Assignment 1

Computer Vision
Elements of Projective Geometry

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2 Points in Homogeneous Coordinates

Exercise 1.

The 2D Cartesian coordinates of the point with homogeneous coordinates

$$x_1 = \begin{bmatrix} 6 \\ -2 \\ 2 \end{bmatrix}$$

is

$$\begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

The 2D Cartesian coordinates of the point with homogeneous coordinates

$$x_2 = \begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$$

is

$$\begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

The 2D Cartesian coordinates of the point with homogeneous coordinates

$$x_3 = \begin{bmatrix} 4\lambda \\ -2\lambda \\ 2\lambda \end{bmatrix}$$

where $\lambda \neq 0$ is

$$\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

The interpretation of the point with homogeneous coordinates

$$x_4 = \begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix}$$

is that the point x_4 is infinitely far away in the $[4, -2]^T$ direction. This is called vanishing point. Every vanishing point fulfills $z = 0$. The line $z = 0$ is called the vanishing line or line of infinity.

Computer Exercise 1.

See the m-files- compEx1.m & pflat.m.

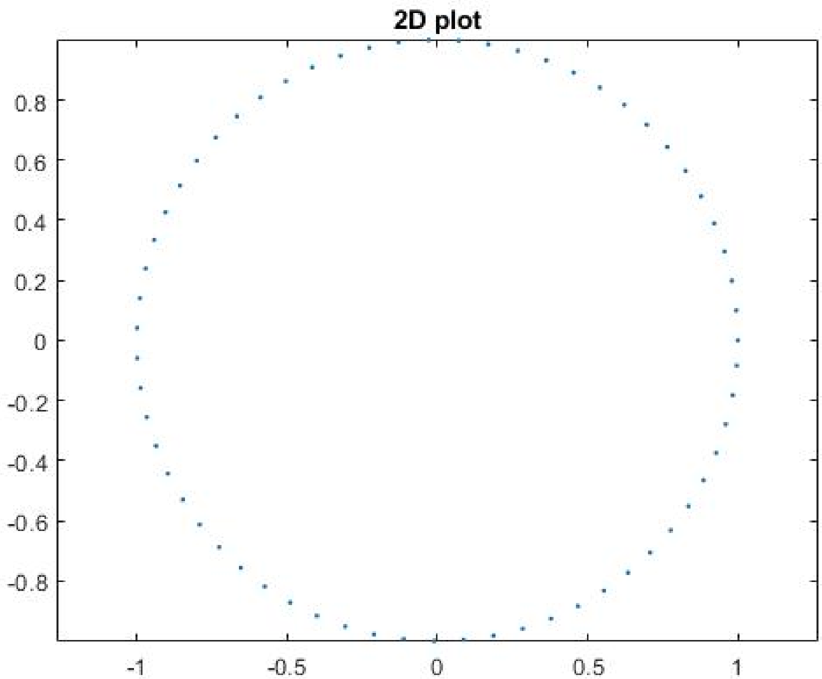

Figure. 1

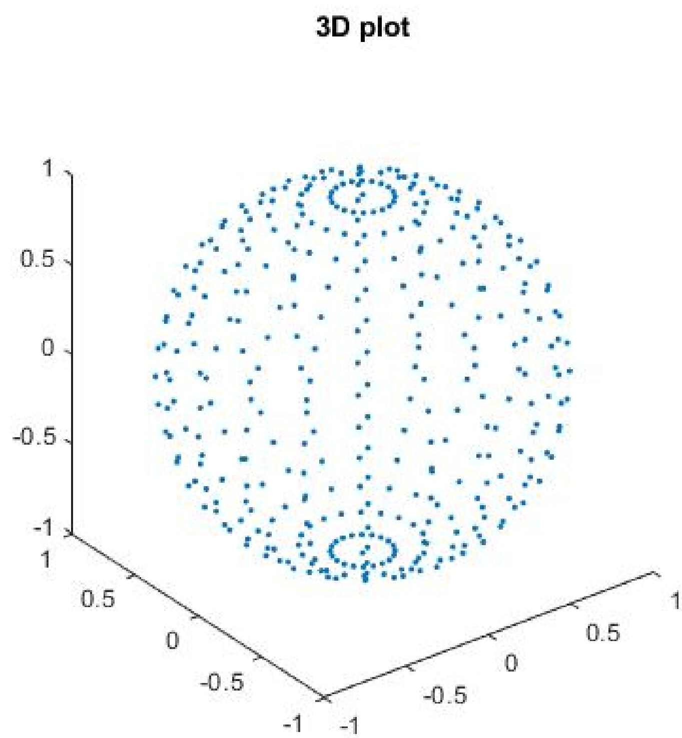

Figure. 2

3 Lines

Exercise 2.

