

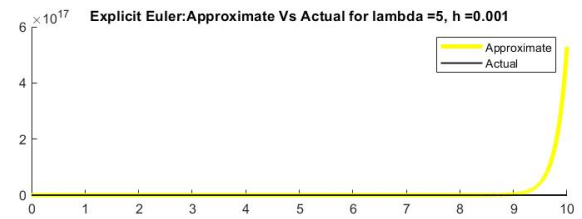
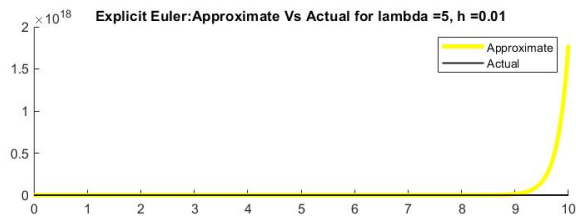
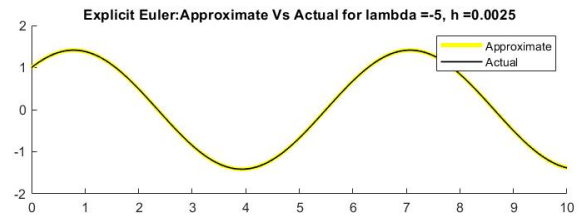
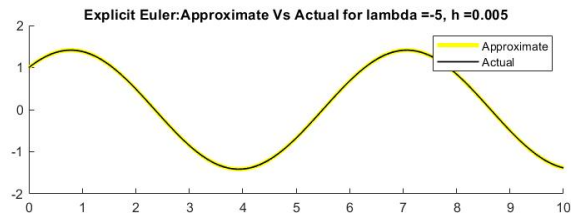
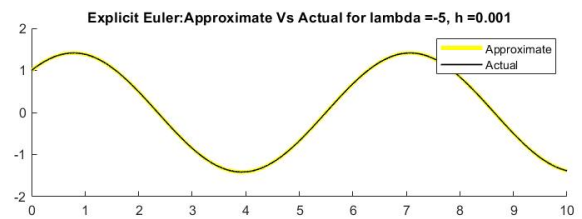
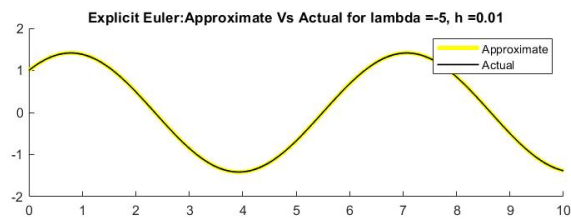
# LAB 07

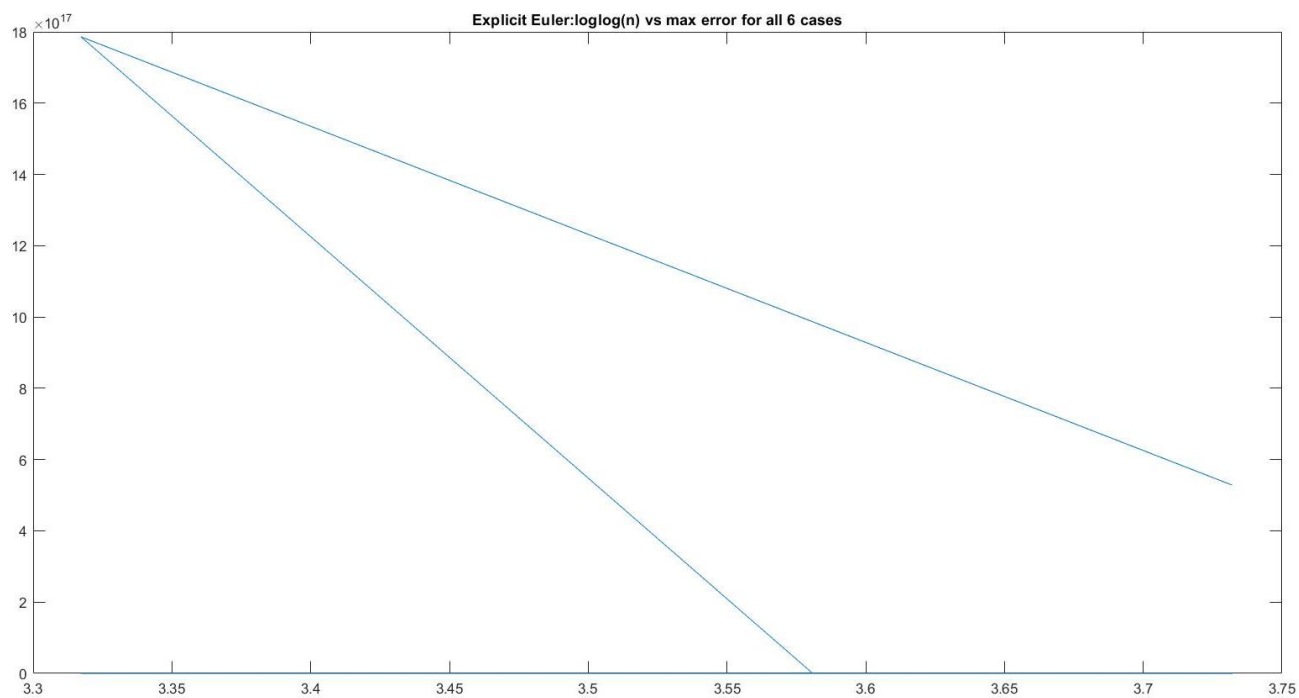
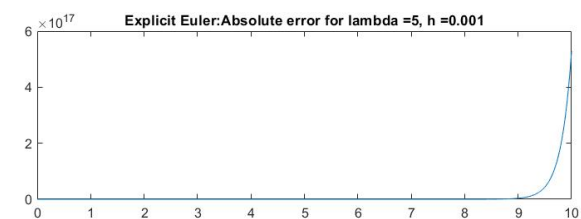
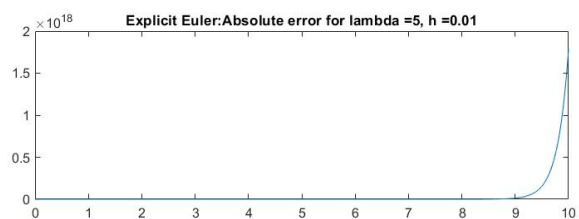
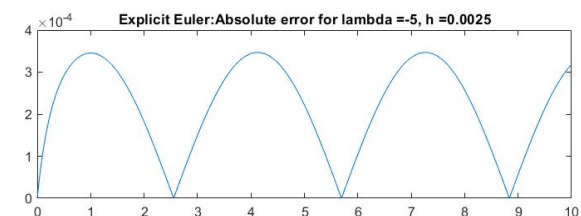
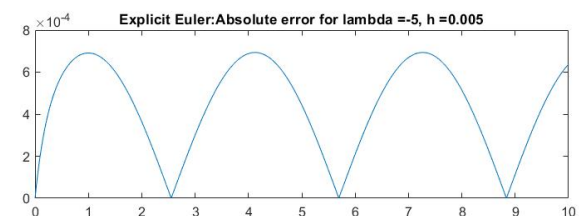
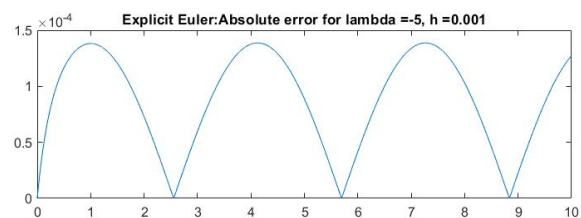
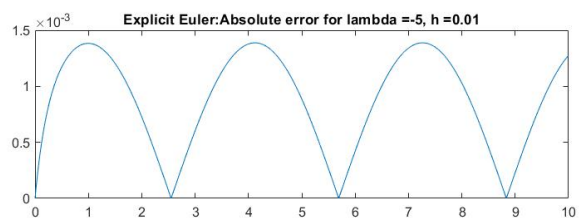
## Q1)

### Using Explicit Euler

Max error table for all 6 cases

lambda	h	n	max error	$\log_2(E_n/E_{n+1})$
-5	1.000000e-02	1001	0.00139	3.32317
-5	1.000000e-03	10001	0.00014	-2.32248
-5	5.000000e-03	2001	0.00069	1.00035
-5	2.500000e-03	4001	0.00035	-72.12534
5	1.000000e-02	1001	1786216577493959680.00000	1.75770
5	1.000000e-03	10001	528219716411543424.00000	0.00000

