M-1 QUESTION BANK(IMP)

UNIT-I

MATRICES

Short answer questions

1. Define symmetric,Skew-symmetric and Orthogonal matrices
2. Define Hermitian,Skew-Hermition and Unitary matrices
3. Define rank of a matrix
4. Define echelon form of a Matrix.
5. Define normal form of a matrix
6. State the conditions for consistency of the system of equations AX=B
7. State the conditions for consistency of the system of equations AX=0
8. Define linearly dependent and linearly independent vectors
9. Find the value of k such that rank of A= $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & k & 1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ is 2
10. Define elementary matrix with an example

LONG ANSWER QUESTIONS

1. Find rank of the following matrices by reducing into Echelon form:

i) $\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$ ii) $\begin{bmatrix} 2 & -4 & 3 & -1 & 0 \\ 1 & -2 & -1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$

2. Find rank of the following matrices by reducing into Normal form:

i) $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ ii) $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$

3. Determine the non-singular matrices P and Q such that PAQ is in Normal form and find its rank:

i) $\begin{bmatrix} 2 & 1 & -3 & -6 \\ 2 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ ii) $\begin{bmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{bmatrix}$

4. Test for consistency and solve if the equations are consistent:

i) $x + 2y + 2z = 2\,, 3x - 2y - z = 5\,, 2x - 5y + 3z = -4\,,\; x + 4y + 6z = 0$

ii) $x + y + z = 3\,,\; 3x - 5y + 2z = 8\,,\; 5x - 3y + 4z = 14.$

5. Solve the system of equations:

i) $x + y + w = 0\,,\ y + z = 0\,, x + y + z + w = 0\,,\ x + y + 2z = 0$

ii) $2x - y + 3z = 0\,,\ 3x + 2y + z = 0\,,\ x - 4y + 5z = 0$

6. Determine whether the following equations will have non-trivial solutions, if so solve them:

i) $x + 3y - 2z = 0\,,\ 2x - y + 4z = 0\,,\ x - 11y + 14z = 0$

ii) $4x + 2y + z + 3w = 0\,, 6x + 3y + 4z + 7w = 0\,, 2x + y + w = 0$

7. For what values of λ and μ the system of equations:
$x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$ has

i) No solution

ii) Unique solutions

iii) Infinite number of solutions

8. Find the value of λ for which the system of equations
$3x - y + 4z = 3\,, x + 2y - 3z = -2\,, 6x + 5y + \lambda z = -3$ will have infinite number of solutions and solve them with that value of λ.

9. Find the values of a and b for which the system of equations
$x + y + z = 3\,, x + 2y + 2z = 6\,, x + 9y + az = b$ will have

i) No solution

ii) Unique solution

iii) Infinitely many solutions

10. Solve the following system using Gauss Elimination method

i) $x - 8y + z = -5\,, x - 2y + 9z = 8\,, 3x + y - z = -8$

ii) $3x + y + 2z = 3\,, 2x - 3y - z = -3\,, x + 2y + z = 4$

11. Solve the following system of equations using Gauss Jordan method:
$10x + y + z = 12\,, 2x + 10y + z + 13\,, x + y + 5z = 7$

UNIT-II
EIGEN VALUES,EIGEN VECTORS AND QUADRATIC FORMS

Short answer questions

1. Define Eigen values and eigen vectors of a matrix
2. State the condition of diagonalizability of a matix
3. State Caley –Hamilton theorem and write its applications
4. Write any five properties of eigen values
5. If A= $\begin{bmatrix} 5 & 2 & 3 \\ 0 & -1 & 4 \\ 0 & 0 & 6 \end{bmatrix}$, find eigen values of i) 5A ii)A^4 iii) adjA iv) A^{-1}
6. Prove that Eigen values of unitary matrix are of constant modulus
7. Define Modal matrix and Spectral matrix
8. i) If λ is an eigen values of an Orthogonal matrix then $1/\lambda$ is also its Eigen value
   ii) Prove that if λ is an Eigen value of a matrix A then λ+KI is Eigen value of A+KI
9. Define Quadratic form, canonical form, index, signature and nature of quadratic form
10. Find index, signature and nature of quadratic form 2xy+2yz+2zx
11. Define linear Transformation
12. If λ is an Eigen value of a matrix A then prove that λ^2 is an Eigen value of A^2
13. Define Algebraic multiplicity and Geometric multiplicity of an Eigen value of a matrix
14. Show that Eigen values of unitary matrix are of unit modulus.

