LAB 06

Q1 to Q3 :

A) f(x) = 2x/(x² - 4)

Rectangle Method

Value: -4.000000e-01, Error Bound: 1.138889e+00, Actual Error: 3.339692e-01

Difference in error bound and actual error: 8.049197e-01

Rectangle Mid-point Method

Value: -6.753247e-01, Error Bound: 2.808642e-01, Actual Error: 5.864450e-02

Difference in error bound and actual error: 2.222197e-01

Trapezoidal Method

Value: -8.666667e-01, Error Bound: 5.617284e-01, Actual Error: 1.326975e-01

Difference in error bound and actual error: 4.290309e-01

Corrected Trapezoid Method

Value: -7.101852e-01, Error Bound: 1.687757e-01, Actual Error: 2.378399e-02

Difference in error bound and actual error: 1.449917e-01

Simpsons Method

Value: -7.391053e-01, Error Bound: 6.328018e-02, Actual Error: 5.136164e-03

Difference in error bound and actual error: 5.814401e-02

B) f(x) = e^{3x} sin (2x)

Rectangle Method

Value: 0, Error Bound: 9.674078e+00, Actual Error: 2.588629e+00

Difference in error bound and actual error: 7.085449e+00

Rectangle Mid-point Method

Value: 1.803915e+00, Error Bound: 1.149403e+00, Actual Error: 7.847139e-01

Difference in error bound and actual error: 3.646888e-01

Trapezoidal Method

Value: 4.143260e+00, Error Bound: 2.298805e+00, Actual Error: 1.554631e+00

Difference in error bound and actual error: 7.441743e-01

Corrected Trapezoid Method

Value: 2.619014e+00, Error Bound: 1.865954e-01, Actual Error: 3.038514e-02

Difference in error bound and actual error: 1.562102e-01

Simpsons Method

Value: 2.583696e+00, Error Bound: 1.267866e-01, Actual Error: 4.932229e-03

Difference in error bound and actual error: 1.218544e-01

C) f(x) = sin(x)² - 2xsin (x) + 1

Rectangle Method

Value: 2.431953e-01, Error Bound: 3.187007e-01, Actual Error: 2.635721e-01

Difference in error bound and actual error: 5.512859e-02

Rectangle Mid-point Method

Value: -1.189526e-02, Error Bound: 1.222040e-02, Actual Error: 8.481537e-03

Difference in error bound and actual error: 3.738865e-03

Trapezoidal Method

Value: -3.702425e-02, Error Bound: 2.444081e-02, Actual Error: 1.664746e-02

Difference in error bound and actual error: 7.793349e-03

Corrected Trapezoid Method

Value: -2.079523e-02, Error Bound: 2.244364e-03, Actual Error: 4.184296e-04

Difference in error bound and actual error: 1.825934e-03

Simpsons Method

Value: -2.027159e-02, Error Bound: 1.147818e-04, Actual Error: 1.052061e-04

Difference in error bound and actual error: 9.575712e-06

D) f(x) = 1/(x ln(x))