The homogeneous coordinates of the intersection (in P^2) of the lines

$$l_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \& l_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

is

$$\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \lambda$$

where $\lambda \neq 0$.
The corresponding point in R^2 is

$$\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

The intersection (in P^2) of the lines

$$l_3 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \& l_4 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

is

$$\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \lambda$$

where $\lambda \neq 0$.
The geometric interpretation in R^2 is that the point is infinitely far away in the $[-2, 1]^T$ direction.
The line that goes through the points with Cartesian coordinates

$$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \& x_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

is

$$\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \lambda$$

where $\lambda \neq 0$. Here we reuse the calculations from the line intersection of l_1 & l_2.

Exercise 3.

To prove: The intersection point (in homogeneous coordinates) of l_1 and l_2 (from Exercise 2) is in the null space of the matrix

$$M = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Now,

$$Mx = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \lambda = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

where $\lambda \neq 0$. Therefore, the intersection point of l_1 and l_2 ($\in R^3$) is in the null space of the 2×3 matrix M. Hence the proof.
There is no other point in the null space besides the intersection point.

Computer Exercise 2.

See the m-file- compEx2.m.
The computed distance between the first line and the intersection point is 8.27.

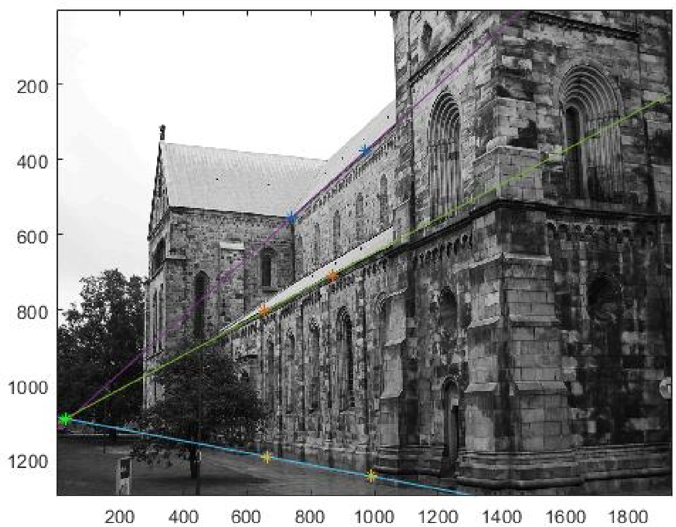

Figure. 3

4 Projective Transformations

Exercise 4.

The transformation

$$y_1 \sim Hx_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \& y_2 \sim Hx_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$Hx_1 = [1, 0, 0]^T$ is a point which is infinitely far away in the $[1, 0]^T$ direction. This is a vanishing point.
The line l_1 containing x_1 and x_2 is

$$l_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

The line l_2 containing y_1 and y_2 is

$$l_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

Now,

$$(H^{-1})^T l_1 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

Therefore, $(H^{-1})^T l_1 \sim l_2$.
Since the projection x belongs to l_1,

$$l_1^T x = 0 \Rightarrow l_1^T H^{-1} H x = 0$$

Now,

$$l_2 \sim (H^{-1})^T l_1$$

Then, the projection y belongs to l_2 since

$$l_2^T y \sim ((H^{-1})^T l_1)^T H x = l_1^T H^{-1} H x = 0$$

Since H is invertible, l_2 is well defined. Therefore, projective mappings preserve lines. Hence the proof.

Computer Exercise 3.

See the m-file- compEx3.m.

