times f(n) \in O(n^3)$.

(d) If $f(n) \in \Theta(n \log n)$ and $g(n) \in \Theta(n \log n)$, then $f(n) \in \Theta(g(n))$.

(e) If $f(n) \in O(n^2)$ and $g(n) \in O(n^2)$, then $f(n) \in O(g(n))$.