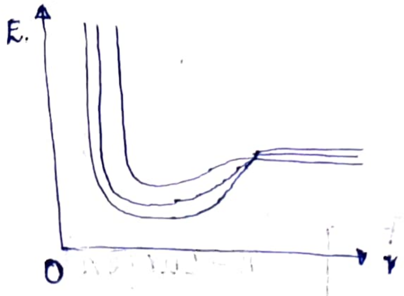
5. Solid State Physics.



$Na(11) - 2, 8, 1.$

Free electron theory :-

$Na(11) - 2, 8, 1.$

→ The Outermost e^-s of the metal atoms are known as valence electrons. These valence e^-s are weakly bound with the atoms.

→ In a Solid, the valence e^-s interact & form a 'gas' of e^-s. The gas of e^-s move with relative freedom throughout the resulting assembly of metal ions.

→ When metal atoms are seperated the total energy is higher than when they are bound together.

Na Na
Isolated aroma

$K_1, U_1 \rightarrow$ P.E

Na Na K_2, V_2 kinetic energy

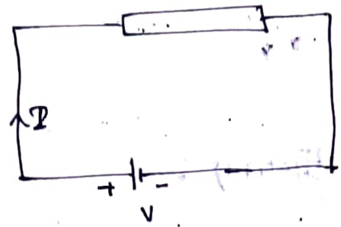
$$U = \frac{-1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

→ The e^- P.E is reduced in metal crystal tha[n] in isolated atoms.

→ The K.E of the e^- increases in the metal crystal.

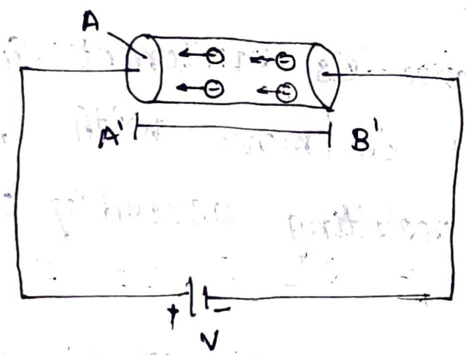
→ <u>Ohm's Law</u> :-

$$\boxed{I = \frac{V}{R}}$$



I - Current
V - voltage
R - Resistance.

→ where R → Resistance of the Conductor, it depends on composition, dimension & temperature of the conductor.



V_F - Electron velocity ($10^6 m/s$)

V_d - Drift velocity ($1 mm/s$)

→ The velocity of the electron in presence of potential diff. or electric field is known as drift velocity ($1 mm/s$).

<u>Electron velocity</u> :- velocity of electron in absence of electric field ($10^6 m/s$).

Mean free Path :- Distance covered by the electron b/w two successive collisions.

Time taken to cover distance - λ.

Mean free path (λ).

Relaxation Time (τ).

Electron velocity is given by (in absence of $\bar{E}$)

$$V_F = \frac{\lambda}{\tau}.$$

$$\Rightarrow \tau = \frac{\lambda}{V_F}.$$

$\because$ Here, V_F is the e^- velocity which corresponds to the fermi energy.

Fermi Energy :- Energy of the e^- at the top most filled energy level at 0 kelvin

$$E_F = \frac{1}{2} m V_F^2.$$

For, W (approx) $\Rightarrow E_F = 7.04$ ev.

$$V_F = \sqrt{\frac{2E_F}{M}}$$

$$= \sqrt{\frac{2 \times 7.04 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

$$\boxed{V_F = 1.57 \times 10^6 m/s}$$

Electric Field :

$$E = \frac{V}{l} \rightarrow (2)$$

Acceleration of the e^-,

$$a = \frac{F}{m} = \frac{eE}{m} \rightarrow (3)$$

e- charge of the e^-.

Drift velocity is given by,

$$V_d = \frac{\bar{x}}{T} \rightarrow (4)$$

where, $\bar{x} = \frac{1}{2} a \overline{\Delta t}^2$

$\because u = 0$

$\overline{\Delta t}^2 = 2\tau^2$

$$\bar{x} = \frac{1}{2} a (2\tau^2)$$

$$\boxed{\bar{x} = a\tau^2} \rightarrow (5)$$

$\because S = ut + \frac{1}{2} a t^2$

From (4)

$$V_d = \frac{a\tau^{\cancel{2}}}{\cancel{\tau}} \Rightarrow \boxed{V_d = a\tau} \rightarrow (6)$$

$$V_d = \left(\frac{eE}{m}\right) \cdot \left(\frac{\lambda}{V_p}\right)$$ from (2) & (3)

$$V_d = \frac{eV}{Lm} \cdot \frac{\lambda}{V_p}$$ from (1)

$E = \frac{V}{L}$.

