Assignment-5

Group 22

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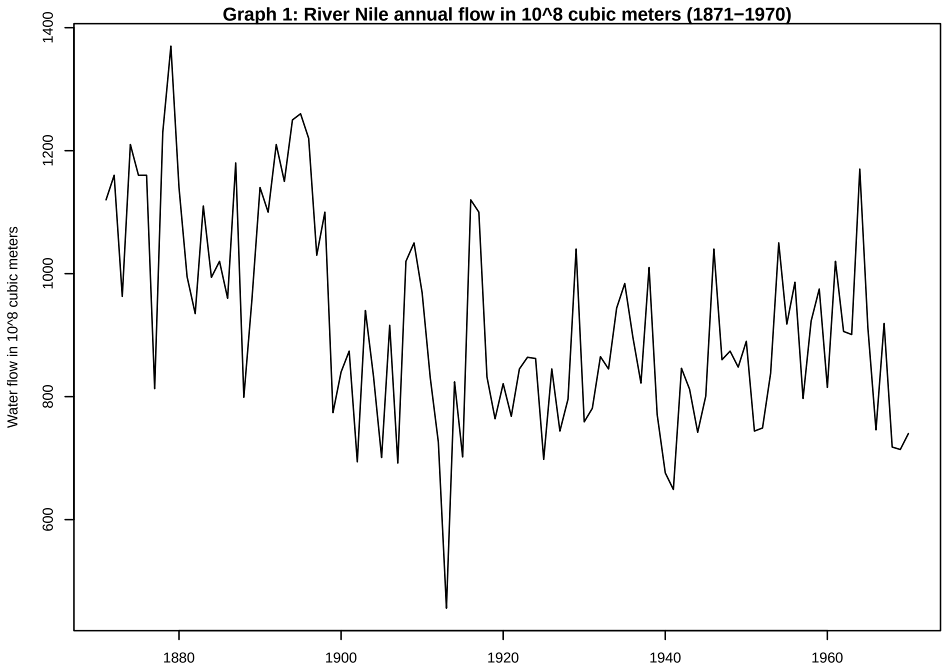
Question 1 - Filtered Estimates

The following plot shows the annual flow in the river Nile, the data we are going to model using a dynamic linear model

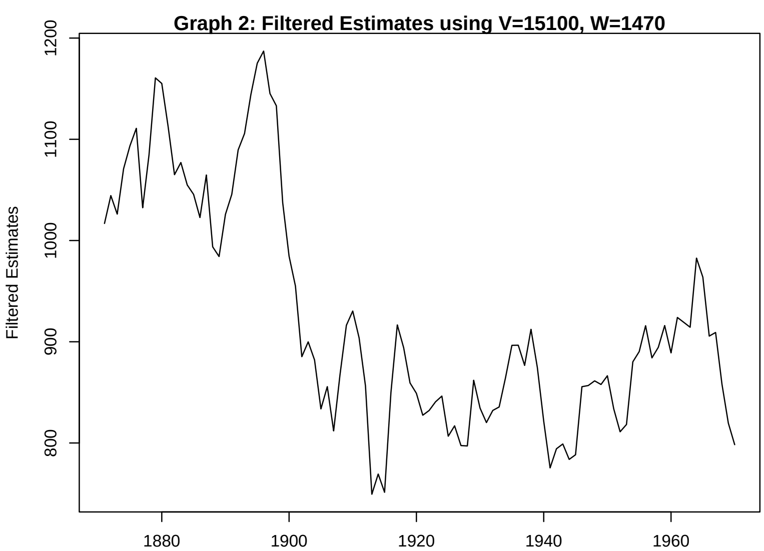


Let us consider the following random walk plus noise model to be applied to this Nile data:

