LAB 02
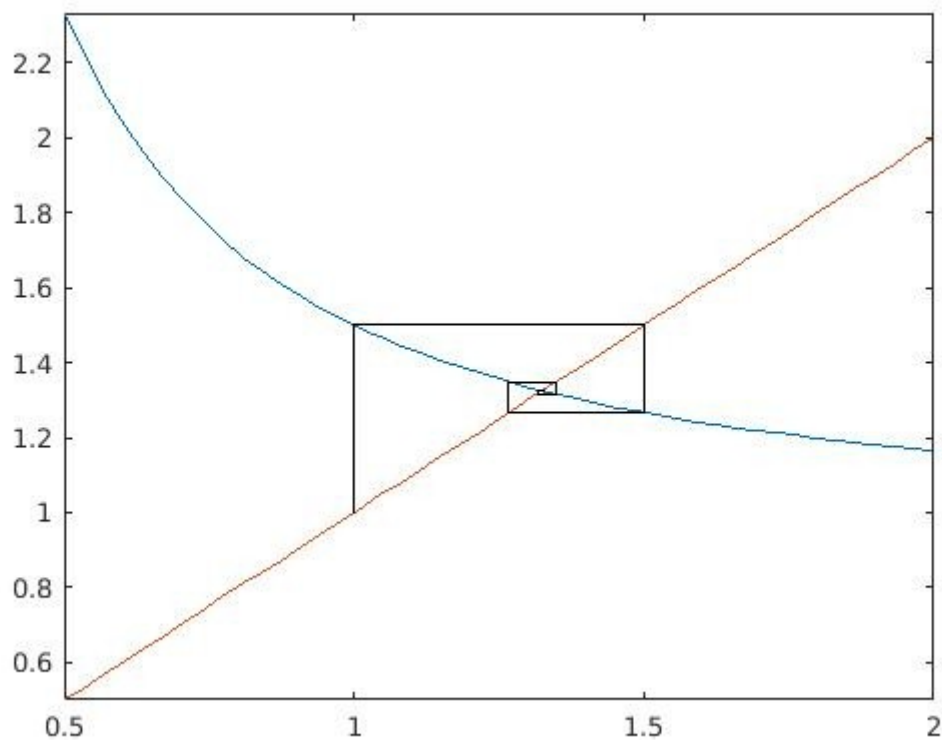
Q1) Fixed Point Iteration
Root of $x^3 - x - 1 = 0$ in [1, 2]
$x_0 = 1$
Solution $= 1.32$ (accurate to within 10^{-2})
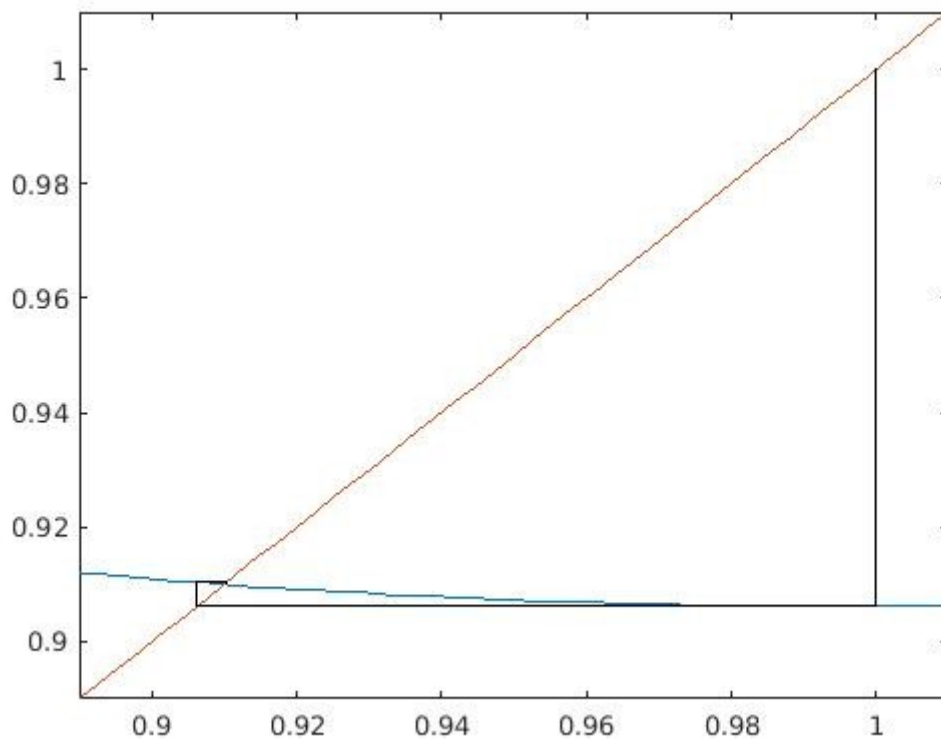
iteration	x	f(x)
1	1	1.500000e+00
2	1.500000e+00	1.266667e+00
3	1.266667e+00	1.348297e+00
4	1.348297e+00	1.315836e+00
5	1.315836e+00	1.328164e+00
6	1.328164e+00	1.323396e+00
7	1.323396e+00	1.325227e+00
8	1.325227e+00	1.324522e+00
9	1.324522e+00	1.324793e+00

