Assignment 5

Computer Vision
Local Optimization and Structure from Motion

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2 Maximum Likelihood Estimation for Structure from Motion Problems

Exercise 1.

Let the 2D-point $x_{ij} = (x_{ij}^1, x_{ij}^2)$ be an observation of the 3D-point X_j in camera P_i. Let us assume that the observations are corrupted by Gaussian noise, that is $(x_{ij}^1, x_{ij}^2) = (\frac{P_i^1 X_j}{P_i^3 X_j}, \frac{P_i^2 X_j}{P_i^3 X_j}) + \epsilon_{ij}$ where P_i^1, P_i^2, P_i^3 are the rows of the camera matrix P_i and ϵ_{ij} is normally distributed with covariance σI.
The probability density function is given by $p(\epsilon_{ij}) = \frac{1}{2\pi\sigma} e^{-\frac{1}{2\sigma^2}||\epsilon_{ij}||^2}$.
Assuming that ϵ_{ij} are independent, $p(\epsilon) = \Pi_{i,j} p(\epsilon_{ij})$.
Taking the log-likelihood of p,
$\ln p(\epsilon) = \ln \Pi_{i,j} p(\epsilon_{ij})$
$= \sum_{i,j} \ln p(\epsilon_{ij})$
$= \sum_{i,j} \ln(\frac{1}{2\pi\sigma} e^{-\frac{1}{2\sigma^2}||\epsilon_{ij}||^2})$
$= \sum_{i,j} \ln(\frac{1}{2\pi\sigma}) - \frac{1}{2\sigma^2} \sum_{i,j} ||\epsilon_{ij}||^2$
$= \sum_{i,j} \ln(\frac{1}{2\pi\sigma}) - \frac{1}{2\sigma^2} \sum_{i,j} ||(x_{ij}^1 - \frac{P_i^1 X_j}{P_i^3 X_j}, x_{ij}^2 - \frac{P_i^2 X_j}{P_i^3 X_j})||^2$.
To maximize the log-likelihood, we need to find
$\min \sum_{ij} ||(x_{ij}^1 - \frac{P_i^1 X_j}{P_i^3 X_j}, x_{ij}^2 - \frac{P_i^2 X_j}{P_i^3 X_j})||^2$
$= \min \sum_{i=1}^n \sum_{j=1}^m ||(x_{ij}^1 - \frac{P_i^1 X_j}{P_i^3 X_j}, x_{ij}^2 - \frac{P_i^2 X_j}{P_i^3 X_j})||^2$.

3 Calibrated Structure from Motion and Local Optimization

Exercise 2. (OPTIONAL.)

$f(v) = ||r(v_0) + J(v_0)\delta v||^2 = (r(v_0) + J(v_0)\delta v)^T (r(v_0) + J(v_0)\delta v)$
$\nabla f(v) = 2\nabla(r(v_0) + J(v_0)\delta v)^T (r(v_0) + J(v_0)\delta v) = 2J(v_0)^T (r(v_0) + J(v_0)\delta v)$ where $\delta v = v - v_0$.
$\nabla f(v_0) = 2J(v_0)^T r(v_0)$
$d = -\nabla f(v_0) = -2J(v_0)^T r(v_0)$.

Exercise 3. (OPTIONAL.)

Since $d = -2J(v_0)^T r(v_0)$ & $\nabla f(v_0) = 2J(v_0)^T r(v_0)$, $\nabla f(v_0)^T d = (2J(v_0)^T r(v_0))^T (-2J(v_0)^T r(v_0)) = -4||J(v_0)^T r(v_0)||^2 < 0$. Therefore, the direction d in equation (8) is a descent direction for the original function (5) at the point v_0.
Since $d = -M\nabla f(v)$ & $v^T M v > 0$ for any v such that $||v|| \neq 0$, $\nabla f(v)^T d = -\nabla f(v)^T M \nabla f(v) < 0$. Therefore, d is a descent direction.
$d = -\frac{(J(v_0)^T J(v_0) + \lambda I)^{-1}}{2}(2J(v_0)^T r(v_0)) = -M\nabla f(v_0)$ where $M = \frac{(J(v_0)^T J(v_0) + \lambda I)^{-1}}{2}$ is a positive definite matrix. Therefore, this is a step in a descent direction.

Computer Exercise 1.