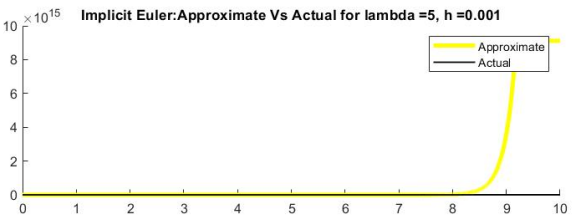
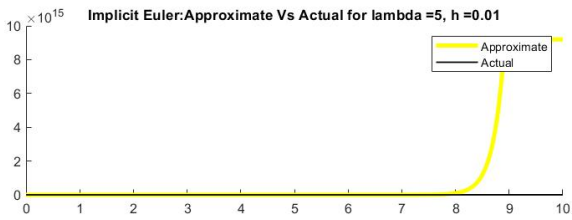
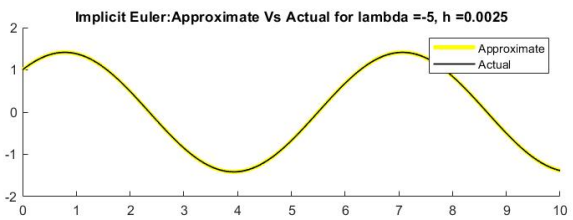
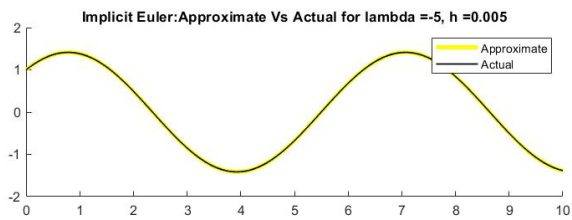
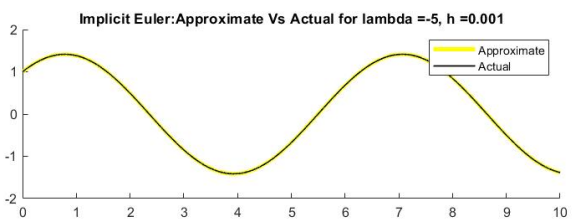
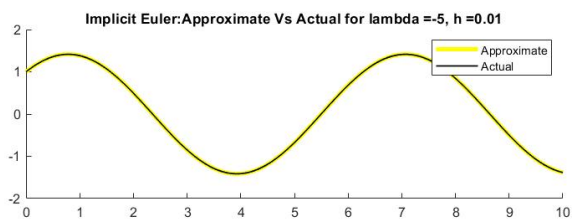


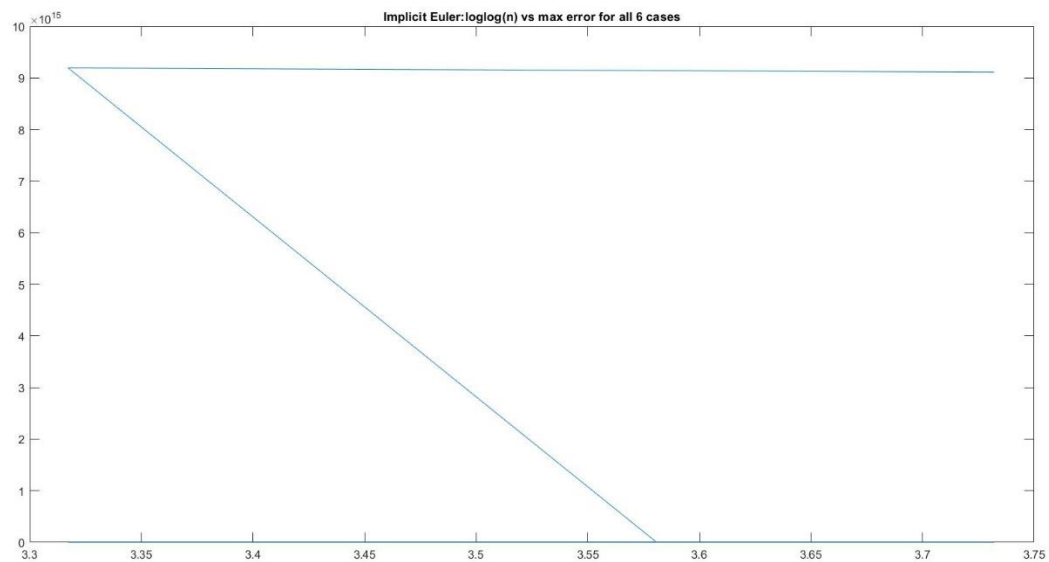
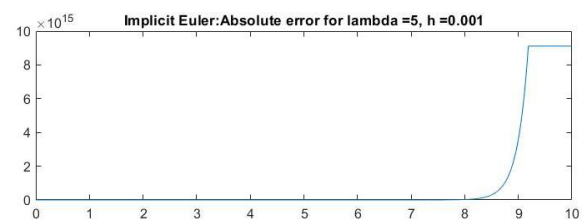
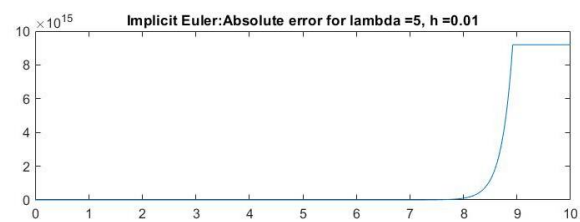
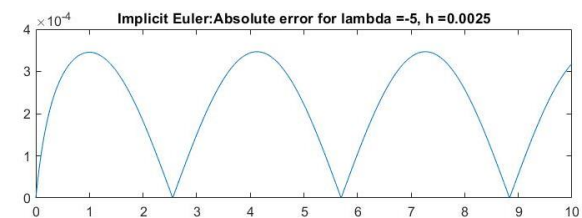
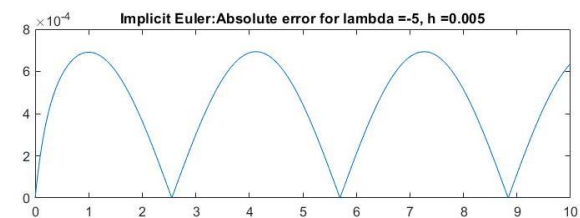
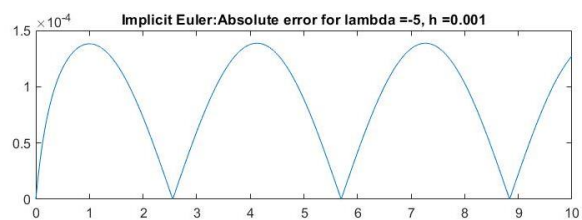
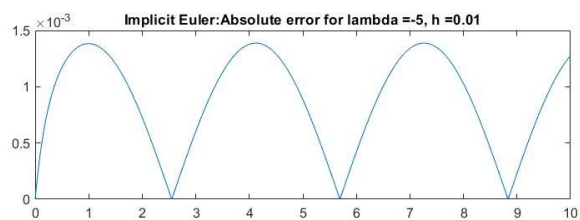


Using Implicit Euler

Max error table for all 6 cases

lambda	h	n	max error	log2(En/En+1)
-5	1.000000e-02	1001	0.00139	3.32317
-5	1.000000e-03	10001	0.00014	-2.32248
-5	5.000000e-03	2001	0.00069	1.00035
-5	2.500000e-03	4001	0.00035	-64.52329
5	1.000000e-02	1001	9193689159757810.00000	0.01279
5	1.000000e-03	10001	9112558679176762.00000	0.00000





Q2)

Using Simple Euler

table summary for h =0.05

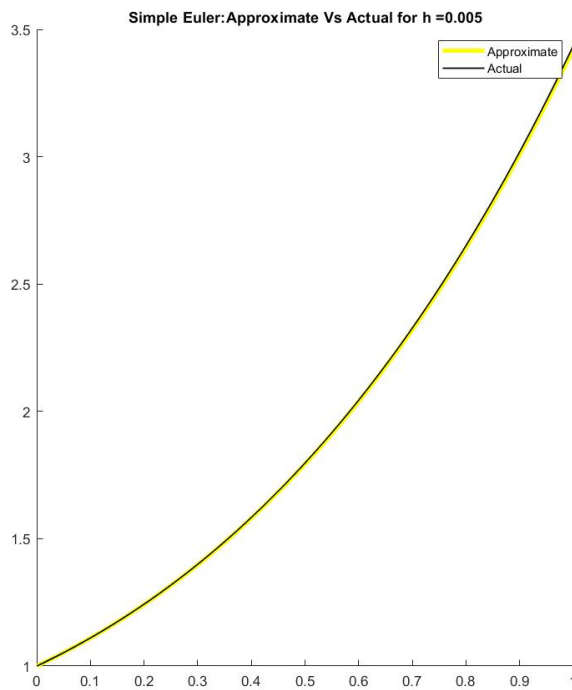
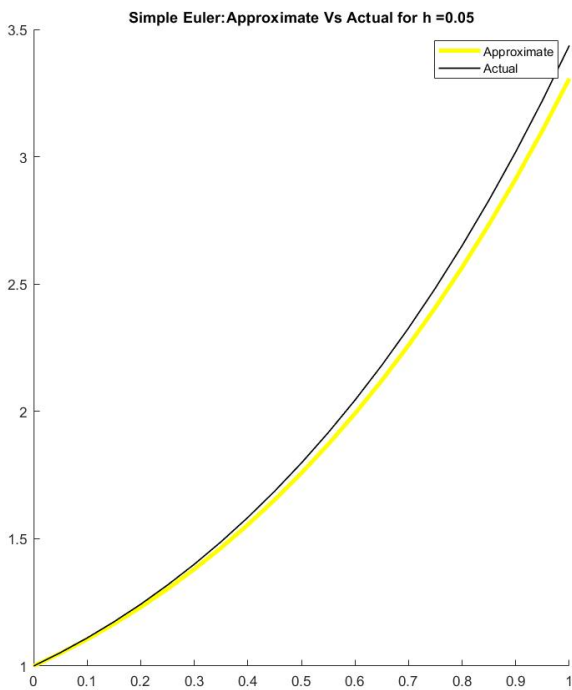
t	Approximate Y	Exact Y	Error
00000	1.00000	1.00000	0.00000
5.00000e-02	1.05000	1.05254	0.00254
9.50000e-01	3.10390	3.22142	0.11752
00001	3.30660	3.43656	0.12997