But Current is given by,

$$i = neAV_d = neA \frac{eV\lambda}{LmV_F}$$

$$\boxed{i = \frac{V}{R}}.$$

where,

$$\frac{1}{R} = \frac{ne^2A\lambda}{mV_F L}.$$

$$R = \frac{mV_F}{ne^2\lambda} \cdot \frac{L}{A}.$$

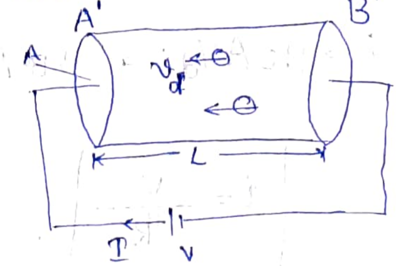
$$\boxed{R = \rho \frac{L}{A}}$$

where $\rho = \frac{mV_F}{ne^2\lambda}$.

$n \rightarrow$ no. of e^- in unit volume.

→ Free electron Theory :-

1. valence e^-s are weakly bound.
2. E^- gas → move freely.
3. $E_1 > E_2$.
4. $k_1 < k_2$ $\quad U_1 > U_2$.



$$\boxed{\tau = \frac{\lambda}{v_F}} \rightarrow (1)$$

v_d → ~~drift~~ drift velocity.

$$a = \frac{F}{m} = \frac{eE}{m} \rightarrow (2)$$

$$E = \frac{V}{L} \rightarrow (3)$$

$$\boxed{t^2 = 2\tau^2}.$$

↓

drift

$$\bar{x} = ut + \frac{1}{2}at^2$$

$$\bar{x} = \frac{1}{2}a\tau^2$$

$$\tau = \frac{\bar{x}}{v_d}$$

$$v_d = \frac{\bar{x}}{\tau} = \frac{a_\tau}{\tau}$$

$$\boxed{v_d = a\tau}$$

we can use, $v = u + at$.

$$v_d = \left(\frac{eE}{m}\right)\left(\frac{\lambda}{v_F}\right) = \frac{eV\lambda}{mv_F L}$$

$$\boxed{I = neAv_d} \quad \rightarrow \boxed{I = \frac{V}{R}}$$

volume of cylinder (fig.) = AL.

$$\rightarrow \boxed{R = \rho\frac{L}{A}}$$

$1 \longrightarrow n$.

$AL \longrightarrow nAL$.

charge $(q) = (nAL)e$.

$$I = \frac{q}{t} = \frac{(nAL)e}{t}$$

$$= neAV_d.$$

Reasons for resistance in matter :-

1) The scattering of free e^- waves in a ~~matter~~ metal.

2) The scattering caused by structural defects & by ions ~~at~~ out of place as they vibrate.

Q) Find the V_d (drift vel.) of the e^-s in a copper wire whose cross-sectional area is $1\,mm^2$ when the wire carries a current of 1 Ampere. Assume that each copper atom contributes one e^- to the e^- gas.

Sol. $V_d = a\tau$.

$$d = 8.94 \times 10^3\ kg/m^3.$$

Atomic mass of Cu = 63.5 amu.

$$1\ amu = 1.66 \times 10^{-27}\ kg.$$

$$I = neAV_d.$$

$$e = 1.6$$

[N - no. of e^-s]

$$n = \frac{N}{V} = \frac{N}{\left(\frac{mass}{density}\right)} = \frac{N \times 8.94 \times 10^3}{63.5 \times 1.66 \times 10^{-27}}$$

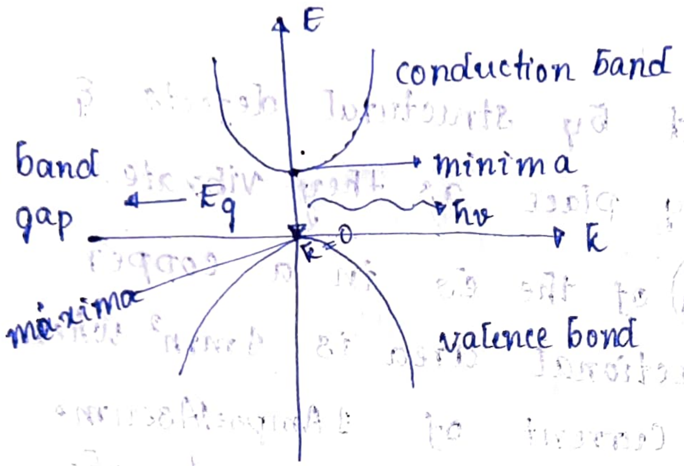
$$= 8.48 \times 10^{28} N\ e^-/m$$

$v_d = 7.4 \times 10^{-4}$ m/s.

