Name:

Self-Test for Graduate-Level Machine Learning Course

Q1. The weatherman has predicted rain tomorrow. In recent years, it has rained only 73 days each year. When it actually rains, the weatherman correctly forecasts rain 70% of the time. When it does not rain, he incorrectly forecasts rain 30% of the time. What is the probability that it will rain tomorrow?

Hint: Bayes Rule formula is $P(X|E) = \frac{P(E|X)P(X)}{P(E)}$

Q2. We are given that the probability density function (pdf) of a continuous random variable X is

$$p(x) = \begin{cases} 4x & 0 \leq x \leq \frac{1}{2} \\ -4x + 4 & \frac{1}{2} \leq x \leq 1 \end{cases}$$

What is the equation for the corresponding cumulative density function (cdf) P(*x*)?

Q3. Calculate the expected value of *X*, *E*[*X*], where *X* is a random variable representing the outcome of a roll of a trick die. Use the sample space $x \in \{1, 2, 3, 4, 5, 6\}$ and let

$$p(X = x) = \begin{cases} \frac{1}{2} & x = 1 \\ \frac{1}{10} & x \neq 1 \end{cases}$$

Q4. Use the properties of expectation to show that the variance of a random variable *X* (defined: $Var[X] = E[(X - \mu)^2]$) can be rewritten as $Var[X] = E[X^2] - (E[X])^2$.

Q5. Consider the following system of equations:

$$\begin{gathered} 2x_1 + x_2 + x_3 = 3 \\ 4x_1 + 2x_3 = 10 \\ 2x_1 + 2x_2 = -2 \end{gathered}$$

(a) Write the system as a matrix equation of the form *Ax*=*b*.

(b) Write the solution of the system as a column *S* and verify by matrix multiplication that *S* satisfies the equation *Ax*=*b*.

(c) Write *b* as a linear combination of the columns in *A*.

Q6. Consider the following matrix:

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$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

(a) Is the matrix invertible? (Hint: Think about singularity and determinants.)

(b) What's the rank of the matrix?

Q7. The eigenvalues of the matrix $A = \begin{bmatrix} 3 & 6 \\ 1 & 4 \end{bmatrix}$ are λ=6 and λ=1. Which of the following is an eigenvector for λ=1?

a. $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

b. $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

c. $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

d. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Q8. Find the 0, 1, 2, and ∞ norms of $x = \begin{pmatrix} 2 \\ 1 \\ -4 \\ -2 \end{pmatrix}$

The zero norm: $\|x\|_0 = \lim_{p\to 0}\|x\|_p^p$ = the number of non-zero elements

The gird norm: $\|x\|_1 = \sum_{i=1}^{n}|x|$

The Euclidean norm: $\|x\|_2 = (\sum_{i=1}^{n}|x_i|^2)^{\frac{1}{2}}$

The max norm: $\|x\|_\infty = \lim_{p\to\infty}\|x\|_p = \lim_{p\to\infty}(\sum_{i=1}^{n}|x_i|^p)^{\frac{1}{p}} = \max_i|x_i|$