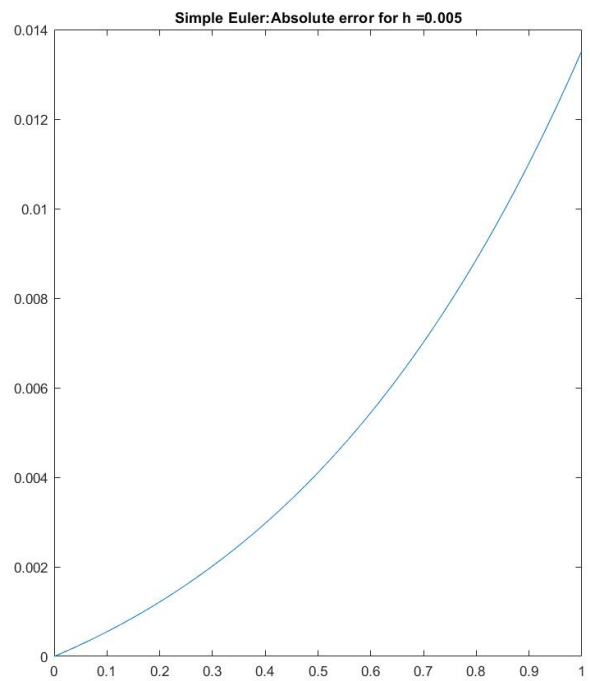
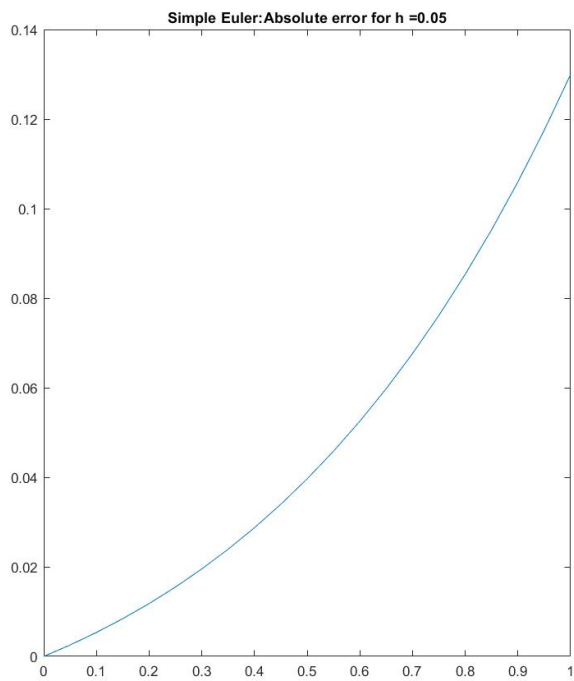
table summary for h =0.005

t	Approximate Y	Exact Y	Error
00000	1.00000	1.00000	0.00000
5.00000e-03	1.00500	1.00503	0.00003
9.95000e-01	3.40105	3.41445	0.01339
00001	3.42303	3.43656	0.01353

Max error table for all cases

h	n	max error
5.000000e-02	21	0.12997
5.000000e-03	201	0.01353





## Using Modified Euler

table summary for  $h = 0.05$

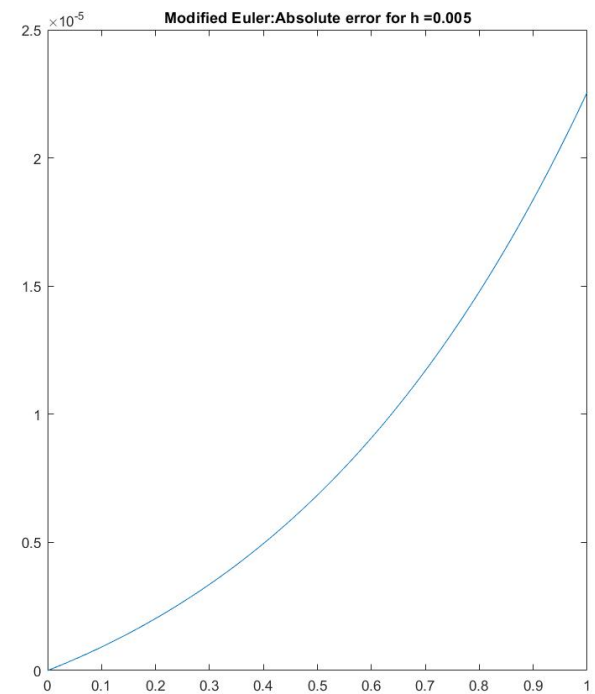
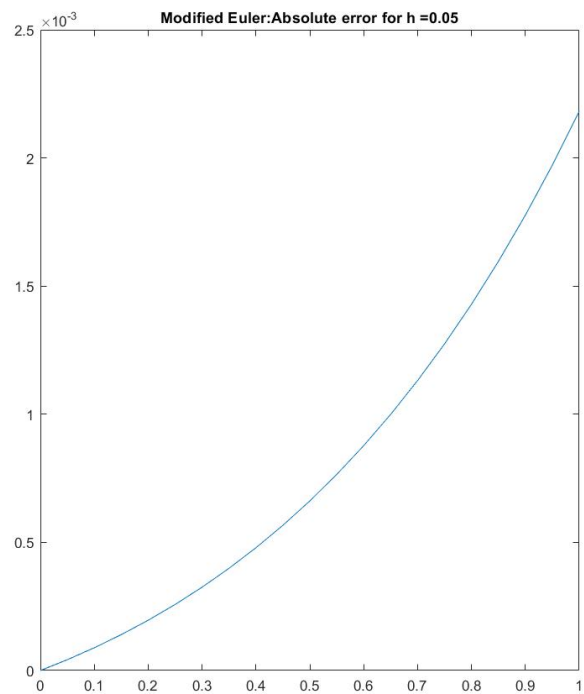
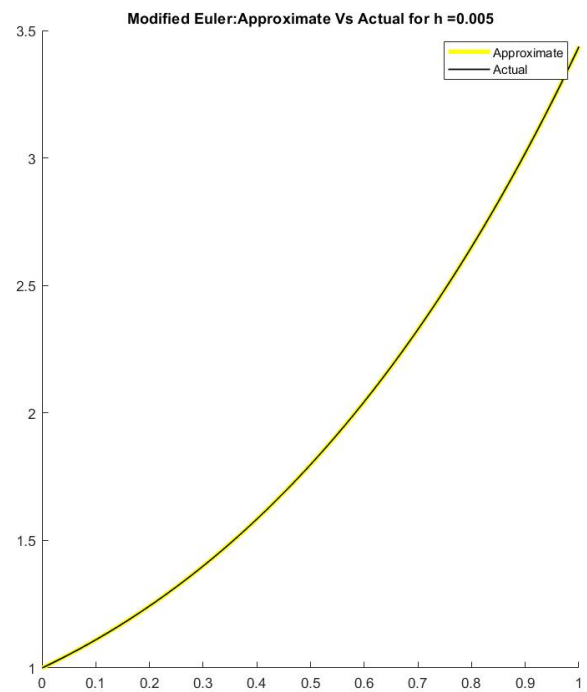
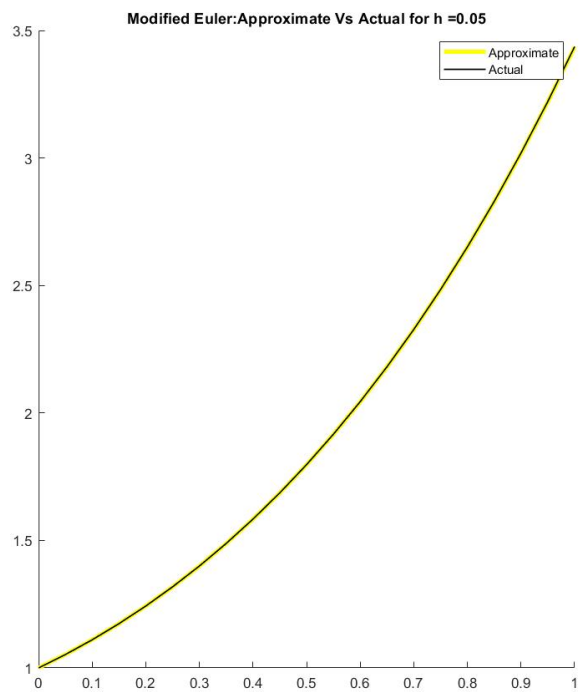
t	Approximate Y	Exact Y	Error
00000	1.00000	1.00000	0.00000
5.00000e-02	1.05250	1.05254	0.00004
9.50000e-01	3.21945	3.22142	0.00197
00001	3.43438	3.43656	0.00218

table summary for  $h = 0.005$

t	Approximate Y	Exact Y	Error
00000	1.00000	1.00000	0.00000
5.00000e-03	1.00503	1.00503	0.00000
9.95000e-01	3.41443	3.41445	0.00002
00001	3.43654	3.43656	0.00002

Max error table for all cases

h	n	max error
5.000000e-02	21	0.00218
5.000000e-03	201	0.00002



**ANSWER:**

Using simple euler for h = 0.05:	
Estimated value of y(1)	3.306595e+00
Absolute error	1.299682e-01
Using simple euler for h = 0.005	
Estimated value of y(1)	3.423034e+00
Absolute error	1.352941e-02
Using modified euler for h = 0.05:	
Estimated value of y(1)	3.434382e+00
Absolute error	2.181548e-03
Using modified euler for h = 0.005	
Estimated value of y(1)	3.436541e+00
Absolute error	2.256753e-05