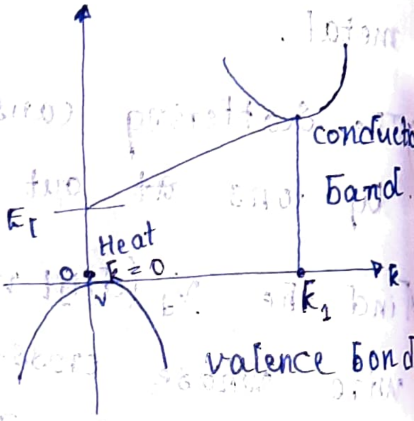
$$v_d = \frac{1}{8.38 \times 10^{28} \times 1.6 \times 10^{-19} \times 1 \times 10^{-6}}$$

$= 7.4 \times 10^{-5}$ m/s.

⇒ Direct & Indirect Semi-conductors :-



Ex → GaAs.



Ex - Si, Ge
1.1ev 0.7ev
band gap.

E.k diagram of direct Semi-conductor

Ex : GaAs.
→ $E_g \approx 1.4$ev

E.k diagram of Indirect Semi-conductor
Ex :- Si, Ge.

→ In direct semi-conductor, conduction band minima & valence band maxima are lying at the same 'k' value.

→ In direct semi-conductor electron from conduction band minima jump to top of the valence band & combine there with their holes.
In this, Process energy is released in the form of light.

which is given by,

$$\boxed{E_g = h\nu = \frac{hc}{\lambda}}$$

E_g - band gap
λ - wavelength of light

∵ This energy is known as re-combination energy. & the Process is known as re-combination process.

Ex:- GaAs.

- In Indirect Semiconductor conduction band minima & valence band maxima have diff. k values.
- In Indirect Semiconductor e̅s from conduction band minima first jump to some default energy level E_r .. then, it will come top of the valence band & combine to the holes.

∴ In this, Process energy is released in the form of heat.

<u>Effective mass of an e̅</u> :-

Kinetic energy of a free electron,

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$\boxed{\because v = \frac{p}{m}}$$

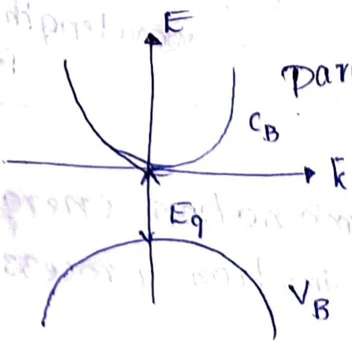
According, to de-Broglie's r/n :

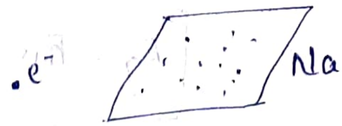
$$\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda}.$$

$$\Rightarrow p = \left(\frac{h}{2\pi}\right)\left(\frac{2\pi}{\lambda}\right)$$

$$\boxed{p = \hbar k}$$ where, $k = \frac{2\pi}{\lambda}$

Now, $$\boxed{E = \frac{\hbar^2 k^2}{2m}}$$ E - k relationship for a free e̅.





→ The $e^{\ominus}$ energy is parabolic with wave vector k.

$$\boxed{m = 9.1 \times 10^{-31}\, kg.} \rightarrow \text{Isolated mass of an } e^-.$$

→ The mass of $e^{\ominus}$ in the crystal is known as effective mass of an $e^{\ominus}$.

