LAB 11

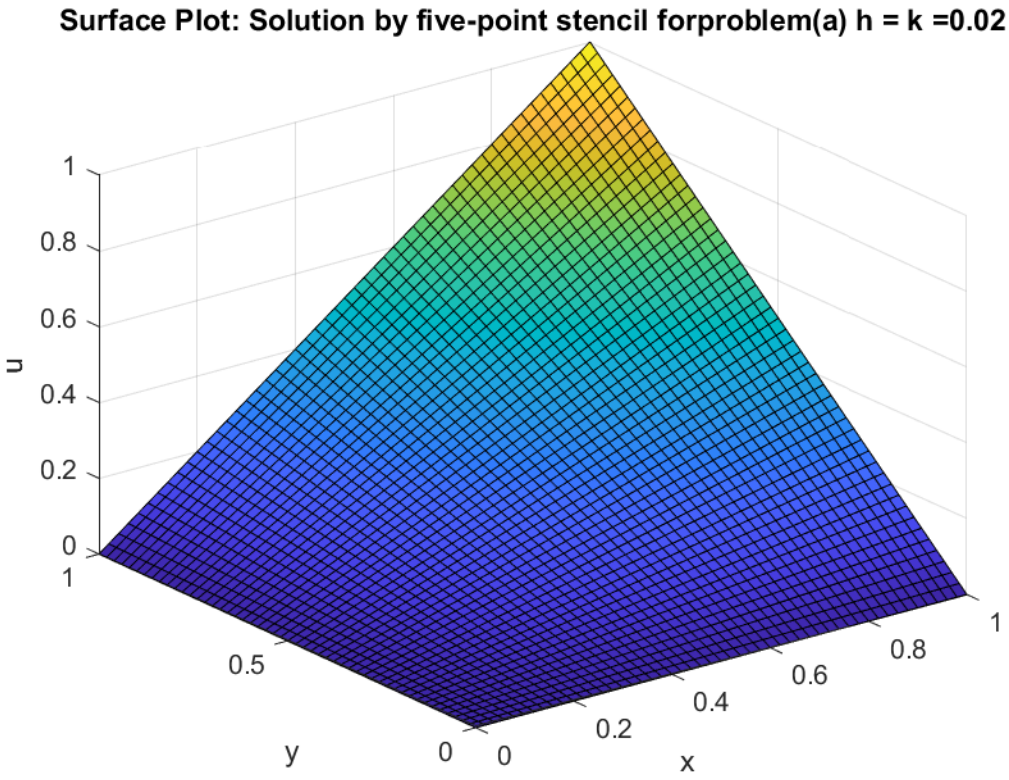
1) Five point stencil scheme

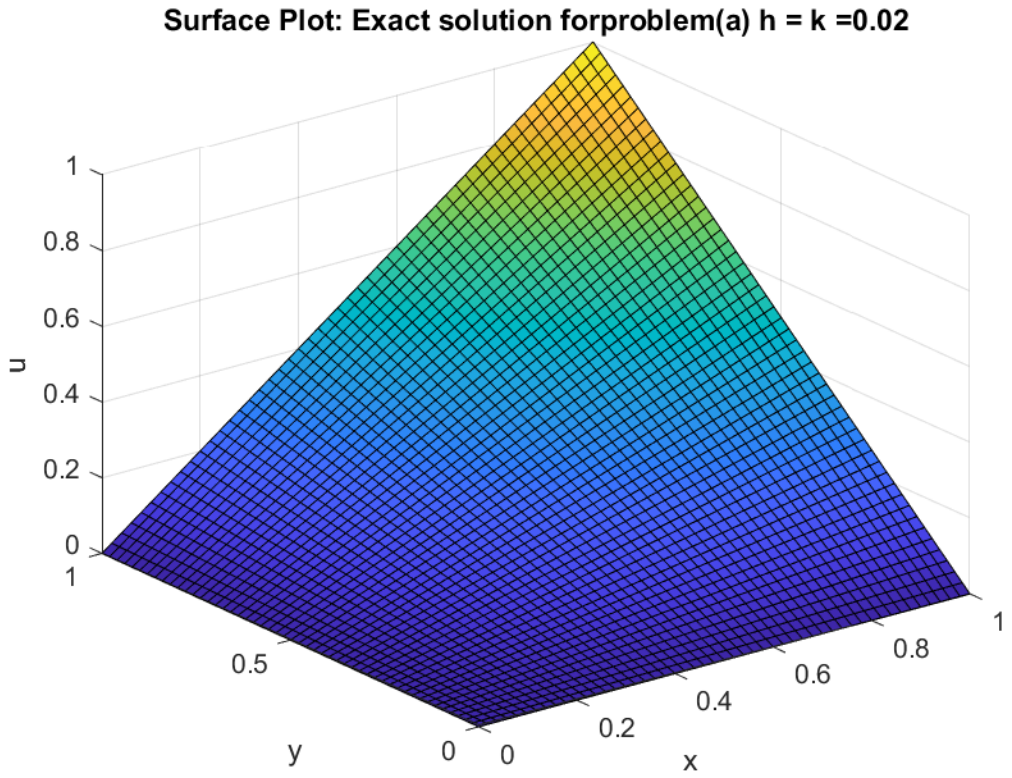
a)

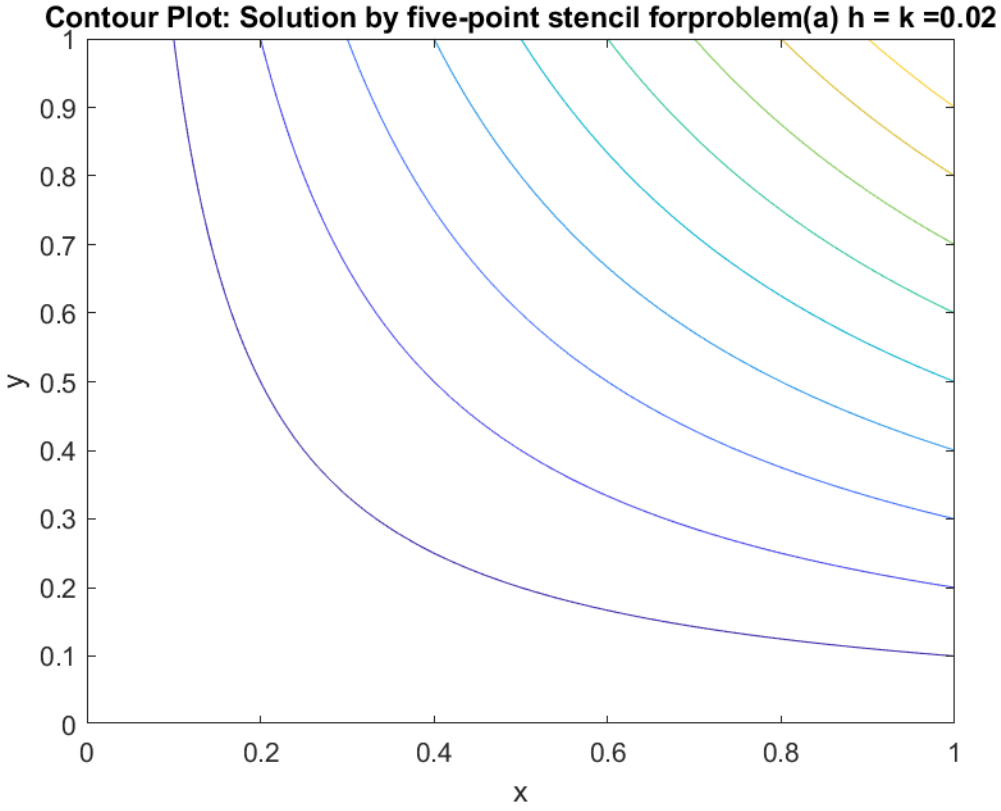
(a)

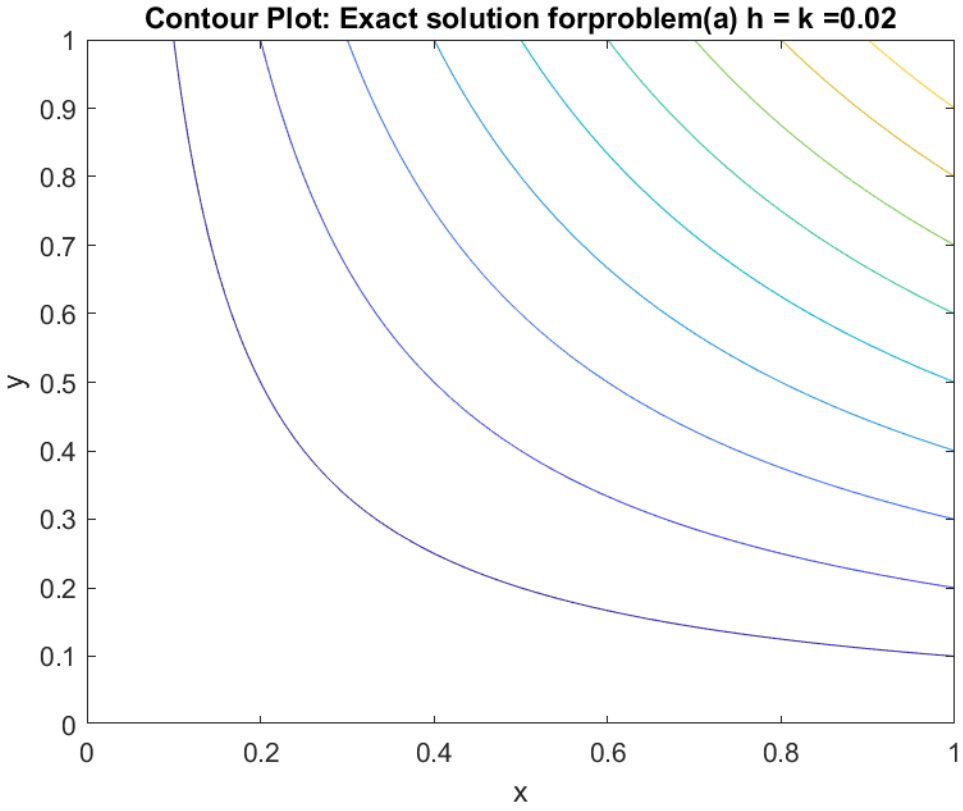
$$\begin{cases} u_{xx} + u_{yy} = 0, \quad 0 < x < 1,\ 0 < y < 1; \\ u(x,0) = 0, \quad u(x,1) = x,\ 0 \le x \le 1; \\ u(0,y) = 0, \quad u(1,y) = y,\ 0 \le y \le 1. \end{cases}$$

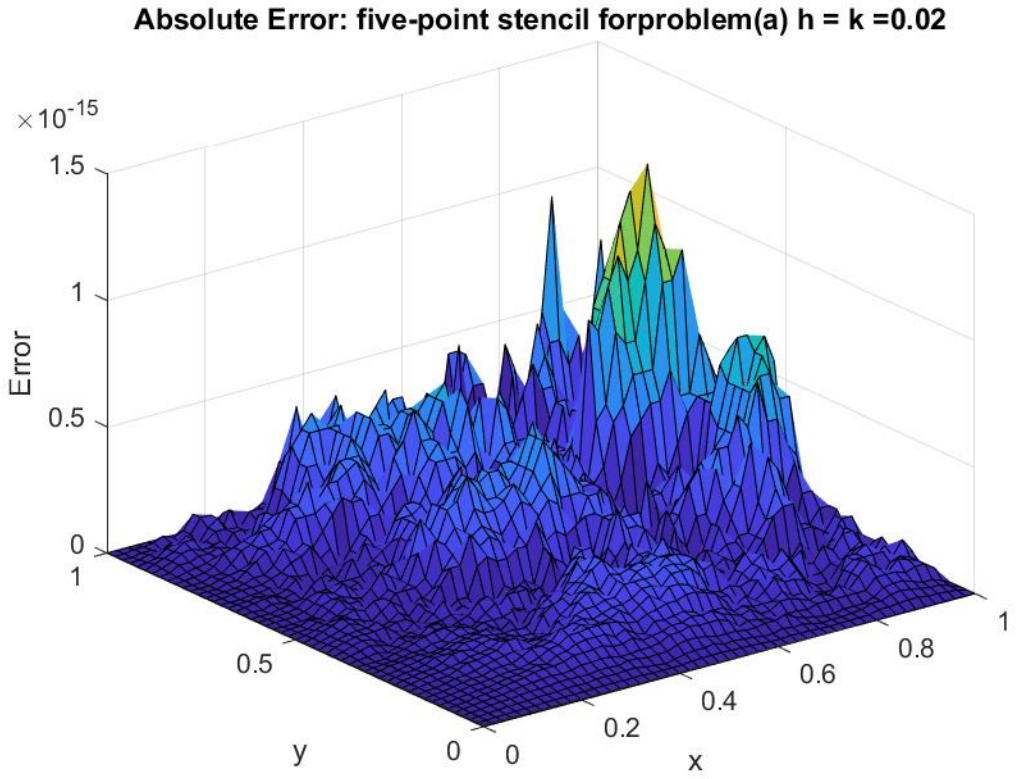
Use $h = k = 0.002$ and compare the results to the exact solution $u(x, y) = xy$.