**Q3)****Using Explicit Euler**

table summary for lambda =-1, h =0.01

t	Approximate Y	Exact Y	Error
00000	0.00000	0.00000	0.00000
1.00000e-02	0.01000	0.01000	0.00000
9.90000e-01	0.78205	0.78037	0.00167
00001	0.78708	0.78540	0.00168

table summary for lambda =-1, h =0.001

t	Approximate Y	Exact Y	Error
00000	0.00000	0.00000	0.00000
1.00000e-03	0.00100	0.00100	0.00000
9.99000e-01	0.78507	0.78490	0.00017
00001	0.78557	0.78540	0.00017

table summary for lambda =-10, h =0.01

t	Approximate Y	Exact Y	Error
00000	0.00000	0.00000	0.00000
1.00000e-02	0.01000	0.01000	0.00000
9.90000e-01	0.78065	0.78037	0.00027
00001	0.78567	0.78540	0.00027

table summary for lambda =-10, h =0.001

t	Approximate Y	Exact Y	Error
00000	0.00000	0.00000	0.00000
1.00000e-03	0.00100	0.00100	0.00000
9.99000e-01	0.78493	0.78490	0.00003
00001	0.78543	0.78540	0.00003

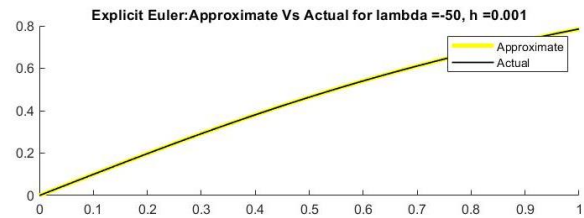
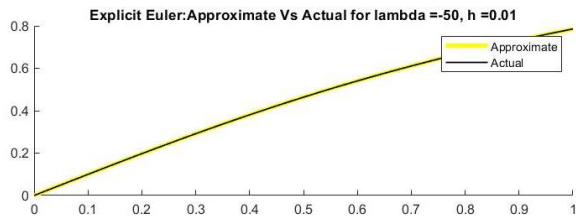
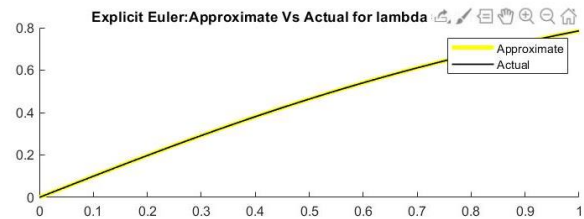
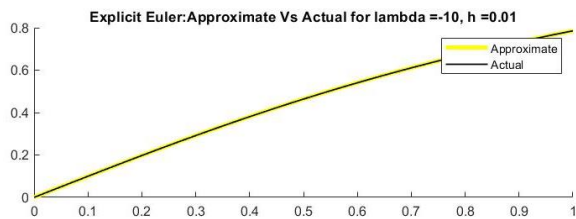
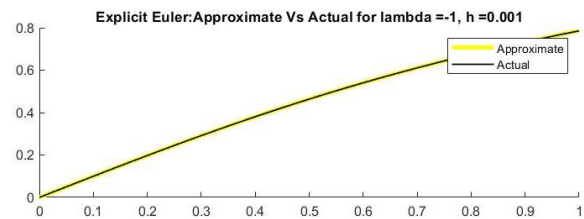
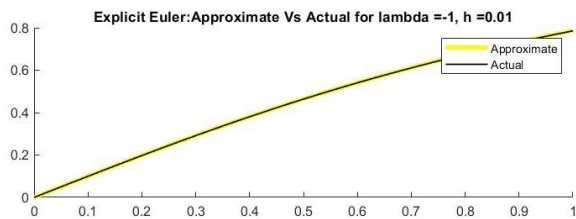


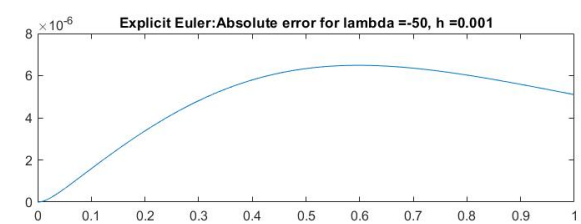
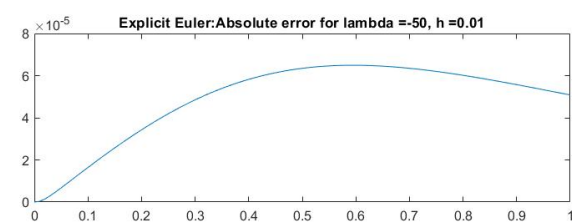
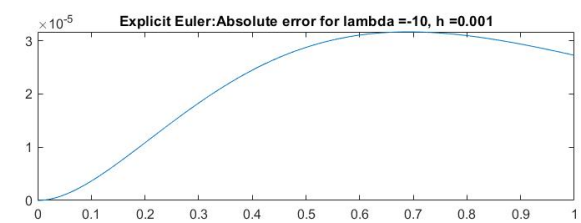
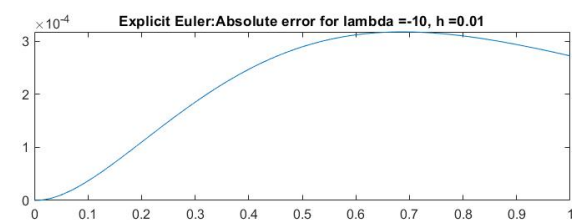
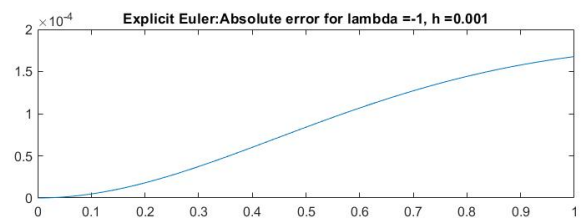
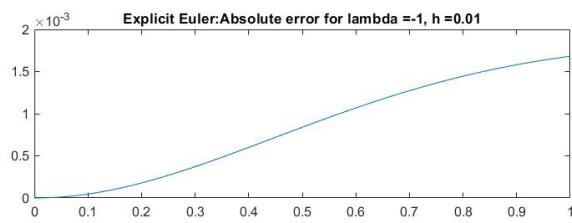
table summary for lambda =-50, h =0.01

t	Approximate Y	Exact Y	Error
00000	0.00000	0.00000	0.00000
1.00000e-02	0.01000	0.01000	0.00000
9.90000e-01	0.78042	0.78037	0.00005
00001	0.78545	0.78540	0.00005

table summary for lambda =-50, h =0.001

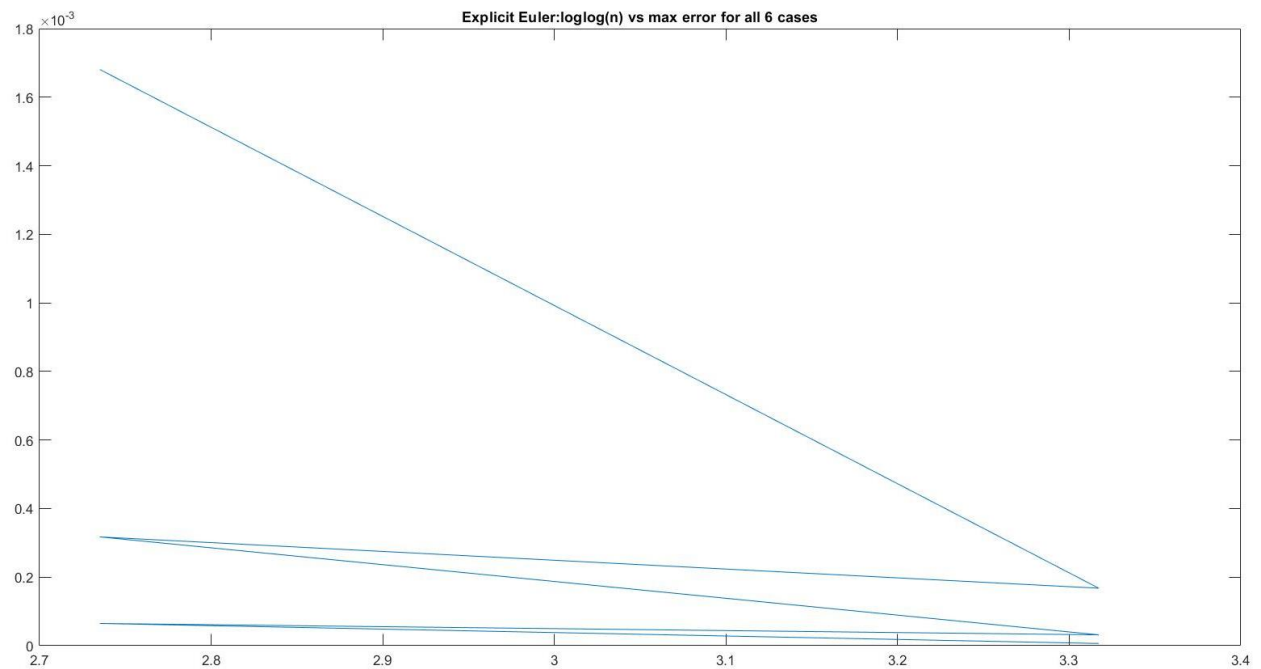
t	Approximate Y	Exact Y	Error
00000	0.00000	0.00000	0.00000
1.00000e-03	0.00100	0.00100	0.00000
9.99000e-01	0.78490	0.78490	0.00001
00001	0.78540	0.78540	0.00001