diff. eq ① twice w.r.t 'k'

$$\frac{d^2E}{dk^2} = \frac{\hbar^2}{m}$$

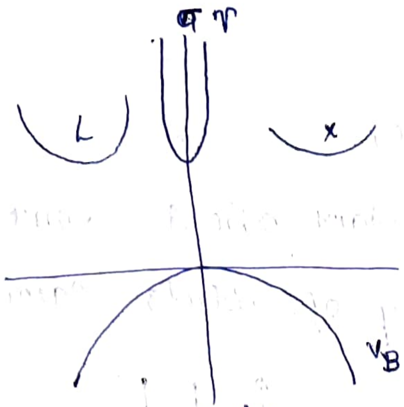
$$\Rightarrow m = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)} = m^*$$

→ Curvature.

$$\boxed{m^* = \hbar^2 \left(\frac{d^2E}{dk^2}\right)^{-1}}$$

Here, m^* → effective mass of an e^-.

$$\Rightarrow \left\{ \cancel{\text{Curvature}} \propto \frac{1}{\cancel{\text{effective mass.}}} \right\}$$



GaAs.

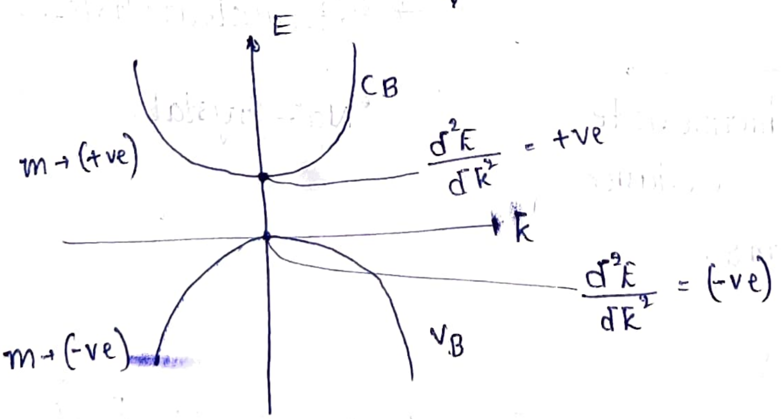
The curvature $\propto \frac{1}{\quad}$

- The e^- effective mass in GaAs is much smaller in the 'Γ'.
- In 'Γ' the ~~cur~~ direct conduction band [strong curvature] larger curvature, will be there.
- In 'GaAs' the effective mass will be maximum.

If the curvature $\left(\frac{d^2E}{dk^2}\right)$ is (+ve), e^- effective mass will be (+ve).

$\left(\frac{d^2E}{dk^2}\right)$ is (−ve) then e^- effective mass will be (−ve)

Directly Proportional.



$\frac{1}{m} \propto$

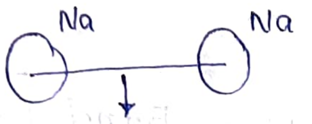
$$\boxed{\left(\frac{d^2E}{dk^2}\right) \propto \frac{1}{m}}$$

Band theory of Solids :-

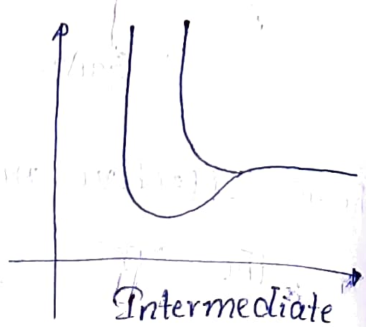
Semiconductor, Insulator.

→ Free e^- theory didn't briefly explain about Semicondu & Insulators the Band theory of Solids came into introduction.

Na metal.



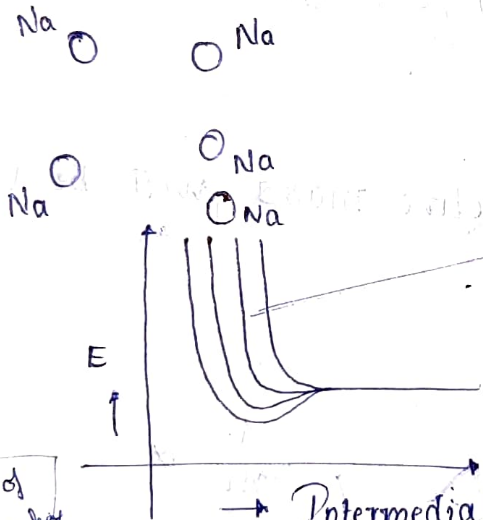
Internuclear distance.