LONG ANSWER QUESTIONS

1. Find the Eigen values and the corresponding Eigen vectors of the matrices: $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ $\begin{bmatrix} 3 & 2 & 2 \\ 1 & 2 & 2 \\ -1 & -1 & 0 \end{bmatrix}$
2. Prove that the Eigen values of Hermitian matrix are all real.
3. Prove that the Eigen values of real symmetric matrix are all real.
4. Diagonalize the matrix A= $\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$
5. Diagonalize the matrix $\begin{bmatrix} 9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ Hence find A^{-1}.
6. If A= $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ verify Caley-Hamiltton theorem, Hence find A^{-1}.
7. Disscuss the nature of the Quadratic form, Find its Rank, Index, and Signature.
   (1) $x^2+4xy+6xz-y^2+2yz+4z^2$
   (2) $2x^2+2y^2+2z^2+2yz$
8. Reduce the Quadratic form $10x^2+2y^2+5z^2-10zx+6yz-4xy$ the Canonical form. Also find the transformation.

9) Reduce the Quadratic form $2x^2+5y^2+3z^2+4xy$ to Canonical form. Also find the transformation.

10) Reduce the Quadratic form $3x^2+2y^2+3z^2-2xy-2zx$ to the Canonical form by Orthogonal transformation. Also find the transformation.

11) Reduce the Quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2xy$ to the Canonical form by Orthogonal transformation. Also find the transformation.

12) Express the Hermitian matrix $A=\begin{bmatrix} 1 & -i & 1+i \\ i & 0 & 2-3i \\ 1-i & 2+3i & 2 \end{bmatrix}$

As P+iQ where P is a Real symmetric matrix and Q is a Real skew-symmetric matrix.

UNIT-IV
CALCULUS

Short answer questions:

1. State Rolle's theorem.
2. Verify Rolle'stheorm for f(x)=$|x|$ in $[\![-1,1]\!]$
3. State Lagrange's mean value theorem
4. Verify mean value theorem for f(x)=$x^{\frac{2}{3}}$ in $[-1,1]$
5. State Cauchy's mean value theorem
6. Verify Cauchy's mean value theorem for f(x)=x^2,g(x)=x^3 in $[1,2]$
7. Using Taylor,s series expand $(x-2)^4-3(x-2)^3+4(x-2)^2$ in powers of x
8. Write Taylor's series expansion of f(x) at x=a
9. Find the volume of the the solid generated by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$=1 about the major axis
10. Write the volume of the solid generated by y=f(x),x=g(y) formulas
11. Write the Surface areas of y=f(x),x=g(y) formulas
12. Define Beta and Gamma functions.

13.Compute the value of $\boldsymbol{\Gamma}(-\frac{11}{2})$

14.Prove that $\int_0^{\infty} e^{-y^{\frac{1}{m}}}dy = m\ \Gamma(m)$

15.Explain geometric interpolation of rolle's theorem

Long answer questions

1. Verify Rolle's theorem for f(x)=$\frac{sinx}{e^x}$ in $[0,\pi]$
2. Verify Rolle's theorem for f(x)=$log\frac{x^2+ab}{x(a+b0}$ in $[a,b]$
3. Verify Rolle's theorem for f(x)=$\frac{x^2-x-6}{x-1}$ in (-2,3)
4. Verify Lagrange's mean value theorem for f(x)=$\log_e x$ in $[1,e]$
5. Prove that $\frac{\pi}{3}-\frac{1}{5\sqrt{3}}$>$\cos^{-1}\frac{3}{5}$>$\frac{\pi}{3}$-$\frac{1}{8}$ using mean value theorem
6. Prove using mean value theorem $|sinu - sinv| \leq |u-v|$
7. Verify Cauchy's mean value theorem for f(x)=e^{-x},g(x)=e^x in $[2,6]$
8. Obtain the Maclaurin's series expansion of $\log_e(1+x)$
9. Verify Taylor's theorem for f(x)=$(1-x)^{\frac{5}{2}}$ with Lagrange's form of remainder upto 2 terms in the interval $[0,1]$
10. Expand logcosx about $\frac{\pi}{3}$ using Taylor's expansion