#### Max error table for all 6 cases

lambda	h	n	max error	log2(En/En+1)
-1	1.000000e-02	101	0.00168	3.32497
-1	1.000000e-03	1001	0.00017	-0.91992
-10	1.000000e-02	101	0.00032	3.32520
-10	1.000000e-03	1001	0.00003	-1.03596
-50	1.000000e-02	101	0.00006	3.32246
-50	1.000000e-03	1001	0.00001	0.00000



## Using Implicit Euler

table summary for lambda =-1, h =0.01

t	Approximate Y	Exact Y	Error
00000	0.00000	0.00000	0.00000
1.00000e-02	0.01000	0.01000	0.00000
9.90000e-01	0.78205	0.78037	0.00167
00001	0.78708	0.78540	0.00168

table summary for lambda =-1, h =0.001

t	Approximate Y	Exact Y	Error
00000	0.00000	0.00000	0.00000
1.00000e-03	0.00100	0.00100	0.00000
9.99000e-01	0.78507	0.78490	0.00017
00001	0.78557	0.78540	0.00017

table summary for lambda =-10, h =0.01

t	Approximate Y	Exact Y	Error
00000	0.00000	0.00000	0.00000
1.00000e-02	0.01000	0.01000	0.00000
9.90000e-01	0.78065	0.78037	0.00027
00001	0.78567	0.78540	0.00027

table summary for lambda =-10, h =0.001

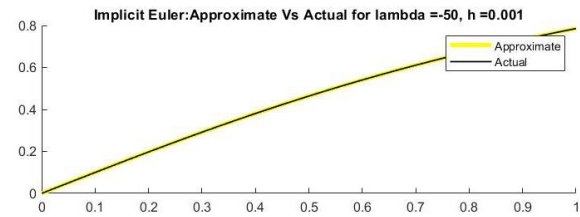
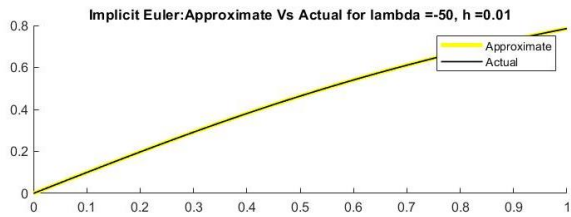
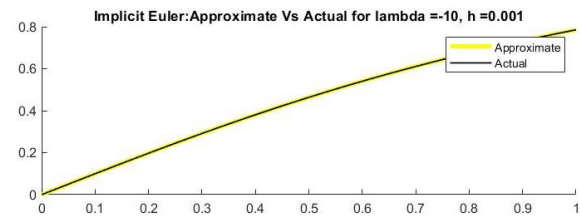
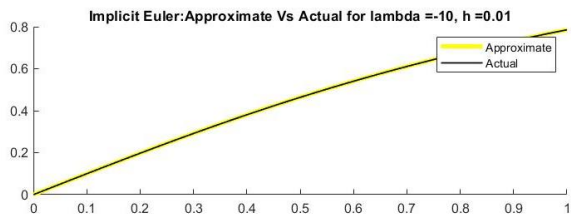
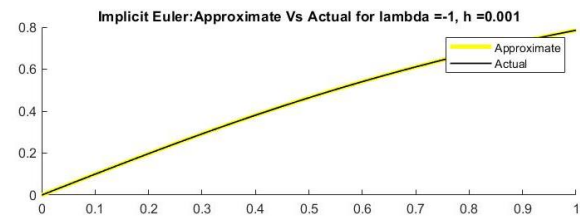
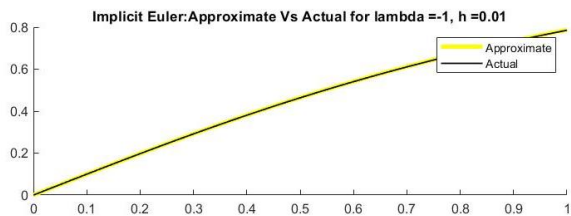
t	Approximate Y	Exact Y	Error
00000	0.00000	0.00000	0.00000
1.00000e-03	0.00100	0.00100	0.00000
9.99000e-01	0.78493	0.78490	0.00003
00001	0.78543	0.78540	0.00003

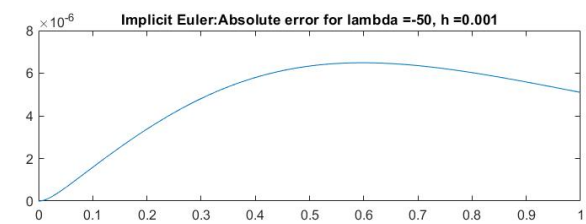
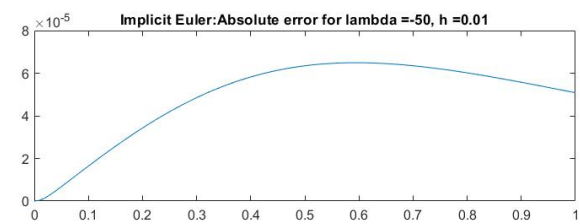
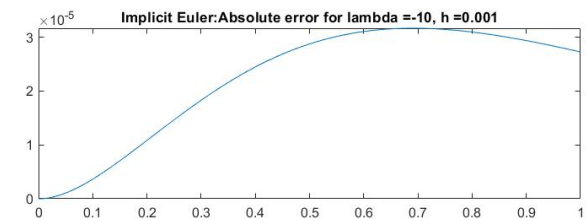
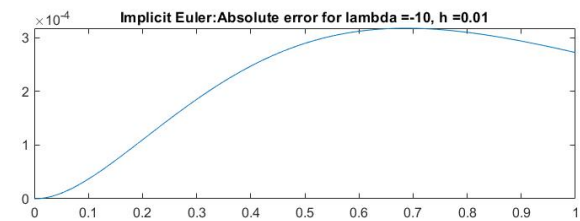
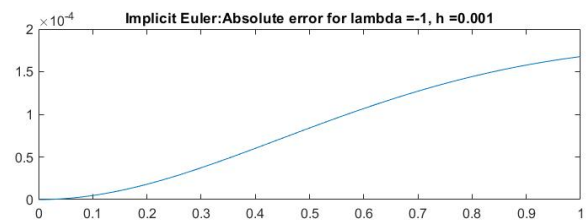
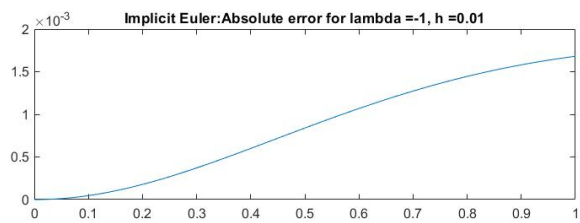
table summary for lambda =-50, h =0.01

t	Approximate Y	Exact Y	Error
00000	0.00000	0.00000	0.00000
1.00000e-02	0.01000	0.01000	0.00000
9.90000e-01	0.78042	0.78037	0.00005
00001	0.78545	0.78540	0.00005

table summary for lambda =-50, h =0.001

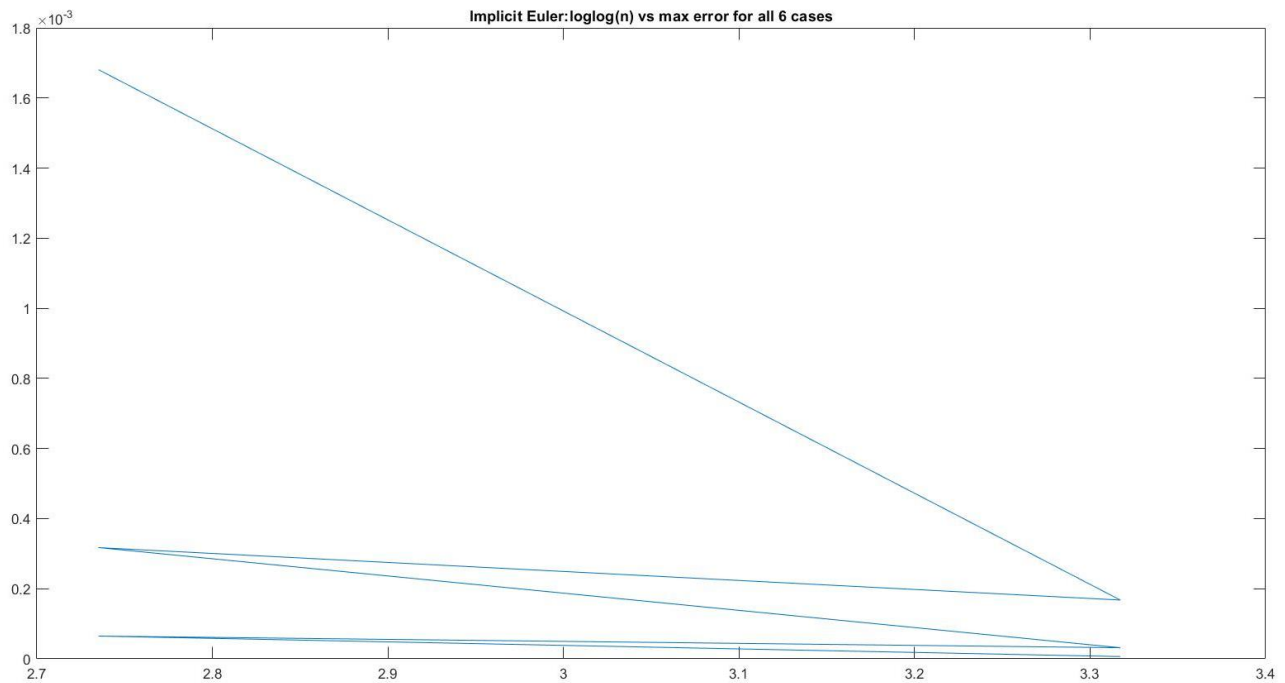
t	Approximate Y	Exact Y	Error
00000	0.00000	0.00000	0.00000
1.00000e-03	0.00100	0.00100	0.00000
9.99000e-01	0.78490	0.78490	0.00001
00001	0.78540	0.78540	0.00001





