<u>**PROJECT 1**</u>

Supply cost optimization of the natural gas distribution network in Belgium by nonlinear programming

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Abstract

Natural gas is transmitted by pipeline network in Belgium. This project presents the study of this gas distribution network in Belgium. Firstly, we define the sets. Next, we state the given parameters of the problem and introduce the variables. Then we construct the objective function which we want to minimize corresponding to a set of constraints. Finally, we formulate a nonlinear optimization model in order to minimize the total supply cost of the gas transmission over different towns of Belgium at some minimal guaranteed pressure.

1 Introduction

Belgium has no natural gas resources of its own. Therefore it has to import all of its gas from two gas producing countries namely, Algeria and Norway. This gas is efficiently transported from producers to consumers through a network of pipelines. The Algerian gas is tranported at Zeebrugge while the Norwegian gas is delivered at Voeren via Netherlands. The gas is then distributed over different towns of Belgium by a transmission network. This project is a survey of this gas transmission network in Belgium. It was implemented in 1989 by the national federation of the gas industry. Their primary objective is to minimize the total supply cost of the gas transmission over different nodes at some minimal guaranteed pressure. Thus we need to develop a nonlinear programming model. Figure 1 shows the schematic diagram of the gas transmission network in Belgium.

The project is classified as follows: §2 encompasses the important features of a nonlinear programming model. In §2.1, we discuss about the sets. Then we mention about the given parameters of the problem and define the variables in §2.2 and §2.3, respectively. Next we construct our objective function in §2.4 and we give an explanation of the constraints in §2.5. The nonlinear programming problem formulation is described in §2.6. Finally, we give our conclusion in §3.