11. The curve y^2(a+x)= $x^2(3a-x)$ revolves about the x-axis.Find the volume of the solid generated
12. Find the surface area generated by the revolution of an arc of the catenary y=ccosh$\frac{x}{c}$ about the x-axis from x=0 to x=c
13. Find the surface area of the solid generated by the revolution of the ellipse $\frac{x^2}{a^2}$+$\frac{y^2}{b^2}$=1 about the x-axis
14. State and prove the relation between Beta and Gamma functions
15. To prove that $\Gamma(n)\ \ \Gamma(1-n)=\frac{\pi}{sinn\pi}$
16. Prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$
17. Evaluate $\int_0^{\infty} x^6 e^{-2x}\, dx$
18. Evaluate $\int_0^{\frac{\pi}{2}} sin^5\theta cos^{\frac{7}{2}}\theta d\theta,$
19. 19.Prove B(m, $\frac{1}{2}$) $=2^{2m-1}B(m,m)$

UNIT-V
PARTIAL DIFFERENTIATION

Short answer questions:

1.State Euler's theorem.

2.Write chain rule of partial of differentiation if z=f(x,y) where x= u(r,s) and y= v(r,s)

3.Define Homogeneous function.

4.Define Jacobian and functional dependence

5.If x=uv , y=u/v then find J

6.If x=u(1+v),y=v(1+u) then find $J(\frac{(X,Y)}{(U,V)})$

7.If $u = x^2 - 2y, v = x + y + z, w = x - 2y + 3y$ find $J(\frac{(u,v,w)}{(x,y,z)})$

8.State and prove the necessary and sufficient conditions for extreme of a function `f' of two variables

9.Find the points on the surface z^2=xy+1 that are nearest to the origin.

10.Find the stationary points of u (x, y) = Sin x Sin y Sin (x + y) where 0 < x <Π,0 < y <Π and find the maximum u

Long answer questions

1. State and prove Euler' theorem of Homogeneous function
2. If $u = f(r, s, t)$, where $r = \frac{x}{y}, s = \frac{y}{z}$ and $t = \frac{z}{x}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$
3. If u=$\tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$=sin2u
4. If $x^x y^y z^z$= c, then show that at x = y = z, $\frac{\partial^2 z}{\partial x \partial y} = -\{x\log(ex)\}^{-1}$
5. If μ = log (x^3+y^3+z^3-3xyz) prove that

   $\mu_x + \mu_y + \mu_z = 3(x+y+z)^{-1}$ and $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 \mu = \frac{-9}{(x+y+z)^2}$
6. Verify chain rule for Jacobian for the following function $x = u, y = u\tan v, z = w$
7. If $u = \frac{yz}{x}; v = \frac{zx}{y}; w = \frac{xy}{z}$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4.$
8. If $x = r\sin\theta\cos\emptyset, y = r\sin\theta\sin\emptyset, z = r\cos\theta$, show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)} = r^2\sin\theta$
9. If $u = x^2 - 2y,\ v = x + y + z, w = x - 2y + 3z$ Find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$
10. If $x = u(1 - v), y = uv$ prove that $JJ^* = 1$

11. If $x + y + z = u , y + z = uv and z = uvw$ then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = u^2 v$

12. If $u = x^2 - y^2, v = 2xy where x = r\cos\theta, y = r\sin\theta$. Show that $\frac{\partial(u,v)}{\partial(r,\theta)} = 4r^3$

13. If u = f(r,s,t) where r = x / y, s = y / z and t = z / x show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$

14. Show that the functions $u = x + y + z$, $v = x^2+y^2+z^2-2xy-2zx-2yz$ and $w = x^3+y^3+z^3-3xyz$ are functionally related. Find the relation between them.

15. Show that the functions $u = xy + yz + zx$, $v = x^2+y^2+z^2$ and $w = x+y+z$ are functionally related. Find the relation between them.

16. Divide a given positive number n into three parts such that their product is maximum.

17. Locate the stationary points and examine their nature of the following functions :
$u=x^4 + y^4 - 2x^2 + 4xy - 2y^2$, $(x > 0, y > 0)$.

18. A rectangular box open at the top is to have a volume of 32 cubic ft. Find the dimensions of the box, requiring least material for its construction.

19. Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

20. Given that $x + y + z = a$, find the maximum value of $x^m y^n z^p$.

21. Find the minimum value of $x^2 + y^2 + z^2$ given $x + y + z = 3a$.

22. Find the maximum value of $x^2 y^3 z^4$ subject to the condition $2x + 3y + 4z = a$.