Rectangle Method

Value: 3.678794e-01, Error Bound: 1.353353e-01, Actual Error: 9.536556e-02

Difference in error bound and actual error: 3.996972e-02

Rectangle Mid-point Method

Value: 2.658386e-01, Error Bound: 1.452123e-02, Actual Error: 6.675288e-03

Difference in error bound and actual error: 7.845940e-03

Trapezoidal Method

Value: 2.863342e-01, Error Bound: 2.904246e-02, Actual Error: 1.382029e-02

Difference in error bound and actual error: 1.522216e-02

Corrected Trapezoid Method

Value: 2.718629e-01, Error Bound: 3.561374e-03, Actual Error: 6.510291e-04

Difference in error bound and actual error: 2.910345e-03

Simpsons Method

Value: 2.726705e-01, Error Bound: 5.334208e-04, Actual Error: 1.565719e-04

Difference in error bound and actual error: 3.768489e-04

Q4) $f(x) = 4/(1+x^2)$

Rectangle Method

Value: 4, Error Bound: 1.299018e+00, Actual Error: 8.584073e-01

Difference in error bound and actual error: 4.406103e-01

Rectangle Mid-point Method

Value: 3.200000e+00, Error Bound: 3.333333e-01, Actual Error: 5.840735e-02

Difference in error bound and actual error: 2.749260e-01

Trapezoidal Method

Value: 3, Error Bound: 6.666667e-01, Actual Error: 1.415927e-01

Difference in error bound and actual error: 5.250740e-01

Corrected Trapezoid Method

Value: 3.166667e+00, Error Bound: 1.037229e-01, Actual Error: 2.507401e-02

Difference in error bound and actual error: 7.864891e-02

Simpsons Method

Value: 3.133333e+00, Error Bound: 3.333333e-02, Actual Error: 8.259320e-03

Difference in error bound and actual error: 2.507401e-02

Simpson's Three-Eighth Method

Value: 3.138462e+00, Error Bound: 1.481481e-02, Actual Error: 3.131115e-03

Difference in error bound and actual error: 1.168370e-02

Comment

 Accuracy Order (according to above observation): Rectangle rule < Trapezoid rule < Corrected Trapezoidal < Simpson one-third < Simpson three-eighth.

Q5)

A) Value of integral using composite trapezoidal: 0.421582

B) Value of integral using composite trapezoidal: 0.440345

C) Value of integral using composite trapezoidal: 3.159476

D) Value of integral using composite trapezoidal: -0.489323

Q6)

A) Value of integral using composite trapezoidal: 0.421582

Value of integral using composite midpoint: 0.423296

Value of integral using composite simpsons: 0.422725

B) Value of integral using composite trapezoidal: 0.440345

Value of integral using composite midpoint: 0.438609

Value of integral using composite simpsons: 0.439187

C) Value of integral using composite trapezoidal: 3.159476

Value of integral using composite midpoint: 3.084203

Value of integral using composite simpsons: 3.109294

D) Value of integral using composite trapezoidal: -0.489323

Value of integral using composite midpoint: -0.488867

Value of integral using composite simpsons: -0.489019

Q7)

A)

n	Actual	Simpson for n partitions	Error
1	3.14159265358979	3.13333333333333	0.00825932025645981
2	3.14159265358979	3.14156862745098	2.40261388126939e-05

3   3.14159265358979   3.14159178093604   8.72653750150221e-07

Number of iterations vs log(Error)
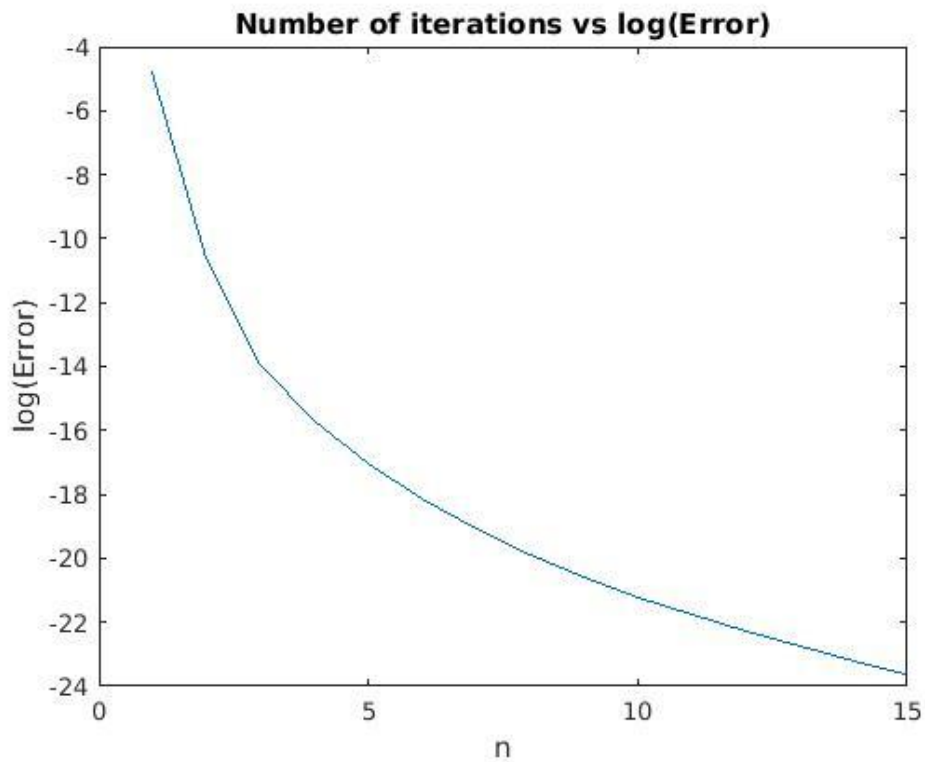
B)