Q2) Fixed point iteration, Convergence interval $|f'(x)|<1$

(a) $3x^2 - e^x = 0$, estimated interval (approx) : $[0, 0.4]$
using $x_0 = 1$

iteration	x	f(x)
1	1	9.060939e-01
2	9.060939e-01	9.103683e-01
3	9.103683e-01	9.099752e-01
4	9.099752e-01	9.100105e-01
5	9.100105e-01	9.100073e-01

(b) $x - \cos x = 0$ estimated interval : [-pi/2, 0], using $x_0 = 0$

iteration	x	f(x)
1	0	1
2	1	5.403023e-01
3	5.403023e-01	8.575532e-01
4	8.575532e-01	6.542898e-01
5	6.542898e-01	7.934804e-01
6	7.934804e-01	7.013688e-01
7	7.013688e-01	7.639597e-01
8	7.639597e-01	7.221024e-01
9	7.221024e-01	7.504178e-01
10	7.504178e-01	7.314040e-01
11	7.314040e-01	7.442374e-01
12	7.442374e-01	7.356047e-01
13	7.356047e-01	7.414251e-01
14	7.414251e-01	7.375069e-01
15	7.375069e-01	7.401473e-01
16	7.401473e-01	7.383692e-01
17	7.383692e-01	7.395672e-01
18	7.395672e-01	7.387603e-01
19	7.387603e-01	7.393039e-01
20	7.393039e-01	7.389378e-01
21	7.389378e-01	7.391844e-01
22	7.391844e-01	7.390183e-01
23	7.390183e-01	7.391302e-01
24	7.391302e-01	7.390548e-01
25	7.390548e-01	7.391056e-01
26	7.391056e-01	7.390714e-01
27	7.390714e-01	7.390944e-01
28	7.390944e-01	7.390789e-01
29	7.390789e-01	7.390893e-01
30	7.390893e-01	7.390823e-01
31	7.390823e-01	7.390870e-01
32	7.390870e-01	7.390838e-01
33	7.390838e-01	7.390860e-01
34	7.390860e-01	7.390845e-01
35	7.390845e-01	7.390855e-01
36	7.390855e-01	7.390849e-01
37	7.390849e-01	7.390853e-01
38	7.390853e-01	7.390850e-01
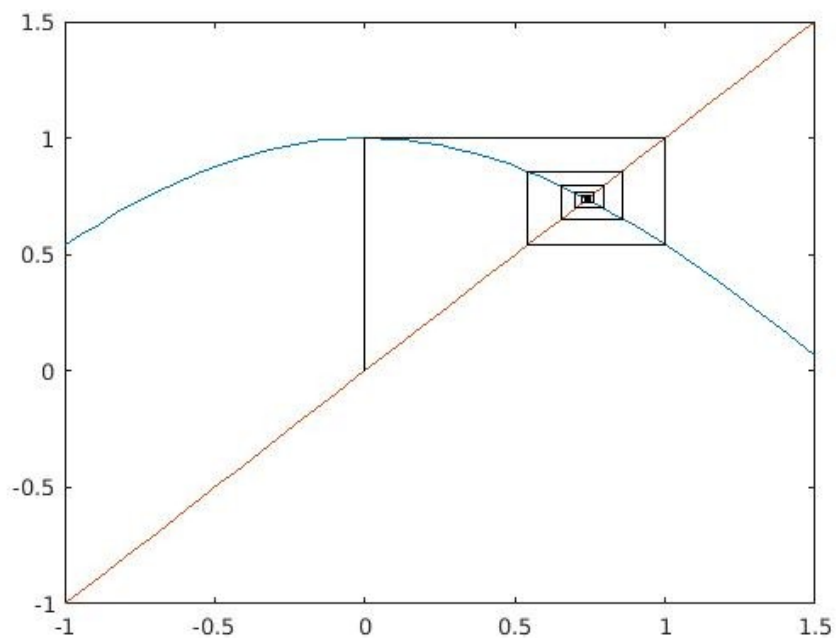
Q3) Fixed Point Iteration
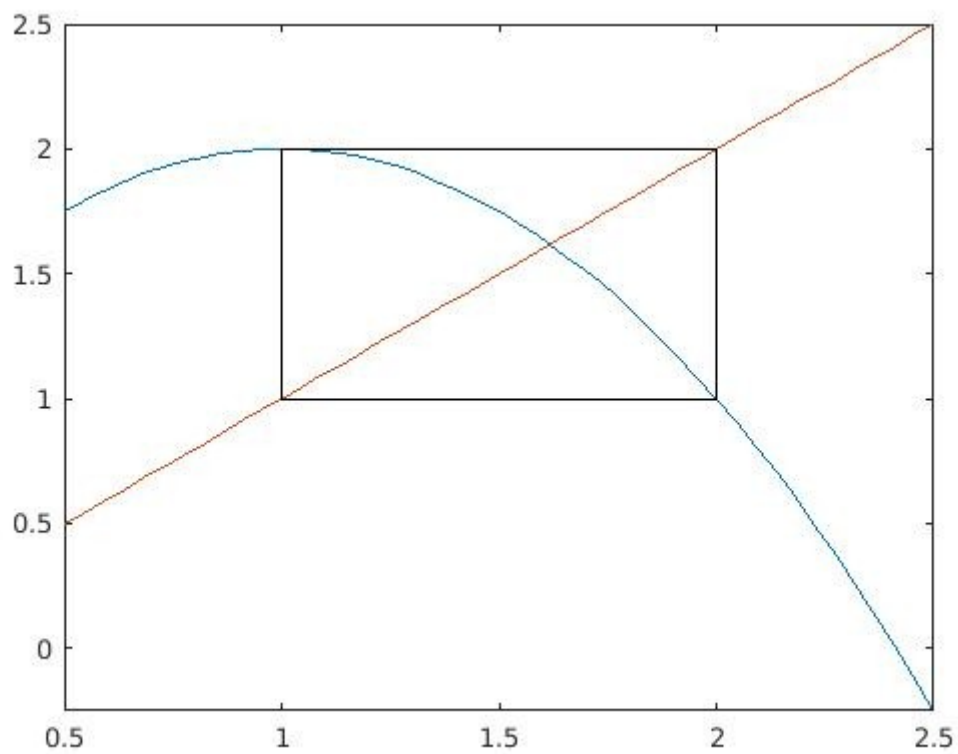Root of $x^3 - x - 1 = 0$ in $[1, 2]$
$x_0 = 1$
a) $x = x^2 - 1$

