LAB 03

1) Newton's method on a system of non-linear equations

a) starting point : (1, 1)

First 2 iterations for given system:

iter	x	y	f1	f2
1	1	1	0	0
2	1	1	0	0

b) starting point : (0, 0)

First 2 iterations for given system:

iter	x	y	f1	f2
1	7.804613e+00	6.129992e-01	-9.447522e+00	5.461192e+07
2	-5.991641e+05	5.691036e-01	5.991635e+05	3.257373e+06

2)Newton's method on a system of non-linear equations

Solution of the given system:

n	x1	x2	x2	f1	f2	f3
0	0	0	0	-3	1.929563e+00	3.141593e+01
1	5.000000e-01	-1.861423e-01	-5.235988e-01	9.491694e-03	1.247190e-01	2.926195e-01
2	4.981578e-01	-1.996068e-01	-5.288264e-01	7.887441e-05	-1.269028e-06	-1.484140e-05
3	4.981447e-01	-1.996059e-01	-5.288260e-01	4.098943e-13	6.805556e-11	5.615419e-11
4	4.981447e-01	-1.996059e-01	-5.288260e-01	-2.220446e-16	1.110223e-16	-3.552714e-15

Solution : x1= 4.981447e-01 x2= -1.996059e-01 x3=-5.288260e-01

3)Eliminating y from the first equation to get x by Newton's Method then calculate y using the obtained x

iter	x	y	f1	f2

0	1	136.358482e-01	-7.773067e+00	0
1	2.545187e+00	132.339952e-01	-5.135555e+00	0
2	2.830910e+00	131.774726e-01	5.769216e-01	0
3	2.804987e+00	131.817455e-01	5.793825e-03	-4.440892e-16
4	2.804721e+00	131.817900e-01	5.991336e-07	0
5	2.804721e+00	1.817900e-01	-3.552714e-15	0

Solution : x = 2.804721e+00 y = 1.817900e-01

4)Newton Vs Modified Newton

A) using newtons: Stopping criteria = (x(i+1)-x(i) < 1e-6)

iter	x	f(x)
1	-1.500000e+00	2.256092e-06
2	-1.478551e+00	7.138014e-07
3	-1.462465e+00	2.258441e-07
4	-1.450402e+00	7.145721e-08
5	-1.441355e+00	2.260928e-08
6	-1.434569e+00	7.153677e-09
7	-1.429480e+00	2.263461e-09
8	-1.425664e+00	7.161719e-10
9	-1.422801e+00	2.266010e-10
10	-1.420654e+00	7.169798e-11
11	-1.419044e+00	2.268552e-11
12	-1.417836e+00	7.177814e-12
13	-1.416931e+00	2.271183e-12
14	-1.416251e+00	7.185363e-13
15	-1.415742e+00	2.272627e-13
16	-1.415360e+00	7.183143e-14
17	-1.415074e+00	2.275957e-14
18	-1.414860e+00	7.105427e-15
19	-1.414703e+00	2.220446e-15
20	-1.414589e+00	7.771561e-16
21	-1.414501e+00	1.110223e-16
22	-1.414473e+00	0
22	-1.414473e+00	0

Solution : x =-1.414473e+00 Iterations: 22

Using Modified Newtons: Stopping criteria = (x(i+1)-x(i) < (1e-6/p))

Here p is the multiplicity which is 3 in this case.

iter	x	f(x)
0	-1.500000e+00	2.256092e-06
1	-1.435652e+00	8.801917e-09

2	-1.419573e+00	3.437972e-11
3	-1.415553e+00	1.341149e-13
4	-1.414550e+00	3.330669e-16
5	-1.414392e+00	0
6	-1.414392e+00	0

Solution : x =-1.414392e+00 Iterations: 6

B) using newtons: Stopping criteria = (x(i+1)-x(i) < 1e-6)