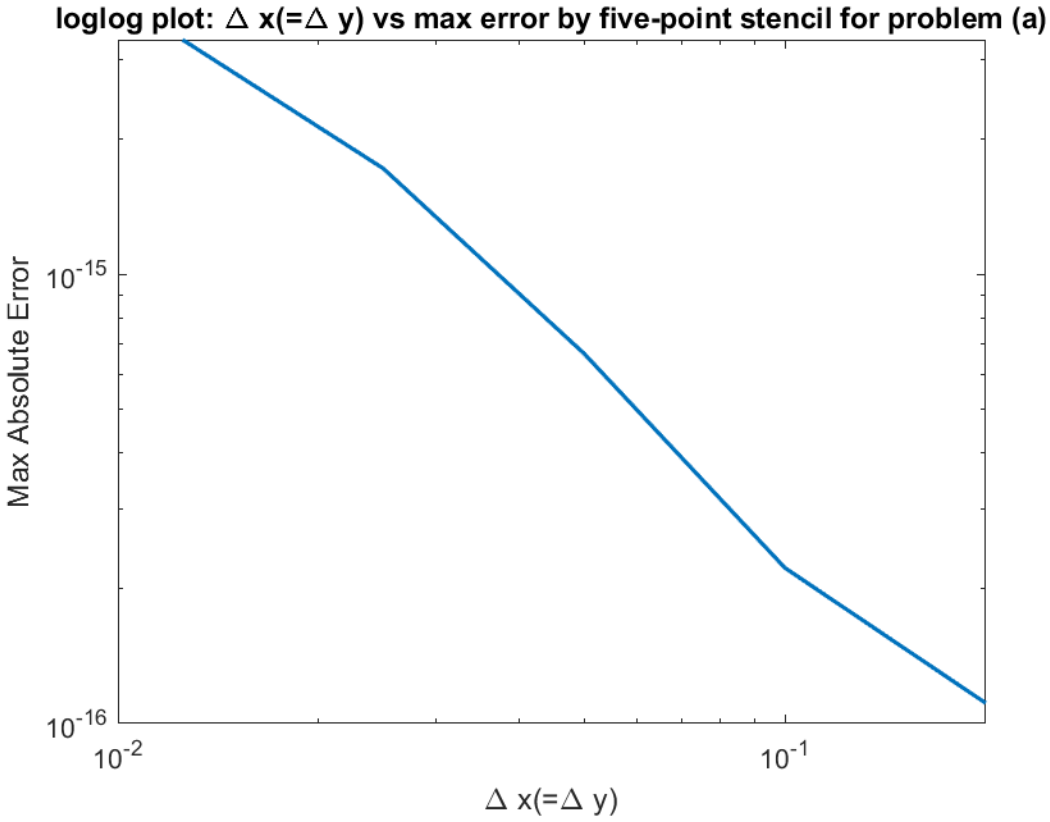



Surface Plot: Exact solution forproblem(a) h = k =0.02
u
1
0.8
0.6
0.4
0.2
0
1
0.5
y
0
0
0.2
0.4
0.6
0.8
1
x


Contour Plot: Solution by five-point stencil forproblem(a) h = k =0.02
y
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
x


Contour Plot: Exact solution forproblem(a) h = k =0.02
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1


Absolute Error: five-point stencil forproblem(a) h = k =0.02
×10^-15
Error
y
x
0
0.5
1
1.5
0
0.5
1
0
0.2
0.4
0.6
0.8
1



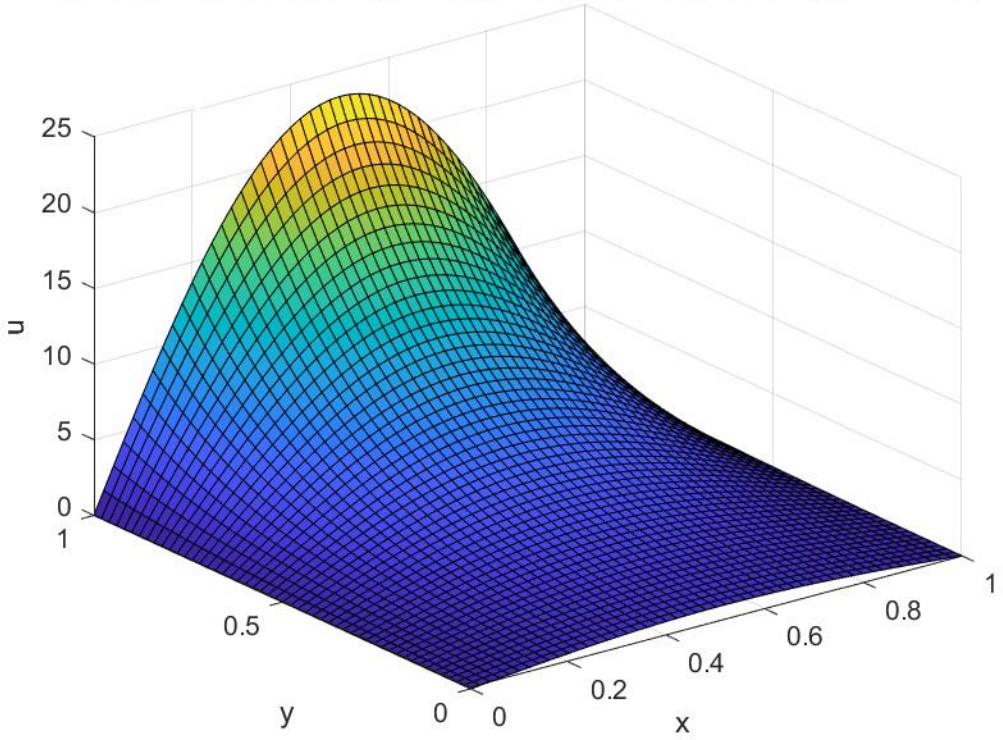
b)

(b)

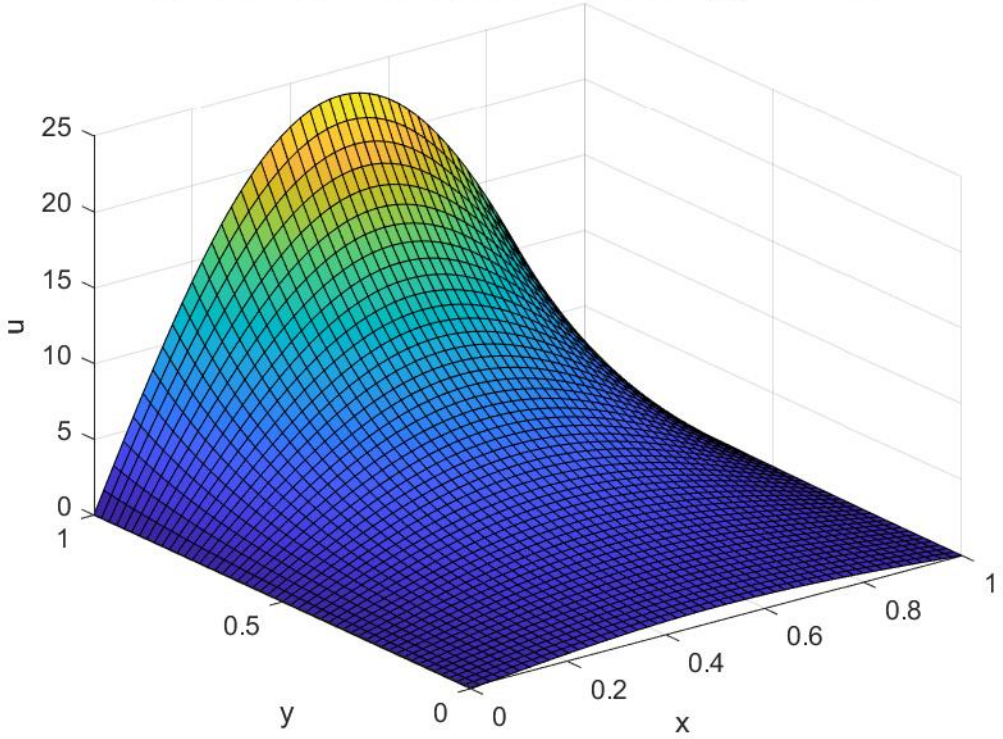
$$\begin{cases} u_{xx} + u_{yy} = x^2 + y^2, \quad 0 < x < 1,\ 0 < y < 1; \\ u(x,0) = 0, \quad u(x,1) = \dfrac{x^2}{2},\ 0 \le x \le 1; \\ u(0,y) = \sin(\pi y), \quad u(1,y) = e^{\pi}\sin(\pi y) + \dfrac{y^2}{2},\ 0 \le y \le 1. \end{cases}$$

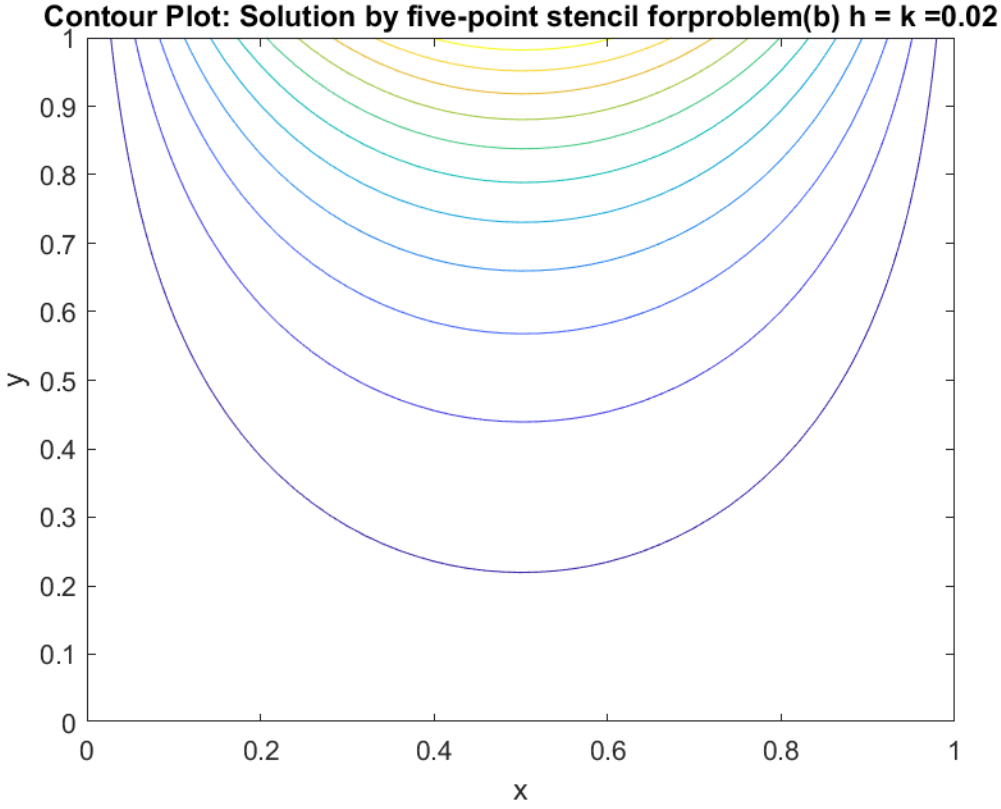
Use $h = k = 0.002$ and compare the results to the exact solution $u(x,y) = e^{\pi x}\sin(\pi y) + \dfrac{(xy)^2}{2}$.