iteration	x	f(x)
1	1	0
2	0	-1
3	-1	0
4	0	-1
5	-1	0
6	0	-1
7	-1	0
8	0	-1
9	-1	0
10	0	-1
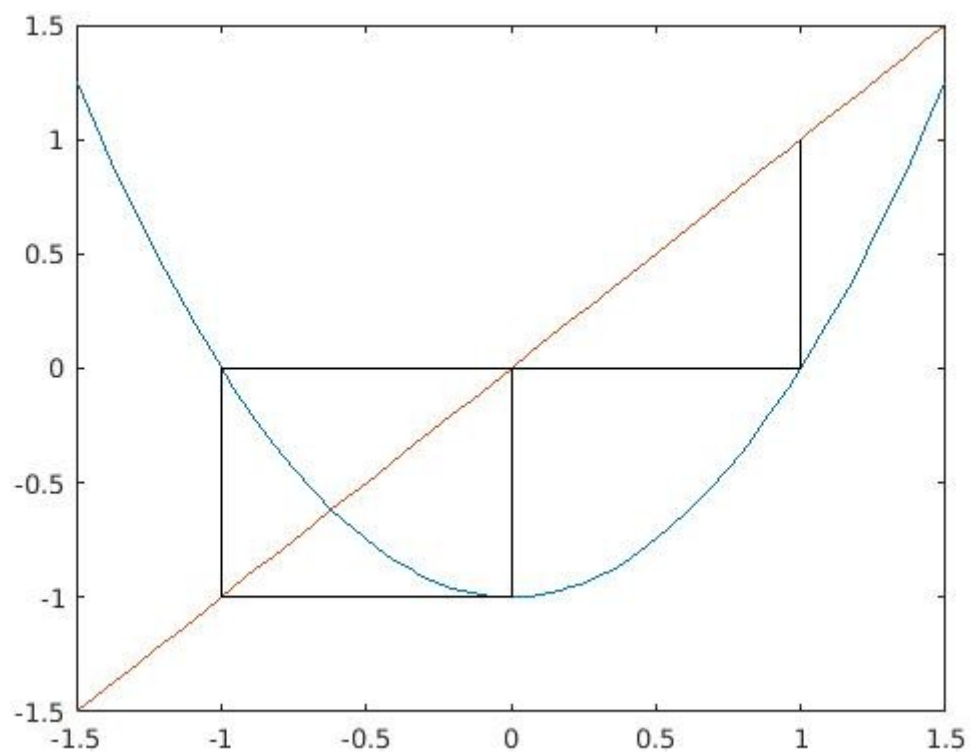
b) $x = 1 + 2x - x^2$

