# Assignment 3

**Computer Vision**
**Epipolar Geometry**

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# 2 The Fundamental Matrix

## Exercise 1.

If $P_1 = [I, 0]$ and $P_2 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, then the fundamental matrix is

$$F = [e_2]_\times A = \begin{bmatrix} 0 & -e_2(3) & e_2(2) \\ e_2(3) & 0 & -e_2(1) \\ -e_2(2) & e_2(1) & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -2e_2(3) & e_2(2) \\ e_2(3) & e_2(3) & -e_2(1) \\ -e_2(2) & -e_2(2) + 2e_2(1) & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & -2 & 0 \end{bmatrix}.$$

The epipolar line in the second image generated from $x$ is

$$l = Fx = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}.$$

The epipolar constraint is written as $\bar{x}^T l = 0$.

$$\begin{bmatrix} 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix} = 0$$

$$\begin{bmatrix} 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix} = 4$$

Therefore, the points (2, 0) and (2, 1) are projections of the same point X into $P_2$.

## Exercise 2.

If $P_1 = [I, 0]$ and $P_2 = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix} = [A, t]$, the camera centers are

$$C_1 = \begin{bmatrix} -I^{-1}O \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} -A^{-1}t \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix}.$$

The epipoles, by projecting the camera centers are computed as

$$e_1 \sim P_1 C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix},$$

$$e_2 \sim P_2C_1 = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}.$$

The fundamental matrix is

$$F = [e_2]_\times A = \begin{bmatrix} 0 & -e_2(3) & e_2(2) \\ e_2(3) & 0 & -e_2(1) \\ -e_2(2) & e_2(1) & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{bmatrix}.$$

$|F| = 0 + 0 + 2(0 - 0) = 0.$

$$e_2^T F = \begin{bmatrix} 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$Fe_1 = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

## OPTIONAL

For a general camera pair $P_1 = \begin{bmatrix} I & O \end{bmatrix}$ and $P_2 = \begin{bmatrix} A & t \end{bmatrix}$, the camera centers are $C_1 = \begin{bmatrix} -I^{-1}O \\ 1 \end{bmatrix}$ and $C_2 = \begin{bmatrix} -A^{-1}t \\ 1 \end{bmatrix}$.

The epipoles are computed, by projecting the camera centers

$$e_1 = P_1C_2 = P_1 \begin{bmatrix} -A^{-1}t \\ 1 \end{bmatrix} = \begin{bmatrix} I & O \end{bmatrix} \begin{bmatrix} -A^{-1}t \\ 1 \end{bmatrix} = -A^{-1}t$$

$$e_2 = P_2C_1 = P_2 \begin{bmatrix} -I^{-1}O \\ 1 \end{bmatrix} = P_2 \begin{bmatrix} O \\ 1 \end{bmatrix} = \begin{bmatrix} A & t \end{bmatrix} \begin{bmatrix} O \\ 1 \end{bmatrix} = t$$

For the fundamental matrix $F = [t]_\times A$,

$e_2^T F = t^T [t]_\times A = 0,$

$Fe_1 = [t]_\times A(-A^{-1}t) = -[t]_\times (AA^{-1})t = -[t]_\times It = -[t]_\times t = 0$

By rank nullity theorem, rank(F) +nullity(F) = n = no_of_columns(F). As $Fe_1 = 0$ (or $e_2^T F = 0$), the nullity of F is one and the number of columns of F is 3. Therefore, the rank of F is 2 which means that F is rank deficient and hence the fundamental matrix F has to have determinant 0.

## Exercise 3.

$\tilde{x_1} \sim N_1 x_1$ and $\tilde{x_2} \sim N_2 x_2$

$0 = \tilde{x_2}^T \tilde{F} \tilde{x_1} \sim (N_2x_2)^T \tilde{F}(N_1x_1) = x_2^T(N_2^T \tilde{F} N_1)x_1$

$\therefore F = N_2^T \tilde{F} N_1$

## Computer Exercise 1.

See the m-files - compEx1.m.

