CPMA 573 — Homework #4

Exercise 1: Recognizing burn-in and autocorrelation. Let X and Y have jointly normal density $f(x, y)$ given by:

$$f(x, y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \cdot \exp\left\{-(x^2 + y^2 - 2\rho xy)/(2(1-\rho^2))\right\} .$$

Write a program that uses Gibbs sampling to produce realizations from this joint density, where $\rho = 0.75$. Your program should be flexible enough to accommodate a choice of

- Initial values (x_0, y_0)
- Number of desired (x_i, y_i) realizations. N
- Number of iterations to skip (lag) in order to avoid serial correlation.

Use this program to complete the following:

a. Saving every iteration, simulate 500 realizations from starting values $x_0 = 80$ and $y_0 = 80$. Plot the first 50 realizations in **R**, and connect-the-points in the order you generated them using the "arrows" command:

```
plot(x[1:50],y[1:50])
arrows(x[1:49],y[1:49],x[2:50],y[2:50])
```

b. Construct the same plot as in **a**, only this time, use all 500 realizations. This type of plot gives an idea of the necessary "burn-in" iterations, for given starting values.

c. When using $x_0 = 80$ and $y_0 = 80$ as initial values, how many iterations (roughly) does it take for the Gibbs sampler to converge to its stationary distribution? Reference the two trace plots (Use `ts.plot` once for the x realizations, and once for the y. Try using the argument `xlab=''Iterations''` to change the x-axis label to something more appropriate.)

d. Now create two autocorrelation plots (`acf`): One for the 500 x-values, and one for the 500 y-values. Based on these plots, what lag should be used to eliminate autocorrelation?

e. Using the lag from your answer to **d**, use your Gibbs sampling program to draw 10000 independent realizations from the marginal density of X. Confirm, using the mean, variance, and quartiles, that these realizations are indeed from a standard normal density.

Exercise 2: The influence of ρ. Use your Gibbs sampling program from Exercise 1 to generate 15000 independent (x, y) realizations. Then, use these realizations to find the following:

a. $P(X > 0 \text{ and } Y > 0)$, for $\rho = \{-0.9, -0.8, \ldots, 0.8, 0.9\}$. Then plot these probabilities as a function of ρ. Set the vertical axis limits to $(0, 1)$.

b. $P(X < 0 \text{ and } Y > 0)$, for $\rho = \{-0.9, -0.8, \ldots, 0.8, 0.9\}$. Then plot these probabilities as a function of ρ. Set the vertical axis limits to $(0, 1)$.

c. $P(X > 0)$, for $\rho = \{-0.9, -0.8, \ldots, 0.8, 0.9\}$. Then plot these probabilities as a function of ρ. Set the vertical axis limits to $(0, 1)$.

↳ should be 1/2