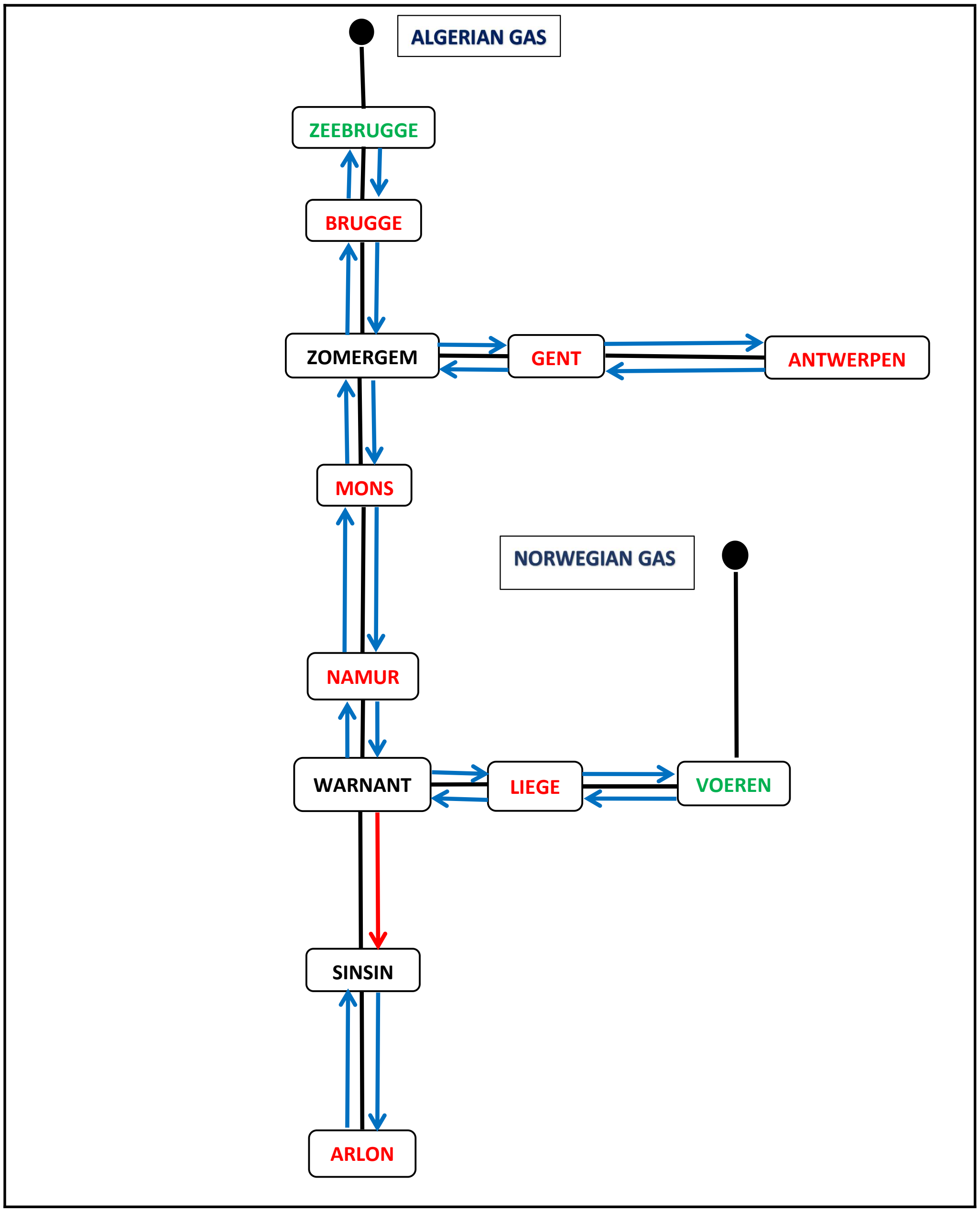

Figure 1: Schematic diagram of the gas transmission network. Here natural gas producing countries, supply nodes, intermediate nodes and demand nodes are labeled in blue, green, black and red, respectively. Blue arrows represent passive arcs and red arrow represents active arc.

2 Nonlinear Programming Model

An optimization problem is the problem of finding the maxima and the minima of functions, possibly subject to constraints. The structure of an optimization model is given below in Figure 2.

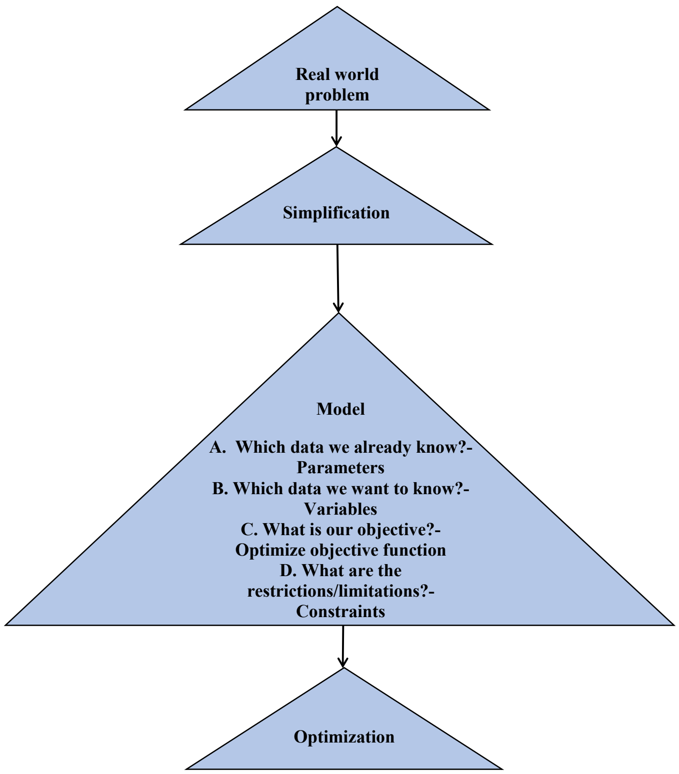

Figure 2: Optimization model.