The fundamental matrix for the original (un-normalized) points is

$$F = \begin{bmatrix} -3.39011033711507e-08 & -3.72005592040353e-06 & 0.00577231569015152 \\ 4.66737185593653e-06 & 2.89360844681799e-07 & -0.0266821124271029 \\ -0.00719360661620428 & 0.0262957198475886 & 1 \end{bmatrix}$$

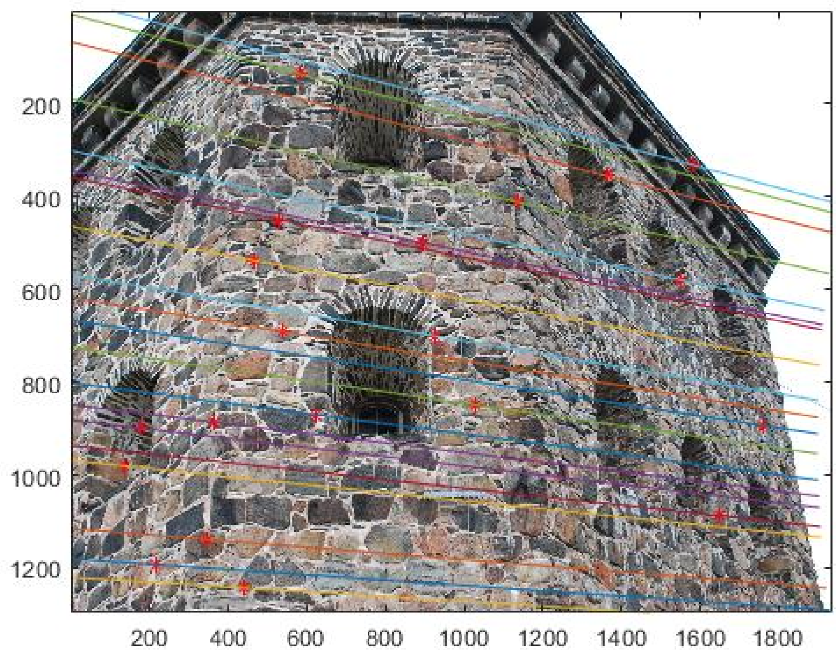


**Figure. 1**

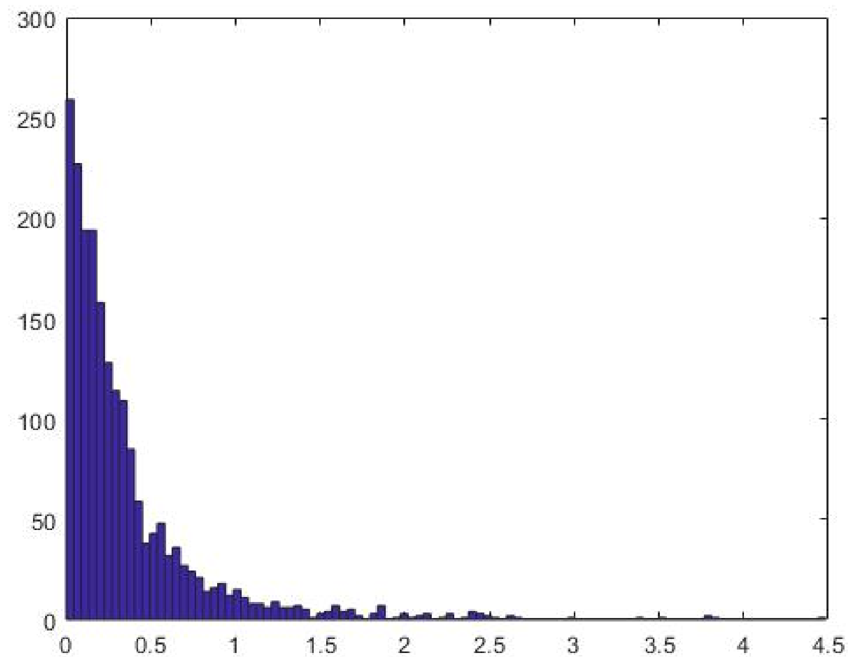


**Figure. 2**

## Exercise 4.

$F = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$P_1 = [I|O]$ and $P_2 = [[e_2]_\times F|e_2]$

$x_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

$F^T e_2 = 0 \Rightarrow \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}^T \begin{bmatrix} e_2(1) \\ e_2(2) \\ e_2(3) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_2(1) \\ e_2(2) \\ e_2(3) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow e_2(2) = 0, e_2(1) + e_2(3) = 0.$

$e_2 = a \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ where a $\in$ R.