n	Actual	Simpson for n partitions	Error
1	0.392699081698724	0.392143015375190	0.000556066323534499
2	0.392699081698724	0.392648364982518	5.07167162063227e-05
3	0.392699081698724	0.392687808568934	1.12731297903790e-05
4	0.392699081698724	0.392695327079649	3.75461907536900e-06

Number of iterations vs log(Error)
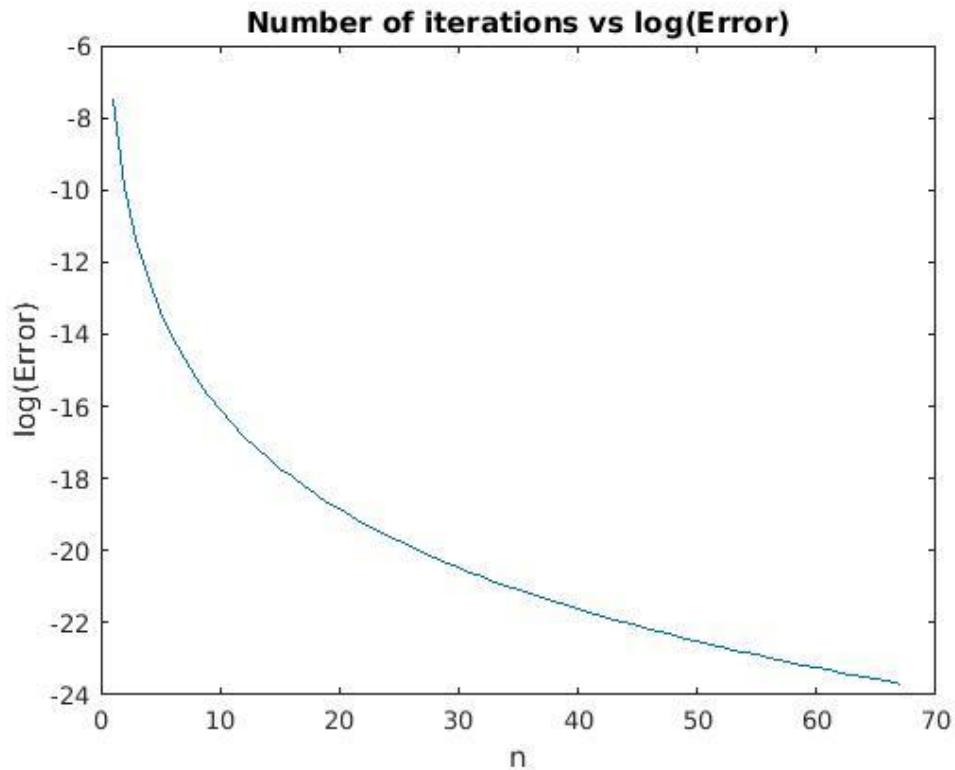
Q8)

The approximate value of the integral using the composite trapezoidal rule: 7.125

Q9)

Composite Trapezoidal rule

Few iterations:

n	Actual	Approximation	Error
1	0.636294361119891	0.693147180559945	0.0568528194400547
2	0.636294361119891	0.650672421361096	0.0143780602412054
3	0.636294361119891	0.642699772257435	0.00640541113754423
75	0.636294361119891	0.636304629934089	1.02688141987439e-05
76	0.636294361119891	0.636304361480817	1.00003609260213e-05

