Assignment 4

Computer Vision
Model Fitting

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2 Plane Fitting

Exercise 1.

In RANSAC, the size of the sample set depends on the degrees of freedom of the model. The model have 3 degrees of freedom to fit a 3D plane to a set of points.
Number of sample sets $N = \frac{log(1-p)}{log(1-(1-\epsilon)^k)} = \frac{log(1-0.99)}{log(1-0.8^3)} \approx 7$

Exercise 2. (OPTIONAL.)

min $\sum_{i=1}^{m}(ax_i + by_i + cz_i + d)^2$
such that $a^2 + b^2 + c^2 = 1$
Taking the derivative with respect to d, we get
$\sum_{i=1}^{m} 2(ax_i + by_i + cz_i + d) = 0$
$\Rightarrow 2m(a\bar{x} + b\bar{y} + c\bar{z} + d) = 0$
$\Rightarrow d = -(a\bar{x} + b\bar{y} + c\bar{z})$ where $(\bar{x}, \bar{y}, \bar{z}) = \frac{1}{m}\sum_{i=1}^{m}(x_i, y_i, z_i)$
Now, min $\sum_{i=1}^{m}(ax_i + by_i + cz_i + d)^2$
$= \min \sum_{i=1}^{m}(ax_i + by_i + cz_i - a\bar{x} - b\bar{y} - c\bar{z})^2$
$= \min \sum_{i=1}^{m}(a(x_i - \bar{x}) + b(y_i - \bar{y}) + c(z_i - \bar{z}))^2$
$= \min \sum_{i=1}^{m}(a\tilde{x}_i + b\tilde{y}_i + c\tilde{z}_i)^2$
such that $1 - (a^2 + b^2 + c^2) = 0$
$M = \sum_{i=1}^{m} \begin{bmatrix} \tilde{x_i}^2 & \tilde{x}_i\tilde{y}_i & \tilde{x}_i\tilde{z}_i \\ \tilde{y}_i\tilde{x}_i & \tilde{y_i}^2 & \tilde{y}_i\tilde{z}_i \\ \tilde{z}_i\tilde{x}_i & \tilde{z}_i\tilde{y}_i & \tilde{z_i}^2 \end{bmatrix}$
Thus, min $f(t) = t^T M t$
such that $g(t) = 1 - t^T t = 0$ where $t = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$
$\Delta f(t) + \lambda \Delta g(t) = 0 \Rightarrow 2Mt + \lambda(-2t) = 0 \Rightarrow Mt = \lambda t$

Computer Exercise 1.

See the m file- comp1.m.
The RMS distance between the 3D-points and the plane is 0.51677.
No. of inliers is 736. The RMS distance between the plane obtained with RANSAC and the distance to the 3D points is 0.5680. There is no improvement.
The RMS distance between the 3D-points and the new plane is 0.5611.
They are located in the wall.
The ones on the wall seems to be correct.