A minimization problem is an optimization problem which is of the form

$$minimize_x \quad f(x)$$

subject to

$$\begin{cases} g_i(x) \leq 0, i \in \mathcal{I}, \\ g_i(x) = 0, i \in \mathcal{E}, \\ x \in X \end{cases}$$

where $\mathcal{I}$=inequality constraints and $\mathcal{E}$=equality constraints.

An nonlinear minimization problem is a minimization problem if some f, g_i, i $\in \mathcal{I} \cup \mathcal{E}$ are nonlinear functions.

In this project, we wish to formulate a nonlinear minimization problem for total supply cost optimization at some minimal guaranteed pressure. Here the gas is injected into the gas transmission system through the towns- Zeebrugge and Voeren and then the gas flows within the towns- Zomergem, Warnant and Sinsin and lastly, the gas flows out of the system through the towns- Antwerpen, Arlon, Brugge, Gent, Liège, Mons and Namur.

2.1 Sets

In this project, we need to define the following sets:

$\mathcal{N}$ = set of nodes = supply nodes $\cup$ demand nodes $\cup$ intermediate nodes where

$\mathcal{N}_S$ = set of supply nodes = towns where the gas is injected into the system,

$\mathcal{N}_D$ = set of demand nodes = towns where the gas flows out of the system and

$\mathcal{N}_I$ = set of intermediate nodes = towns where the gas is rerouted,

$\mathcal{A}_{passive}$ = set of passive arcs,

$\mathcal{A}_{active}$ = set of active arcs.

2.2 Parameters

We use the following parameters which are already given in the problem:

$q_i^{(min)}$ = Minimum quantity of gas flow in 10^6 m^3/day at node i, i $\in \mathcal{N}$,

$q_i^{(max)}$ = Maximum quantity of gas flow in 10^6 m^3/day at node i, i $\in \mathcal{N}$,

$p_i^{(min)}$ = Minimum gas pressure measured in bar at node i, i $\in \mathcal{N}$,

$p_i^{(max)}$ = Maximum gas pressure measured in bar at node i, i $\in \mathcal{N}$,

P_i = Price of gas flow in \$/MBTU at node i, i $\in \mathcal{N}$,

D_{ij} = Interior diameter of the pipe in mm from node i to node j, (i,j) $\in \mathcal{A}_{active} \cup \mathcal{A}_{passive}$,

ε = Absolute pipe roughness in mm,

$\lambda_{ij} = [2\log_{10}(\frac{3.7D_{ij}}{\varepsilon})]^{-2}$ = Constant that depends on the interior diameter of the pipe and the absolute pipe roughness, (i,j) $\in \mathcal{A}_{active} \cup \mathcal{A}_{passive}$,

z = Gas compressibility factor,

T = Gas temperature in K,

L_{ij} = Length of the pipe in km from node i to node j, (i,j) $\in \mathcal{A}_{active} \cup \mathcal{A}_{passive}$,

δ = Density of the gas relative to air,

$C_{ij} = [96.074830 \cdot 10^{-15} \frac{D_{ij}^5}{\lambda_{ij} z T L_{ij} \delta}]^{\frac{1}{2}}$ = Constant that depends on the length, the interior diameter, the absolute rugosity of pipe, and the gas composition, (i,j) $\in \mathcal{A}_{active} \cup \mathcal{A}_{passive}$,

c = Constant that converts MBTU to m^3.

2.3 Variables

We define the variables as:

f_{ij} = Flow of gas from node i to node j, (i,j) $\in \mathcal{A}_{active} \cup \mathcal{A}_{passive}$,

d_i = Supply Demand of gas flow in 10^6 m^3/day at node i, i $\in \mathcal{N}$,

p_i = Exact pressure measured in bar at node i, i $\in \mathcal{N}$.

