

# 1.

$\{10, 99, 23, 12, 0, 5, 9, 8\}$   
 $\{10, 99, 23, 12\} \quad \{0, 5, 9, 8\}$   
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 $\{10\} \quad \{99\} \quad \{23\} \quad \{12\} \quad \{0\} \quad \{5\} \quad \{9\} \quad \{8\}$   
 $\{10, 99\} \quad \{12, 23\} \quad \{0, 5\} \quad \{8, 9\}$   
 $\{10, 12, 23, 99\} \quad \{0, 5, 8, 9\}$   
 $\{0, 5, 8, 9, 10, 12, 23, 99\}$

2. The running time is  $O(N)$ . Because if we have a loop to insert same elements, the loop will terminates right at first try . So the running time should be  $O(N)$  which depends on how many elements are there to sort.

## 3. 1)

$$\begin{aligned}
 F(n) &= 2F(n/2) + n \\
 F(n) &= 2(2F(n/4) + n/2) + n = 4F(n/4) + 2n \\
 F(n) &= 2^k F(n/2^k) + kn \\
 \text{when } n/2^k &= 1 \quad k = \log_2 n \\
 \text{so } F(n) &= 2^{\log_2 n} F(n/2^{\log_2 n}) + n \log_2 n \\
 &= nF(1) + n \log_2 n \\
 &= n + n \log_2 n
 \end{aligned}$$

## 2)

$$\begin{aligned}
 T(n) &= n \log_2 n + 2n - 1 \quad F(n) = n + n \log_2 n \\
 \text{proof: need to show: } &n + n \log_2 n = O(n \log_2 n + 2n - 1) \\
 &n + n \log_2 n = \Omega(n \log_2 n + 2n - 1)
 \end{aligned}$$

Let  $C_1 = 2$  when  $n \geq 1$   $C_1(n \log_2 n + 2n - 1) = n \log_2 n + n \log_2 n + 4n - 2 > n \log_2 n + n$   
 Thus, choose  $n_0 = 1$  we have  $n \log_2 n + 2n - 1 \geq n + n \log_2 n$  when  $n_0 \geq n$   
 Let  $C_1 = 0.5$  when  $n \geq 1$   $C_1(n \log_2 n + 2n - 1) = 0.5n \log_2 n + 4n - 2 < n \log_2 n + n$   
 Thus, choose  $n_0 = 1$  we have  $n \log_2 n + 2n - 1 \leq n + n \log_2 n$  when  $n_0 \leq n$   
 Therefore  $F(n) = \theta(T(n))$

## 4. Report

	InsertionSort	QuickSort
10	9.53674e-07	9.53674e-07
20	2.86102e-06	5.00679e-06
100	1.88351e-05	1.19209e-05
200	5.19753e-05	1.78814e-05
400	2.44856e-04	4.57764e-05
800	2.29692e-03	1.60933e-04
1600	3.26395e-03	1.78099e-04
3200	0.0304101	0.000496864
6400	0.0869331	0.00106311