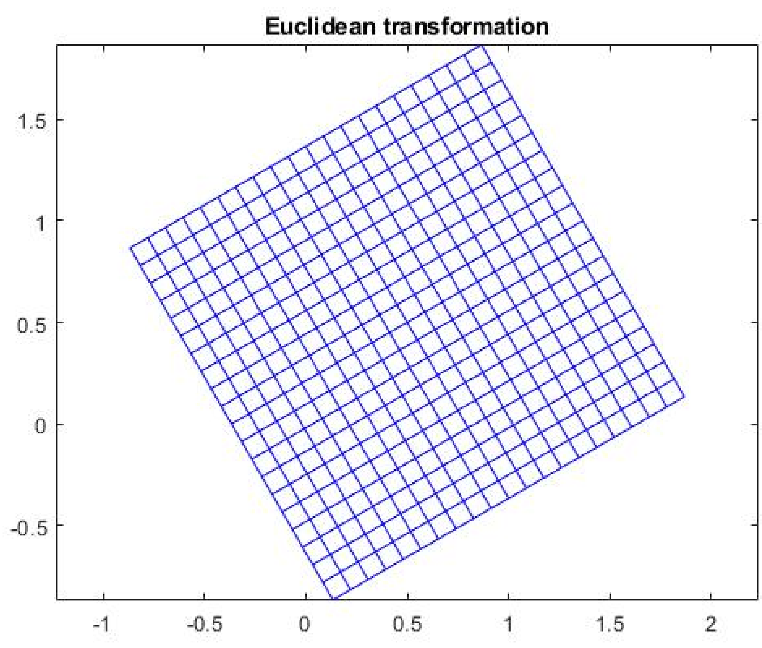

Figure. 4

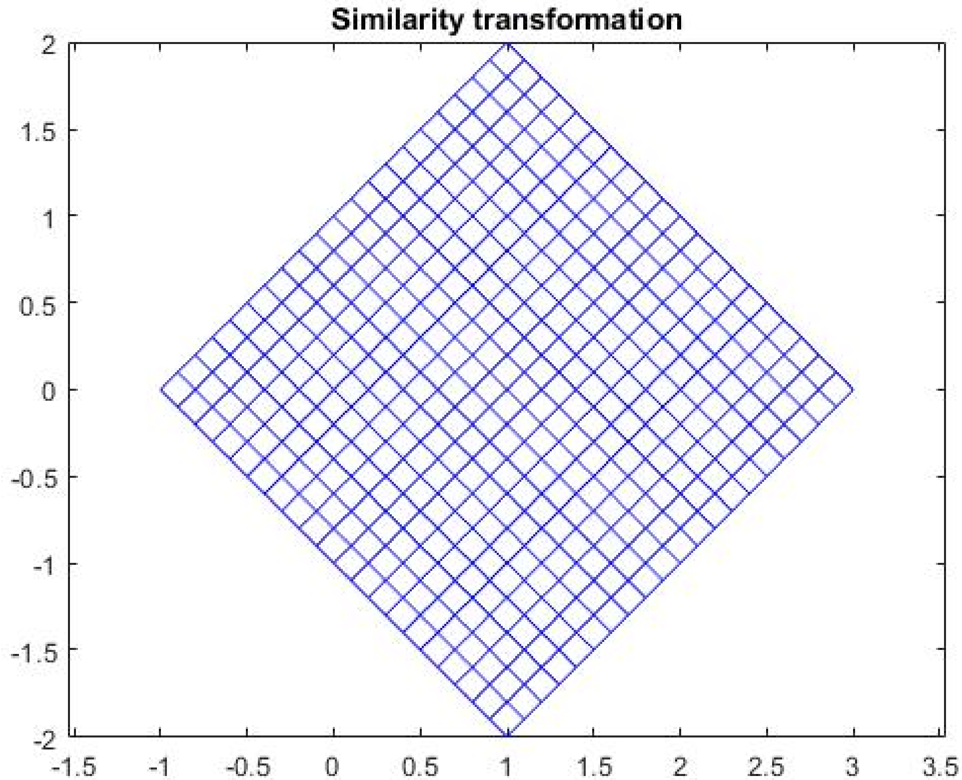

Figure. 5

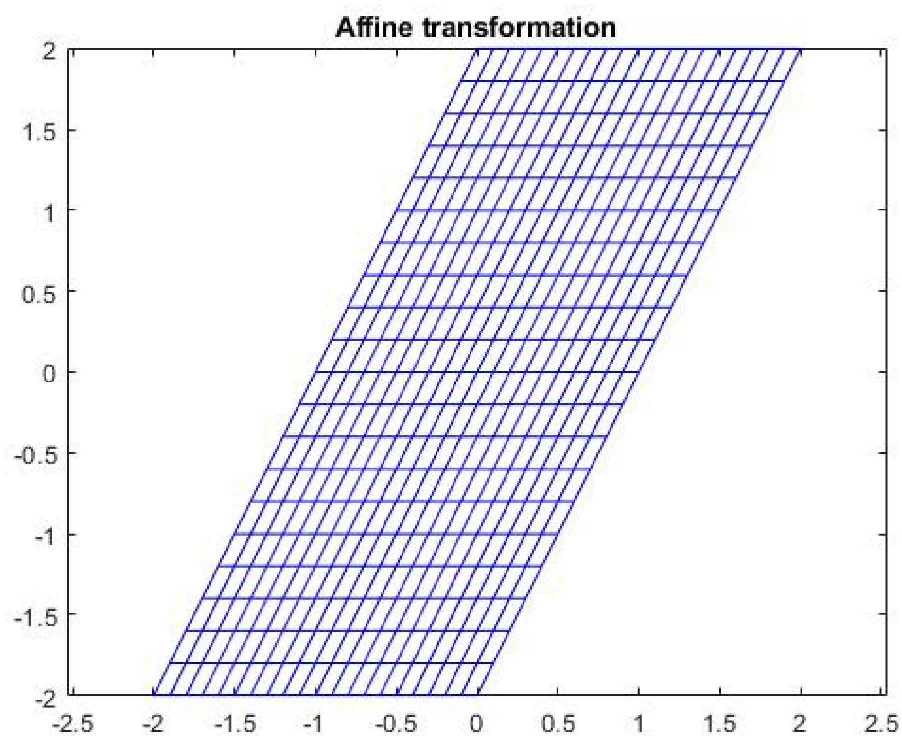

Figure. 6

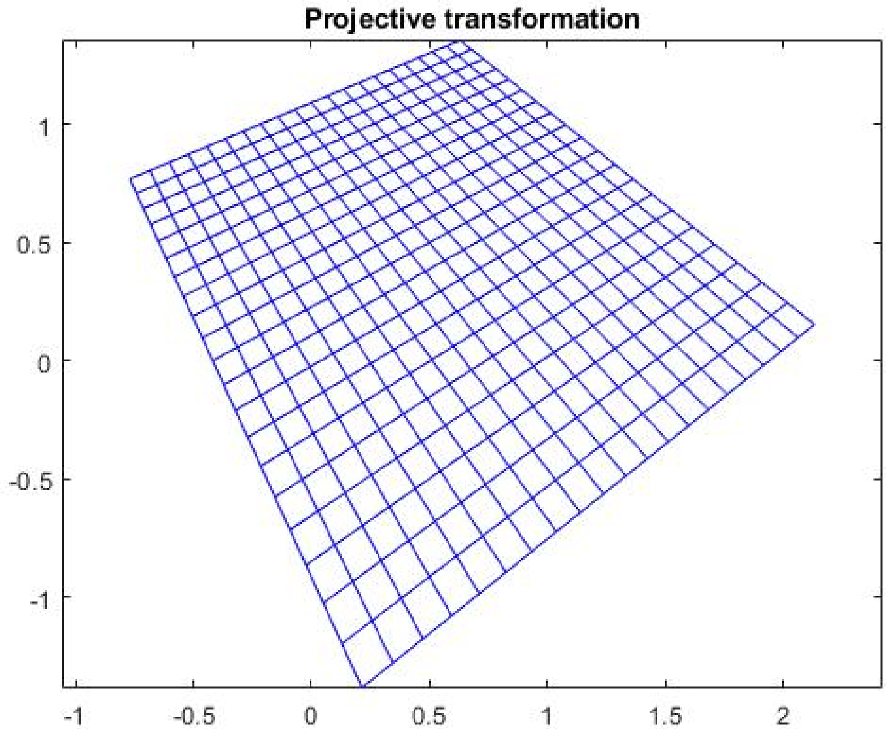

Figure. 7

First transformation preserves length between points. It is a euclidean transformation.
Second transformation preserves angle between lines. It is a similarity transformation.
Third transformation maps parallel lines to parallel lines. It is a affine transformation.
Fourth transformation is a projective transformation.

5 The Pinhole Camera

Exercise 5.

The projection of the 3D points with homogeneous coordinates

$$X_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

in the camera with camera matrix P is

$$\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

The projection of the 3D points with homogeneous coordinates

$$X_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

in the camera with camera matrix P is

$$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

The projection of the 3D points with homogeneous coordinates

$$X_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

in the camera with camera matrix P is

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

The camera center (position) of the camera is given by

$$C = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

The principal axis (viewing direction) is given by

$$V = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Computer Exercise 4.

See the m-file- compEx4.m.
The camera centers in Cartesian coordinates are $C1 = [0, 0, 0]^T$ & $C2 = [6.6352, 14.8460, -15.0691]^T$ and the principal axes are $V1 = [0.3129, 0.9461, 0.0837]^T$ & $V2 = [0.0319, 0.3402, 0.9398]^T$.

