PROJECT 2

Supply cost minimization analysis of the natural gas distribution network in Belgium by AMPL programming

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Abstract

This project investigates some important aspects of the nonlinear optimization model for the gas transmission network in Belgium using AMPL programming. Firstly, we obtain the optimal solution data of this optimization model. Next, we perform the data analysis for the model. Finally, we deal with sensitivity analysis of the optimal solution.

1 Introduction

Project 1 represents the study of the gas distribution network in Belgium. In project 1, we formulate a nonlinear optimization model in order to minimize the total supply cost of the gas transmission over different towns of Belgium at some minimal guaranteed pressure. This project is the extension of the project 1. It analyses the solution of the optimization model using AMPL programming.

AMPL is a computer programming language developed by Robert Fourer, David M. Gay, and Brian W. Kernighan. It is used for solving optimization problems. AMPL has a familiar algebraic and interactive command environment, mainly designed to easily formulate optimization models, communicate with different types of solvers, and hence examine solutions of the respective optimization models.

The project is classified as follows: §2 consists of important attributes of the nonlinear programming model for the gas distribution network in Belgium using AMPL programming. In §2.1, we mainly carry out the numerical analysis for the nonlinear model. Then we analyse the solution data in §2.2. Next in §2.3, we focus on sensitivity analysis. Finally, we give our conclusion in §3.

2 Results and discussions

In this section, we discuss about some significant results of the nonlinear optimization model using AMPL programming. The source codes can be found in Appendix A.

2.1 Group I - Numerical analysis

Here we use AMPL programming to obtain the optimal solution of the nonlinear optimization model.

2.1.1 Total supply cost of gas in Belgium

The total cost of the gas supplying the demands in Belgium is $1921170.

2.1.2 Amount of gas sent from Zomergem to Gent

The amount of gas which should be sent from Zomergem to Gent is 9.29×10^6 m^3/day.

2.1.3 Amount of Algerian gas purchased

We want to find the amount of Algerian gas purchased which is the amount of gas sent from the supply node Zeebrugge to the demand node Brugge. Therefore, the amount of Algerian gas which should be purchased is 8.87×10^6 m^3/day.

2.1.4 Amount of gas transported from Warnant to Sinsin due to the compressor working on the active pipeline

We want to find the amount of gas sent from Warnant to Sinsin which has an active pipe type. The relation between the flow of gas f_{ij} in the arc(i,j) and the pressures p_i and p_j at node i and node j for an active arc associated with a pipeline with a compressor is given by

$$sign(f_{ij})f_{ij}^2 \geq C_{ij}^2(p_i^2 - p_j^2). \tag{1}$$

Now gas will flow from Warnant to Sinsin only when the above condition holds. The data for the gas flow from Warnant to Sinsin can be found in table 1 in appendix B. Since the slack (which is -1.80322 $\times$ 10^{-8}) is approximately equal to zero, the compressor is inactive in this case.

2.2 Group II - Data analysis

Here we analyse the solution data which we obtain from AMPL programming.

2.2.1 Local optimal and global optimal solutions

In this project, the solution is the supply demand. We have to verify whether it is local optimal or global optimal.
Consider a problem (P)
minimize f(x)
subject to $x \in S$, where $S \neq \phi \subseteq \mathbb{R}^n$.
Then $x^* \in S$ is a global minimum in problem (P) if f(x^*) $\leq$ f(x) $\forall$ x $\in$ S and $x^* \in S$ is a local minimum in problem (P) if $\exists$ $\epsilon > 0$ such that f(x^*)$\leq$ f(x) $\forall$ x $\in$ S $\cap$ B_ϵ (x^*) where

$$B_\epsilon(x^*) = \{x \in \mathbb{R}^n \mid ||x - x^*|| < \epsilon\}.$$

For our minimization problem, the solution is global minima and hence local minima by definition. The reason is that the transmission company takes the minimum quantity of gas at the supply nodes Zeebrugge and Voeren which in turn, implies that the supply demand at the supply nodes is equal to the minimum quantity of gas at the supply nodes. For this particular case, we can easily prove that the solution is global minimum. But in general, it is hard to find global minimum. This cannot be an approach to verify optimality in general cases beyond this exercise.

2.2.2 Pressure-related decision variables

The pressure-related decision variables are not exactly the pressures at the nodes. These decision variables are square of pressure at the nodes measured in bar^2.

