DEPARTMENT OF MATHEMATICS
IIT GUWAHATI

MA 332 **Scientific Computing** **Lab - XI**

1. Approximate the solutions to the following elliptic PDEs by using the five–point stencil finite difference scheme,

(a)

$$\begin{cases} u_{xx} + u_{yy} = 0, \quad 0 < x < 1,\ 0 < y < 1; \\ u(x,0) = 0, \quad u(x,1) = x,\ 0 \leq x \leq 1; \\ u(0,y) = 0, \quad u(1,y) = y,\ 0 \leq y \leq 1. \end{cases}$$

Use $h = k = 0.002$ and compare the results to the exact solution $u(x,y) = xy$.

(b)

$$\begin{cases} u_{xx} + u_{yy} = x^2 + y^2, \quad 0 < x < 1,\ 0 < y < 1; \\ u(x,0) = 0, \quad u(x,1) = \dfrac{x^2}{2},\ 0 \leq x \leq 1; \\ u(0,y) = \sin(\pi y), \quad u(1,y) = e^{\pi}\sin(\pi y) + \dfrac{y^2}{2},\ 0 \leq y \leq 1. \end{cases}$$

Use $h = k = 0.002$ and compare the results to the exact solution
$u(x,y) = e^{\pi x}\sin(\pi y) + \dfrac{(xy)^2}{2}$.

(c)

$$\begin{cases} u_{xx} + u_{yy} + u = 2x - y, \quad 0 < x < 1,\ 0 < y < 1; \\ u(x,0) = 2x, \quad u(x,1) = 2x - 1,\ 0 \leq x \leq 1; \\ u_x(0,y) + u(0,y) = 2 - y, \quad u(1,y) = 2 - y,\ 0 \leq y \leq 1. \end{cases}$$

Use $h = k = 0.002$ and compare the results to the exact solution
$u(x,y) = 2x - y$.

(d)

$$\begin{cases} u_{xx} + u_{yy} + u_x + u_y + u = e^x(2\cos y - \sin y), \quad 0 < x < 1,\ 0 < y < 1; \\ u(x,0) = e^x, \quad u(x,1) = e^x\cos(1),\ 0 \leq x \leq 1; \\ u(0,y) = \cos(y), \quad u(1,y) = e\cos(y),\ 0 \leq y \leq 1. \end{cases}$$

Use $h = k = 0.002$ and compare the results to the exact solution $u(x,y) = e^x\cos(y)$.

2. Solve the system of linear algebraic equations of the above elliptic BVPs by *Gauss-Seidel iterative method* and *Jacobi iterative method*.

Provide the following:

(a) Draw the surface plot of the exact and numerical solutions

(b) Draw the contour plot of the exact and numerical solutions

(c) Draw the surf plot of the absolute error.

(d) Plot $\triangle x (= \triangle y)$ versus $Max. Error$ in loglog scale.

3. Solve the following 1D hyperbolic PDE by the forward explicit and implicit schemes (FTFS and BTFS):

$$\begin{cases} u_t - 2u_x = 0, \quad x \in (0,1),\, t > 0 \\ u(x,0) = 1 + \sin(2\pi\, x), \quad x \in [0,1] \\ u(1,t) = 1.0 \end{cases}$$

Take $\Delta\, x = h = 0.005$, and $\Delta\, t = k = 0.001$. Determine the solution for three time-levels.

4. Solve the following 1D hyperbolic PDE by the *Lax-Wendroff scheme*:

$$\begin{cases} u_t + 2u_x = 0, \quad x \in (0,1),\, t > 0 \\ u(x,0) = 1 + \sin(2\pi\, x), \quad x \in [0,1] \\ u(0,t) = 1.0 \end{cases}$$

Take $\Delta\, x = h = 0.0005$, and $\Delta\, t = k = 0.001$. Determine the solution for three time-levels. Use the following numerical boundary conditions at $x = 0$:

(i) $u_0^n = 1.0$.

(ii) $u_0^n = u_1^n$.

(iii) $u_0^{n+1} = u_0^n - \frac{a\Delta\, t}{\Delta\, x}(u_1^n - u_0^n)$.

(iv) $u_0^{n+1} + u_1^{n+1} + \frac{a\Delta\, t}{\Delta\, x}(u_1^{n+1} - u_0^{n+1}) = u_0^n + u_1^n + \frac{a\Delta\, t}{\Delta\, x}(u_1^n - u_0^n)$