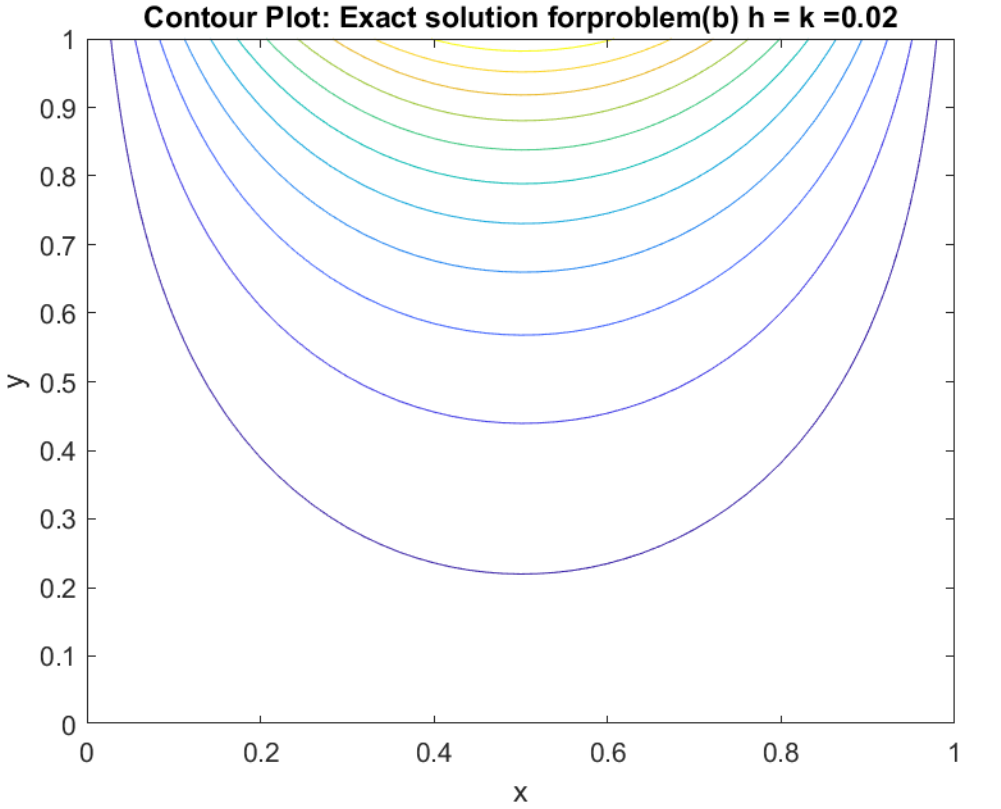
Surface Plot: Solution by five-point stencil forproblem(b) h = k =0.02

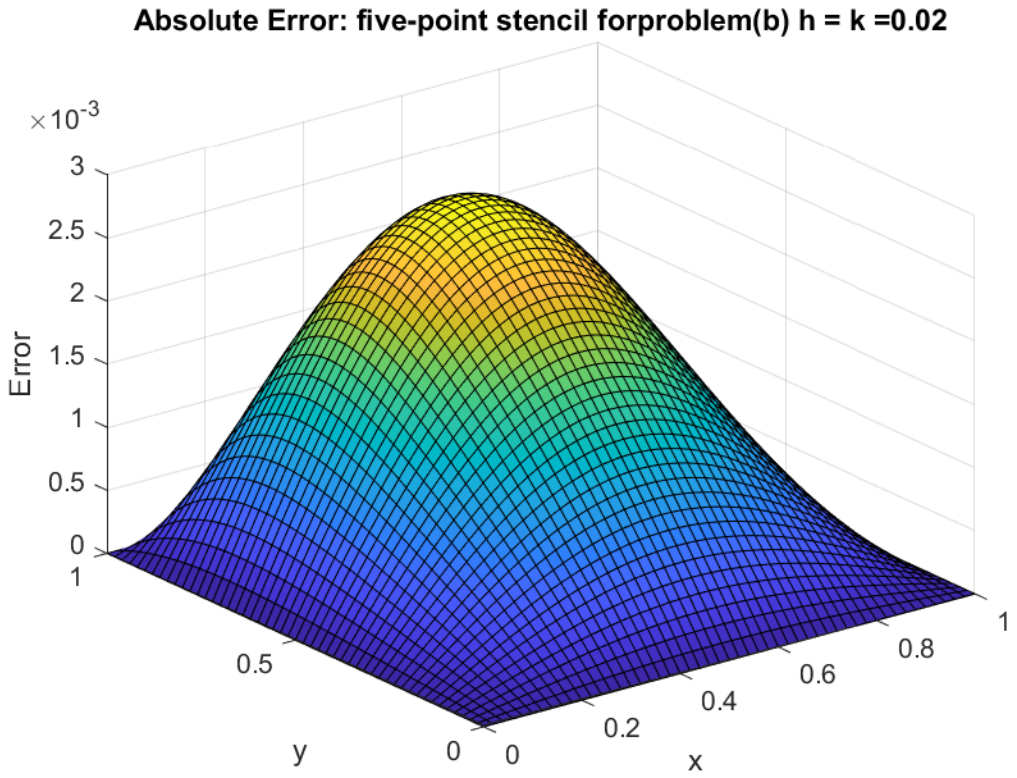


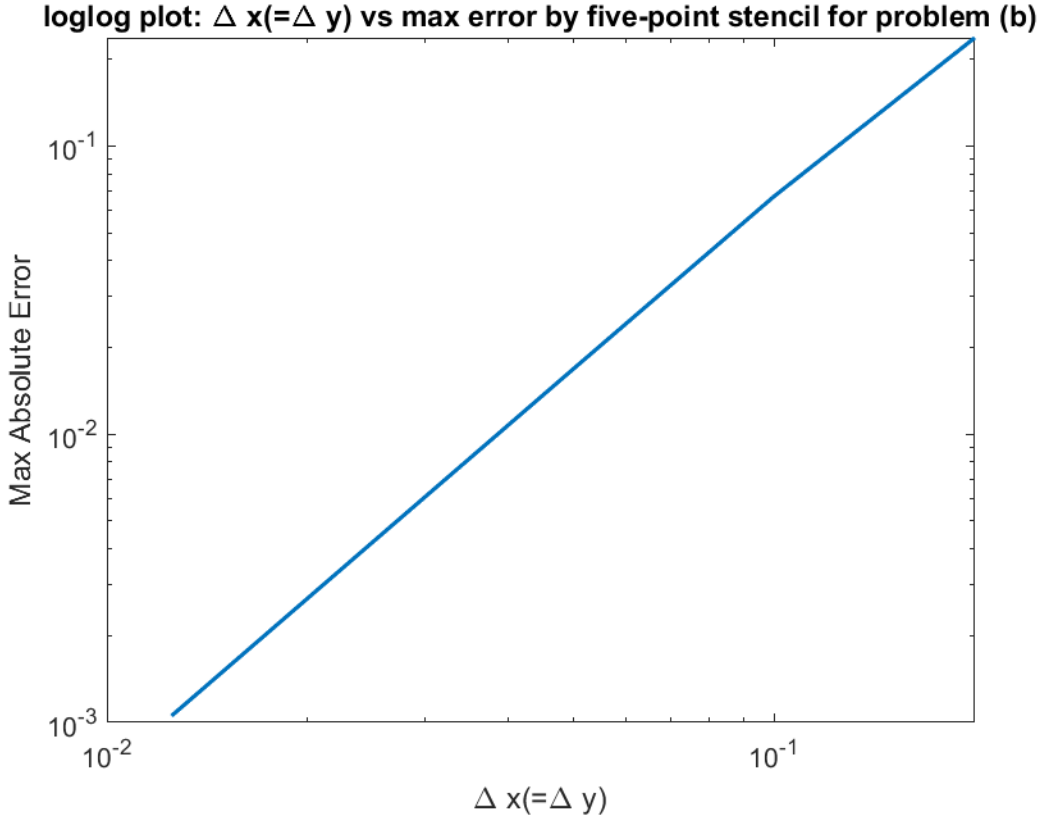
Surface Plot: Exact solution forproblem(b) h = k =0.02




Contour Plot: Solution by five-point stencil forproblem(b) h = k =0.02
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1


Contour Plot: Exact solution forproblem(b) h = k =0.02
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1


Absolute Error: five-point stencil forproblem(b) h = k =0.02
×10^-3
3
2.5
2
1.5
1
0.5
0
Error
1
0.5
y
0
0
0.2
0.4
0.6
0.8
1
x

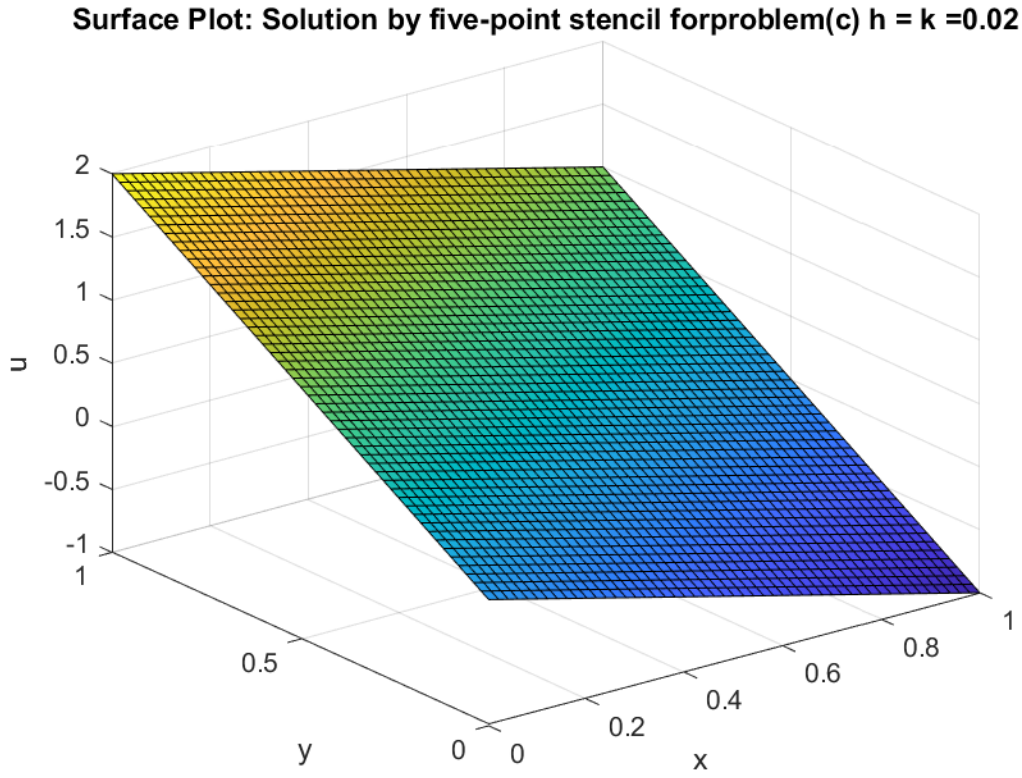

loglog plot: Δ x(=Δ y) vs max error by five-point stencil for problem (b)
10^-1
10^-2
10^-3
Max Absolute Error
10^-2
10^-1
Δ x(=Δ y)