Intermediate distance

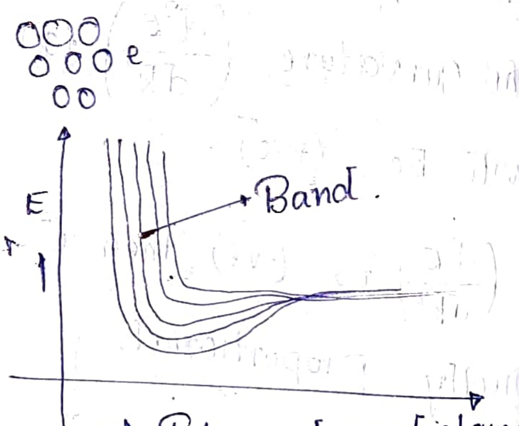
2-Na atoms.

$\underset{2,8,(3)}{\underset{\downarrow}{Na}}$ $\underset{2,8,(3)}{\underset{\downarrow}{Na}}$



→ Intermediate distance.

5-Na atoms.



→ Internuclear distance

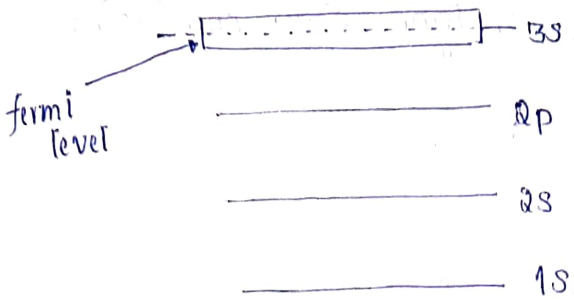
"Na" – Crystal.

Concept of Modern phy
Orthabizer

→ What is Band ?

→ Closely packed energy levels are known as band.

→ Na (11) - $1s^2 2s^2 2p^6 3s^1$



Q. why Na is a conductor ?

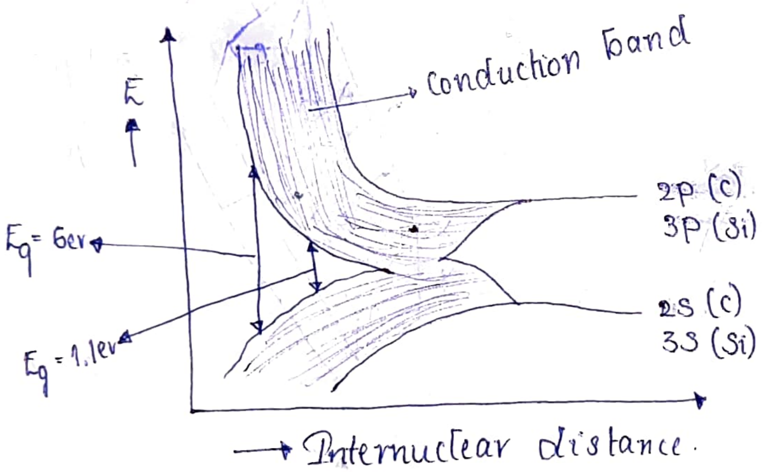
Q. Why Mg is a Conductor ? → E.C → $1s^2 2s^2 2p^6 \underline{3s^2 3p^0} 3d^0$ ← these bands are overlapping

Q. why Si is Semi-conductor ?

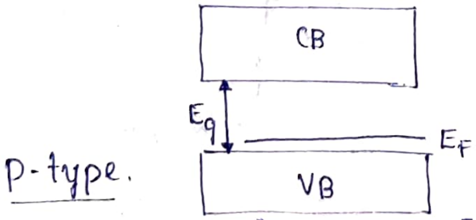
$\underline{Si}$ (14) → $1s^2 2s^2 2p^6 3s^2 3p^2$.

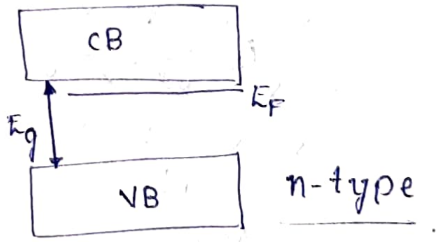
$\underline{diamond}$ is an allotrope of C.

C (6) → $1s^2 2s^2 2p^2$.