77   0.636294361119891   0.636304103418653   -9.74229876227195e-06

No. Of iterations required were: 77

h= 1/n = 0.01298

Composite Simpson's rule

n	Actual	Approximation	Error
1	0.636294361119891	0.636514168294813	0.000219807174922204
2	0.636294361119891	0.636309829796949	1.54686770587054e-05
3	**0.636294361119891**	**0.636297500790914**	**3.13967102305579e-06**

No. Of iterations required were: 3

h= 1/n = 0.33333

Composite Midpoint rule

Few iterations:

n	Actual	Approximation	Error
1	0.636294361119891	0.608197662162247	0.0280966989576440
2	0.636294361119891	0.629128534014876	0.00716582710501468
3	0.636294361119891	0.633096365057653	0.00319799606223736
52	0.636294361119891	0.636283680352335	1.06807675553933e-05
53	0.636294361119891	0.636284079593344	1.02815265462519e-05
54	**0.636294361119891**	**0.636284456860641**	**9.90425925007088e-06**

No. Of iterations required were: 54

h=1/n = 0.01851

log(Error) vs Number of iterations
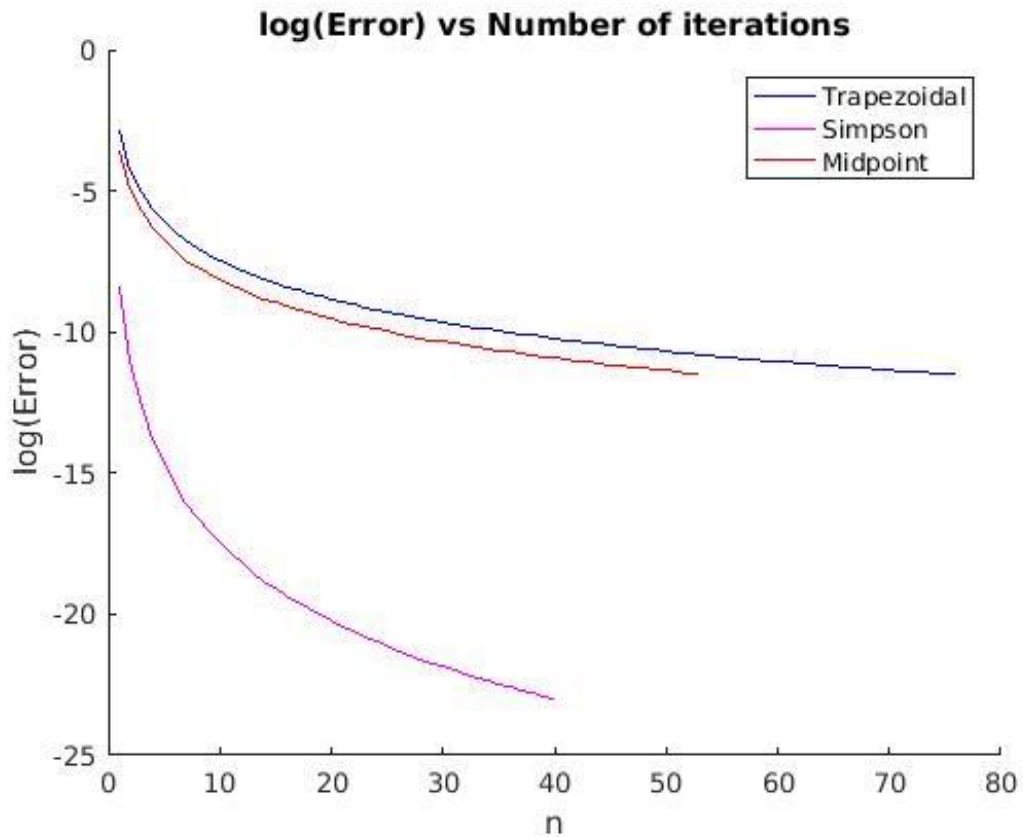
Q10)

Approximate value of the of the ellipse arc's length evaluated using Simpson rule: 15.8654.

Q11)

Approximate value of track length evaluated using Simpson rule: 9858 feet