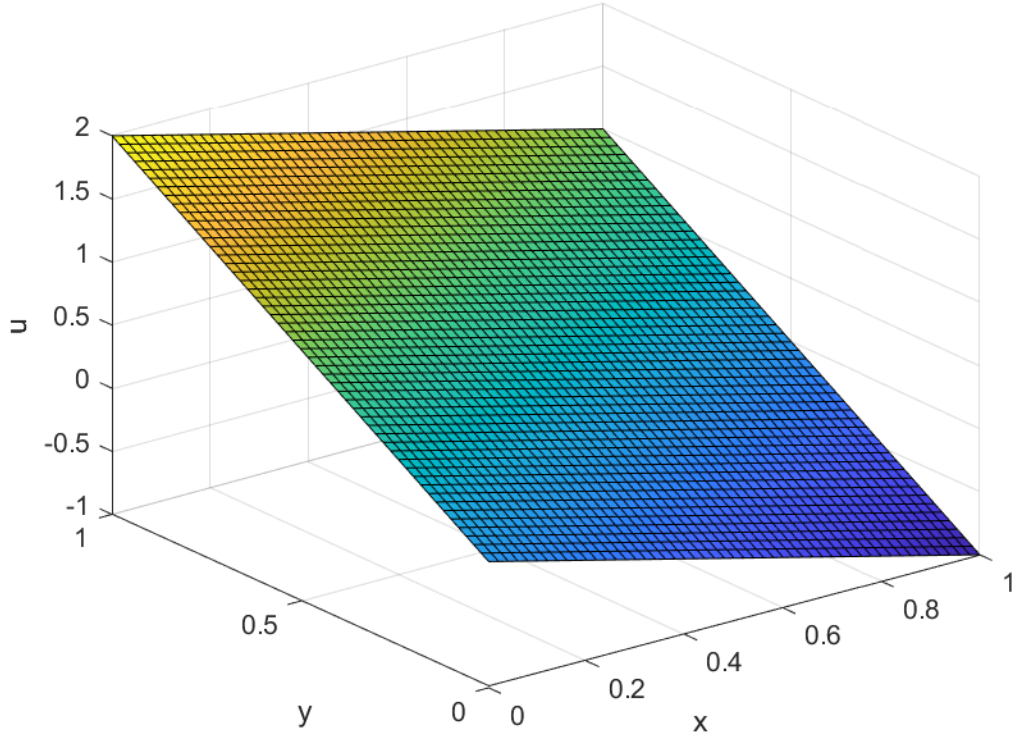
c)

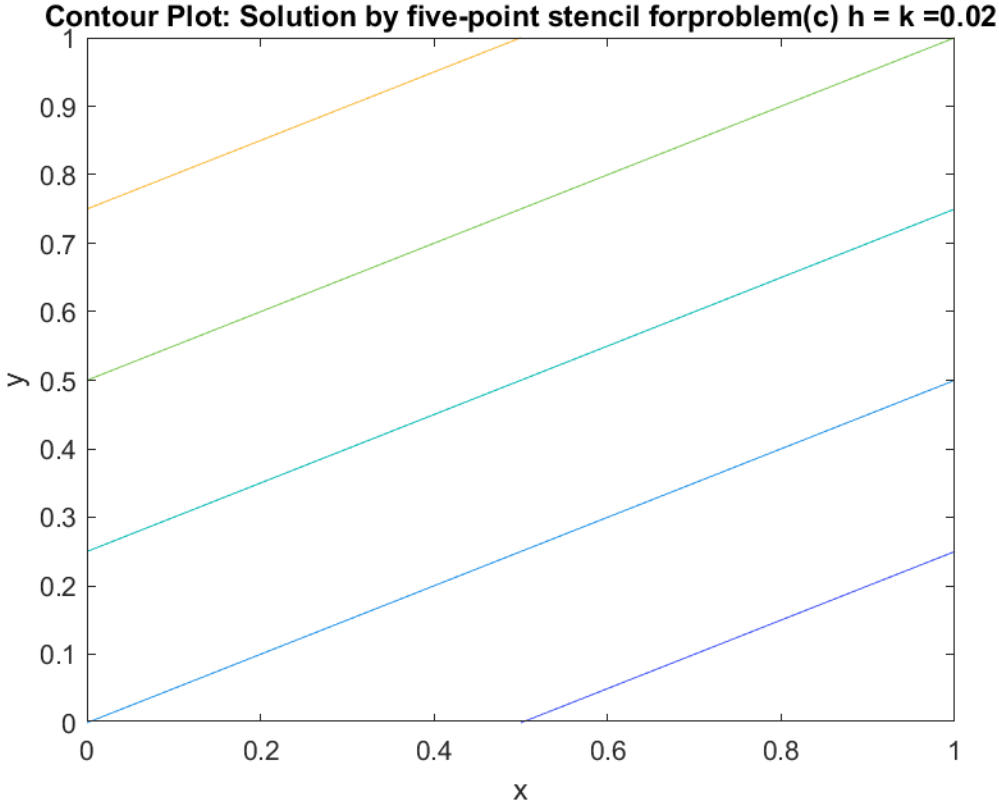
(c)

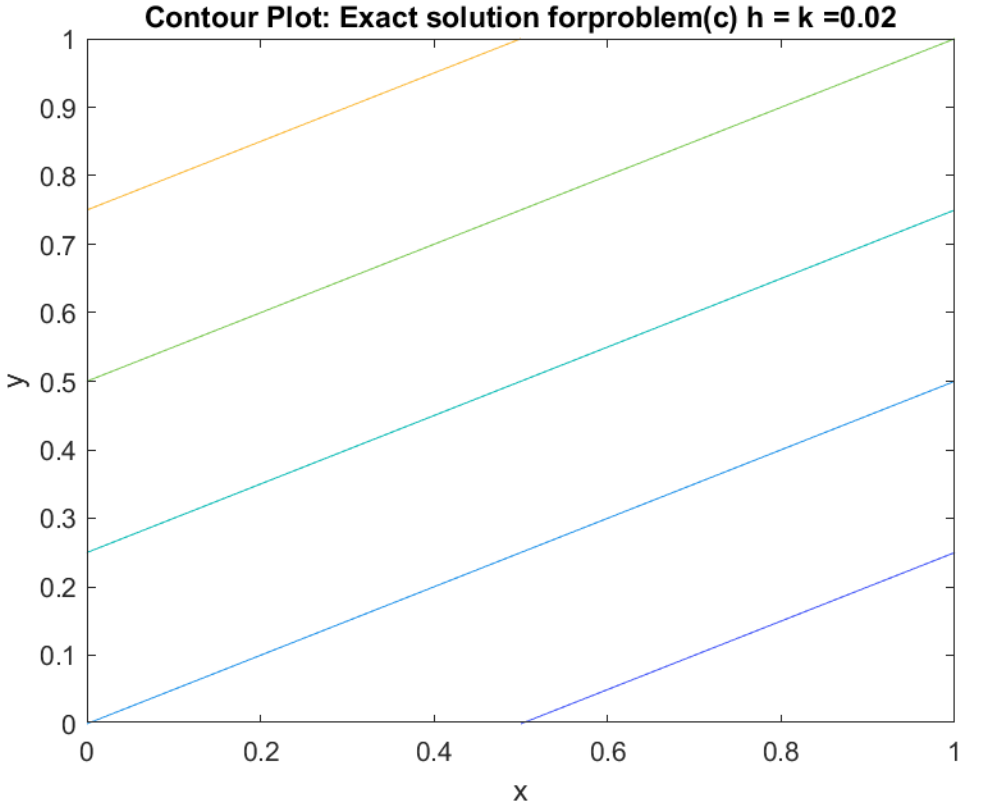
$$\begin{cases} u_{xx} + u_{yy} + u = 2x - y, \quad 0 < x < 1,\ 0 < y < 1; \\ u(x,0) = 2x, \quad u(x,1) = 2x - 1,\ 0 \le x \le 1; \\ u_x(0,y) + u(0,y) = 2 - y, \quad u(1,y) = 2 - y,\ 0 \le y \le 1. \end{cases}$$

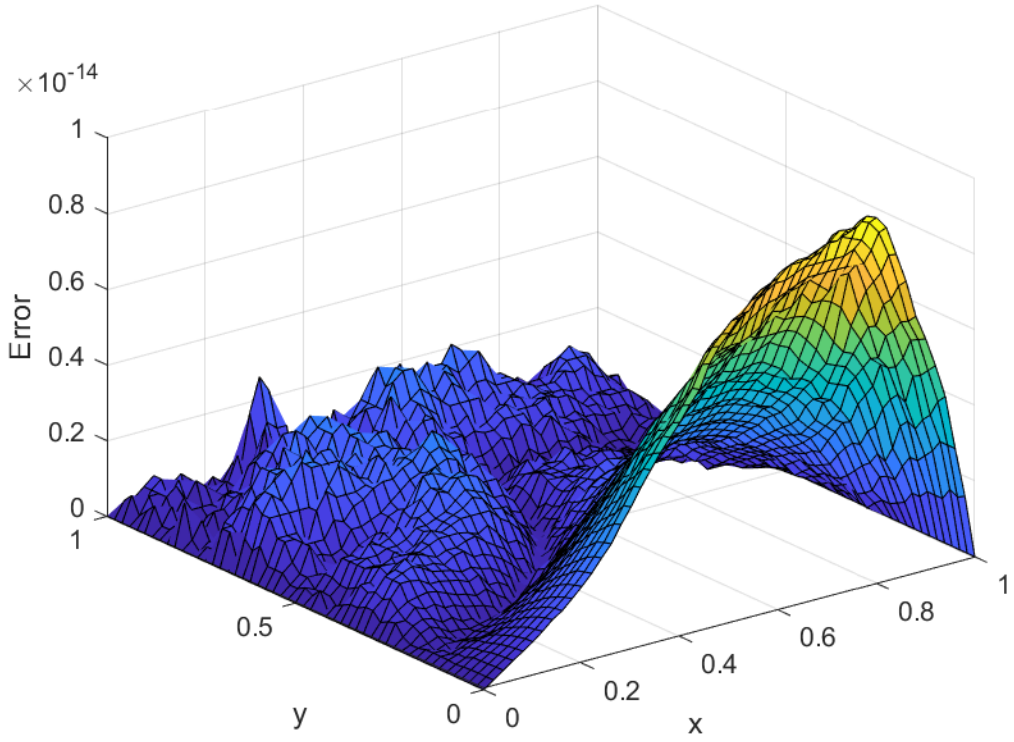
Use $h = k = 0.002$ and compare the results to the exact solution $u(x,y) = 2x - y$.