Extrinsic Semi-conductor :-



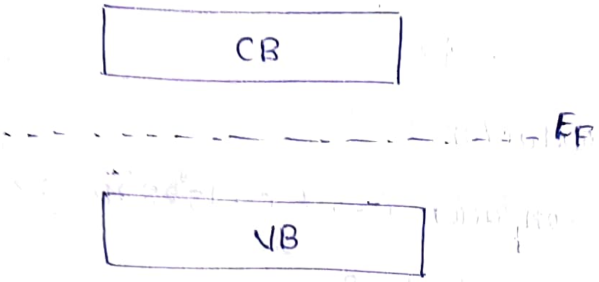


→ Top most filled energy level at 0°K is known as fermi energy level.

→ The fermi level lies close to the valence band in p-type Semi-conductor.

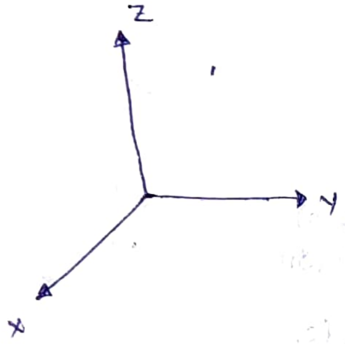
→ Fermi level will lie in the bottom of the Semi conduction band in n-type Semi-conductor.

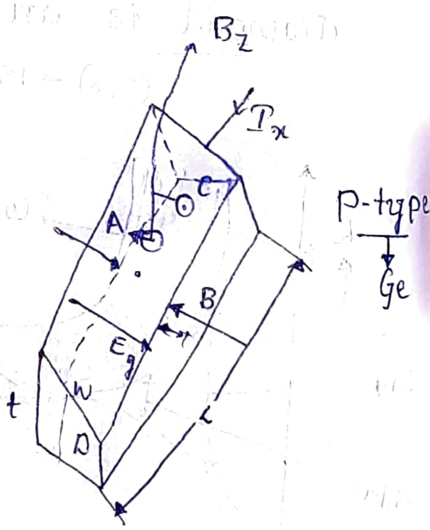
Intrinsic Semi-conductor :-

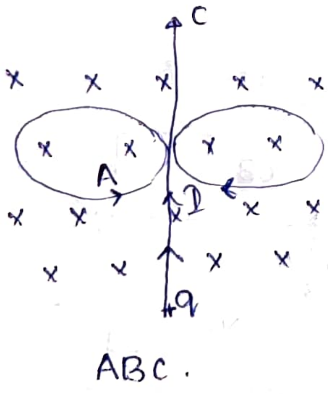


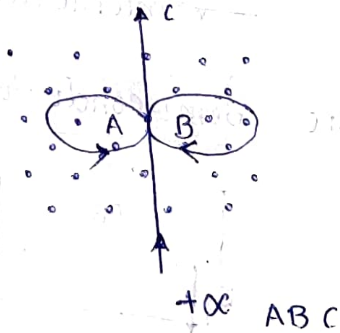
E_F → At the middle of the band gap.

→ Hall effect :-









→ This establition of e^- is called Hall-effect.

→ majority charge carriers are holes.

$\boxed{F = q v B \sin\theta}$

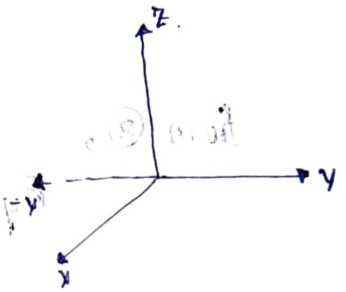
- '+q' is will effect due to the drude-Lorentz forces.

- Production of electric field due to the effect on charge particles is known Hall-effect.

  [Gauss-meter

$q v_x B_z \sin 90 \longleftarrow (+q) \longrightarrow q E_y$.

- The total force on single hole in electric & magnetic field given by,



$$\vec{F} = q(\bar{v} \times \bar{B} + \bar{E}) \longrightarrow (1)$$

In the y-direction, force is given.

$$\cancel{F_y = q[v_x B_z \sin 90 - E_y]}$$

$$F_y = q[E_y - v_x B_z \sin 90] \longrightarrow (2)$$

- unless, an electric field E_y is established along the width of the bar each hole will experience a net force in the $(-y)$ direction, due to $q v_x B_z$.

∴ To maintain, steady flow of holes down the length of the bar electric force must balance the magnetic force.

→ i.e., $F_y = 0$

$$\Rightarrow E_y = v_x B_z \longrightarrow (3).$$

→ The establishment of electric field is known as hall-effect & the resulting voltage be AB is

$$\boxed{V_{AB} = E_y W}$$

known as Hall-voltage.

