

**DEPARTMENT OF MATHEMATICS
IIT GUWAHATI**

MA 332

Scientific Computing

Lab - X

1. For various Δt and Δx , solve the following parabolic initial-boundary-value problem, numerically, by

- i) forward-time and central space (FTCS) discretization scheme,
- ii) backward-time and central space (BTCS) discretization scheme,
- iii) Crank-Nicolson scheme.

(a)

$$\begin{cases} \frac{\partial u}{\partial t} - \left(\frac{4}{\pi^2}\right) \frac{\partial^2 u}{\partial x^2} = 0, & (x, t) \in (0, 4) \times (0, 1), \\ u(x, 0) = \sin\left(\frac{\pi}{4}x\right) \left(1 + 2\cos\left(\frac{\pi}{4}x\right)\right), & x \in (0, 4), \\ u(0, t) = u(4, t) = 0, & t \in (0, 1]. \end{cases}$$

The exact solution is given by

$$u(x, t) = e^{-t} \sin\left(\frac{\pi}{2}x\right) + e^{(-t/4)} \sin\left(\frac{\pi}{4}x\right).$$

(b)

$$\begin{cases} \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0, & (x, t) \in (0, 1) \times (0, 1), \\ u(x, 0) = \sin(\pi x), & x \in (0, 1), \\ u(0, t) = u(1, t) = 0, & t \in (0, 1]. \end{cases}$$

The exact solution is given by

$$u(x, t) = e^{-\pi^2 t} \sin(\pi x).$$

(c)

$$\begin{cases} \frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0, & (x, t) \in (0, 1) \times (0, 1), \\ u(x, 0) = \sin\left(\frac{\pi x}{2}\right) + \frac{1}{2} \sin(2\pi x), & x \in (0, 1), \\ u(0, t) = 0, \quad u(1, t) = e^{-\pi^2 t/4}, & t \in (0, 1]. \end{cases}$$

The exact solution is given by

$$u(x, t) = e^{-\pi^2 t/4} \sin\left(\frac{\pi x}{2}\right) + \frac{1}{2} e^{-4\pi^2 t} \sin(2\pi x).$$

Provide the following:

1. Plot the exact and numerical solutions at the final time level in different colors with some symbols.
 2. Draw the surface plot of the exact and numerical solutions.
 3. Plot N versus $Max. Error$ in loglog scale.
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