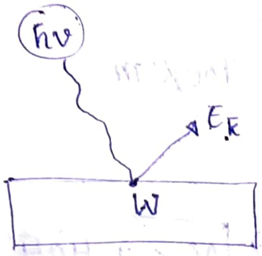
Passing of diff. ordered signals doesn't reach in same time.

- change in wavelength also causes distortion.

Unit-4 Quantum Mechanics.

De-Broglie concept : Matter waves.

└ there will be group of waves associated



Photons.

$h\nu = W + E_K$

$h\nu = h\nu_0 + E_K$

$E_K = \frac{1}{2} m v_{max}^2$

$$\boxed{\frac{1}{2} m v_{max}^2 = h(\nu - \nu_0)}$$

h - plancks const.
ν_0 - threshold frequency.
W - work func.
E_K - kinetic energy

→ Black Body radiation.

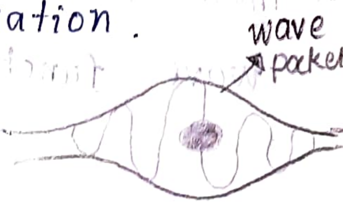
→ P.E.E

→ Crompton Effect.

De-Broglie's wavelength → m ○ → v $p = mv$

$\lambda = \frac{h}{p}$ → particle nature.

particle nature	Wave nature
- P.E.E	- Interference
- Compton Effect	- Diffraction
- Black Body radiation	- Polarisation.



$\Rightarrow \lambda = \frac{h}{p}$ $p = mv$.

→ momentum of the particle.

For relativistic particle $v \simeq c$

$p = \gamma mv$.

Where, γ - relativistic factor.

$$\boxed{p = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$$

⇒ <u>Different forms of De-Broglie relations</u> :-

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE_k}} = \frac{h}{\sqrt{2mqV}} = \frac{h}{\sqrt{3mkT}}$$

$$E_k = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE_k}$$

$E_k = \frac{3}{2}kT$

$E_k = \frac{1}{2}kT$

$E_k = qV$
↓
kinetic energy.

q → charge
V - potential diff.
k - Boltzmann cons.

<u>Matter wave</u> :- wave associated with material particles.

+ <u>Schröndinger's Equation</u> :- (Ψ) wave function.

+ Mathematical representation of ~~wave~~ matter waves associated with particles is known as wave f^n.

→ The quantity whose variations make up matter waves, known as wave f^n.

→ There is no physical Significance of wave function.

$$\psi = A + iB$$

A → real

iB → imaginary part.

$$|\psi^2| = \psi^* \psi = (A - iB)(A + iB) = A^2 + B^2$$

$= +ve$, real.

$|\psi^2|$ → probability density

$$|\psi^2|^2 dv = |\psi|^2 dx\, dy\, dz$$

dv → Volume element

→ probability of finding the particle at any point in space at any time.

⇒ Properties of well-behaved wave function :- { Should follow 3 steps.

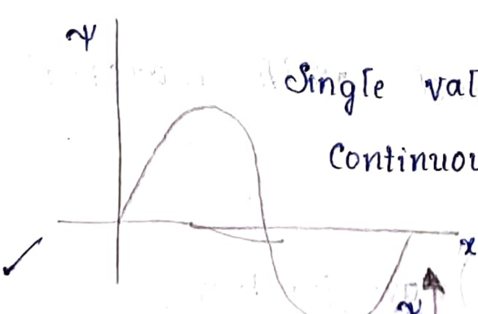
1. ψ must be normalizable.

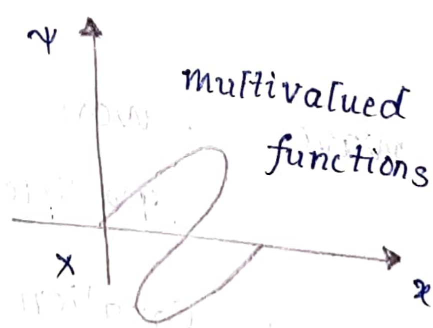
$$\int_{-\infty}^{+\infty} |\psi|^2 dx = 1$$

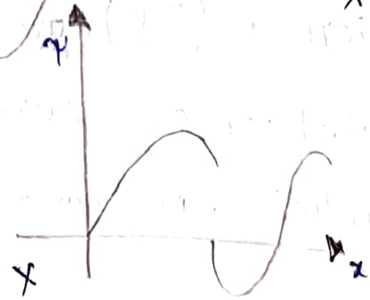
⇒ $\int_{-\infty}^{+\infty} \psi_1 \psi_2 = 0$

→ ψ_1 & ψ_2 are orthogonal

2. ψ must be continuous & Single value.