→ Drift velocity of holes is given by,

$$\boxed{\cancel{I = n_e A V_d}}$$

$$J_x = p q V_x$$

$$\boxed{I = neAV_d}$$

Hall velocity $\left[J = \frac{I}{A}\right]$

∴ where, p is conservation of holes.

$$V_x = \frac{J_x}{pq}.$$

from ③,

$$E_y = \frac{J_x B_z}{pq}.$$

$$E_y = R_H J_x \cdot B_z \longrightarrow ④$$

∴ where, $R_H = \frac{1}{pq}$, Hall coefficient.

Hole concentration is given by,

$$P = \frac{1}{R_H q} \quad \text{from ⑤}$$

$$P = \frac{J_x \cdot B_z}{E_y \cdot q} \quad \text{from ④}$$

$$\boxed{p = \frac{\left(\frac{I_x}{W \times t}\right) B_z}{\left(\frac{V_{AB}}{W}\right) \times q}}$$

Resistivity of the Semi-conductor,

$$\rho = \frac{RA}{l} = \frac{R(t \times W)}{l} = \frac{\left(\frac{V_{CD}}{I_x}\right)(t \times W)}{l}$$

Conductivity of the Semi-conductor,

$$\sigma = \frac{1}{\rho} = \frac{l}{\left(\frac{V_{CD}}{I_x}\right)(W \times t)}$$

Mobility of the holes is given by,

$$\sigma = q \mu_p \cdot p .$$

$$\boxed{\mu_p = \frac{\sigma}{qp}} = \frac{\frac{1}{\rho}}{\cancel{q}\left(\frac{1}{R_H q}\right)} = \frac{R_H}{\cancel{q}\rho}$$

$$\boxed{\mu_p = \frac{R_H}{\rho}}$$

→ Fermi-Dirac distribution :-

n-number of particles.

distinguishable

Ex :- gas molecules

→ classical statistics

→ Maxwell-Boltzmann distribution.

$$f(E) = e^{-E/kT}$$

Indistinguishable

Ex : Electrons, photons

→ Quantum statistics

Fermi-Drag Distribution

→ follow pauli's exclusion principle

→ Election ← ~~proton~~ fermions

Bose-Einstein Distribution

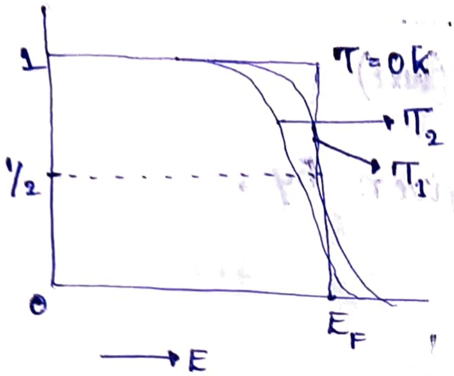
photons ↑ Bosons

$\rightarrow$ $f(E) = \dfrac{1}{1+e^{(E-E_f)kT}}$

$\downarrow$

distribution function.

$E_f \rightarrow$ Fermi energy.



$T_2 > T_1$

(A) $f(E) = ?$ $E > E_F$ & $T=0$

(B) $f(E) = ?$ $E < E_F$ & $T=0$

(C) $f(E) = ?$ $E = E_F$ & $T>0$

a) 0   b) ∞   c) 1/2   d) 1.

A) $f(E) = \dfrac{1}{1+e^{(+ive)/0}} = \dfrac{1}{1+e^{+\infty}}$

$= \dfrac{1}{\infty} = 0.$

B) $f(E) = ?$

$f(E) = \dfrac{1}{1+e^{-/0}} \Rightarrow \dfrac{1}{1+e^{-\infty}}$

$\dfrac{1}{1+e^{1/\infty}} = \dfrac{1}{1+0} = 1.$

c) $f(E) = 1/2.$

Numericals :-

Q.1) Caluculate the energy difference between the ground state and first excited state for an electron in a box of length 1A°.

Sol.
$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

$$E_2 - E_1 = ?$$

$$E_2 - E_1 = \frac{\pi^2\hbar^2}{2mL^2}[4-1]$$

$$= \frac{3\pi^2\hbar^2}{2mL^2}$$

$h = 6.63 \times 10^{-34}$ J.S.

$\frac{h}{2\pi} = \hbar \Rightarrow$ $L = 1A^\circ$
$= 1 \times 10^{-10} m$.