iteration	x	f(x)
1	1	2
2	2	1
3	1	2
4	2	1
5	1	2
6	2	1
7	1	2
8	2	1
9	1	2
10	2	1

c) $x = 1/2 \, (1 + 3x - x^2)$

iteration	x	f(x)
1	1	1.500000e+00
2	1.500000e+00	1.625000e+00
3	1.625000e+00	1.617188e+00
4	1.617188e+00	1.618134e+00
5	1.618134e+00	1.618022e+00
6	1.618022e+00	1.618035e+00
7	1.618035e+00	1.618034e+00
8	1.618034e+00	1.618034e+00
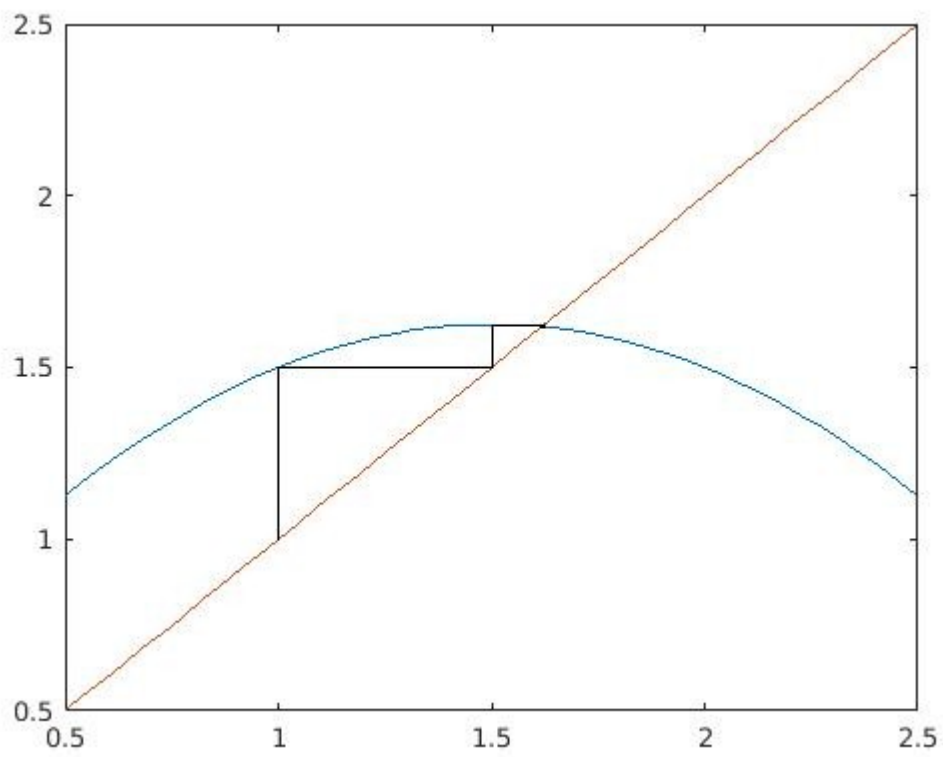
$x_0 = 2$