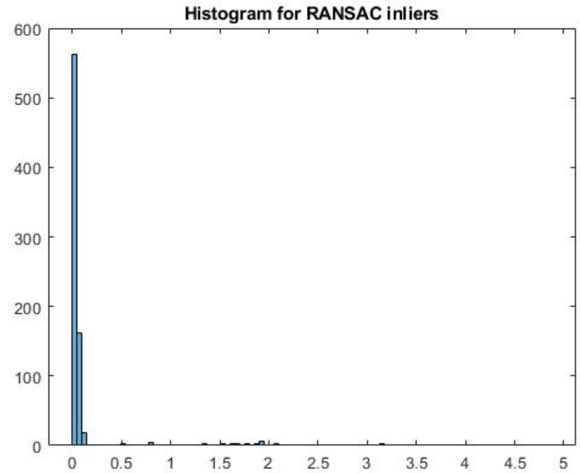

Figure. 1

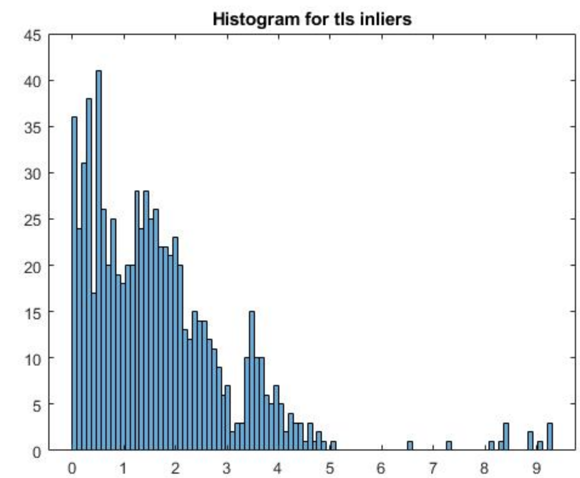

Figure. 2

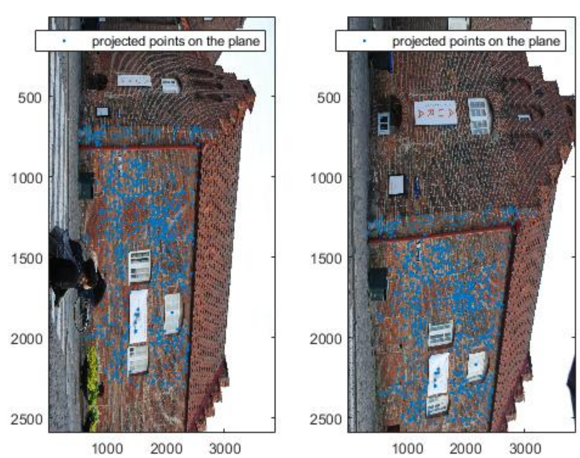

Figure. 3

3 Robust Homography Estimation and Stitching

Exercise 3.

Since $P_1 = [A_1|t_1]$ and $P_2 = [A_2|t_2]$ have same camera centers, $C_1 = C_2 \Rightarrow -A_1^{-1}t_1 = -A_2^{-1}t_2 \Rightarrow t_2 = A_2A_1^{-1}t_1$.
Now, $x_1 = A_1X + t_1$ & $x_2 = A_2X + t_2$
Then, $X = A_1^{-1}(x_1 - t_1)$.
$x_2 = A_2A_1^{-1}(x_1 - t_1) + t_2 = A_2A_1^{-1}x_1 . \therefore H = A_2A_1^{-1}$. Hence the proof.

Exercise 4.

A homography has 8 degrees of freedom.
The minimal number of point correspondences to determine the homography is 4 since each point has two equations.
Number of iterations $N = \frac{log(1-p)}{log(1-(1-\epsilon)^k)} = \frac{log(1-0.98)}{log(1-(0.9)^4)} \approx 4$

Computer Exercise 2.

See the m file- comp2.m.
There are 900 SIFT features in each image. 204 matches.
No. of inliers is 152.

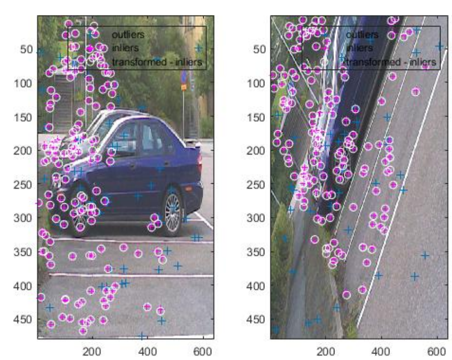

Figure. 4

Exercise 5. (OPTIONAL.)

$x^2 + 2y^2 - 6 = 0$
$xy - 2 = 0 \Rightarrow x = \frac{2}{y}$
Roots are (-2, -1), (2, 1), ($\sqrt{2}$, $\sqrt{2}$) & (-$\sqrt{2}$, -$\sqrt{2}$).
$1 \longrightarrow x$
$x \longrightarrow x^2$
$y \longrightarrow xy$
$y^2 \longrightarrow xy^2$
$1 \longrightarrow x_0$
$x_0 \longrightarrow x_0^2$
$y_0 \longrightarrow x_0y_0$

Figure. 5

$y_0^2 \longrightarrow x_0 y_0^2$

$q(x_0, y_0) = c_p^1 x_0 + c_p^2(6 - 2y_0^2) + c_p^3 2 + c_p^4 2y_0 = (6c_p^2 + 2c_p^3) + c_p^1 x_0 + 2c_p^4 y_0 - 2c_p^2 y_0^2$

$$\begin{bmatrix} c_q^1 \\ c_q^2 \\ c_q^3 \\ c_q^4 \end{bmatrix} = \begin{bmatrix} 6c_p^2 + 2c_p^3 \\ c_p^1 \\ 2c_p^4 \\ -2c_p^2 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_p^1 \\ c_p^2 \\ c_p^3 \\ c_p^4 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 6 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & -2 & 0 & 0 \end{bmatrix}$$

$$m(x_0, y_0) = \begin{bmatrix} 1 \\ x_0 \\ y_0 \\ y_0^2 \end{bmatrix}$$

$$m(-2, -1) = \begin{bmatrix} 1 \\ -2 \\ -1 \\ 1 \end{bmatrix}$$

$$m(2, 1) = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}$$

$$m(-\sqrt{2}, -\sqrt{2}) = \begin{bmatrix} 1 \\ -\sqrt{2} \\ -\sqrt{2} \\ 2 \end{bmatrix}$$

$$m(\sqrt{2}, \sqrt{2}) = \begin{bmatrix} 1 \\ \sqrt{2} \\ \sqrt{2} \\ 2 \end{bmatrix}$$