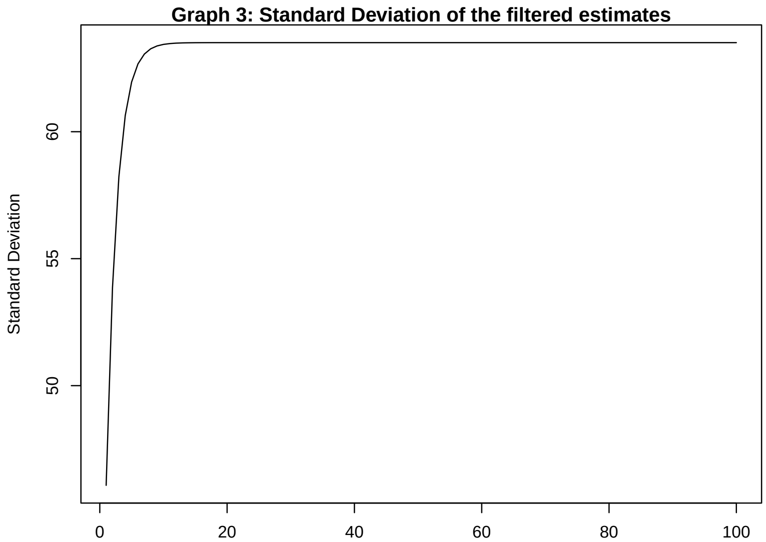
$$
\begin{aligned}
Y_t &= \theta_t + v_t \qquad v_t \overset{i.i.d.}{\sim} N(0, V) \\
\theta_t &= \theta_{t-1} + w_t \qquad v_t \overset{i.i.d.}{\sim} N(0, W)
\end{aligned}
$$

We will set V = 15100 and W = 1470 and the initial distribution $\theta_0 \sim N(1000, 1000)$ for our model

Plotting the filtered estimates we get

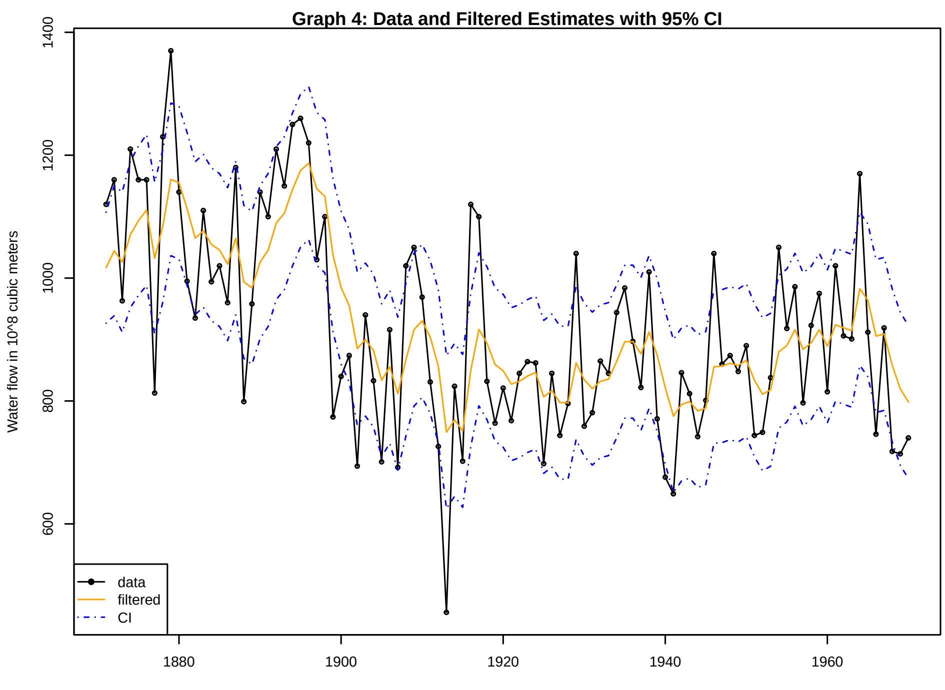


We can compute the variance as well. It is plotted as follows



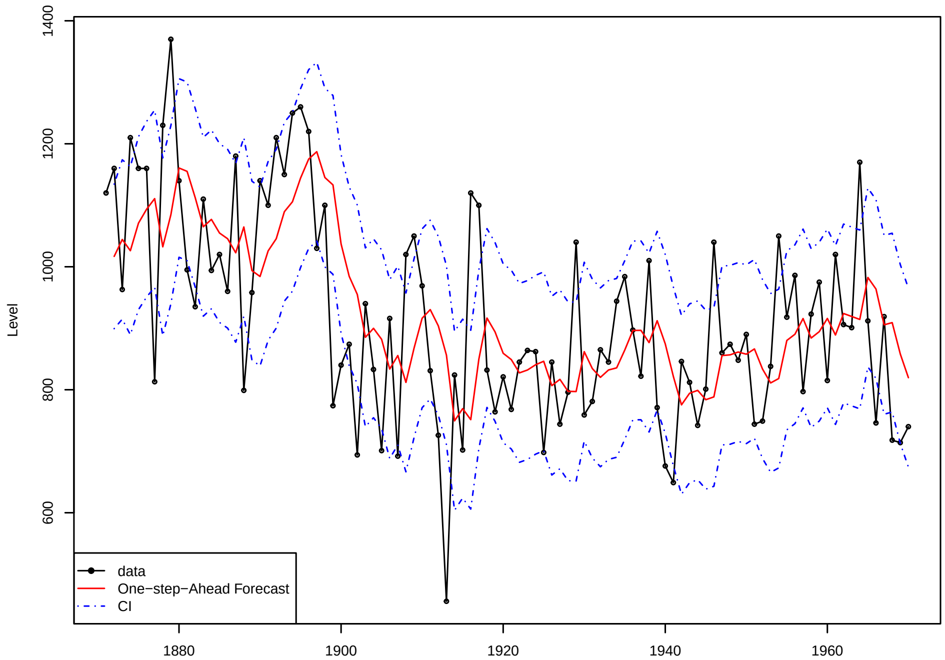
As we can see from the graph above, the standard deviations of the filtering estimates decrease as time passes by, and converges to a value above 60. This might reflect the low guess of the standard deviation C0 (sqrt(1000) = 31) made at the beginning . As new observations arrive the standard deviation converges to the stable value.