If the decision variables are written as squares of pressure at the nodes, then the nonlinearity of the problem is reduced. Most of the solvers works efficiently for a less nonlinear problem. Thus we can use these decision variables because of the reduced nonlinearity of the problem.

We want to use them since it makes the problem simpler and hence easier to solve.

2.2.3 Method to find a feasible solution without using any nonlinear programming solver

Our present constrained minimization problem is a nonlinear problem. Most of the nonlinear solvers has already a method for finding an initial feasible solution. But in some of the nonlinear solvers, the user is required to provide an initial feasible solution. For a nonlinear problem, we can choose an interior point of the feasible region as an initial feasible point. But finding an interior point is harder than finding a feasible one. Here we present a method to find an initial feasible solution for our nonlinear programming problem without using any nonlinear programming solver.

Firstly, we assign a supply node for each demand node. For example, let Zeebrugge supply Brugge and Gent, and let Voeren supply the other demand nodes. From here, we determine the flow directions on the pipelines and we let the flow quantity of a pipeline to be the sum of the demands of all downstream nodes. For example, the pipeline from Zomergen to Gent carries the sum of the demands by Gent and Antwerpen. Thus the flow quantities of gas on the pipelines is fixed. Since the flow is fixed, we use the flow-pressure relation to compute the differences of square of pressures between the two end nodes of the pipelines. Using the lower and upper bounds of the square of pressures, we formulate a linear program to solve for the pressures at each node. Since the present form of our problem becomes linear, we can use the phase I simplex method to generate an initial feasible solution for our problem.

2.3 Group III - Sensitivity Analysis

Here we determine the sensitivity of the optimal solution after changing the data values.

2.3.1 Effect of reduction of Norwegian gas supply

Suppose that Norwegian gas supply is reduced. In particular, both the minimum and maximum quantities of the supply from Voeren are reduced to 80% of their original values.

The total amount of Algerian gas purchased is increased by 2.2834×10^6 m^3/day and the total amount of Norwegian gas purchased is decreased by 2.7344×10^6 m^3/day.

Now Norwegian gas is cheaper than Algerian gas. So when we want to minimize the total supply cost, the gas company will take the maximum reduced quantity of Norwegian gas and the rest Algerian gas. After reduction of Norwegian gas, the total amount of Norwegian gas purchased which is equal to the maximum reduced quantity of Norwegian gas decreases from the original amount of gas purchased at Norway and the total amount of Algerian gas purchased increases from the original amount of gas purchased at Algeria. This is the reason behind the above changes in the amount of gas purchased from Norway and

Algeria. The complete data can be found in table 2 and table 3 in Appendix B.

2.3.2 Data manipulation to make the pipe type from Warnant to Sinsin active

To make the compressor on the pipeline from Warnant to Sinsin active, the pressure at Sinsin should be higher than the pressure at Warnant. If we fix the minimum pressure at Sinsin to 45 bar, then the pressure at Sinsin is higher than the pressure at Warnant. The compressor is indeed active since it satisfies equation (1) and the slack is 1.10963.

2.3.3 Feasibility of optimization model when the demands at demand nodes are doubled keeping the supply at the supply nodes same.

We are presently working with the snopt solver. So when the demands at all demand nodes are doubled, while the supply at each supply node remains the same, the optimization model becomes infeasible. The reason is that the total demand of gas at demand nodes is greater than the total supply of gas from supply nodes. The data can be found in table 4 in Appendix B. For the present case, we will present the feasibility of the optimization model using different solvers in table A.

Table A: In this table, we present the different solvers and their feasibility.

Solvers	**Feasibility**
baron, conopt, gurobi, knitro, lgo, loqo, minos, snopt, xpress	infeasible

2.3.4 Feasibility of optimization model when both the demands at demand nodes and the supply at supply nodes are doubled.

While working with the present solver snopt, when the demands at all demand nodes are doubled, and the maximum supplies at all supply nodes are also doubled, then this optimization model is infeasible since it violates the pressure requirements. The data can be found in table 5 in Appendix B. We will present the feasibility of the optimization model for this case using different solvers in table B.

Table B: In this table, we present the different solvers and their feasibility.