$[e_2]_\times F = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & -2 \\ 1 & 0 & 0 \end{bmatrix}$

$P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$

The scene point (1, 2, 3)

$X_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$

$x_1 = P_1 X_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$x_2 = P_2 X_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -10 \\ 0 \end{bmatrix}$

$x_2^T F x_1 = \begin{bmatrix} 2 & -10 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 & -10 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \\ 5 \end{bmatrix} = 10 - 10 = 0$

The scene point (3, 2, 1)

$X_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}$

$x_1' = P_1 X_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

$$x_2' = P_2X_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -2 & -2 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -6 \\ 2 \end{bmatrix}$$

$$x_2^T F x_1 = \begin{bmatrix} 4 & -6 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 & -6 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} = 12 - 18 + 6 = 0$$

The camera center of $P_2$ is at infinity.

## Computer Exercise 2.

See the m-files - compEx1.m.

The camera matrices are $P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $P_2 = \begin{bmatrix} -0.0016 & 0.0057 & 0.2163 & 0.9763 \\ 0.0070 & -0.0257 & -0.9763 & 0.2163 \\ 0.0000 & 0.0000 & -0.0273 & 0.0001 \end{bmatrix}$.

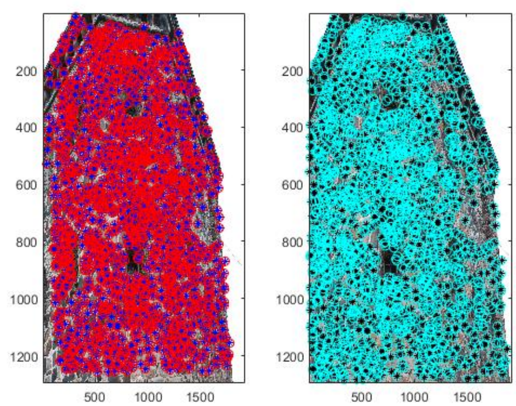


**Figure. 3**

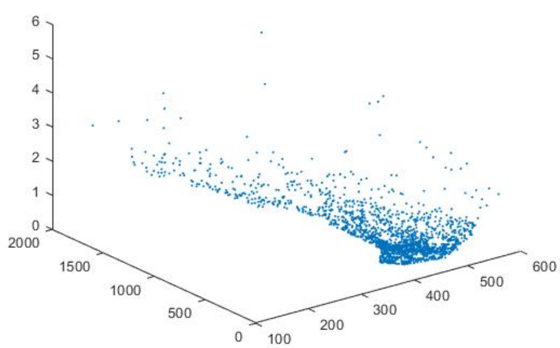


**Figure. 4**

# 3 The Essential Matrix

## Exercise 5.

## OPTIONAL

$[t]_\times = USV^T$.
Thus, U & V are orthogonal ($UU^T = U^TU = I$ & $VV^T = V^TV = I$) and S is diagonal ($S^TS = S^2$).
$[t]_\times^T[t] = (USV^T)^T(USV^T) = VS^TU^TUSV^T = VS^TISV^T = VS^TSV^T = VS^2V^T$.
Therefore, the eigenvalues of $[t]_\times^T[t]$ are the squared singular values. Hence the proof.
Since $[t]_\times$ is skew symmetric ($[t]_\times^T = -[t]_\times$),
$-t \times (t \times w) = \lambda w \Rightarrow -[t]_\times[t]_\times w = \lambda w \Rightarrow [t]_\times^T[t]_\times w = \lambda w$
Using $-t \times (t \times w) = \lambda w \Rightarrow -(t.w)t + (t.t)w = \lambda w$ and putting $w = t$, we get $\lambda = 0$. Thus there is one eigenvector corresponding to $\lambda = 0$.
Again using $-t \times (t \times w) = \lambda w \Rightarrow -(t.w)t + (t.t)w = \lambda w$ and putting $t.w = 0$, we get $\lambda = ||t||^2$. Thus there are two eigenvectors corresponding to $\lambda = ||t||^2$.
These are the three eigenvectors.
Since the eigenvalues of $[t]_\times^T[t]_\times$ are 0, $||t||^2$ & $||t||^2$ and the eigenvalues of $[t]_\times^T[t]_\times$ are the squared singular values, the singular values of $[t]_\times$ are 0, $||t||$ & $||t||$.
If $E = [t]_\times R$ and $[t]_\times$ has the SVD ($USV^T$), the SVD of E is $USV^TR$. The singular values of E are same as that for $[t]_\times$ which are 0, $||t||$ & $||t||$ since rotating $[t]_\times$ by R does not change the singular values.