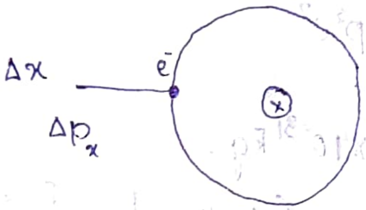


3. Derivative of wave function i.e $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z}$. must be Continuous & Singled values.

4. $\psi \to 0$ at $\left.\begin{matrix} x \\ y \\ z \end{matrix}\right\} \longrightarrow \pm\infty$.

These all four Properties must follow the wave function.

→ Uncertainity Principle :-



$$\underbrace{\Delta x}_{\text{uncertainity in position}} \; \overbrace{\Delta p_x}^{\text{uncertainity in momentum}} \geq \frac{\hbar}{2}$$

where, $\hbar = \frac{h}{2\pi}$.

$$\Delta x \cdot \Delta p_y = 0$$

$$\Delta z \cdot \Delta p_z \geq \frac{\hbar}{2}$$

$$\Delta y \cdot \Delta p_y \geq \frac{\hbar}{2}.$$

Electron can not exist inside the nucleaus :-

Let, Size of the nucleaus,

$$\Delta x = 10^{-14} m$$

Use uncertainity Principle,

$$\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2}$$

$$\Delta p_x \geq \frac{\hbar}{2\Delta x} \qquad \left[\hbar = \frac{h}{2\pi}\right]$$

$$\geq \frac{\hbar}{4\pi \cdot \Delta x}$$

$$\geq \frac{6.6 \times 10^{-34} \ J.S}{4 \times 3.14 \times 10^{-14}}$$

$$= 0.25 \times 10^{-20} \ kg \times m/s.$$

$$\simeq 10^{-20} \ Kg \ m/sec$$

→ Energy of the electron inside nucleaus :-

$$E = \sqrt{m_e^2 c^4 + p^2 c^2}$$

$$m_e = 9.1 \times 10^{-31} \ kg.$$

$$E = pc = (\Delta p_x) \cdot c \rightarrow \cancel{[m_e^2 c^4 \approx 0]} \quad [m_e^2 c^4 \approx 0].$$

$$= 10^{-20} \cdot 3 \times 10^{8} \ Joules.$$

$$= \frac{10^{-20} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 10^{6}} \ M_e V.$$

$$= 18.7 \ M_e V.$$

→ The energy of B-particle coming from the nucle~~a~~us is very small $(\sim 0.51 \ M_e V)$.

~~The B-Par~~ → B-particle is equivalent to electron.

→ Electron cannot exist in the nucle~~a~~us due to very small energy by B-particle.

→ proton/Neutron can exist in the nucleus :-

Let, Size of nucleus,

$$\Delta x = 10^{-14} m$$

use uncertainity principle,

$$\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2}$$

[ħ → h cross]

$$\hbar = \frac{h}{2\pi}$$

$$\Delta p_x \geq \frac{\hbar}{2\Delta x}$$

$$\geq \frac{h}{4\pi \Delta x}$$

$$\geq \frac{6.6 \times 10^{-34} J.s}{4 \times 3.14 \times 10^{-14}} = 0.525 \times 10^{-20} kg\,m/s$$

$$\approx 10^{-30} kg\,m/sec$$

→ Uncertainity Principle :-

Energy of the proton inside nucleus,

$$E = \sqrt{m_p^2 c^4 + p^2 c^2}$$

$$E = \sqrt{(1.67 \times 10^{-27})^2 (3 \times 10^8)^4 + (10^{-20})^2 (3 \times 10^8)^2}$$

$$E = 15.03 \times 10^{-11} \text{ Joule}$$

$$\frac{15.03 \times 10^{-11}}{1.6 \times 10^{-19} \times 10^6} = 9.39 \times 10^2$$

$$= 939 \text{ MeV}.$$

Q. Calucculate rest mass energy of e^-, p, n ?

Proton :-

$$E = mc^2.$$

$$= 1.67\times10^{-27}\times(3\times10^8)^2 \text{ Joule}.$$

$$= \frac{1.67\times10^{-27}\times9\times10^{16}}{1.6\times10^{-19}\times10^6} \text{ MeV}.$$

$$= \frac{15.03\times10^{-11}}{1.6\times10^{-19}\times10^6} \text{ MeV} = 937 \text{ MeV}.$$

∴ This energy is equivalent to rest mass energy of proton. So, proton can exist in nucleus.