2.4 Objective function

Since we want to minimize the supply cost of the gas transmission in Belgium, the objective function depends on the price of gas outflow at supply nodes and the demand of gas outflow from supply nodes. Therefore the objective function takes the form:

$$\sum_{i \in \mathcal{N}_S} cP_i d_i \tag{1}$$

2.5 Constraints

Since the transmission company lifts a quantity inbetween a lower bound and an upper bound on the contracted quantity at node i,

$$q_i^{(min)} \leqslant d_i \leqslant q_i^{(max)}, \forall i \in \mathcal{N} \tag{2}$$

Since the pressure of gas ranges between the minimum pressure and the maximum pressure at node i,

$$p_i^{(min)} \leqslant p_i \leqslant p_i^{(max)}, \forall i \in \mathcal{N} \tag{3}$$

To maintain equilibrium gas flow between the node i and the node j,

$$\sum_{j \in \mathcal{N}} f_{ij} = d_i + \sum_{j \in \mathcal{N}} f_{ji}, \forall i \in \mathcal{N} \tag{4}$$

For passive arcs associated with pipelines, the relation between the flow of gas f_{ij} in the arc(i,j) and the pressures p_i and p_j at the node i and the node j is given by

$$sign(f_{ij}) f_{ij}^2 = C_{ij}^2 (p_i^2 - p_j^2), \forall (i,j) \in \mathcal{A}_{passive} \tag{5}$$

For an active arc associated with a pipeline with a compressor, the relation between the flow of gas f_{ij} in the arc(i,j) and the pressures p_i and p_j at the node i and the node j is given by

$$sign(f_{ij}) f_{ij}^2 \geq C_{ij}^2 (p_i^2 - p_j^2), \forall (i,j) \in \mathcal{A}_{active} \tag{6}$$

The flow of gas between the node i and the node j is given by-

$$f_{ij} = 0, \forall (i,j) \notin \mathcal{A}_{active} \cup \mathcal{A}_{passive} \tag{7}$$

The positive flow condition on active arc between the node i and the node j is given by

$$f_{ij} \geq 0, \forall (i,j) \in \mathcal{A}_{active} \tag{8}$$

Since the pressure of gas at node i is nonnegative,

$$p_i \geqslant 0, \forall i \in \mathcal{N} \tag{9}$$

2.6 Formulation of the problem

Using (1)-(9), the non linear programming problem is formulated as

$$\sum_{i \in \mathcal{N}_S} cP_i d_i$$

subject to

$$\begin{cases} q_i^{(min)} \leqslant d_i \leqslant q_i^{(max)}, \forall i \in \mathcal{N}, \\ p_i^{(min)} \leqslant p_i \leqslant p_i^{(max)}, \forall i \in \mathcal{N}, \\ \sum_{j \in \mathcal{N}} f_{ij} = d_i + \sum_{j \in \mathcal{N}} f_{ji}, \forall i \in \mathcal{N}, \\ sign(f_{ij}) f_{ij}^2 = C_{ij}^2 (p_i^2 - p_j^2), \forall (i,j) \in \mathcal{A}_{passive}, \\ sign(f_{ij}) f_{ij}^2 \geq C_{ij}^2 (p_i^2 - p_j^2), \forall (i,j) \in \mathcal{A}_{active}, \\ f_{ij} = 0, \forall (i,j) \notin \mathcal{A}_{active} \cup \mathcal{A}_{passive}, \\ f_{ij} \geq 0, \forall (i,j) \in \mathcal{A}_{active}, \\ p_i \geqslant 0, \forall i \in \mathcal{N}. \end{cases}$$

3 Conclusion

A real case study is solved as a nonlinear programming problem. This project provides a nonlinear minimization model of the pipeline network used to deliver natural gas in different nodes at some minimal guaranteed pressure in Belgium.

References

[1] Andréasson Niclas, Evgrafov Anton, Patriksson Michael, Gustavsson Emil and Önnheim Magnus,(2016). An Introduction to Continuous Optimization. Studentlitteratur AB.