## Computer Exercise 3.

See the m-files - compEx3.m.

$$E = \begin{bmatrix} -8.88845452067019 & -1005.80666398263 & 377.078253597591 \\ 1252.52308320841 & 78.3677159723631 & -2448.17425754168 \\ -472.788838981850 & 2550.19169646444 & 1 \end{bmatrix}$$

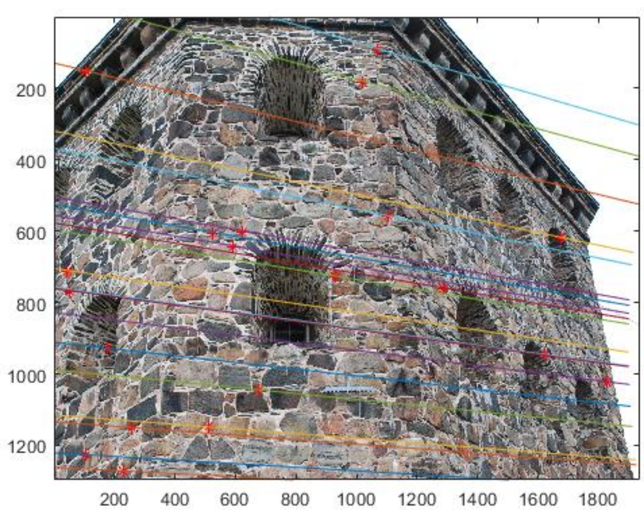


**Figure. 5**

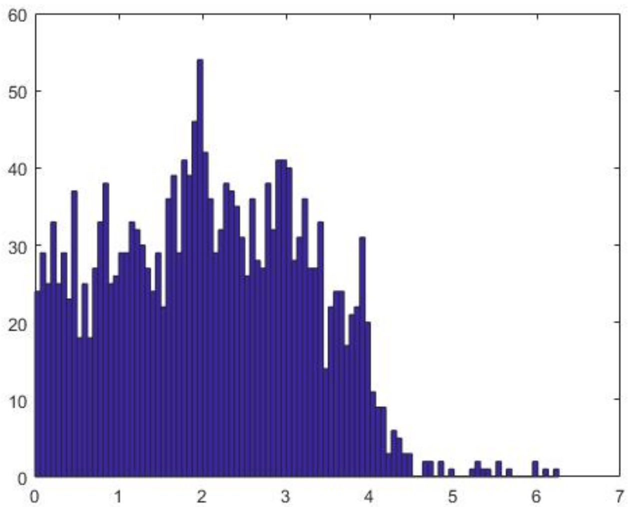


**Figure. 6**

## Exercise 6.

$$UV^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & -1 & 0 \end{bmatrix}$$

$det(UV^T) = \frac{1}{\sqrt{2}}(0 + \frac{1}{\sqrt{2}}) - \frac{1}{\sqrt{2}}(-\frac{1}{\sqrt{2}} - 0) = \frac{1}{2} + \frac{1}{2} = 1.$

$$E = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 \end{bmatrix}$$

$$x_2^T E x_1 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} = 0$$

Therefore, $x_1 = (0, 0)$ (in camera 1) and $x_2 = (1, 1)$ (in camera 2) is a plausible correspondence.