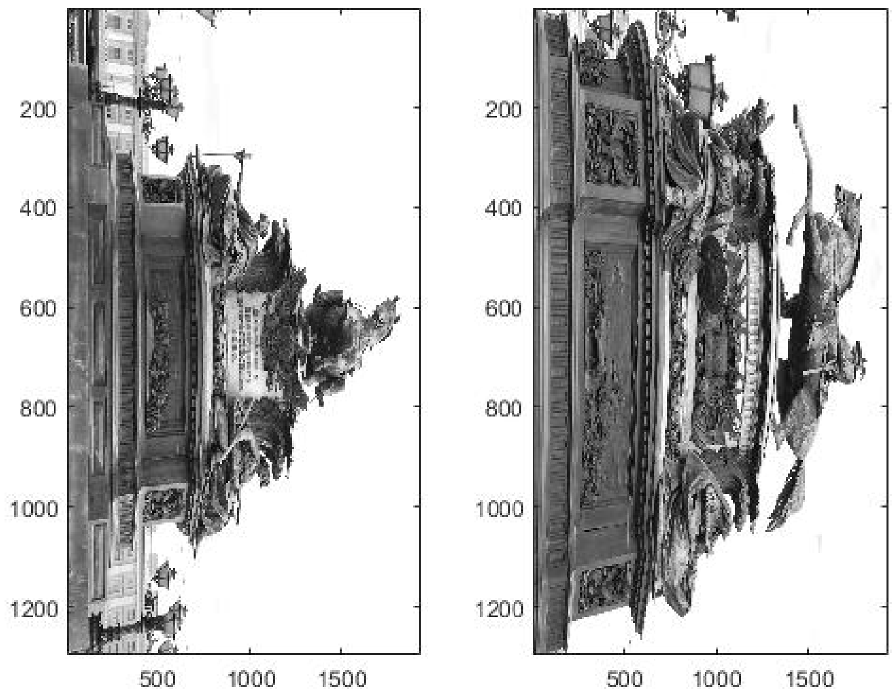

Figure. 8

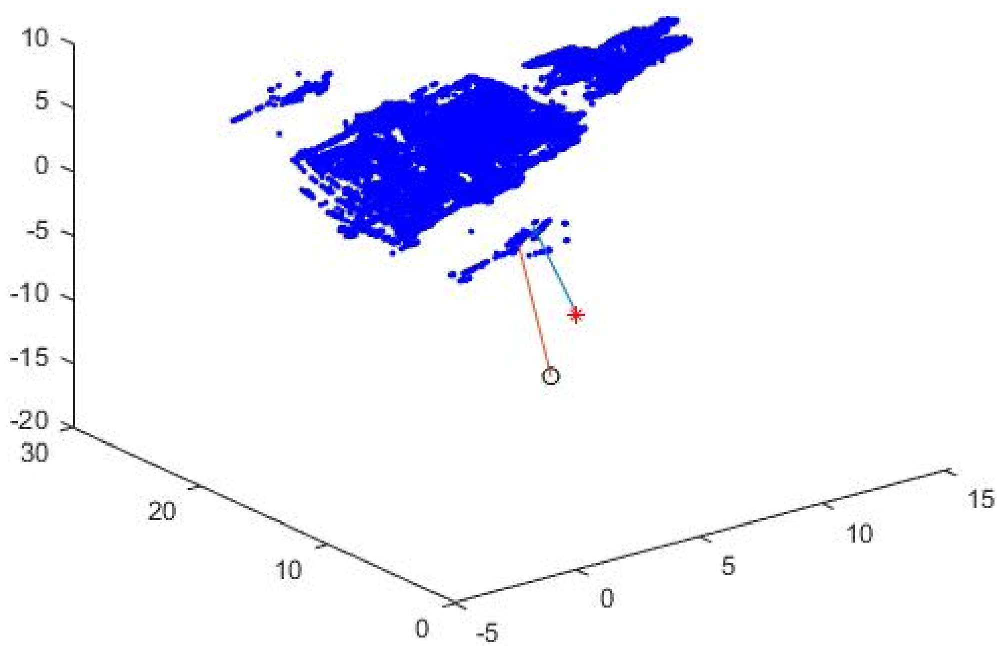

Figure. 9

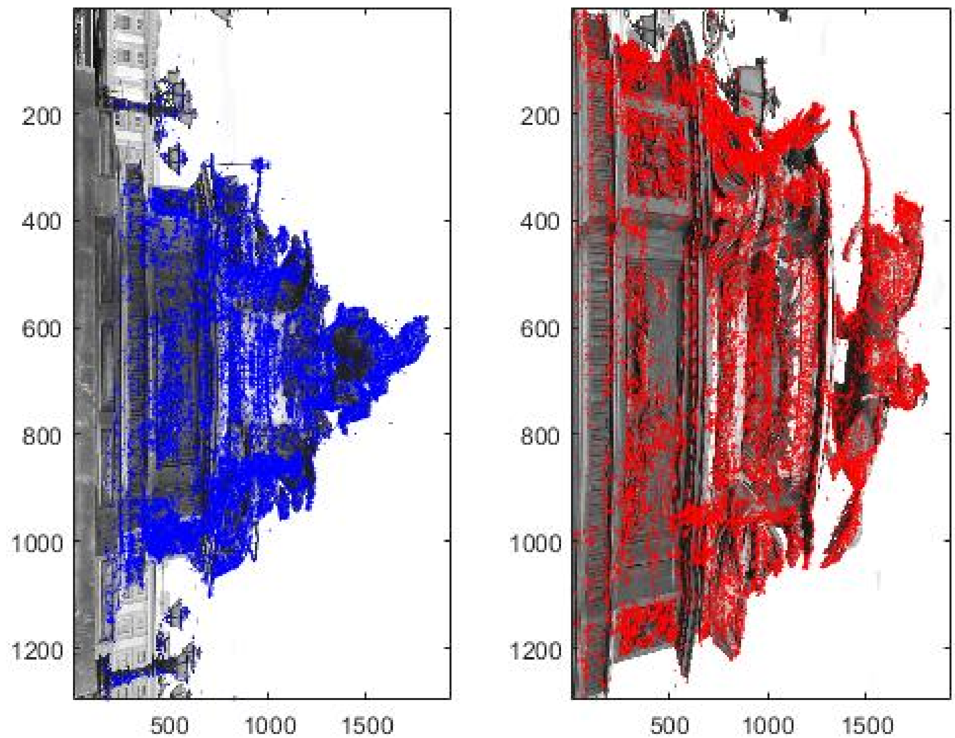

Figure. 10

Exercise 6. (OPTIONAL.)

The projection is

$$P_1U = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} x \\ s \end{bmatrix} = x$$

The collection of points is a multipoint object. No, it is not possible to determine s using only information from P_1.
Since U(s) belongs to the plane Π and

$$\Pi = \begin{bmatrix} \pi \\ 1 \end{bmatrix} \& U \sim \begin{bmatrix} x \\ s \end{bmatrix}$$

$$\Pi^T U(s) = 0$$

$$\Rightarrow \begin{bmatrix} \pi^T 1 \end{bmatrix} \begin{bmatrix} x \\ s \end{bmatrix} = 0 \Rightarrow \pi^T x + s = 0 \Rightarrow s = -\pi^T x$$

Using

$$P_2U(s) = \begin{bmatrix} R & t \end{bmatrix} \begin{bmatrix} x \\ s \end{bmatrix} = Rx + ts$$

Now,

$$y \sim P_2U(s) = Rx + t(-\pi^T x) = (R - t\pi^T)x \equiv Hx$$

Therefore, $H \equiv R - t\pi^T$ maps x to y.

Computer Exercise 5. (OPTIONAL.)

See the m-file- compEx5.m.
The origin of the image coordinate system in figure. 11 is located at (0,0).
The origin of the image coordinate system in figure. 12 is located at (-0.2132,-0.3944).
Yes, figure. 13 looks reasonable.
The result shown in figure. 14 looks as expected when moving the camera like this.
Projecting the 3D points into the same image using the camera matrix gives the same result which is shown in figure. 15.
The transformed image is shown in figure. 16.