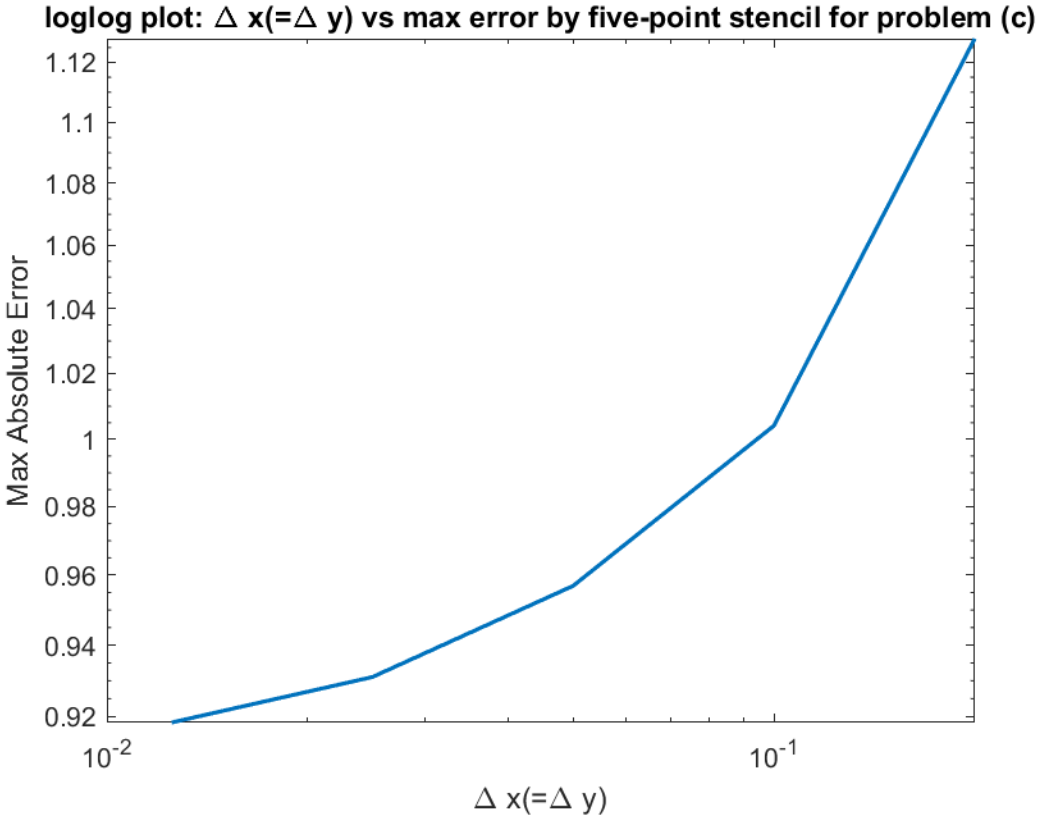

Surface Plot: Solution by five-point stencil forproblem(c) h = k =0.02
u
2
1.5
1
0.5
0
-0.5
-1
1
0.5
0
y
0
0.2
0.4
0.6
0.8
1
x


Surface Plot: Exact solution forproblem(c) h = k =0.02
u
2
1.5
1
0.5
0
-0.5
-1
1
0.5
0
y
0
0.2
0.4
0.6
0.8
1
x


Contour Plot: Solution by five-point stencil forproblem(c) h = k =0.02
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1


Contour Plot: Exact solution forproblem(c) h = k =0.02
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1


Absolute Error: five-point stencil forproblem(c) h = k =0.02
×10^-14
1
0.8
0.6
0.4
0.2
0
Error
1
0.5
0
y
0
0.2
0.4
0.6
0.8
1
x

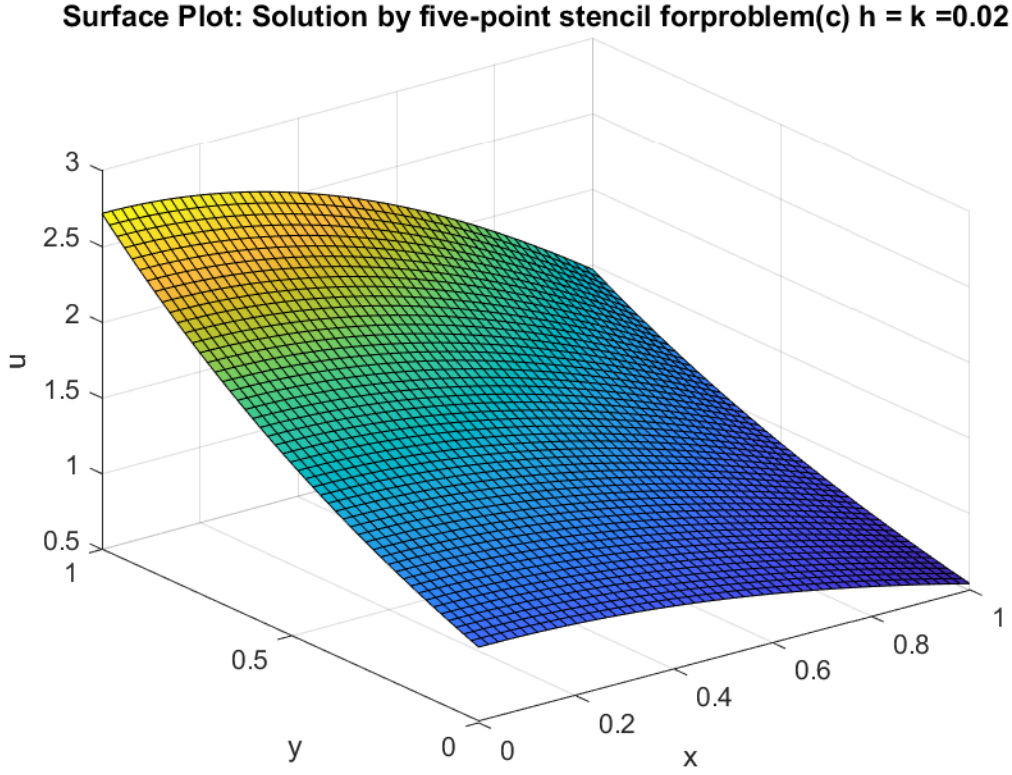

loglog plot: Δ x(=Δ y) vs max error by five-point stencil for problem (c)
1.12
1.1
1.08
1.06
1.04
1.02
1
0.98
0.96
0.94
0.92
Max Absolute Error
10^-2
10^-1
Δ x(=Δ y)

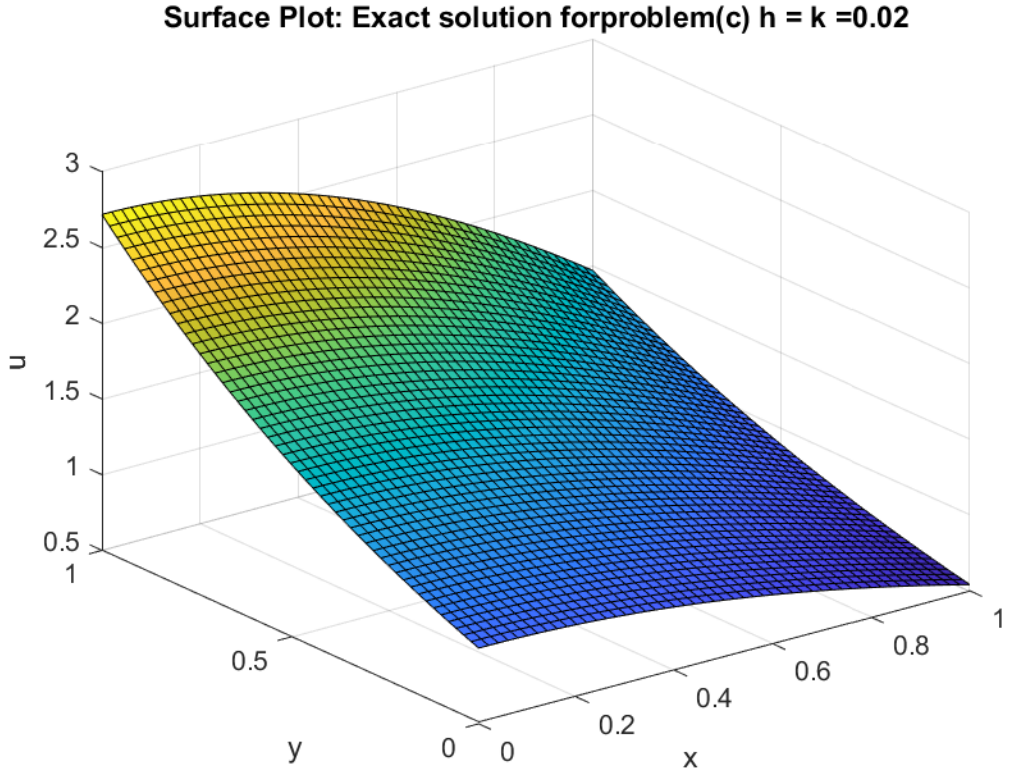
d)

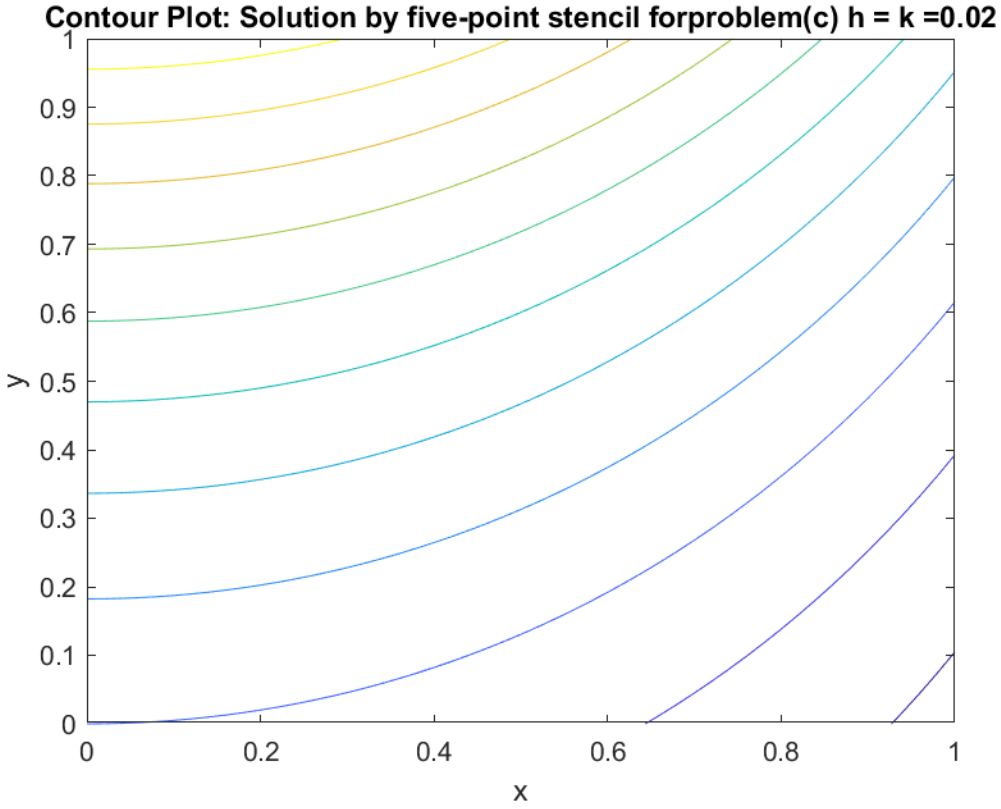
(d)

