

# Chem 30324, Spring 2020, Homework 9

**Due April 10, 2020**

## The two-state system.

Consider a closed system containing  $N$  objects, each of which can be in one of two energy states, of energy either 0 or  $\epsilon$ . The total internal energy  $U$  of the box is the sum of the energies of the individual objects.



1. Write down all the possible microstates for a box in which  $N = 4$  and the internal energy  $U = 2\epsilon$ .
2. What does the postulate of *equal a priori probabilities* say about the relative likelihood of occurrence of any one of these microstates?
3. What is the entropy of the box? (Thank you, Ludwig Boltzmann.)
4. Suppose two identical such boxes are brought into thermal contact and allowed to come to equilibrium. Calculate the change in internal energy  $\Delta U$  and in entropy  $\Delta S$  associated with this process.

## The canonical ensemble.

The energy spectrum of some molecule is described by the diagram below.



5. Write the partition function  $q$  for the molecular at thermal equilibrium at a temperature  $\beta = 1/k_B T$ .
6. Plot the probability for the molecule to be in each of the three energy states vs. temperature. Be sure to indicate the probabilities in the limits of  $T \rightarrow 0$  and  $T \rightarrow \infty$ .
7. Derive an expression for the energy  $U$  per molecule by summing over the possible microstates weighted by their probabilities. Plot the average energy vs. temperature.
8. Derive an expression for the energy  $U$  per molecule by taking the appropriate derivative of the partition function from problem 5 (*Hint: it is easier to work with the expressions in term of  $\beta$  than in  $T$ .*) Does your result agree with that from problem 7?
9. Derive an expression for the Helmholtz energy  $A$  per molecule from the partition function. Plot  $A$  vs. temperature, assuming  $\epsilon/k_B = 300$  K.
10. Derive an expression for the entropy  $S$  per molecules and plot vs. temperature, again assuming  $\epsilon/k_B = 300$  K.
11. In class we took the First Law as a postulate and demonstrated the Second Law. Look at your results for Problems 6 and 10. Can you use them to rationalize the Third Law? Explain your answer.