Solvers	**Feasibility**
snopt, baron, conopt, knitro, lgo, minos	infeasible

2.3.5 Approximate derivatives of the total cost with respect to changes of the minimal supplies of Norwegian and Algerian gas, evaluated at the original optimal solution.

We consider the marginal reductions of the minimal supplies of Norwegian and Algerian gas. We consider one limit at a time. We reduce the minimum quantity of gas at Voeren by 1% and the minimum quantity of gas at Zeebrugge by 5% after each iteration.

The approximate derivatives of the total cost with respect to changes of the minimal supplies of Norwegian and Algerian gas, evaluated at the original optimal solution is 0. The explanation is given in table

C and table D.

Table C: In this table, we present the minimum quantities of gas, the total supply and their approximate derivatives at Voeren.

No of iterations	**slo**	**total_supply**	**Approximate derivative**
1	20.344	1921170	
2	20.14056	1909100	59329.53205
3	19.9391544	1897150	59333.00762
4	19.73976286	1894410	13741.80642
5	19.54236523	1894410	0
6	19.34694158	1894410	0

Table D: In this table, we present the minimum quantities of gas, the total supply and their approximate derivatives at Zeebrugge.

No of iterations	**slo**	**total_supply**	**Approximate derivative**
1	8.87	1921170	
2	8.4265	1885460	80518.60203
3	8.005175	1876090	22239.36391
4	7.60491625	1867610	21186.29512
5	7.224670438	1859550	21196.81463
6	6.863436916	1851900	21177.43658
7	6.52026507	1849520	6935.300868
8	6.194251816	1849520	0
9	5.884539226	1849520	0

In addition, when we compare the results with the values of the dual variables for the limiting constraints in the optimal solution to the original problem, we observe that the minimum quantity of gas is equal to the supply demand (dual variables) at the supply nodes after each iteration.

3 Conclusion

We find the optimal solution of the nonlinear optimization model for the gas distribution network in Belgium that is described in project 1 by AMPL programming and hence we analyse the sensitivity of the optimal solution.

References

[1] Andréasson Niclas, Evgrafov Anton, Patriksson Michael, Gustavsson Emil and Önnheim Magnus,(2016). An Introduction to Continuous Optimization. Studentlitteratur AB.

Appendix A: AMPL code

The source code to generate the solution of the nonlinear optimization problem is as follows

include belgium.run;

The source code to find the total supply cost is as follows

display total_cost;

The source code to find the flow of gas is as follows

display f;

The source code to find the squares of constant that depends on the length, the interior diameter, the absolute rugosity of pipe, and the gas composition and the squares of pressure is as follows

display c2, pi;

The source code to find the slack in the active arc constraint is as follows

display Active_Equilibrium.slack;

The source code to find the supply demand is as follows

display s;

Appendix B: Tables

Table 1: In this table, we present the flow of gas (f_{ij}), the constant that depends on the length, the interior diameter, the absolute rugosity of pipe, and the gas composition (c2), the square of pressure of gas (p_i^2) at Warnant and the square of pressure of gas (p_j^2) at Sinsin. Here the flow of gas is measured in 10^6 m^3/day and the square of pressure is measured in bar^2.

f_{ij}	c2	p_i^2	p_j^2
0.222	0.00413627	1733.31	1721.4

Table 2: In this table, we present the minimum quantity (slo), the maximum quantity (sup), the minimum reduced quantity (slo$_{red}$) and the maximum reduced quantity (slo$_{dec}$) from Voeren. Here the quantities are measured in 10^6 m^3/day.

slo	sup	slo$_{red}$	sup$_{red}$
20.344	22.012	16.2752	17.6096

Table 3: In this table, we present the demand (s), the new demand (s$_{new}$), the change in demand (s$_{change}$) of gas from Norway and Algeria. Here the quantities are measured in 10^6 m^3/day.

Country	s	s$_{new}$	s$_{change}$
Norway	20.344	17.6096	-2.7344
Algeria	8.87	11.1534	2.2834

Table 4: In this table, we present the total demand, the total supply and the slack. Here the quantities are measured in $10^6\ m^3$/day.

Total demand	Total supply	Slack
57.526	33.606	23.92

Table 5: In this table, we present the total demand, the total supply and the slack. Here the quantities are measured in $10^6\ m^3$/day.

Total demand	Total supply	Slack
57.526	57.526	0