The projection is $P_1 X = \begin{bmatrix} I & O \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$UWV^T = \begin{bmatrix} -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & -1 & 0 \end{bmatrix}$ and $u_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$P_2 = \begin{bmatrix} UWV^T & u_3 \end{bmatrix}$

$$P_2 X = \begin{bmatrix} -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ s \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ s \end{bmatrix}$$

Thus, $\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}s} \\ -\frac{1}{\sqrt{2}s} \end{bmatrix} \Rightarrow s = -\frac{1}{\sqrt{2}}$

$P_2 = \begin{bmatrix} UWV^T & -u_3 \end{bmatrix}$

$$P_2X = \begin{bmatrix} -\frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ s \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ -s \end{bmatrix}$$

Thus, $\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}s} \\ \frac{1}{\sqrt{2}s} \end{bmatrix} \Rightarrow s = \frac{1}{\sqrt{2}}$

$UW^TV^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & -1 & 0 \end{bmatrix}$ and $u_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$P_2 = \begin{bmatrix} UW^TV^T & u_3 \end{bmatrix}$

$$P_2X = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ s \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ s \end{bmatrix}$$

Thus, $\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}s} \\ \frac{1}{\sqrt{2}s} \end{bmatrix} \Rightarrow s = \frac{1}{\sqrt{2}}$

$P_2 = \begin{bmatrix} UW^TV^T & -u_3 \end{bmatrix}$

$$P_2X = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ s \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ -s \end{bmatrix}$$

Thus, $\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}s} \\ -\frac{1}{\sqrt{2}s} \end{bmatrix} \Rightarrow s = -\frac{1}{\sqrt{2}}$

We have four solutions
$s = -\frac{1}{\sqrt{2}}$,
$s = \frac{1}{\sqrt{2}}$,
$s = \frac{1}{\sqrt{2}}$,
$s = -\frac{1}{\sqrt{2}}$.
The point X(s) is in front of the camera 2 for solutions 3 & 4 and it is in front of camera 1 only for solution 3.

## Computer Exercise 4.

See the m-files - compEx3.m

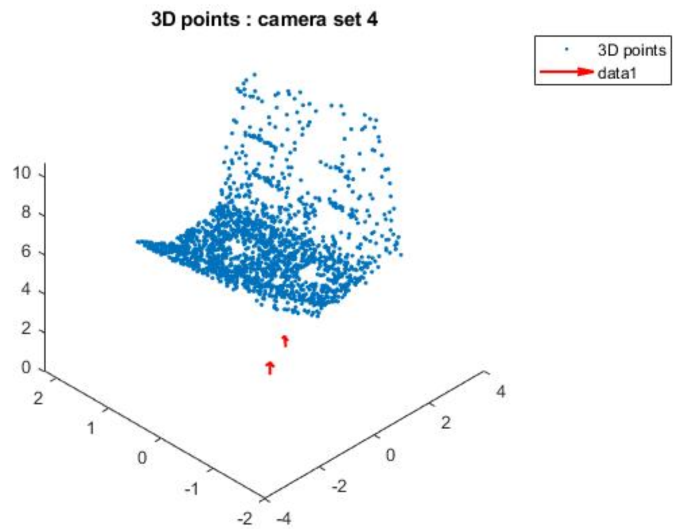


**Figure. 7**

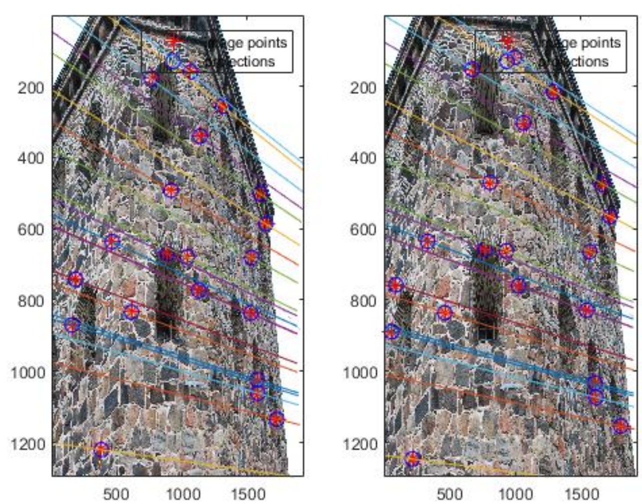


**Figure. 8**