$m = 9.1 \times 10^{-31}$ kg

$\pi = 3.14$

$$= \frac{3(3.14)^2\left[\frac{6.63 \times 10^{-34}}{2(3.14)}\right]^2}{2(9.1 \times 10^{-31})(1 \times 10^{-10})^2}$$

Q.2) An electron is constrained to more in a one dimensional box of width 0.1nm. Find the first three eigen values & corresponding de-Broglie wavelength.

Sol. Ground state $n = 1$.

first excited state $\Rightarrow n = 2$.

$$H\psi = E\psi \rightarrow \text{Eigen fn.}$$

$E \rightarrow$ Eigen values.

$l = 0.1 nm = 0.1 \times 10^{-9} m$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2 m l^2}$$

$E_2 = 4E_1$

$E_3 = 9E_1$.

$$E_1 = \frac{\pi^2 \hbar^2}{2 m l^2}$$

$$= \frac{(3.14)^2 \left(\frac{6.63 \times 10^{-34}}{2 \times 3.14}\right)^2}{2 (9.1 \times 10^{-31}) (0.1 \times 10^{-9})^2}$$

$$= \frac{9.8596 \, (10.4091 \times 10^{-34})^2}{2 (9.1)(0.1) \, 10^{-31} \times 10^{-18}}$$

$$= \frac{9.8596 \times 10^{-68} \times 108.347281}{18.2 \times 10^{-49}}$$

18.2

$\frac{1}{18.2}$

$$= \frac{9.8596 \times 108.347281 \times 10^{-68} \times 10^{49}}{18.2}$$

$$= 0.5417362 \times 108.347281 \times 10^{-19}$$

$$E_1 = \underline{6.04 \times 10^{-18}}$$

$\because E_2 = 4 \times (6.04 \times 10^{-18})$

$= 24.16 \times 10^{-18}$

$\because E_3 = 9 \times 6.04 \times 10^{-18}$

$= 54.36 \times 10^{-18}$.

$\Rightarrow \lambda_n = \frac{h}{p_n} = \frac{h}{\sqrt{2mE_n}}$

$\lambda_1 = \frac{h}{\sqrt{2mE_1}}$

$= 2 \times 10^{-10} m$

$= 2 A^{\circ}$.

Q. In a position of momentum of ~~1 kilo e-volt~~ 1 kev are simultaneously measured if the position is located within $1 A^{\circ}$. What is the percentage of uncertainity in momentum.

position $\sim 1 A^{\circ}$

$\Delta x = 1 \times 10^{-10} m$

$\Delta p_x = ?$

% change $= \frac{\Delta p}{p} \times 100$

$E = 1 \times 10^{3} eV$

$= 1 \times 10^{3} \times 1.6 \times 10^{-19} J$

$p = \sqrt{2mE}$.

$p_n = \sqrt{2 (9.1 \times 10^{-31}) (1 \times 10^{3} \times 1.6 \times 10^{-19})}$

$= \cancel{5.27 \times 10^{-27}} kg\, m/s$.

$\Rightarrow \cancel{\frac{\Delta p}{p} \times 100} = 1.704 \times 10^{-23} kg\, m/s$.

$$\Delta x \cdot \Delta p_x \geq \hbar/2.$$

$$\Delta p_x \geq \frac{\hbar}{2} \cdot \frac{1}{\Delta x}.$$

$$\geq \frac{h}{4\pi \Delta x}.$$

$$\geq \frac{1.055 \times 10^{10} \times 10^{-34}}{2.} = \frac{\overset{0.52}{\cancel{1.055}} \times 10^{-24}}{\cancel{2}}$$

$$\geq 5.27 \times 10^{-27}\ kg/ms.$$ [ans. 3.1 %]

Q. Caluculate drift velocity of an $\bar{e}$ in an aluminium wire of diameter 0.9 mm, Carrying current of 6 Ampere. Assume that $4.5 \times 10^{+28}\ \bar{e}/m^3$. are available for conduction.

Sol. drift velocity $(v_d) = a\tau$.

$$a = \frac{F}{m}, \quad \tau = \frac{\bar{x}}{v_d}.$$

$d = 0.9mm.$

Atomic mass of Al = 26 amu.

$1 amu = \cancel{8}.\ 1.66 \times 10^{-27}\ kg.$

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$$I = neAV_d.$$

$$n = \frac{N}{V}$$