DEPARTMENT OF MATHEMATICS
IIT GUWAHATI

MA 332 **Scientific Computing** **Lab - VII**

1. Consider the linear problem

$$Y'(t) = \lambda Y(t) + (1-\lambda)\cos(t) - (1+\lambda)\sin(t), \quad Y(t) = 0.$$

The true solution is $Y(t) = \sin(t) + \cos(t)$, solve this problem using explicit and implicit Euler's method with several values λ and h, for $0 \le t \le 10$. Comment on the results,

(a) $\lambda = -5$; $h = 0.01, 0.001, 0.005, 0.0025$

(b) $\lambda = 5$; $h = 0.01, 0.001$

2. Solve the differential equation

$$\frac{dy}{dx} = x + y, \quad y(0) = 1$$

by the simple Euler's method and modified Euler's method to estimate $y(1)$ using $h = 0.05$ and $h = 0.005$. Compute errors in both the cases. How do they compare ? Also, compare your rersults with the exact answer given the analytical solution as $y(t) = 2e^x - x - 1$

3. Solve the equation

$$y'(t) = \lambda y(t) + \frac{1}{1+t^2} - \lambda \tan^{-1}(t), \quad y(0) = 0;$$

$y(t) = \tan^{-1}(t)$ is the true solution. Use explicit, implicit Euler's method and the trapezoidal method. Let $\lambda = -1, -10, -50$, and $h = 0.01, 0.001$, Discuss the results.

4. Use the methods mentioned below to approximate the solutions to each of the following initial-value problems, and compare the results to the actual values.

(a) $y' = \dfrac{2-2ty}{t^2+1}$, $0 \le t \le 1$, $y(0) = 1$ with $h = 0.01$; actual solution $y(t) = \dfrac{2t+1}{t^2+1}$.

(b) $y' = \dfrac{y^2+y}{t}$, $1 \le t \le 3$, $y(1) = -2$ with $h = 0.02$; actual solution $y(t) = \dfrac{2t}{1-2t}$.

(c) $y' = 1 + y/t + (y/t)^2$, $1 \le t \le 3$, $y(1) = 0$ with $h = 0.002$; actual solution $y(t) = t\tan(\ln t)$.

(d) $y' = e^(t - y)$, $0 \le t \le 1$, $y(0) = 1$ with $h = 0.005$; actual solution $y(t) = \ln(e^t + e - 1)$.

1. Explicit-Euler method
2. Implicit-Euler method
3. Modified-Euler method
4. Trapezoidal method

5. Approximate the following integrals using Gaussian quadrature with $n = 2, 3, 4, 5$, uniformly spaced data points of the respective intervals:

$$a)\quad \int_1^{1.5} x^2 \ln x dx \qquad b)\quad \int_0^{\pi/4} e^{3x} \sin 2x dx$$

$$c)\quad \int_0^{0.35} \frac{2}{x^2-4} dx \quad d)\quad \int_1^{1.6} \frac{2x}{x^2-4} dx$$