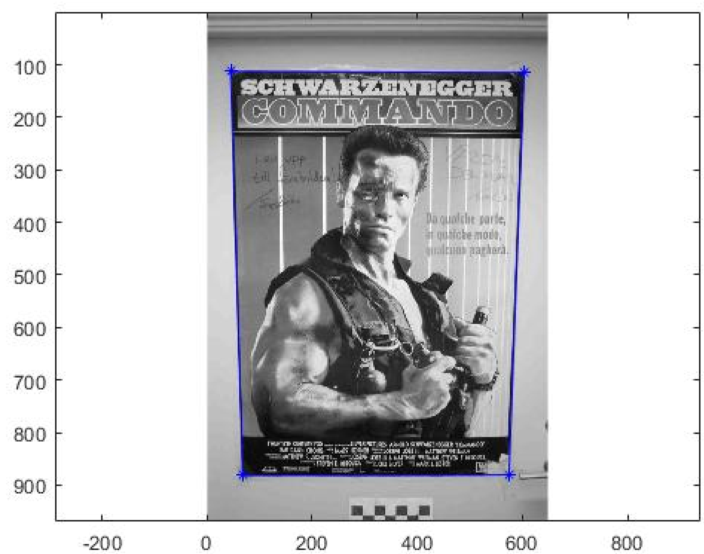

Figure. 11

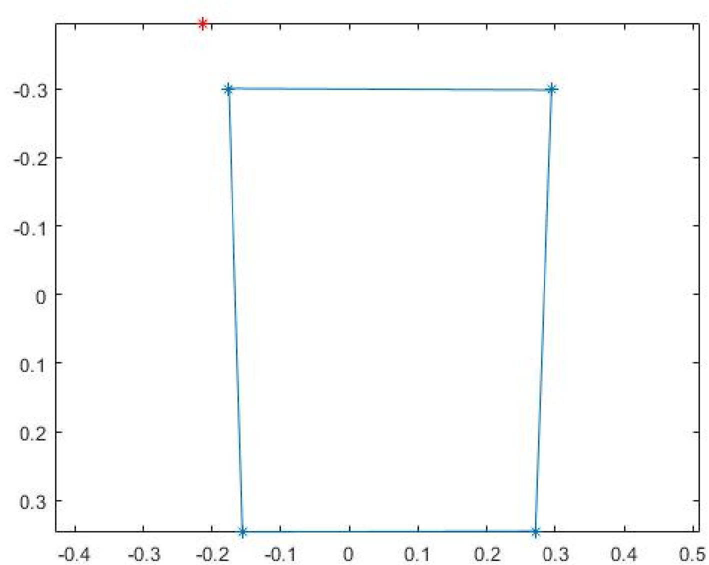

Figure. 12

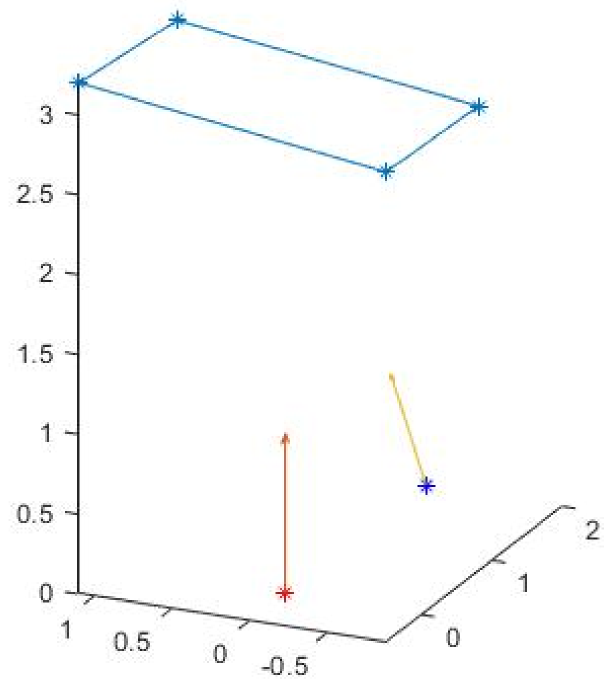

Figure. 13