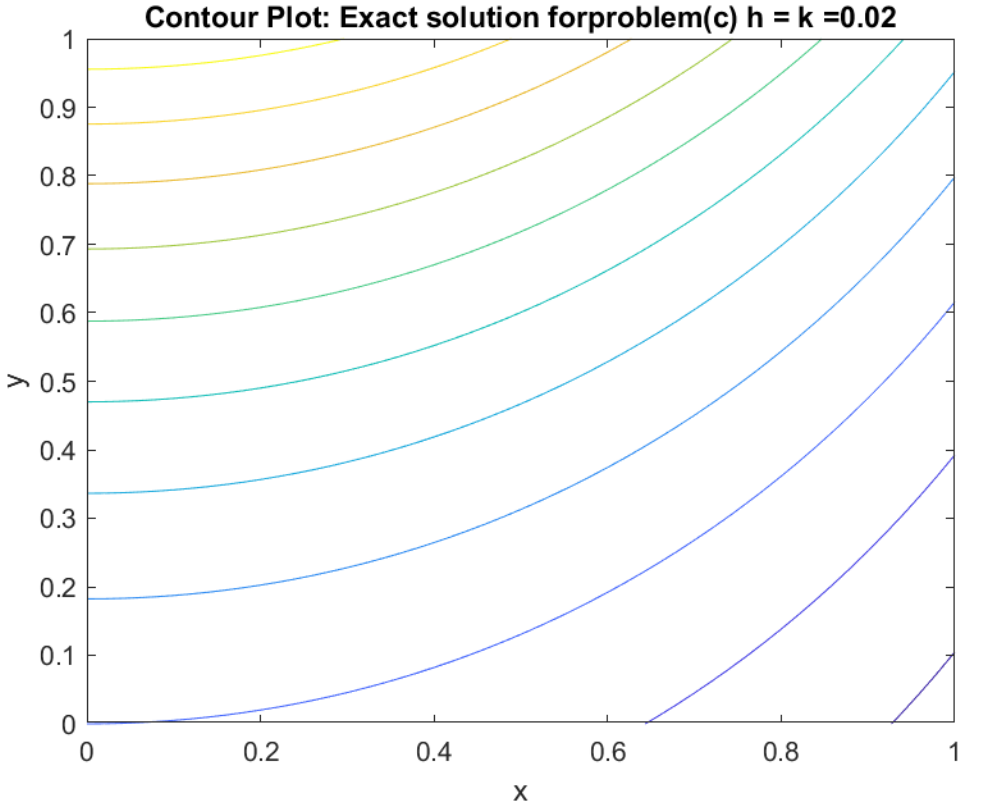
$$\begin{cases} u_{xx} + u_{yy} + u_x + u_y + u = e^x(2\cos y - \sin y), \quad 0 < x < 1,\ 0 < y < 1; \\ u(x,0) = e^x, \quad u(x,1) = e^x \cos(1),\ 0 \le x \le 1; \\ u(0,y) = \cos(y), \quad u(1,y) = e\cos(y),\ 0 \le y \le 1. \end{cases}$$

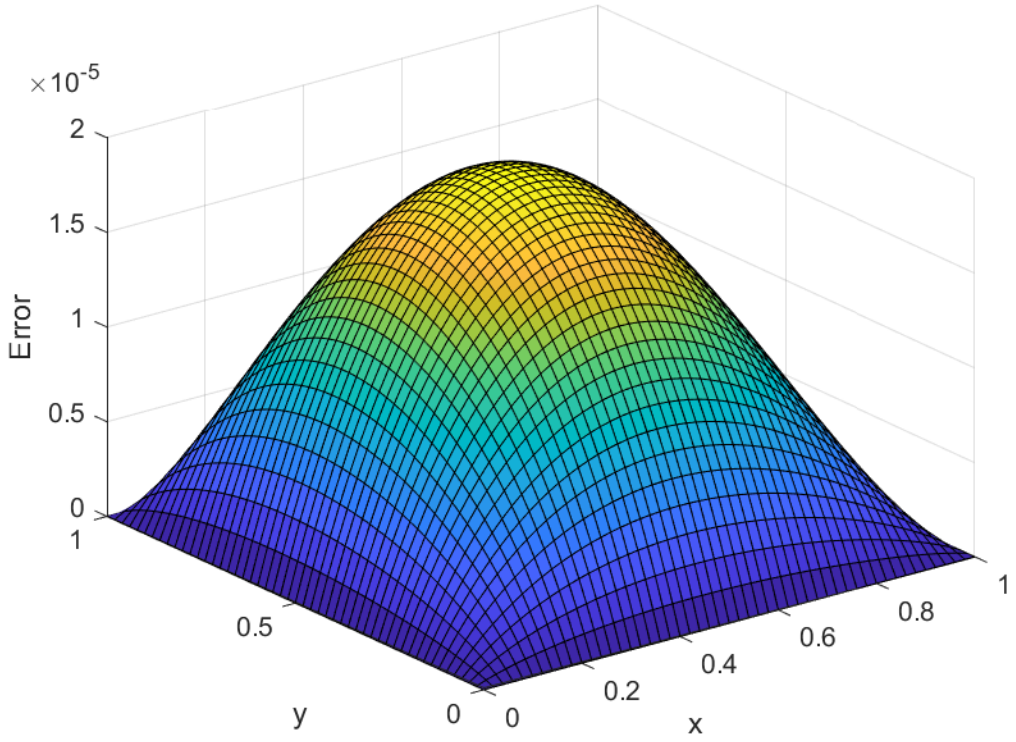
Use $h = k = 0.002$ and compare the results to the exact solution $u(x,y) = e^x \cos(y)$.

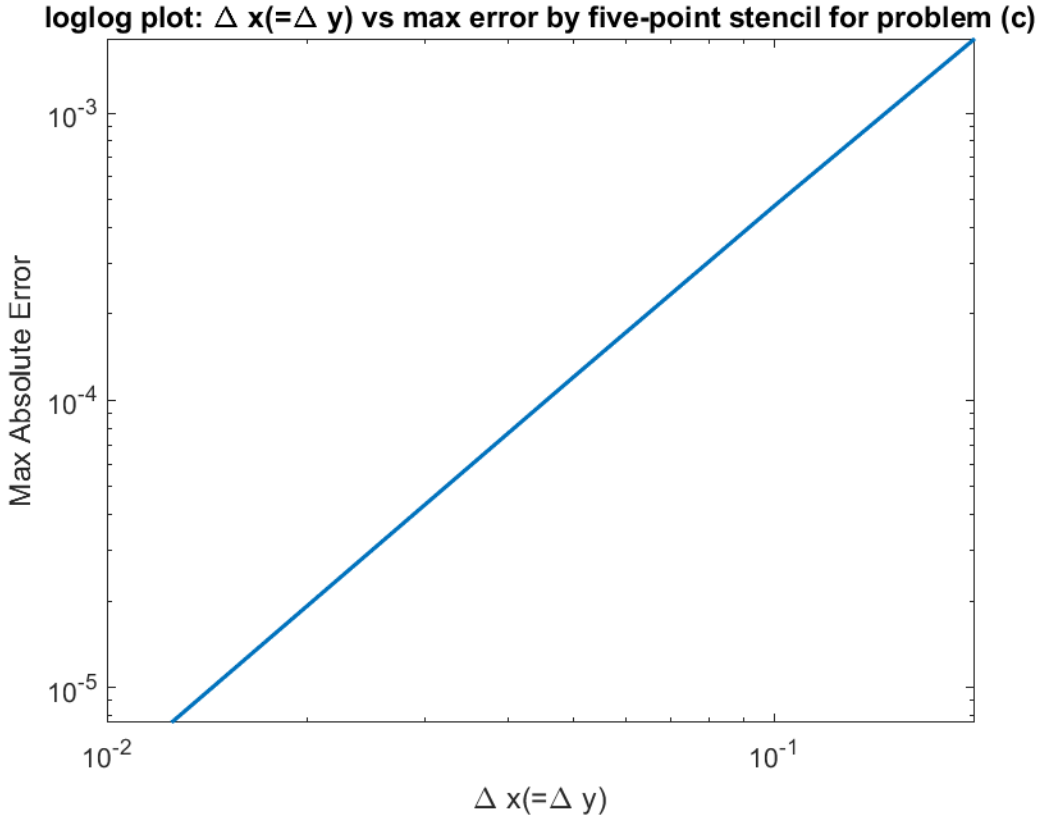

Surface Plot: Solution by five-point stencil forproblem(c) h = k =0.02
u
3
2.5
2
1.5
1
0.5
1
0.5
0
y
0
0.2
0.4
0.6
0.8
1
x


Surface Plot: Exact solution forproblem(c) h = k =0.02
u
3
2.5
2
1.5
1
0.5
1
0.5
0
y
0
0.2
0.4
0.6
0.8
1
x

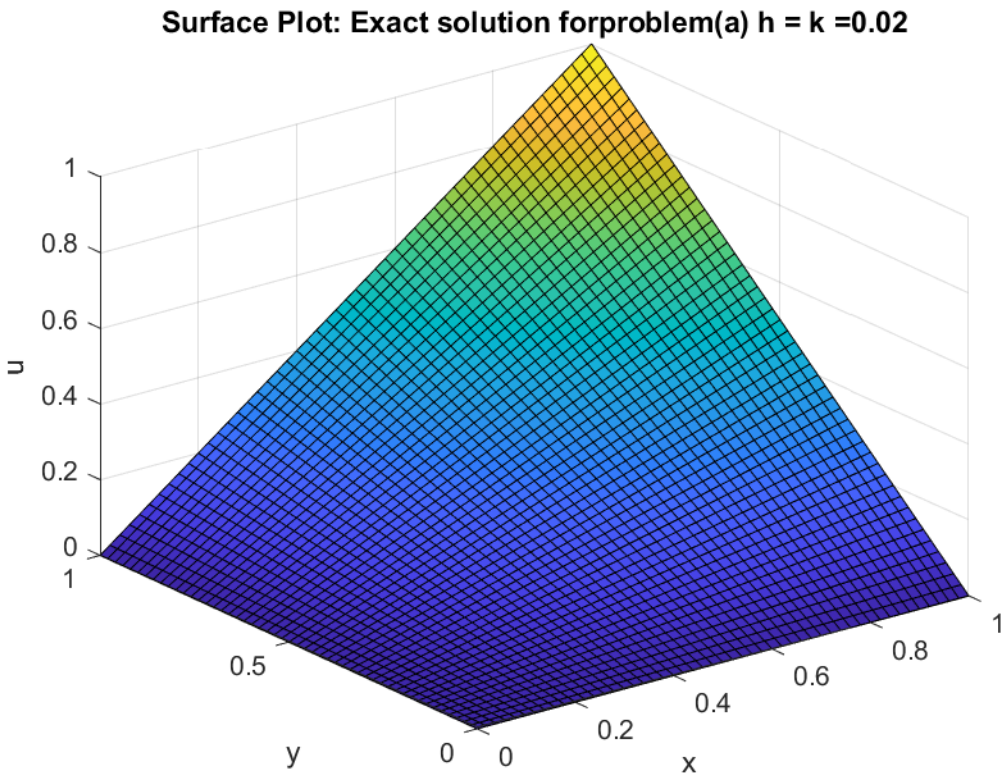

Contour Plot: Solution by five-point stencil forproblem(c) h = k =0.02
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1