iter	x	f(x)
1	-5.000000e-01	-3.441303e-02
2	-3.526384e-01	-7.901454e-03
3	-2.854364e-01	-2.102817e-03
4	-2.476465e-01	-5.875295e-04
5	-2.247398e-01	-1.680793e-04
6	-2.103222e-01	-4.872172e-05
7	-2.010516e-01	-1.423501e-05
8	-1.950131e-01	-4.179540e-06
9	-1.910478e-01	-1.231011e-06
10	-1.884303e-01	-3.633115e-07
11	-1.866968e-01	-1.073678e-07
12	-1.855460e-01	-3.175777e-08
13	-1.847811e-01	-9.398912e-09
14	-1.842721e-01	-2.782736e-09
15	-1.839332e-01	-8.240953e-10
16	-1.837075e-01	-2.440937e-10
17	-1.835571e-01	-7.230766e-11
18	-1.834568e-01	-2.142125e-11
19	-1.833900e-01	-6.346423e-12
20	-1.833455e-01	-1.880440e-12
21	-1.833158e-01	-5.571654e-13
22	-1.832960e-01	-1.650347e-13
23	-1.832828e-01	-4.890532e-14
24	-1.832740e-01	-1.448841e-14
25	-1.832682e-01	-4.385381e-15
26	-1.832642e-01	-1.054712e-15
27	-1.832619e-01	-6.106227e-16
28	-1.832594e-01	-1.665335e-16
29	-1.832570e-01	5.551115e-17
30	-1.832843e-01	-5.723200e-14
31	-1.832750e-01	-1.693090e-14
32	-1.832688e-01	-5.273559e-15
33	-1.832645e-01	-1.387779e-15
34	-1.832619e-01	-6.106227e-16
35	-1.832593e-01	5.551115e-17
35	-1.832602e-01	5.551115e-17

Solution : x =-1.832602e-01 Iterations: 35

Using Modified Newtons: Stopping criteria = (x(i+1)-x(i) < (1e-6/p))

Here p is the multiplicity which is 2 in this case.

iter	x	f(x)
0	-5.000000e-01	-3.441303e-02
1	-2.052769e-01	-2.663532e-05
2	-1.902685e-01	-8.994571e-07
3	-1.855609e-01	-3.237819e-08
4	-1.840211e-01	-1.188136e-09a)
5	-1.835109e-01	-4.387041e-11
6	-1.833412e-01	-1.623202e-12
7	-1.832847e-01	-6.011858e-14
8	-1.832659e-01	-2.164935e-15
9	-1.832598e-01	-1.665335e-16
10	-1.832559e-01	-2.775558e-16
11	-1.830321e-01	3.008899e-11
12	-1.831817e-01	1.115386e-12
13	-1.832315e-01	4.124479e-14
14	-1.832481e-01	1.609823e-15
15	-1.832539e-01	-1.665335e-16
16	-1.832476e-01	1.720846e-15
17	-1.832531e-01	-5.551115e-17
18	-1.832519e-01	1.665335e-16

Solution : x= -1.832519e-01 Iterations: 18

Comments:

Using modified newtons method **<u>improves</u>** the speed significantly the number of iterations reduced from 22 to 6 in the first example and from 35 to 18 in the second one.

5)Modified Newton's

A) seed = 1.3, root = 1, multiplicity = 4

iter	x	f(x)	df(x)	en/en-1	log(en/en-1)
0	1.300000e+00	2.673000e-02	3.645000e-01		
1	1.006667e+00	5.939094e-09	3.565432e-06	2.222222e-02	-3.806662e+00
2	1.000004e+00	8.881784e-16	0	5.541831e-04	-7.498015e+00

Solution : 1.000004e+00

B) seed = 1.3, root = 1, multiplicity = 2

iter	x	f(x)	df(x)	en/en-1	log(en/en-1)
0	1.300000e+00	-3.087000e-02	-7.350000e-02		
1	4.600000e-01	-1.065000e+00	6.019121e+00	1.800000e+00	5.877867e-01
2	8.138723e-01	-5.781175e-02	7.674251e-01	3.446809e-01	-1.065136e+00
3	9.645366e-01	-1.396259e-03	8.278890e-02	1.905329e-01	-1.657930e+00
4	9.982671e-01	-3.018411e-06	3.492790e-03	4.886303e-02	-3.018734e+00
5	9.999955e-01	-2.011191e-11	8.969643e-06	2.588064e-03	-5.956845e+00

Solution : 9.999955e-01

C) seed = 3, root = 2, multiplicity = 3

iter	x	f(x)	df(x)	en/en-1	log(en/en-1)
0	3	4	16		
1	2.250000e+00	2.441406e-02	3.320312e-01	2.500000e-01	-1.386294e+00
2	2.029412e+00	2.696134e-05	2.802439e-03	1.176471e-01	-2.140066e+00
3	2.000550e+00	1.663238e-10	9.080134e-07	1.869159e-02	-3.979682e+00
4	2.000000e+00	1.421085e-14	1.705303e-13	4.230418e-04	-7.768040e+00
5	9.999955e-01	-2.011191e-11	8.969643e-06	2.588064e-03	-5.956845e+00

Solution : 2.000000e+00