Electron :-

$$E = mc^2.$$

$$= 9.1\times10^{-31}\times(3\times10^8)^2$$

$$= 9.1\times10^{-31}\times9\times10^{16}$$

$$= \frac{81.9\times10^{-33}}{1.6\times10^{-19}\times10^6} \text{ MeV}.$$

$$= 0.511 \text{ MeV}.$$

Q) Prove that group velocity of the particle is equal to particle velocity and phase velocity of the particle is equal to $v_p = \frac{c^2}{v}$ ⌈ v - particle velocity

⌈ group velocity — $v_g = v$

phase velocity — v_p

c - speed of light ⌋

→ Group velocity (v_g): velocity of group of waves (or) matter waves is known as group velocity.

$\boxed{\omega = 2\pi\nu}$

$\Delta\omega \to$ change in angular frequency

$$v_g = \frac{d\omega}{dk} \quad v_g = \frac{d\omega}{dk} = \frac{\Delta\omega}{\Delta k}$$

$\Delta k \to$ change in wave vector.

$\boxed{v_g = v}$ → particle vel.

~~→ velocity of single wave in a~~ group/ ~~Individual waves~~ ~~of waves is known grou phase velocity~~

→ phase velocity (v_p): velocity of an individual waves of waves in group/matter waves is known as phase velocity.

$$v_p = \frac{c^2}{v}$$

⇒ Group velocity: $\omega = 2\pi\nu$

$$\omega = \frac{2\pi E}{h} \Rightarrow \omega = \frac{2\pi mc^2}{h\sqrt{1-\frac{v^2}{c^2}}} \longrightarrow (3)$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} \quad \because \lambda = \frac{h}{p}$$

$$k = \frac{2\pi mv}{\sqrt{1-v^2/c^2}} \longrightarrow (4)$$

$$p = \gamma m v = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$$

Phase velocity :- $v_p = \frac{\omega}{k} = \frac{c^2}{v}$ from ③ & ④.

Group velocity :- $v_g = \frac{d\omega}{dv} \times \frac{dv}{dk} = \left(\frac{(d\omega/dv)}{(dk/dv)}\right)$.

from ③,

$$\frac{d\omega}{dv} = \frac{2\pi mc^2}{h} \cdot \frac{d}{du}\left(1-\frac{v^2}{c^2}\right)^{-1/2}.$$

$$= \frac{2\pi mc^2}{h}\left(\frac{-1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(0-\frac{2v}{c^2}\right)$$

$$= \frac{2\pi mv}{h}\left(1-v^2/c^2\right)^{-3/2} \longrightarrow ⑥$$

$$\frac{dk}{dv} = \frac{2\pi m}{h} \cdot \frac{d}{dv}\left[v\left(1-\frac{v^2}{c^2}\right)^{-1/2}\right].$$

$$= \frac{2\pi m}{h}\left[v\left(\frac{-1}{2}\right)\cdot\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(0-\frac{2v}{c^2}\right)+\left(1-\frac{v^2}{c^2}\right)^{-1/2}\right]$$

$$= \frac{2\pi m}{h}\left[1-\frac{v^2}{c^2}\right]^{-3/2}\left[\frac{v^2}{c^2}+1-\frac{v^2}{c^2}\right]$$

$$k = \frac{2\pi}{h}$$

$$= \frac{2\pi mv}{h\sqrt{1-v^2/c^2}}$$

$$= \frac{2\pi m}{h}\left[1-\frac{v^2}{c^2}\right]^{-3/2} \longrightarrow ⑦$$

Q. Calculate phase and group velocity of a pa non-relativistic particle

Q. Find the K.E of ~~ph~~ proton whose de-broglie's wave length is 1fm [$1fm = 10^{-15} m$].

Sol. $\lambda = 10^{-15} m$.

proton $\Rightarrow m = 1.67 \times 10^{-27} kg$.