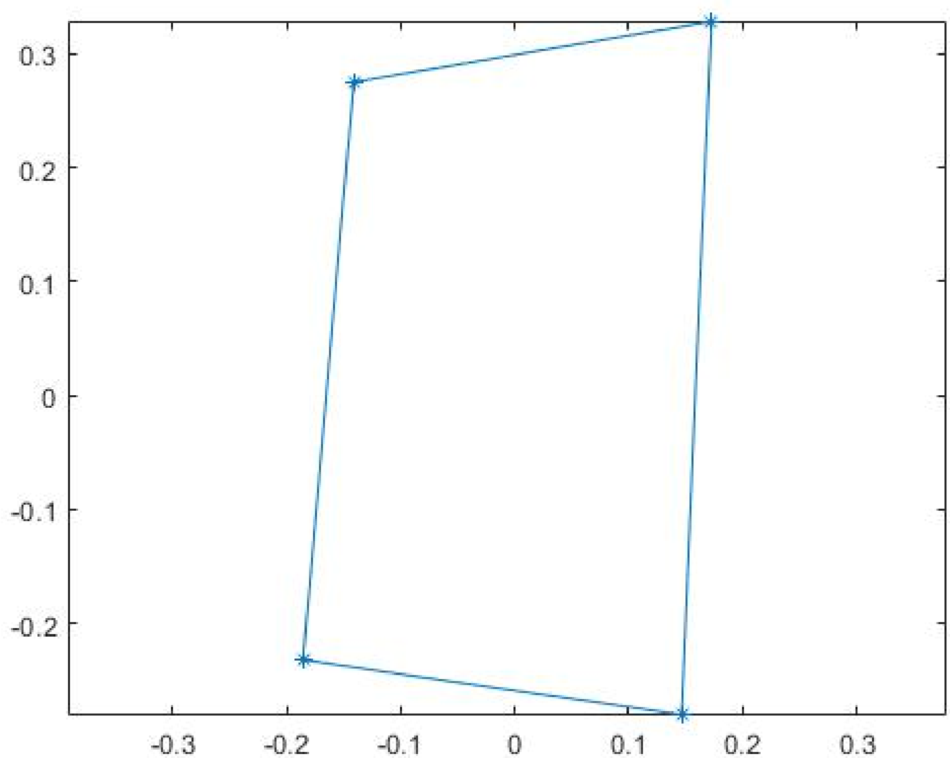

Figure. 14

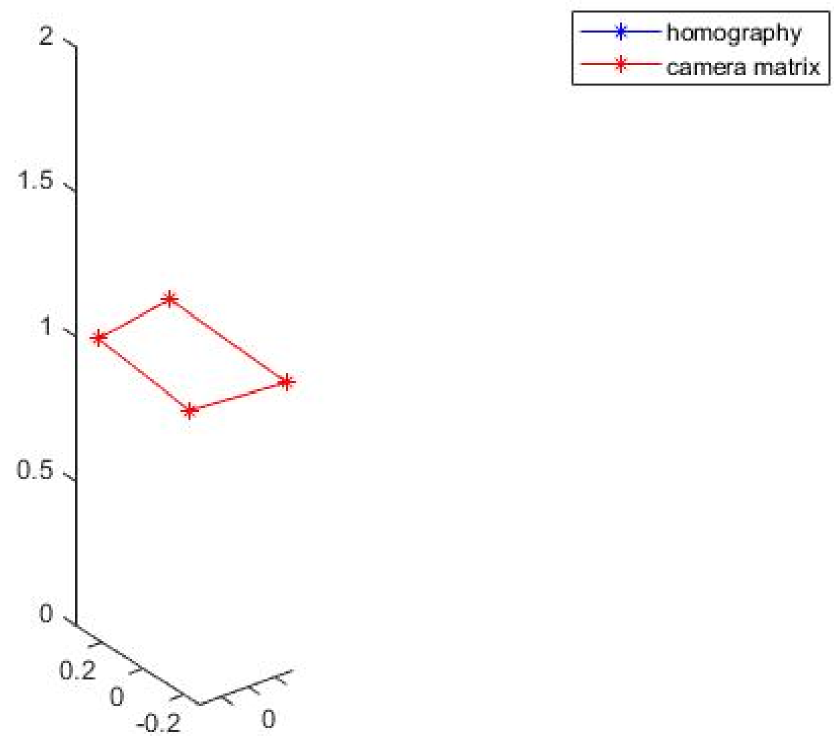

Figure. 15

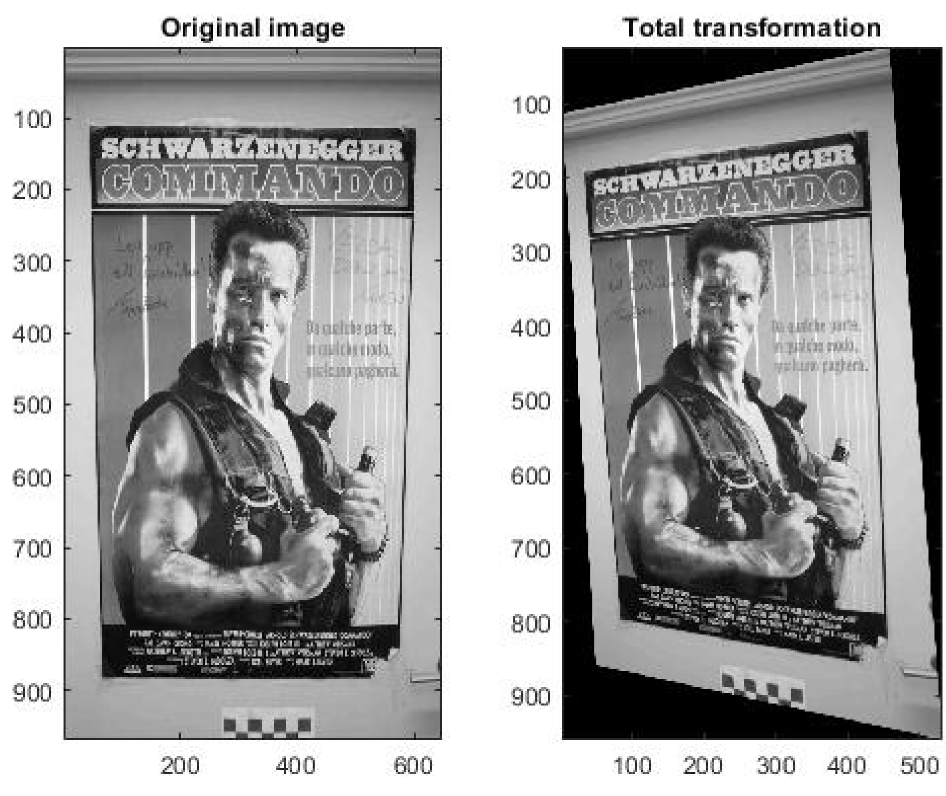

Figure. 16