a) $x = x^2 - 1$

iteration	x	f(x)
1	2	3
2	3	8
3	8	63
4	63	3968
5	3968	15745023
6	15745023	247905749270528
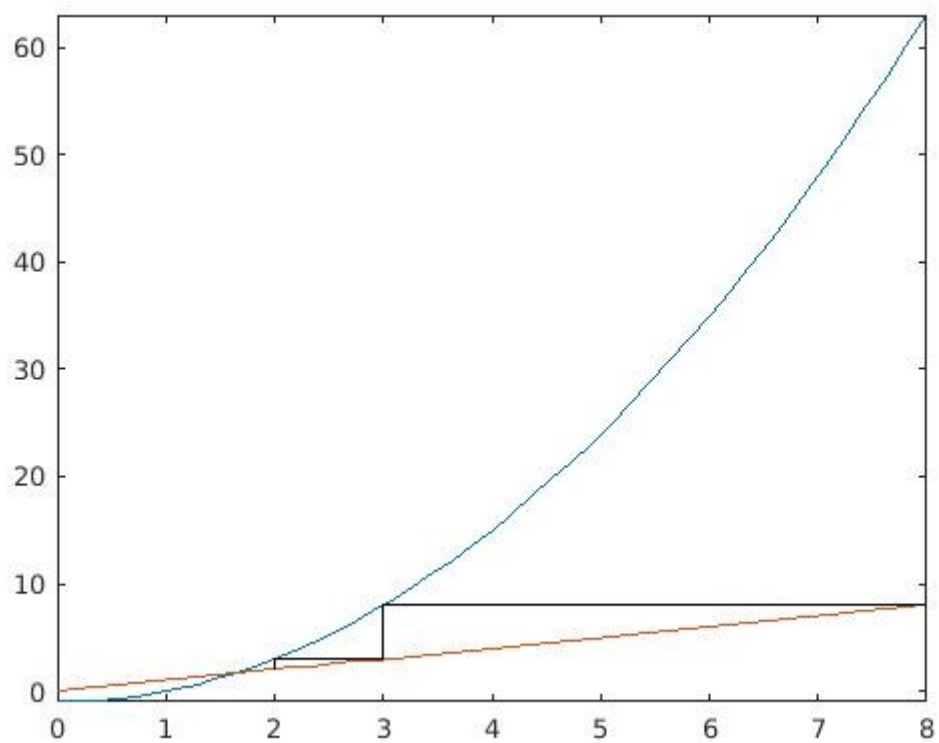
b) $x = 1 + 2x - x^2$

iteration	x	f(x)
1	2	1
2	1	2
3	2	1
4	1	2
5	2	1
6	1	2
7	2	1
8	1	2
9	2	1
10	1	2
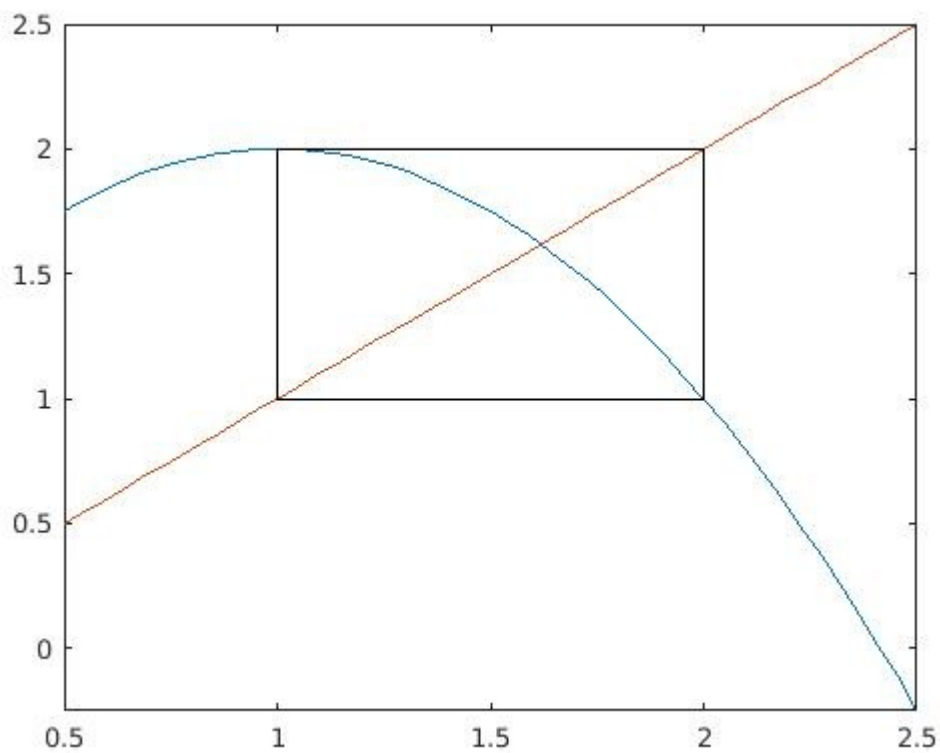
c) $x = 1/2 \, (1 + 3x - x^2)$