#### Max error table for all 6 cases

lambda	h	n	max error	log2(En/En+1)
-1	1.000000e-02	101	0.00168	3.32497
-1	1.000000e-03	1001	0.00017	-0.91992
-10	1.000000e-02	101	0.00032	3.32520
-10	1.000000e-03	1001	0.00003	-1.03596
-50	1.000000e-02	101	0.00006	3.32246
-50	1.000000e-03	1001	0.00001	0.00000



## Using Trapezoidal Euler

table summary for lambda =-1, h =0.01

t	Approximate Y	Exact Y	Error
00000	0.00000	0.00000	0.00000
1.00000e-02	0.01005	0.01000	0.00005
9.90000e-01	0.78271	0.78037	0.00234
00001	0.78774	0.78540	0.00234

table summary for lambda =-1, h =0.001

t	Approximate Y	Exact Y	Error
00000	0.00000	0.00000	0.00000
1.00000e-03	0.00100	0.00100	0.00000
9.99000e-01	0.78513	0.78490	0.00023
00001	0.78563	0.78540	0.00023

table summary for lambda =-10, h =0.01

t	Approximate Y	Exact Y	Error
00000	0.00000	0.00000	0.00000
1.00000e-02	0.01050	0.01000	0.00050
9.90000e-01	0.78316	0.78037	0.00279
00001	0.78816	0.78540	0.00276

table summary for lambda =-10, h =0.001

t	Approximate Y	Exact Y	Error
00000	0.00000	0.00000	0.00000
1.00000e-03	0.00100	0.00100	0.00000
9.99000e-01	0.78518	0.78490	0.00028
00001	0.78568	0.78540	0.00028

table summary for lambda =-50, h =0.01

t	Approximate Y	Exact Y	Error
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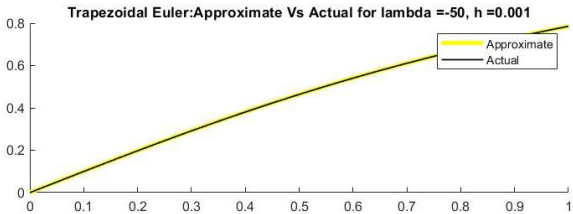
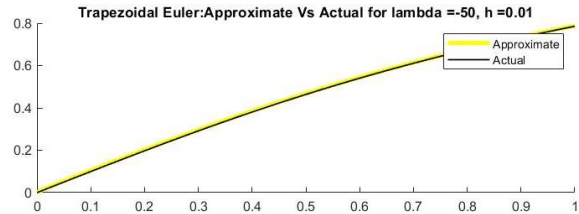
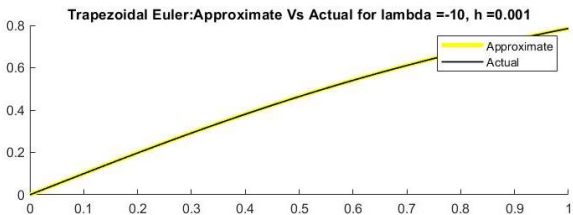
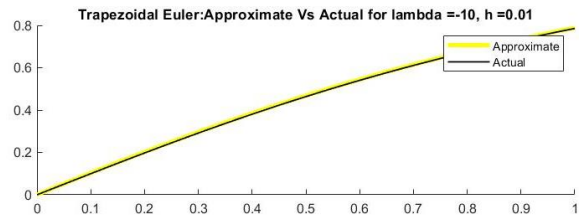
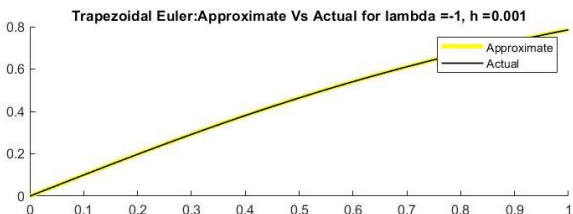
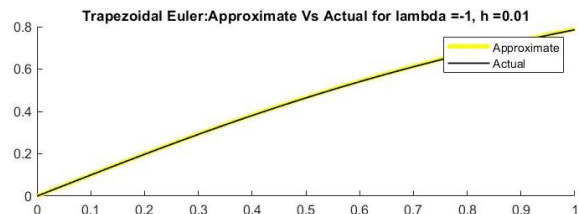
00000	0.00000
1.00000e-02	0.01250
9.90000e-01	0.78294
00001	0.78794

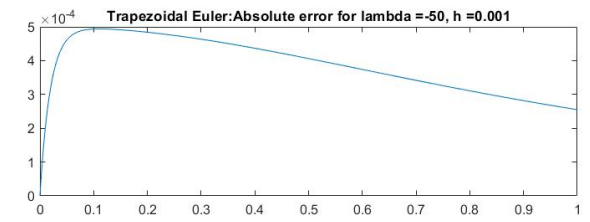
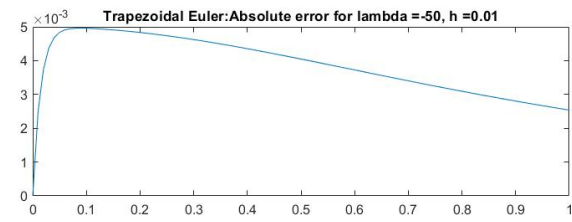
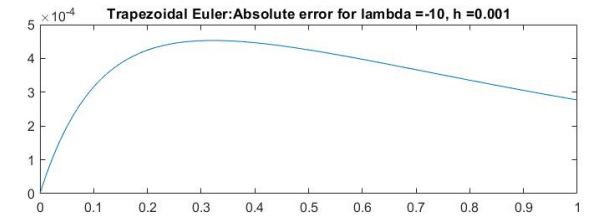
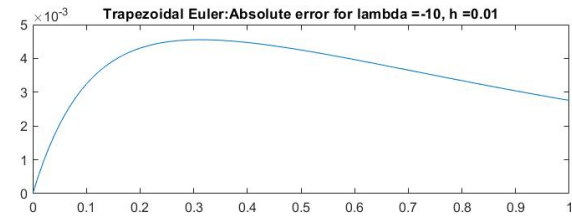
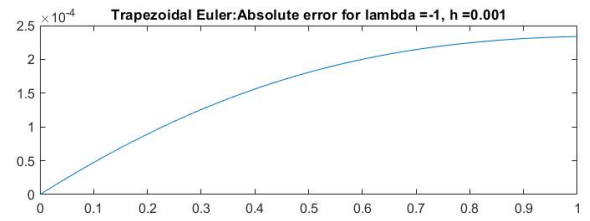
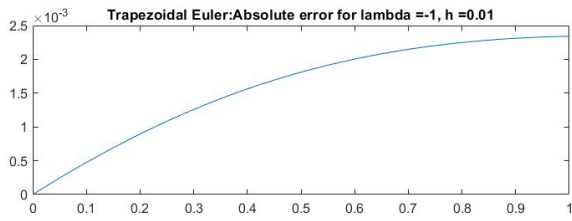
table summary for lambda =-50, h =0.001

t	Approximate Y
00000	0.00000
1.00000e-03	0.00102
9.99000e-01	0.78515
00001	0.78565