Rest mass energy of the proton,

$E = mc^2 = 938 MeV = 0.938 GeV \Rightarrow E = \sqrt{m^2c^4 + p^2c^2}$

$pc = \frac{h}{\lambda} c = \frac{6.63 \times 10^{-34}}{10^{-15}} \times 3 \times 10^8$

$= \cancel{1.2410\ GeV}$

[$1 Ge = 10^{-9} m$]
gega

$= \frac{\quad}{1.6 \times 10^{-19} \times 10^9} = 1.2410\ GeV.$

$pc > E \rightarrow$ Relativistic.

$pc < E \rightarrow$ Non-Relativistic.

Total energy for relativistic particles :-

$E = \sqrt{m^2c^4 + p^2c^2}$

$= \sqrt{(0.938)^2 + (1.2410)^2} = 1.555\ GeV.$

$E = K.E + \text{rest mass energy}$

$1.555 = K.E + \cancel{1.2410}\ 0.938$

$K.E = 1.555 - \cancel{1.2410}\ 0.938$

$\cancel{K.E = 0.314}$ $\boxed{K.E = 0.617\ Ge}$

Q. An electron has a de-Broglie wave length of 2pm find it's K.E, V_g, V_p. $\left[2pm = 10^{-12} \times 2\ m\right]$

↳ pico m

Sol. $pc = \frac{h}{\lambda} \cdot c$

$$= \frac{6.63 \times 10^{-34}}{2 \times 10^{-12}} \cdot 3 \times 10^{8} \text{ Joule}$$

$$= \frac{19.89 \times 10^{-26}}{1.6 \times 10^{-19} \times 10^{6}} \text{ MeV}$$

$\Rightarrow E_0 = 0.51$ MeV $\Rightarrow \boxed{p_c > E_0}$ → Relativistic.

$$mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$E = \frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{E_0}{E}$$

$$1 - \frac{v^2}{c^2} = \left(\frac{E_0}{E}\right)^2$$

$$\frac{v^2}{c^2} = 1 - \left(\frac{E_0}{E}\right)^2$$

$$\boxed{v = c\sqrt{1 - \left(\frac{E_0}{E}\right)^2}}$$

$\therefore$ $v = 0.771c$

$\therefore$ $v_p = 1.30c$.

→ **Operator** :- An Operator tells us what kind of operation we are going to follow on the function that follows it.

Ex :- $\frac{d}{dx} f(x)$

$\frac{d}{dx}$ → Operator.

Free particle wave function :-

$$\psi = A e^{-i/\hbar [Et - px]} \longrightarrow (1)$$

[particle moving along $+x$ axis with energy & momentum by p].

$$y = A \sin(\omega t - kx)$$

$\omega = 2\pi\nu$

$\omega = \frac{2\pi E}{h}$ $E = h\nu$.

$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h}$

$\lambda = \frac{h}{p}$

Differentiate eq (1) ~~with~~ w.r.t time.

$$\frac{d\psi}{dt} = A e^{-i/\hbar [Et - px]} \times \left[\frac{-iE}{\hbar}\right].$$

$$\frac{d\psi}{dx} = \psi \left[\frac{-iE}{\hbar}\right]$$

$$\Rightarrow E\psi = -\frac{\hbar}{i} \cdot \frac{d\psi}{dx} \times \frac{i}{i}.$$

$$E\psi = i\hbar \cdot \frac{d\psi}{dt}.$$

$$\boxed{\hat{E} = i\hbar \frac{d}{dt}}$$

Differentiate eq ① w.r.t 'x':

$$\frac{d\psi}{dx} = Ae^{-i/\hbar[Et - px]}\left[\frac{ip}{\hbar}\right] \longrightarrow ②$$

Diff. eq ② w.r.t 'x'

$$\frac{d^2\psi}{dx^2} = Ae^{-i/\hbar[Et - px]}\left[\frac{ip}{\hbar}\right]^2 \longrightarrow ③$$

from ②

$$\frac{d\psi}{dx} = \psi\left(\frac{ip}{\hbar}\right).$$

$$p\psi = \frac{\hbar}{i}\cdot\frac{d\psi}{dx}\cdot\frac{i}{i}.$$

$$p\psi = -i\hbar.\frac{d\psi}{dx}.$$

$$\boxed{\hat{p} = -i\hbar\frac{d}{dx}}$$ momentum Operator.