iteration	x	f(x)
1	2	1.500000e+00
2	1.500000e+00	1.625000e+00
3	1.625000e+00	1.617188e+00
4	1.617188e+00	1.618134e+00
5	1.618134e+00	1.618022e+00
6	1.618022e+00	1.618035e+00
7	1.618035e+00	1.618034e+00
8	1.618034e+00	1.618034e+00
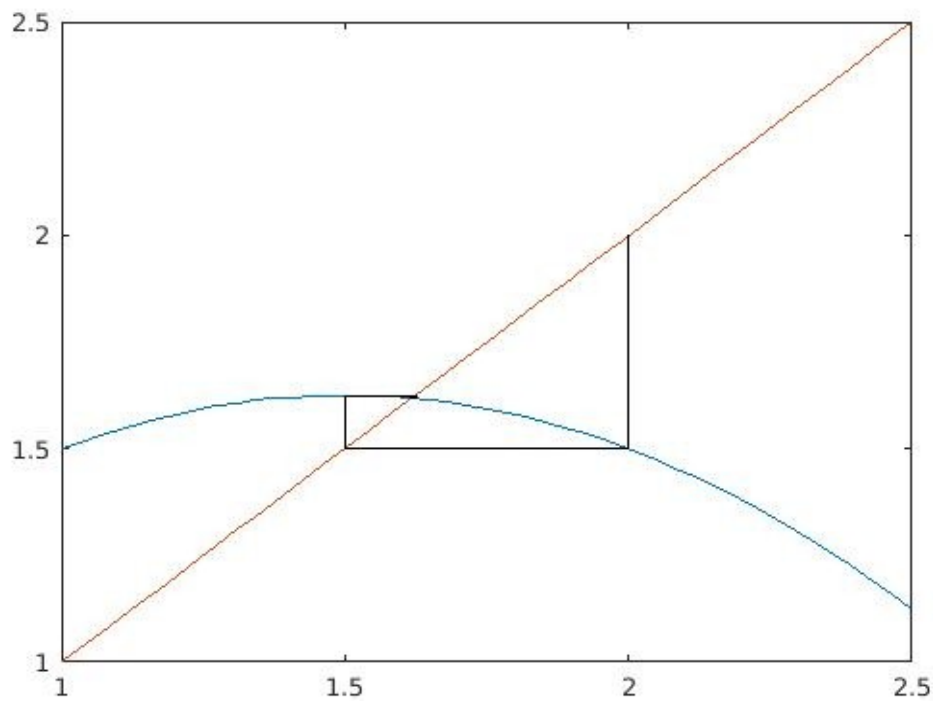
Results Discussion :
- The iterations are not converging using (a) and (b).
- The solution is diverging for x0=2 using (a).
- The solution is converging using (c) for both x0=1 and x0=2.
- The root of this equaltion is 1.618

Q4) Finding the square root of 0.75 by writing $f(x) = x^2 - 0.75$.
Solving $x = x^2 + x - 0.75$ by fixed-point iteration method.