0.00000	0.00000
0.01000	0.00250
0.78037	0.00256
0.78540	0.00254

Exact Y	Error
0.00000	0.00000
0.00100	0.00002
0.78490	0.00026
0.78540	0.00025

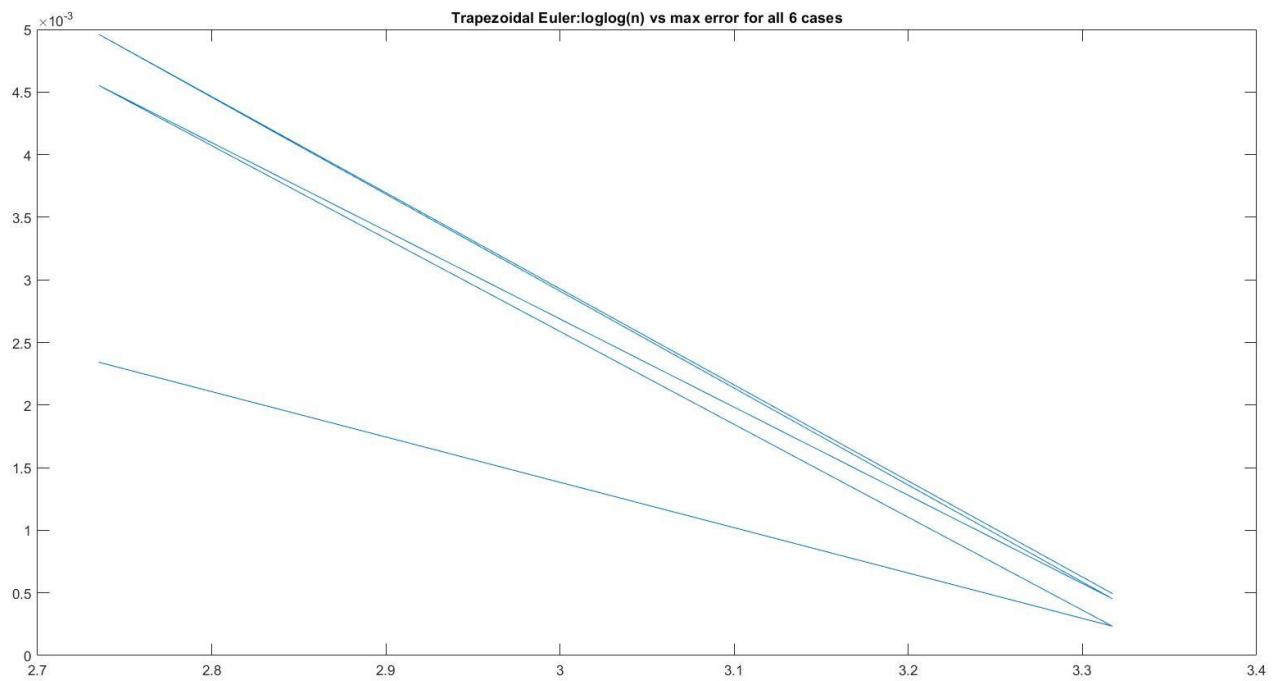




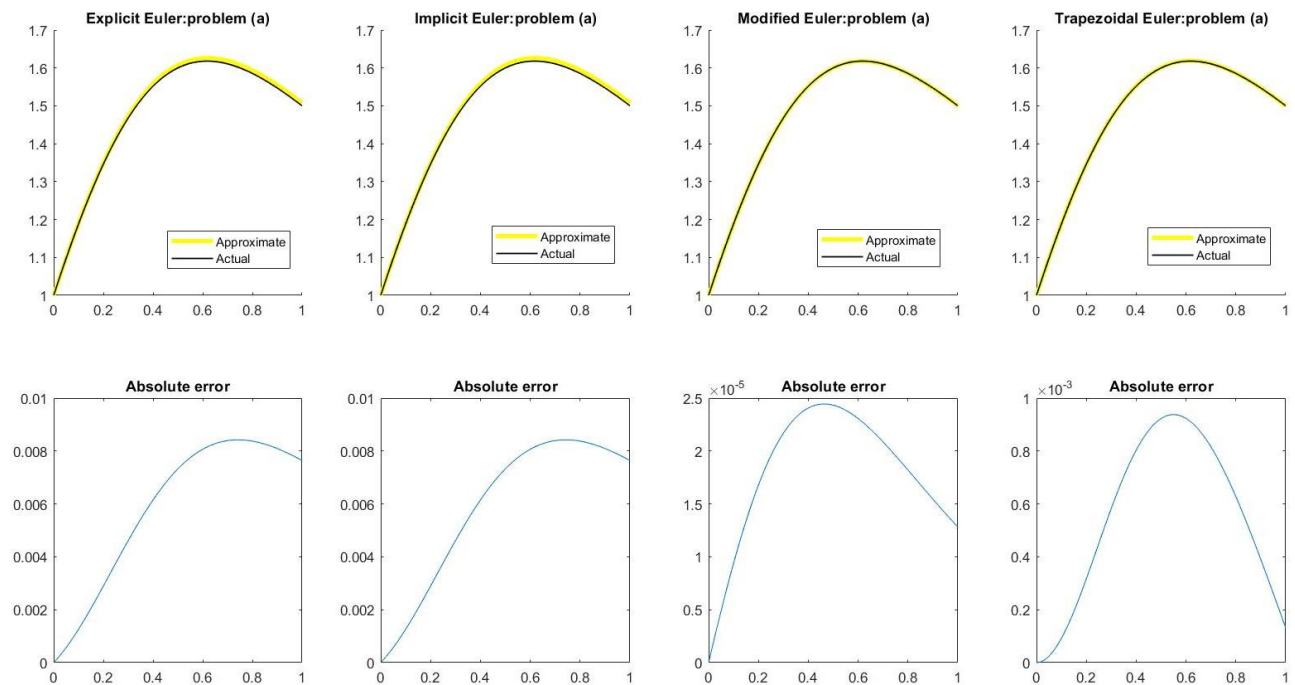
#### Max error table for all 6 cases

lambda	h	n	max error	$\log_2(E_n/E_{n+1})$
-1	1.000000e-02	101	0.00234	3.32457
-1	1.000000e-03	1001	0.00023	-4.28361
-10	1.000000e-02	101	0.00455	3.32907
-10	1.000000e-03	1001	0.00045	-3.45282
-50	1.000000e-02	101	0.00496	3.32808
-50	1.000000e-03	1001	0.00049	0.00000





**Q4)**



Explicit Euler: table summary for problem (a)

t	Approximate Y	Exact Y	Error
00000	1.00000	1.00000	0.00000
1.00000e-02	1.02000	1.01990	0.00010
9.90000e-01	1.51267	1.50497	0.00769
00001	1.50764	1.50000	0.00764

Implicit Euler: table summary for problem (a)

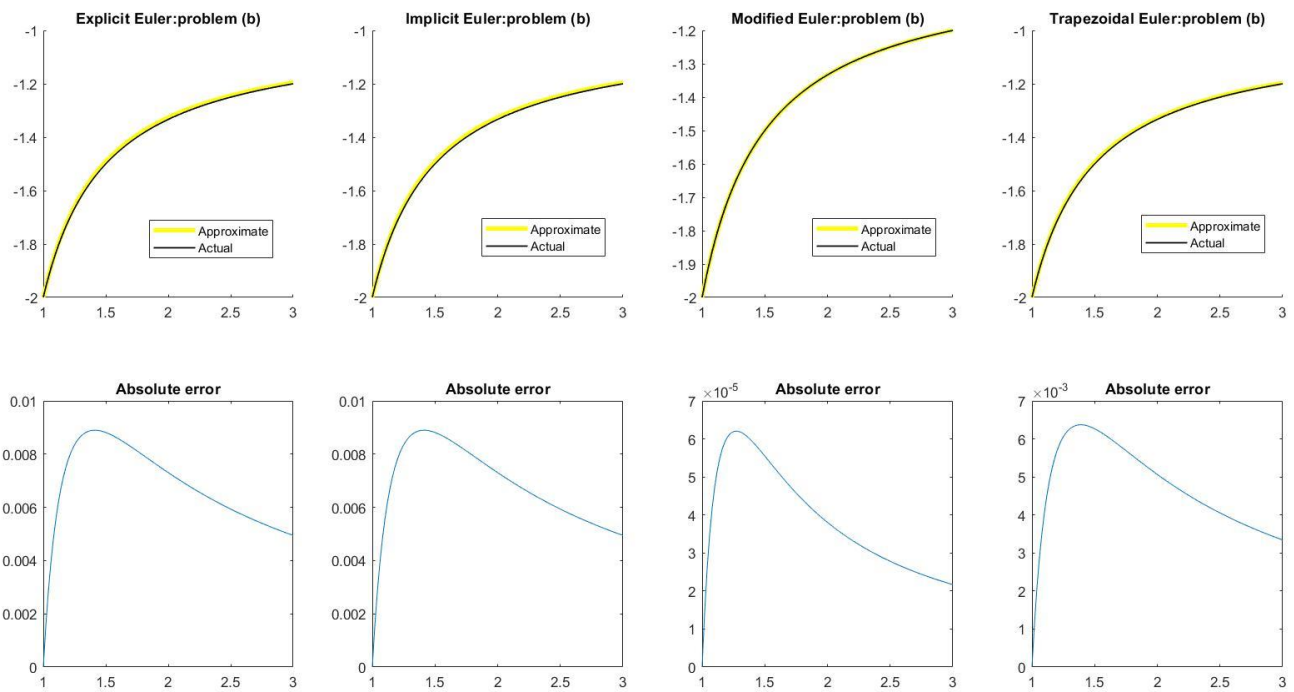
t	Approximate Y	Exact Y	Error
00000	1.00000	1.00000	0.00000
1.00000e-02	1.02000	1.01990	0.00010
9.90000e-01	1.51267	1.50497	0.00769
00001	1.50764	1.50000	0.00764

Modified Euler: table summary for problem (a)

t	Approximate Y	Exact Y	Error
00000	1.00000	1.00000	0.00000
1.00000e-02	1.01990	1.01990	0.00000
9.90000e-01	1.50496	1.50497	0.00001
00001	1.49999	1.50000	0.00001

Trapezoidal Euler: table summary for problem (a)

t	Approximate Y	Exact Y	Error
00000	1.00000	1.00000	0.00000
1.00000e-02	1.01990	1.01990	0.00000
9.90000e-01	1.50513	1.50497	0.00016
00001	1.50013	1.50000	0.00013



Explicit Euler: table summary for problem (b)

t	Approximate Y	Exact Y	Error
00001	-2.00000	-2.00000	0.00000
1.02000e+00	-1.96000	-1.96154	0.00154
2.98000e+00	-1.19663	-1.20161	0.00498
00003	-1.19505	-1.20000	0.00495

Implicit Euler: table summary for problem (b)