See the m files- compEx1a.m & compEx1b.m

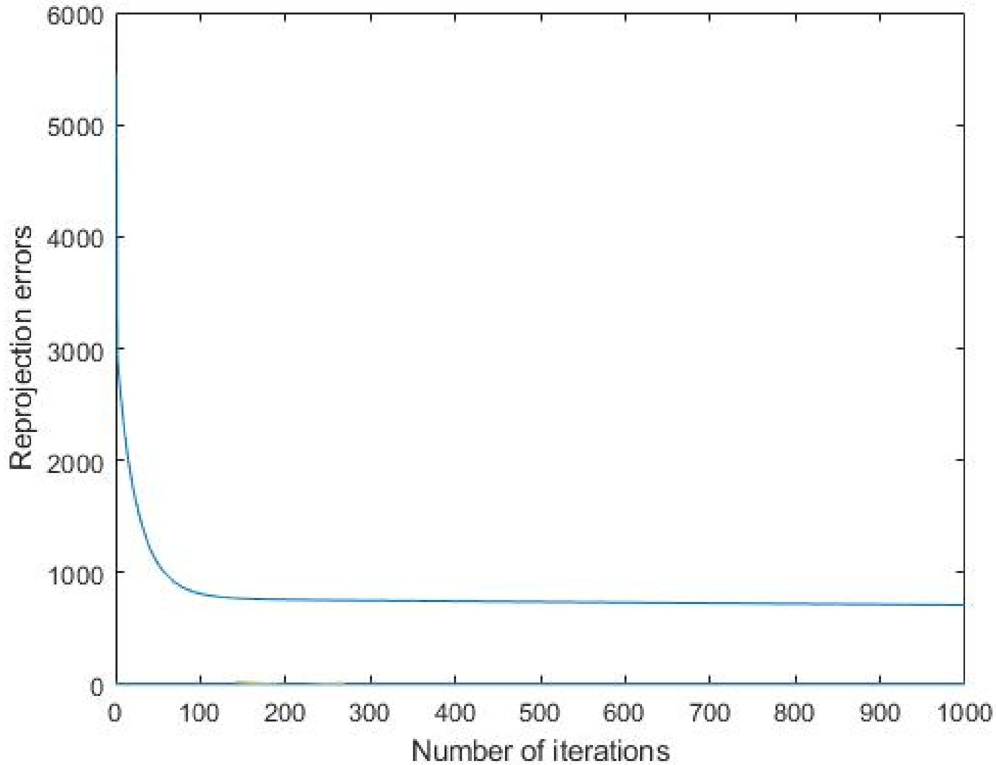

Figure. 1 $\lambda = 1$

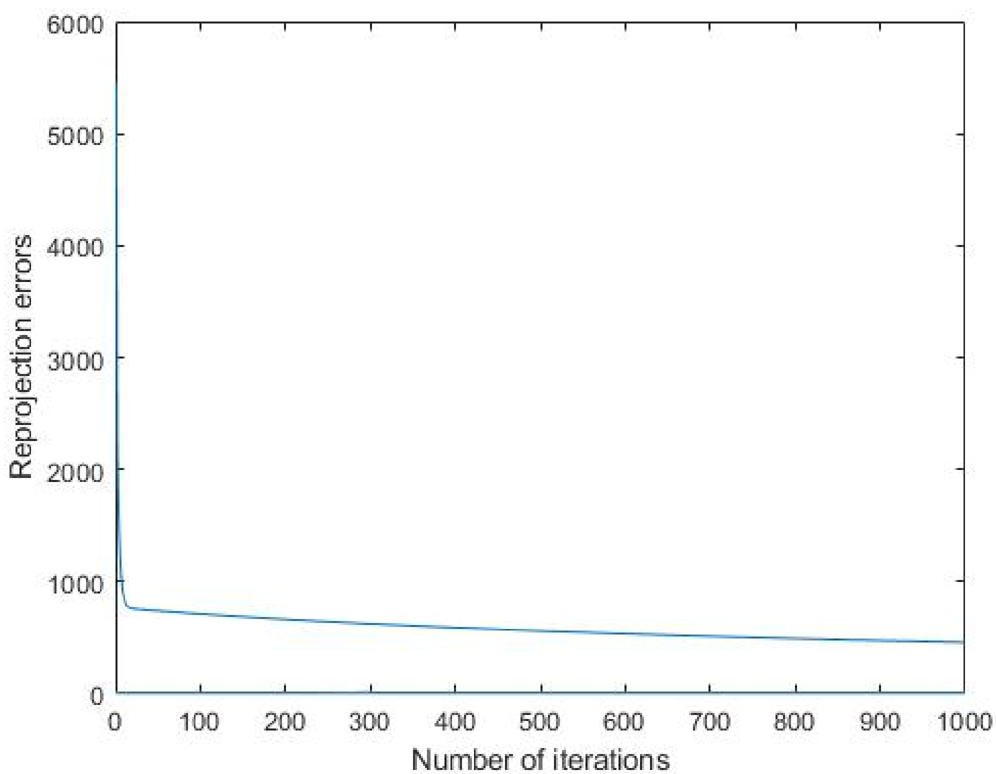

Figure. 2 $\lambda = 0.1$

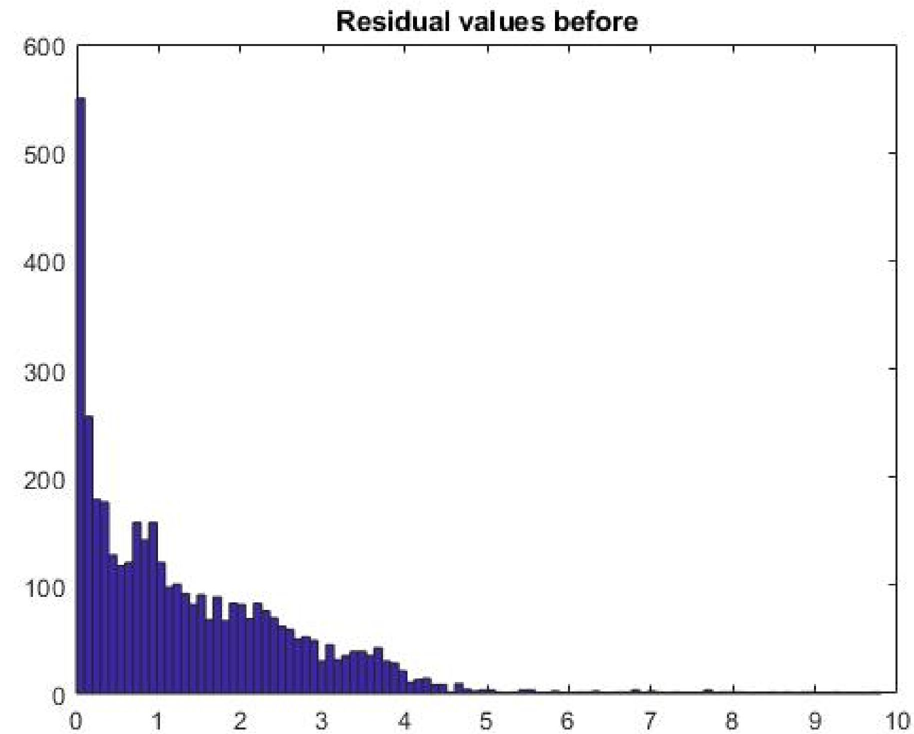

Figure. 3 Histogram of all the residual values before running the Levenberg-Marquardt method