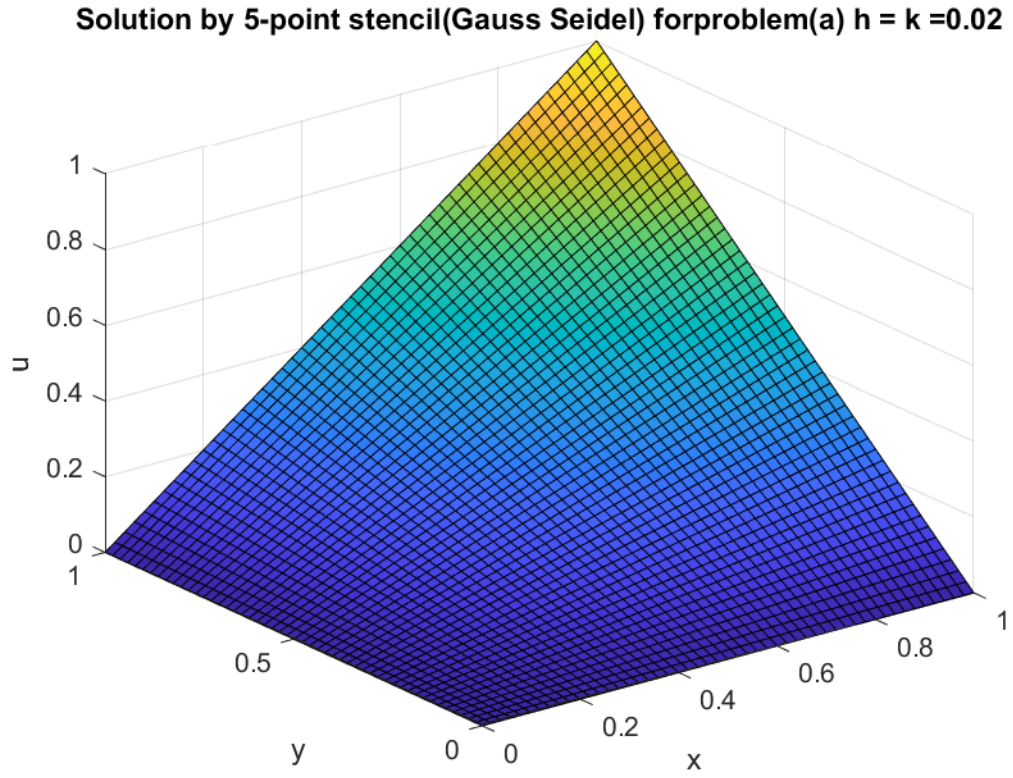

Contour Plot: Exact solution forproblem(c) h = k =0.02
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1

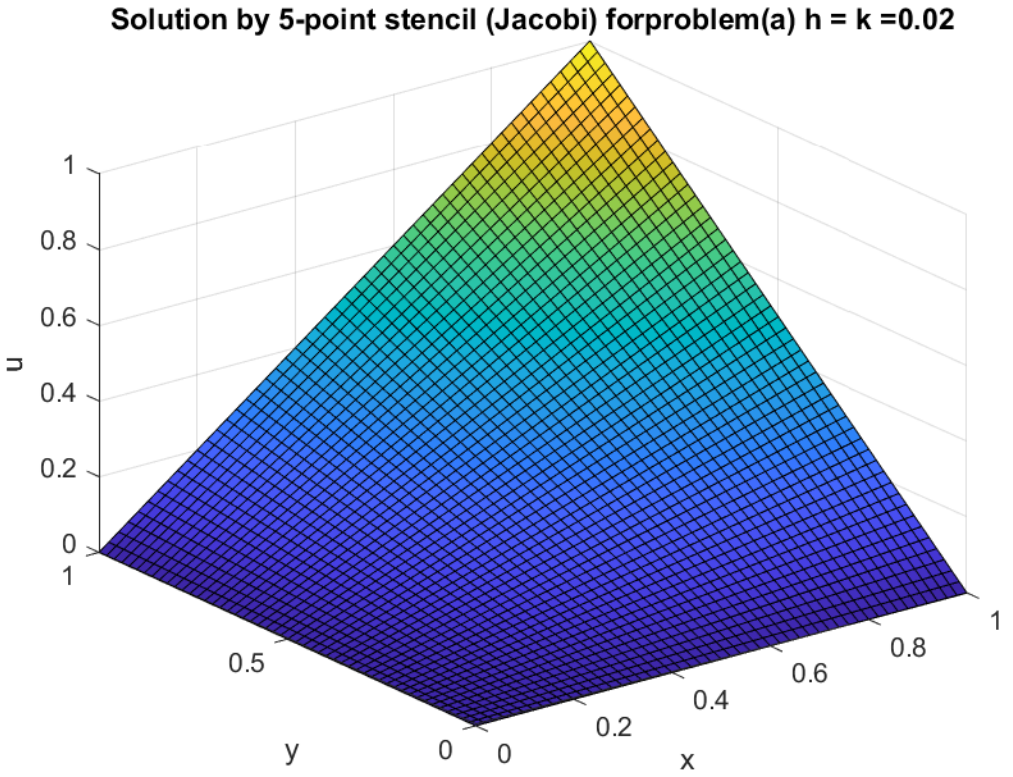

Absolute Error: five-point stencil forproblem(c) h = k =0.02
×10^-5
2
1.5
1
0.5
0
Error
1
0.5
y
0
0
0.2
0.4
0.6
0.8
1
x

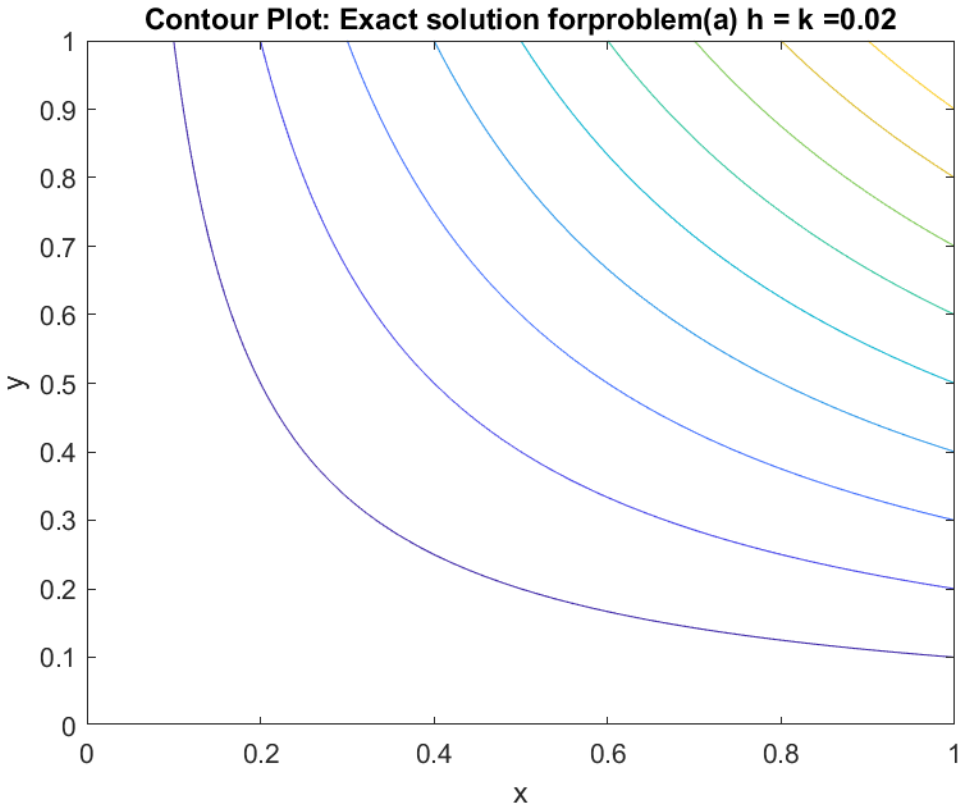

loglog plot: Δ x(=Δ y) vs max error by five-point stencil for problem (c)
10^-3
10^-4
10^-5
Max Absolute Error
10^-2
10^-1
Δ x(=Δ y)

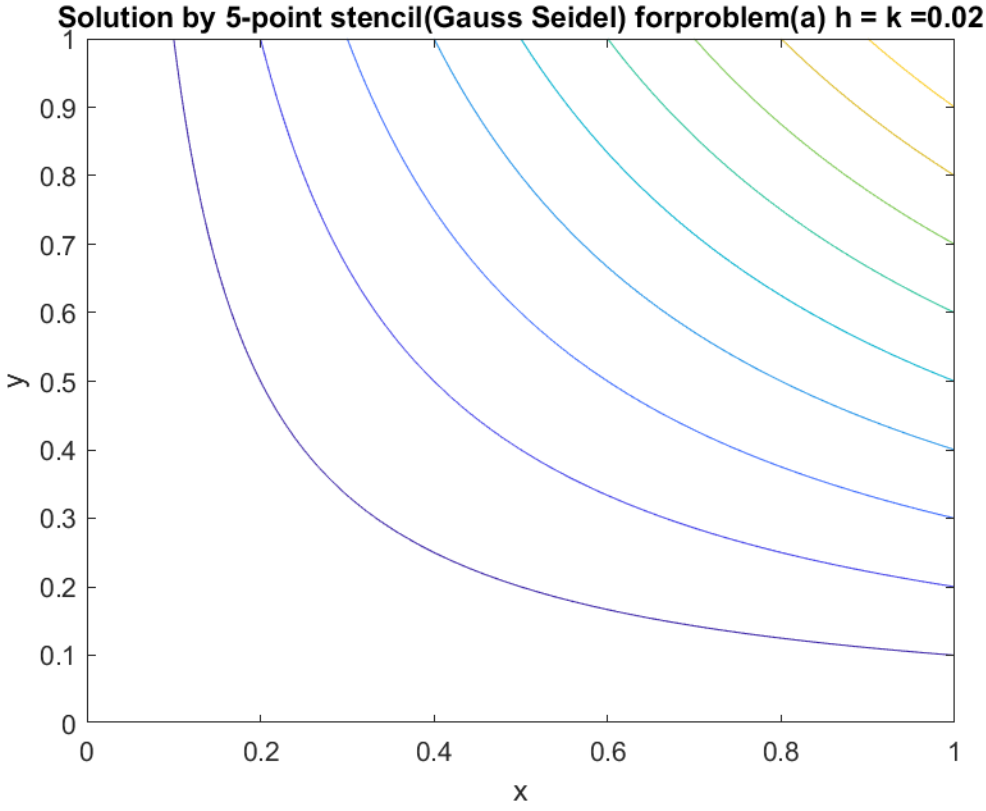
2) a)

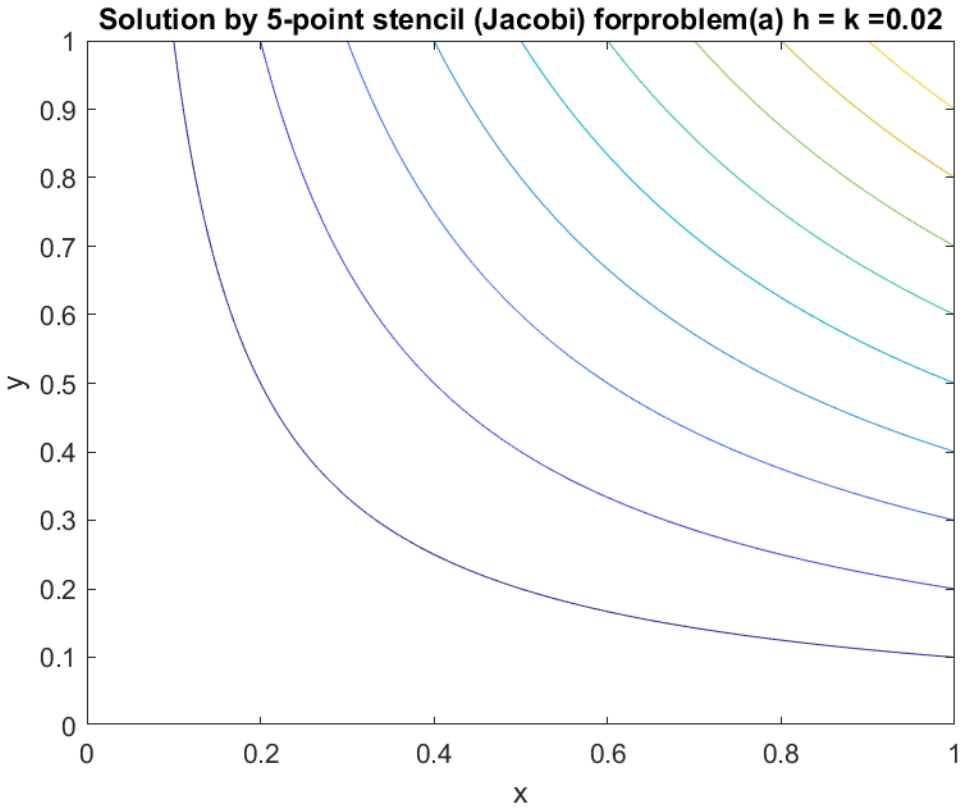

Surface Plot: Exact solution forproblem(a) h = k =0.02
u
1
0.8
0.6
0.4
0.2
0
1
0.5
y
0
0
0.2
0.4
0.6
0.8
1
x