$x_0 = -0.8$

iteration	x	f(x)
1	-8.000000e-01	-9.100000e-01
2	-9.100000e-01	-8.319000e-01
3	-8.319000e-01	-8.898424e-01
4	-8.898424e-01	-8.480229e-01
5	-8.480229e-01	-8.788801e-01
6	-8.788801e-01	-8.564499e-01
7	-8.564499e-01	-8.729435e-01
8	-8.729435e-01	-8.609132e-01
9	-8.609132e-01	-8.697417e-01
10	-8.697417e-01	-8.632911e-01
11	-8.632911e-01	-8.680196e-01
12	-8.680196e-01	-8.645616e-01
13	-8.645616e-01	-8.670949e-01
14	-8.670949e-01	-8.652414e-01
15	-8.652414e-01	-8.665987e-01
16	-8.665987e-01	-8.656054e-01
17	-8.656054e-01	-8.663327e-01
18	-8.663327e-01	-8.658003e-01
19	-8.658003e-01	-8.661901e-01
20	-8.661901e-01	-8.659048e-01
21	-8.659048e-01	-8.661137e-01
22	-8.661137e-01	-8.659608e-01
23	-8.659608e-01	-8.660727e-01
24	-8.660727e-01	-8.659908e-01
25	-8.659908e-01	-8.660508e-01
26	-8.660508e-01	-8.660068e-01
27	-8.660068e-01	-8.660390e-01
28	-8.660390e-01	-8.660155e-01
29	-8.660155e-01	-8.660327e-01
30	-8.660327e-01	-8.660201e-01
31	-8.660201e-01	-8.660293e-01
32	-8.660293e-01	-8.660225e-01
33	-8.660225e-01	-8.660275e-01
34	-8.660275e-01	-8.660239e-01
35	-8.660239e-01	-8.660265e-01
36	-8.660265e-01	-8.660246e-01
37	-8.660246e-01	-8.660260e-01
38	-8.660260e-01	-8.660250e-01
39	-8.660250e-01	-8.660257e-01
40	-8.660257e-01	-8.660252e-01
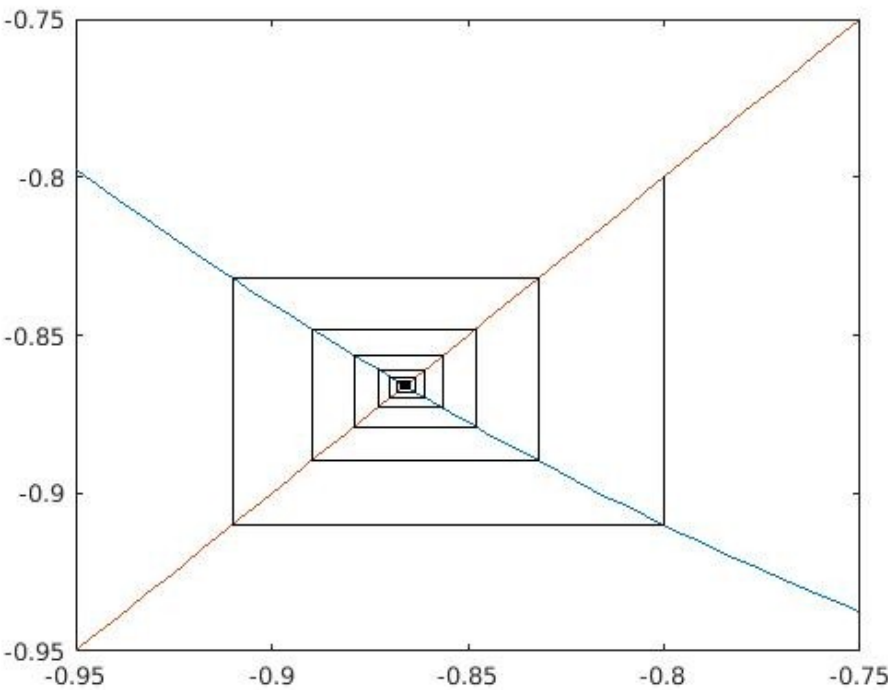
$$x_0 = 0.8$$

iteration	x	f(x)
1	8.000000e-01	6.900000e-01
2	6.900000e-01	4.161000e-01
3	4.161000e-01	-1.607608e-01
4	-1.607608e-01	-8.849168e-01
5	-8.849168e-01	-8.518391e-01
6	-8.518391e-01	-8.762093e-01
7	-8.762093e-01	-8.584666e-01
8	-8.584666e-01	-8.715017e-01
9	-8.715017e-01	-8.619865e-01
10	-8.619865e-01	-8.689658e-01
11	-8.689658e-01	-8.638643e-01
12	-8.638643e-01	-8.676028e-01
13	-8.676028e-01	-8.648682e-01
14	-8.648682e-01	-8.668712e-01
15	-8.668712e-01	-8.654055e-01
16	-8.654055e-01	-8.664788e-01
17	-8.664788e-01	-8.656933e-01
18	-8.656933e-01	-8.662684e-01
19	-8.662684e-01	-8.658474e-01
20	-8.658474e-01	-8.661556e-01
21	-8.661556e-01	-8.659300e-01
22	-8.659300e-01	-8.660952e-01
23	-8.660952e-01	-8.659743e-01
24	-8.659743e-01	-8.660628e-01
25	-8.660628e-01	-8.659980e-01
26	-8.659980e-01	-8.660455e-01
27	-8.660455e-01	-8.660107e-01
28	-8.660107e-01	-8.660361e-01
29	-8.660361e-01	-8.660175e-01
30	-8.660175e-01	-8.660312e-01
31	-8.660312e-01	-8.660212e-01
32	-8.660212e-01	-8.660285e-01
33	-8.660285e-01	-8.660231e-01
34	-8.660231e-01	-8.660271e-01
35	-8.660271e-01	-8.660242e-01
36	-8.660242e-01	-8.660263e-01
37	-8.660263e-01	-8.660248e-01
38	-8.660248e-01	-8.660259e-01
39	-8.660259e-01	-8.660251e-01
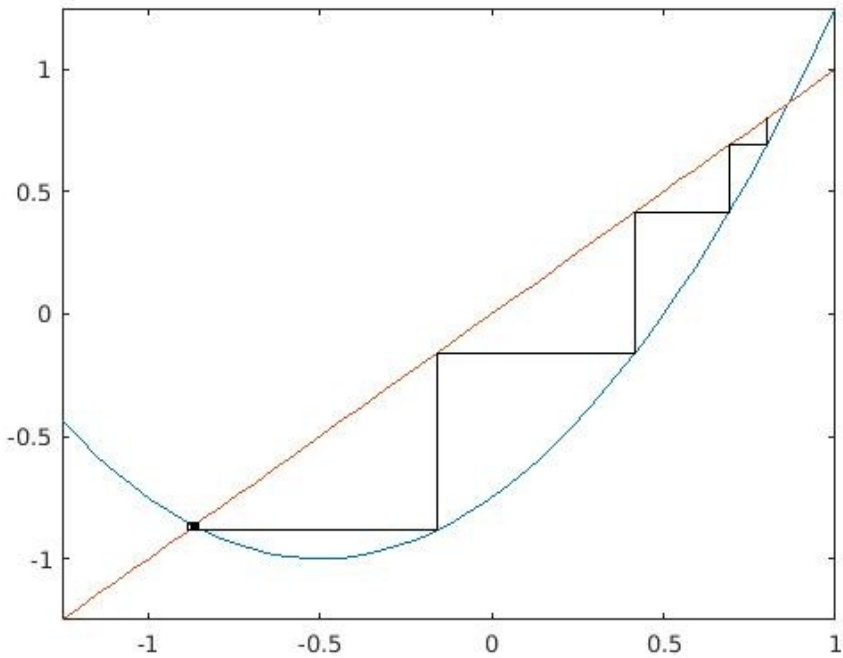
Comments:
The iterations are converging in both the cases with a minute difference in the number of iterations and it is converging to the negative square root = -0.866 in both the cases (x0=0.8, x0=-0.8).