We finally plot the whole data along with the filtered estimates with 95% confidence interval



Question 2 - One-step ahead forecasts

Below is the plot for the one-step ahead forecast with 95% confidence intervals



Question 3 - Signal to noise ratio

The signal-to-noise ratio are an important factor which set the weight of the most recent data point for the filtered estimates, with a larger value putting more weight to the recent data-point.

To test the effect of signal-to-noise ratio on our model we test the following models.

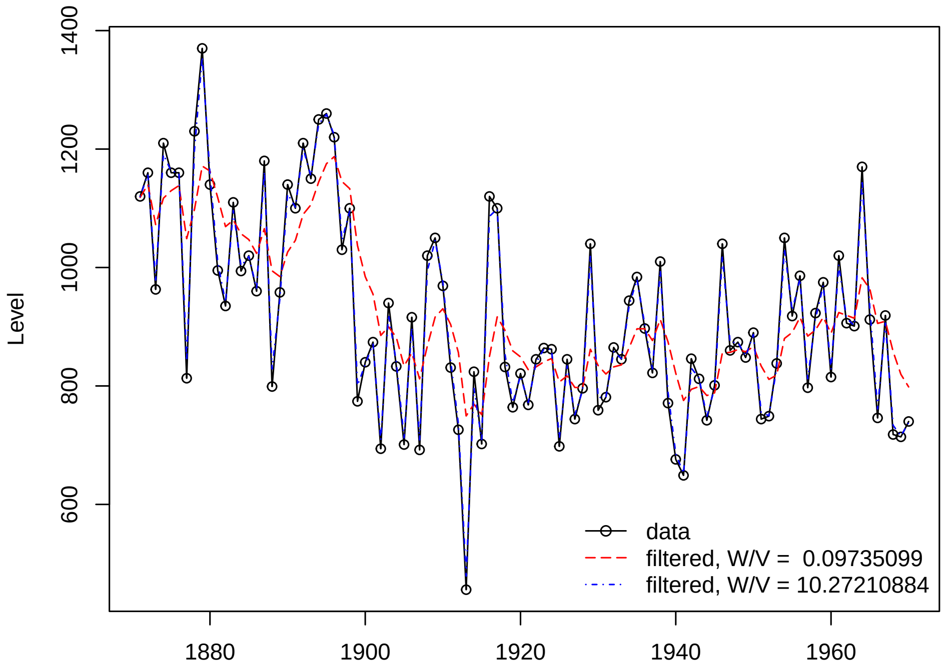
Model 1 (Signal-to-noise ratio = 0.097) -

$$\begin{aligned} Y_t &= \theta_t + v_t \qquad v_t \overset{i.i.d.}{\sim} N(0, 15100) \\ \theta_t &= \theta_{t-1} + w_t \qquad v_t \overset{i.i.d.}{\sim} N(0, 1470) \end{aligned}$$

Model 2 (Signal-to-noise ratio = 10.27)-

$$\begin{aligned} Y_t &= \theta_t + v_t \qquad v_t \overset{i.i.d.}{\sim} N(0, 1470) \\ \theta_t &= \theta_{t-1} + w_t \qquad v_t \overset{i.i.d.}{\sim} N(0, 15100) \end{aligned}$$

Plotting the filtered estimates of both the models we find



We can see from the graph that when we increase the signal to noise ratio (W/V) substantially, the filtered estimates follow the data more closely. This can be explained, because when we increase W/V we increase the weight on the most recent data point. In the plot, given we increase the ratio hundredfold the estimates almost match the most recent points.