from ③,

$$\frac{d^2\psi}{dx^2} = \psi\left(\frac{-p^2}{\hbar^2}\right).$$

$$\Rightarrow p^2\psi = -\hbar^2\frac{d^2\psi}{dx^2}.$$

Kinetic energy of the particle,

$$KE\,\psi = \frac{p^2\psi}{2m}.$$

$$KE\,\psi = \frac{-\hbar^2}{2m}\cdot\frac{d^2\psi}{dx^2}.$$

Kinetic. E Operator. $$\boxed{\hat{KE} = \frac{-\hbar^2}{2m}\times\frac{d^2}{dx^2}}.$$

Hamiltonian is given by.

$$H = K.E + U \qquad [U - P.E].$$

$$H\psi = KE\psi + U\psi$$

$$H\psi = \frac{-\hbar^2}{2m} \cdot \frac{d^2\psi}{dx^2} + U\psi$$

$$\boxed{\hat{H} = \frac{-\hbar^2}{2m} \cdot \frac{d^2}{dx^2} + U} \rightarrow \text{Hamiltonian operator.}$$

$\hat{u} = U$ → potential energy operator.

$\hat{x} = x$ → position operator.

1. $\hat{P} = -i\hbar \frac{d}{dx}$
2. $\hat{E} = i\hbar \frac{d}{dt}$
3. $\hat{KE} = \frac{-\hbar^2}{2m} \cdot \frac{d^2}{dx^2}$
4. $\hat{H} = \frac{-\hbar^2}{2m} \cdot \frac{d^2}{dx^2} + u.$
5. $\hat{u} = u$
6. $\hat{x} = x.$

→ Schrödinger's Equation :- two types.

i. Time independent Schrödinger Eqn.

[∵ stationary state Schrödinger eqn].

ii. Time dependent Schrödinger eqn.

$$H\psi = i\hbar \frac{d\psi}{dt}.$$

$$H\psi = E\psi.$$

Proof :- Hamiltonian is given by,

$$H = K.E + U. = E$$

multiply by wave fn,

$$H\psi = K\psi + U\psi = E\psi.$$

$$\Rightarrow \boxed{\frac{-\hbar^2}{2m} \cdot \frac{d^2\psi}{dx^2} + U\psi = E\psi}$$

↳ Time independent S.E.

But, $E\psi = i\hbar \frac{d\psi}{dt}$.

$$\boxed{\frac{-\hbar^2}{2m} \cdot \frac{d^2\psi}{dx^2} + U\psi = i\hbar \frac{d\psi}{dt}}$$ → Time dependent S.E.

$$\frac{-\hbar^2}{2m} \cdot \frac{d^2\psi}{dx^2} + U\psi = E\psi.$$

$$\frac{d^2\psi}{dx^2} - \frac{2mU\psi}{\hbar^2} = \frac{-2mE}{\hbar^2}\psi.$$

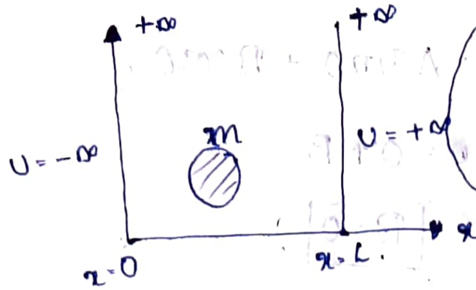
$$\frac{d^2\psi}{dx^2} + \frac{2mU\psi}{\hbar^2} = \frac{-2mE}{\hbar^2}\psi.$$

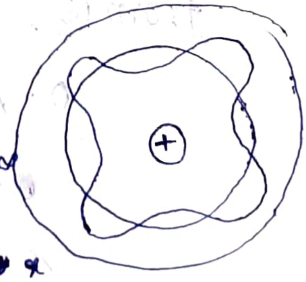
Time independent Schrondinger eqn

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}[E - U]\psi = 0}$$

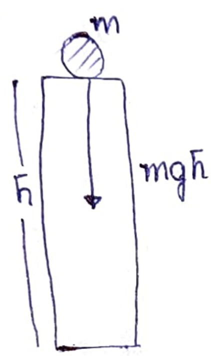
$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - U)\psi = 0.$$

particle in a Box problem :-





- Consider, a particle of mass 'm' confined in a box of size 'L'. The potential is (∞) infinite at the wall of the box as well as outside the box.