Q5) Muller's Method

(a) $x^3 - x - 2 = 0$, x 1 = 1, x 2 = 1.2, x 3 = 1.4

iteration	x	f(x)
1	1.524956e+00	2.131598e-02
2	1.521356e+00	-1.403967e-04
3	1.521380e+00	-1.025336e-08
4	1.521380e+00	0

Solution = 1.521380e+00

(b) $1 + 2x - \tan x = 0$, x 1 = 1.5, x 2 = 1.4, x 3 = 1.3

iteration	x	f(x)
1	1.300200e+00	-4.497738e-03
2	1.299824e+00	1.951893e-06
3	1.299824e+00	-2.002398e-12

Solution =   1.299824e+00

Q6) Muller's Method

(a) $x^2 + e^x = 5$, root near x = 1 and x = −2

iteration	x	f(x)
1	1.242097e+00	5.673481e-03
2	1.241136e+00	-3.950658e-05
3	1.241143e+00	-1.148883e-09

Solution =   1.241143e+00

iteration	x	f(x)
1	-2.211434e+00	-1.573827e-05
2	-2.211438e+00	-1.704707e-10

Solution =   -2.211438e+00

(b) $x^2 = \sin x$, root near x = 0.9

iteration	x	f(x)
1	8.766940e-01	-3.592058e-05
2	8.767263e-01	5.012194e-08
3	8.767262e-01	-2.864375e-14

Solution =   8.767262e-01

Q7) Muller's Method complex root:

(i) $z^4 - 2z^3 - 2iz^2 + 4iz = 0$

iteration	x	f(x)
1	7.208614e-01 + 6.809028e-01 i	-4.647164e-01
2	9.315658e-01 + 8.493513e-01 i	-2.998960e-01
3	1.026505e+00 + 9.669532e-01 i	-2.288692e-01
4	1.000344e+00 + 1.002111e+00 i	7.089028e-03
5	1.000004e+00 + 9.999895e-01 i	-5.993232e-05
6	1.000000e+00 + 1.000000e+00 i	-4.163695e-09
7	1 + 1.000000e+00 i	8.881784e-16

Solution = 1 + i

(ii) $z = e^z$

iteration	x	f(x)
1	2.994784e-01 + 1.364624e+00 i	2.328688e-02
2	3.189639e-01 + 1.337230e+00 i	5.602540e-04
3	3.181305e-01 + 1.337237e+00 i	1.229708e-06
4	3.181315e-01 + 1.337236e+00 i	9.448775e-12

Solution =   3.181315e-01 + 1.337236e+00 i