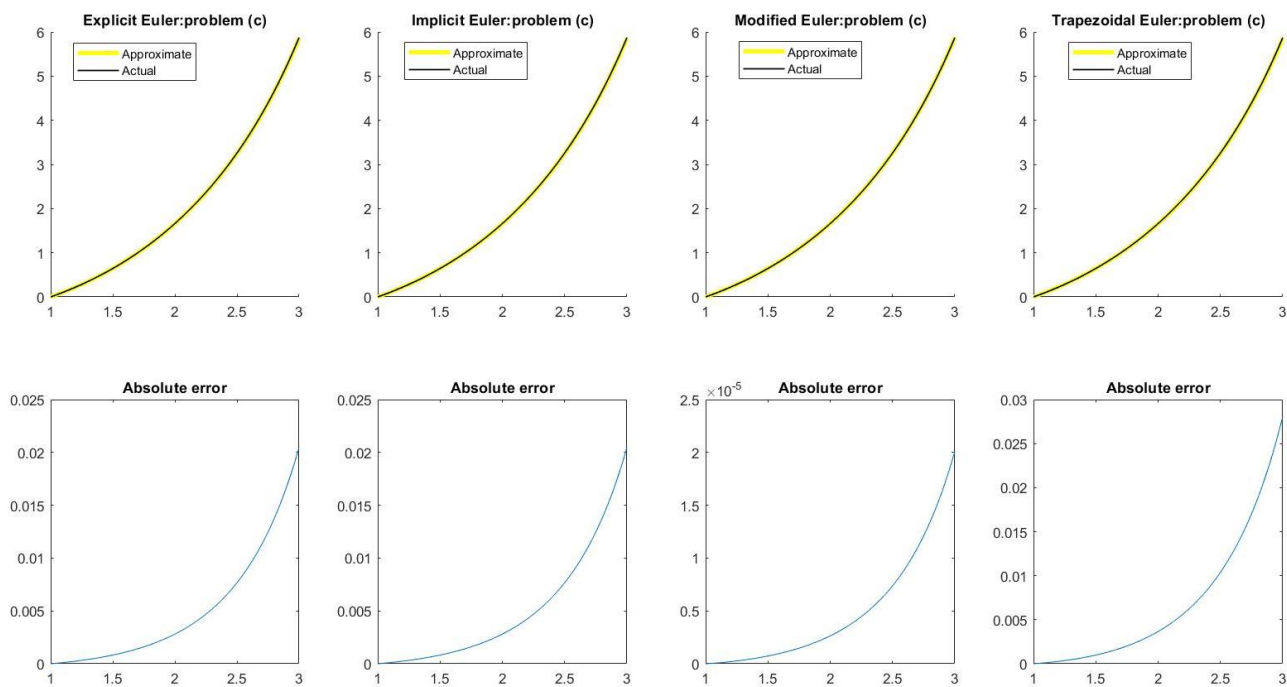
t	Approximate Y	Exact Y	Error
00001	-2.00000	-2.00000	0.00000
1.02000e+00	-1.96000	-1.96154	0.00154
2.98000e+00	-1.19663	-1.20161	0.00498
00003	-1.19505	-1.20000	0.00495

Modified Euler: table summary for problem (b)

t	Approximate Y	Exact Y	Error
00001	-2.00000	-2.00000	0.00000
1.02000e+00	-1.96155	-1.96154	0.00001
2.98000e+00	-1.20163	-1.20161	0.00002
00003	-1.20002	-1.20000	0.00002

Trapezoidal Euler: table summary for problem (b)

t	Approximate Y	Exact Y	Error
00001	-2.00000	-2.00000	0.00000
1.02000e+00	-1.96039	-1.96154	0.00115
2.98000e+00	-1.19825	-1.20161	0.00337
00003	-1.19666	-1.20000	0.00334



Explicit Euler: table summary for problem (c)

t	Approximate Y	Exact Y	Error
00001	0.00000	0.00000	0.00000
1.00200e+00	0.00200	0.00200	0.00000
2.99800e+00	5.84015	5.86053	0.02038
00003	5.85364	5.87410	0.02046

Implicit Euler: table summary for problem (c)

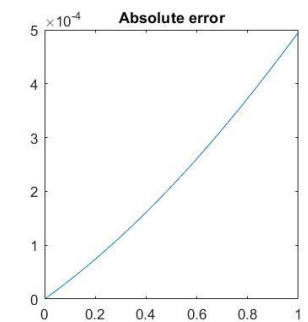
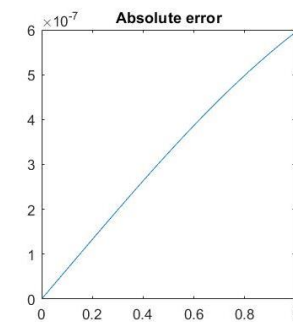
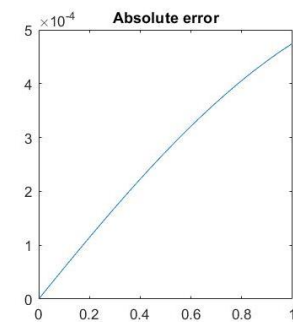
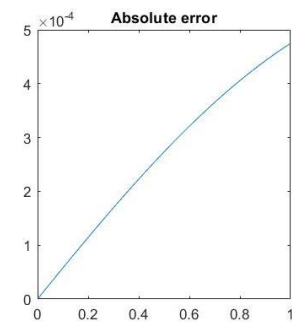
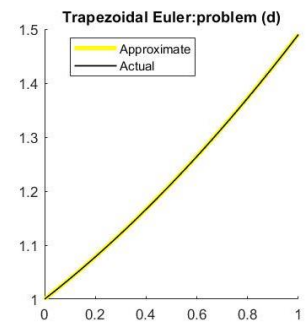
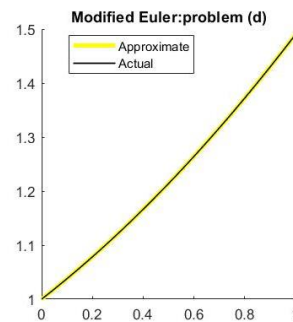
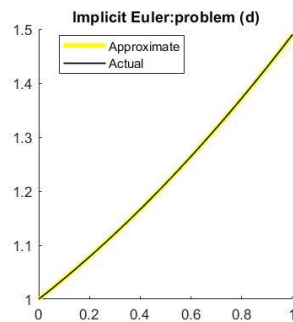
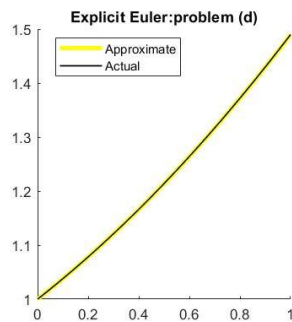
t	Approximate Y	Exact Y	Error
00001	0.00000	0.00000	0.00000
1.00200e+00	0.00200	0.00200	0.00000
2.99800e+00	5.84015	5.86053	0.02038
00003	5.85364	5.87410	0.02046

Modified Euler: table summary for problem (c)

t	Approximate Y	Exact Y	Error
00001	0.00000	0.00000	0.00000
1.00200e+00	0.00200	0.00200	0.00000
2.99800e+00	5.86051	5.86053	0.00002
00003	5.87408	5.87410	0.00002

Trapezoidal Euler: table summary for problem (c)

t	Approximate Y	Exact Y	Error
00001	0.00000	0.00000	0.00000
1.00200e+00	0.00200	0.00200	0.00000
2.99800e+00	5.83258	5.86053	0.02796
00003	5.84603	5.87410	0.02807



Explicit Euler: table summary for problem (d)

t	Approximate Y	Exact Y	Error
00000	1.00000	1.00000	0.00000
5.00000e-03	1.00184	1.00184	0.00000
9.95000e-01	1.48635	1.48682	0.00047
00001	1.48941	1.48988	0.00047

Implicit Euler: table summary for problem (d)

t	Approximate Y	Exact Y	Error
00000	1.00000	1.00000	0.00000
5.00000e-03	1.00184	1.00184	0.00000
9.95000e-01	1.48635	1.48682	0.00047
00001	1.48941	1.48988	0.00047

Modified Euler: table summary for problem (d)

t	Approximate Y	Exact Y	Error
00000	1.00000	1.00000	0.00000
5.00000e-03	1.00184	1.00184	0.00000
9.95000e-01	1.48682	1.48682	0.00000
00001	1.48988	1.48988	0.00000

Trapezoidal Euler: table summary for problem (d)

t	Approximate Y	Exact Y	Error
00000	1.00000	1.00000	0.00000
5.00000e-03	1.00184	1.00184	0.00000
9.95000e-01	1.48731	1.48682	0.00049
00001	1.49038	1.48988	0.00050

**Q5)** Approximating integrals using Gaussian quadrature with  $n = 2, 3, 4, 5$ .

For a.

Degree	Gaussian quadrature	Exact	Error
2	0.1922687064	0.1922593577	0.000009348638122
3	0.1922593773	0.1922593577	0.000000019524083
4	0.1922593578	0.1922593577	0.000000000072067
5	0.1922593577	0.1922593577	0.0000000000000346

For b.

Degree	Gaussian quadrature	Exact	Error
2	2.5913247157	2.5886286325	0.002696083147748
3	2.5892580031	2.5886286325	0.000629370608250
4	2.5886327465	2.5886286325	0.000004114025076
5	2.5886286187	2.5886286325	0.000000013839986

For c.

Degree	Gaussian quadrature	Exact	Error
2	-0.1768189895	-0.1768200201	0.000001030666877
3	-0.1768200179	-0.1768200201	0.000000002235569
4	-0.1768200201	-0.1768200201	0.000000000004894
5	-0.1768200201	-0.1768200201	0.000000000000011

For d.

Degree	Gaussian quadrature	Exact	Error
2	-0.7307230363	-0.7339691751	0.003246138808281
3	-0.7337990223	-0.7339691751	0.000170152823742
4	-0.7339603934	-0.7339691751	0.000008781638310
5	-0.7339687249	-0.7339691751	0.000000450219742