- STM – Scanning tunneling magnetic tube.
- Quantum tunneling magnetic tube

- The ~~particle~~ box height is infinite.

- Schrödinger eqn of particle :-

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}[E-U]\psi = 0.$$

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0. \qquad \boxed{\because U=0}$$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \longrightarrow (1)$$

where, $k^2 = \frac{2mE}{\hbar^2} \longrightarrow (2)$

The general solution of eq(1) is given by,

$$\psi = A\sin kx + B\cos kx \longrightarrow (3)$$

from ③,

$$0 = A\sin 0 + B\cos 0.$$

$$0 = 0 + B$$

$$\boxed{B = 0}.$$

from ③ $\Rightarrow$ $\psi = A\sin kx$.

Now, $\psi = 0$, at $x = \hbar$.

$$0 = A\sin kL.$$

$$\sin kL = 0 = \sin 0.$$

$$kL = n\pi$$

$$\boxed{k = \frac{n\pi}{L}},$$

from eq ②,

$$k^2 = \frac{2mE}{\hbar^2}$$

$$\left(\frac{n\pi}{L}\right)^2 = \frac{2mE}{\hbar^2}.$$

$$\frac{n^2\pi^2}{L^2} = \frac{2mE}{\hbar^2}.$$

$$\boxed{E_n = E = \frac{n^2\pi^2\hbar^2}{2mL^2}}.$$

where, $n = 1, 2, 3, 4, \ldots$

Q. Find the energy of the e^- $(n=1, n=2)$ moving in a infinetely high potential box of width $1 Å$.

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

$$E_1 = \frac{1^2 \pi^2 h^2}{8\pi^2 m L^2} = \frac{h^2}{8mL^2}$$

$$E = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (1 \times 10^{-10})^2}$$

$$= 6.02 \times 10^{-18} J$$

$$E_2 = \frac{2^2 \pi^2 h^2}{8\pi^2 m L^2} = 4\left(\frac{h^2}{8mL^2}\right)$$

$$E_2 = 4 \times E_1 = 4 \times 6.02 \times 10^{-18} J$$

Q. Calucculate the Smallest possible

Q. If an excited state of a hydrogen atom has a life time of $2.5 \times 10^{-14}s$. What is the mimmum error with which the energy of this state can be measured?

Sol. Use uncertainity Principle.

$$\Delta E \cdot \Delta t \geq \hbar/2$$

$$\Delta E \cdot (2.5 \times 10^{-14}) \geq \frac{h}{4\pi}$$

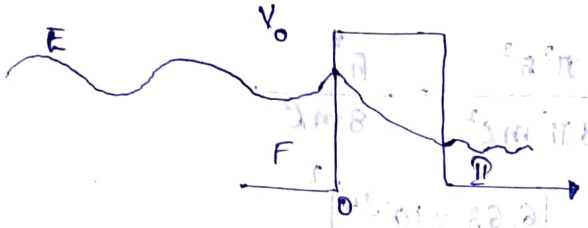
$$\Delta E \geq \frac{h}{4\pi \times 2.5 \times 10^{-14}}$$

$$\geq \frac{6.63 \times 10^{-14}}{4 \times 3.14 \times 2.5 \ldots}$$

$= 2.11 \times 10^{-21} J.$

→ **Quantum Tunneling & its applications :-**

T → transmission probability $\boxed{\text{Nanotechnology}}$



$$T = e^{-2k_2 L}.$$

where $k_2 = \sqrt{\frac{2m(U-E)}{\hbar}}$

If a particle has energy (E) less than barrier energy (V_o) the classically ·

It can not cross the barrier but quantum mechanically even if particle energy is less than barrier energy, the particle can cross the barrier.

STM

Scanning Tunneling Microscope.

$\boxed{\text{U - potential Energy}}$

Arthur Beiser

Modrn phy