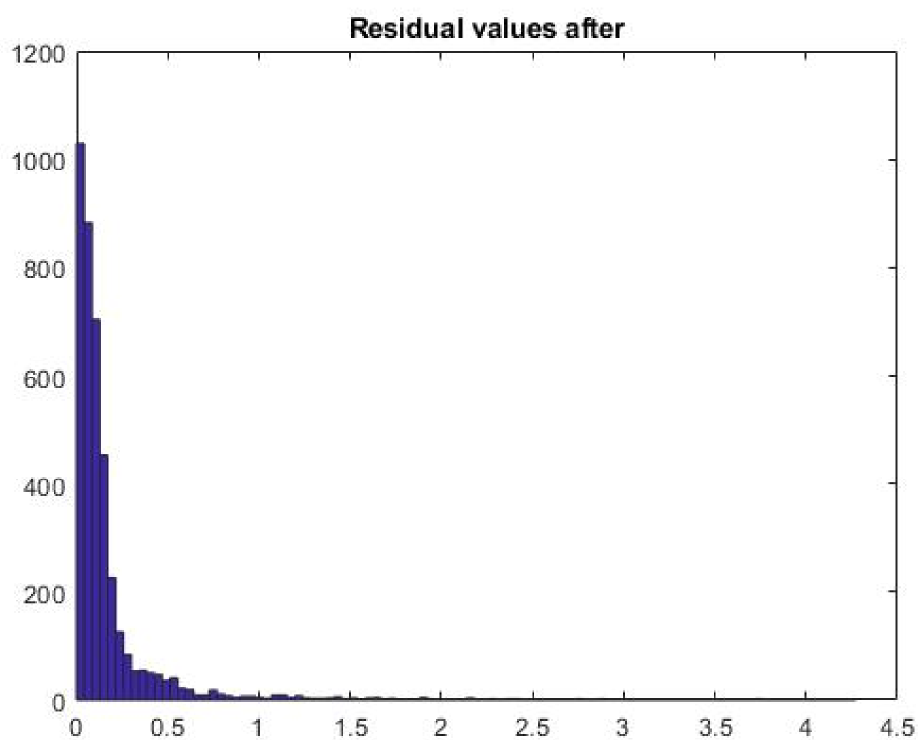

Figure. 4 Histogram of all the residual values after running the Levenberg-Marquardt method

Computer Exercise 2.

See the m file- compEx2.m

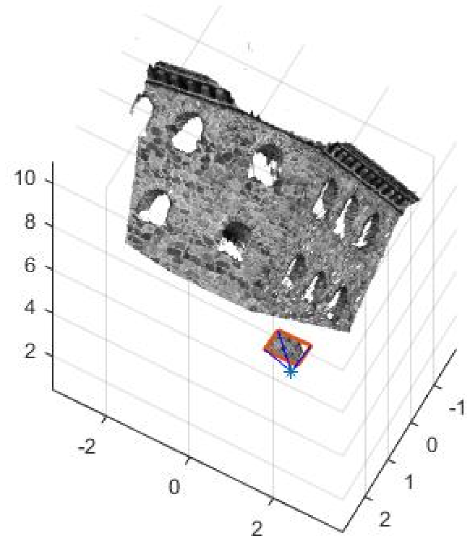

Figure. 5 Final 3D reconstruction