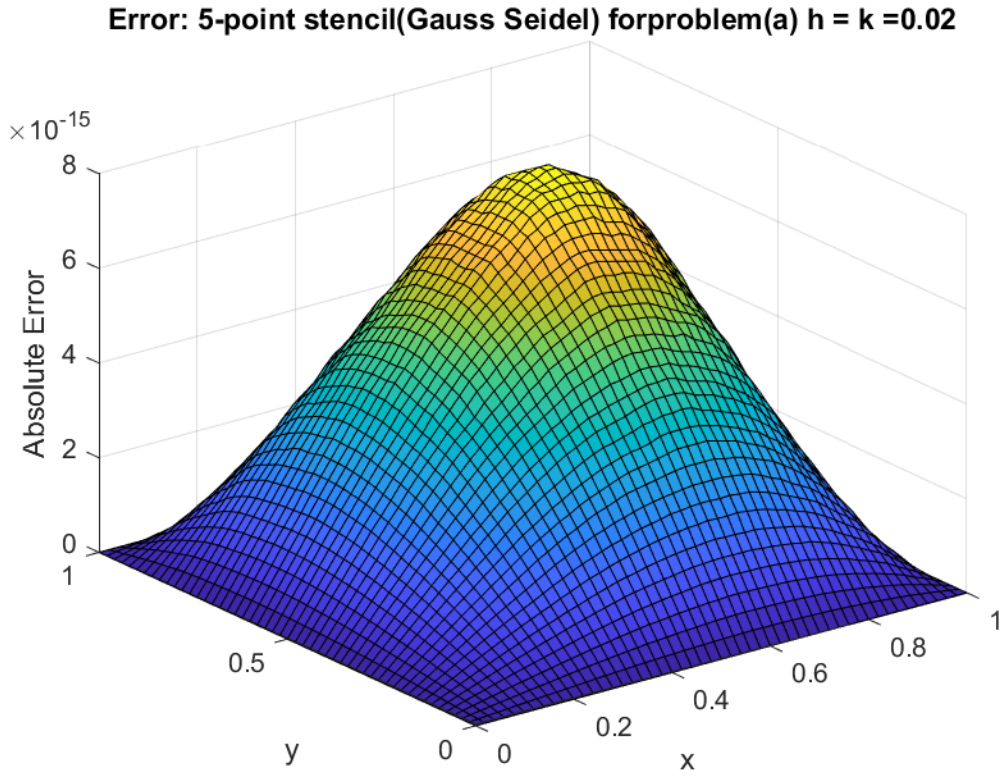

Solution by 5-point stencil(Gauss Seidel) forproblem(a) h = k =0.02
u
1
0.8
0.6
0.4
0.2
0
1
0.5
y
0
0
0.2
0.4
0.6
0.8
1
x

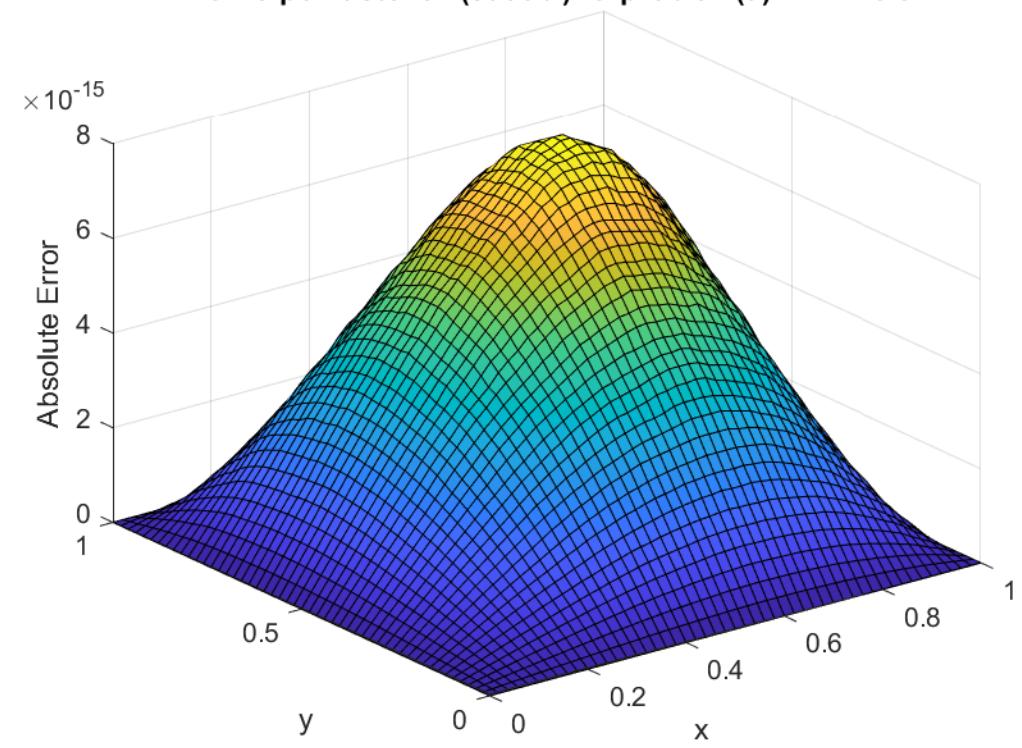

Solution by 5-point stencil (Jacobi) forproblem(a) h = k =0.02
u
1
0.8
0.6
0.4
0.2
0
1
0.5
y
0
0
0.2
0.4
0.6
0.8
1
x

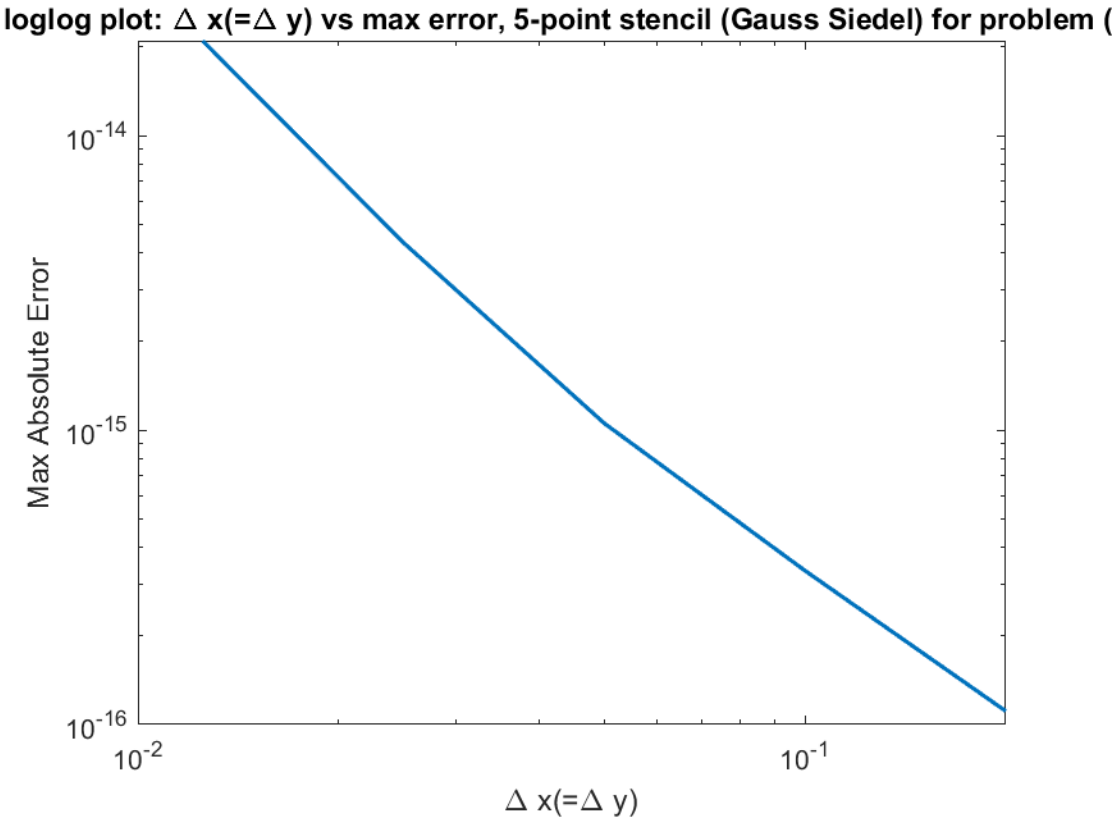

Contour Plot: Exact solution forproblem(a) h = k =0.02
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1

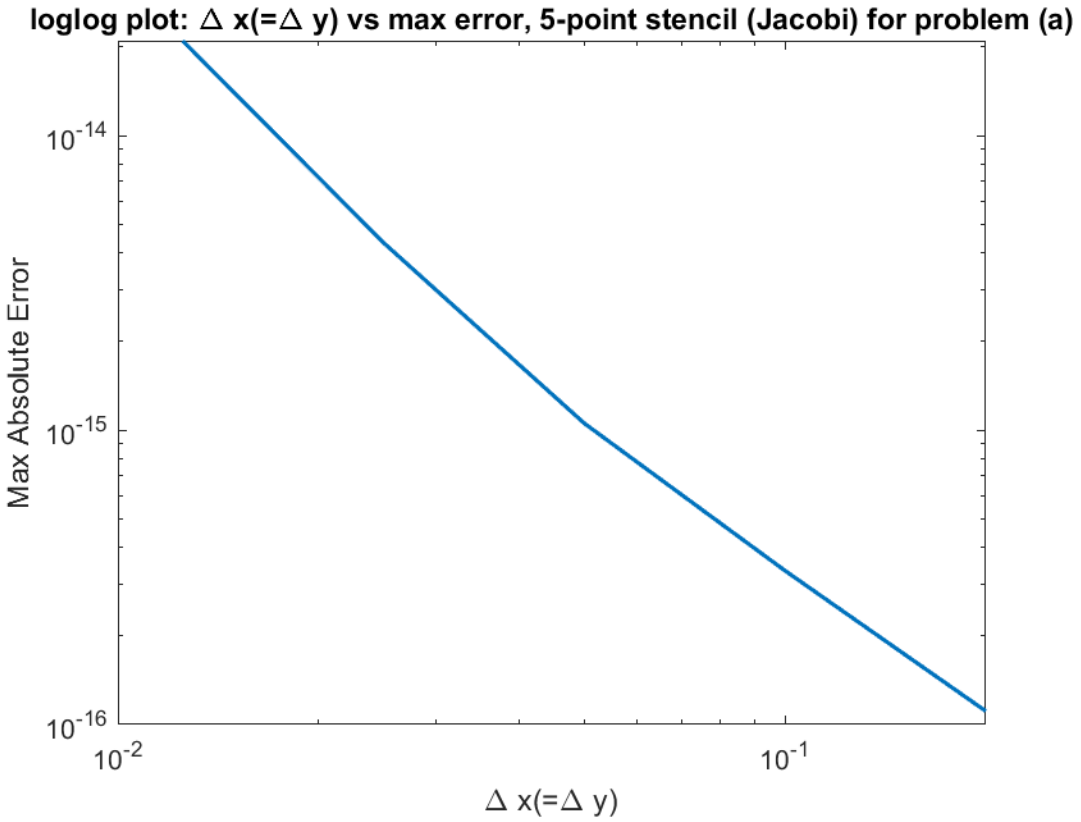

Solution by 5-point stencil(Gauss Seidel) forproblem(a) h = k =0.02
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1


Solution by 5-point stencil (Jacobi) forproblem(a) h = k =0.02
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1

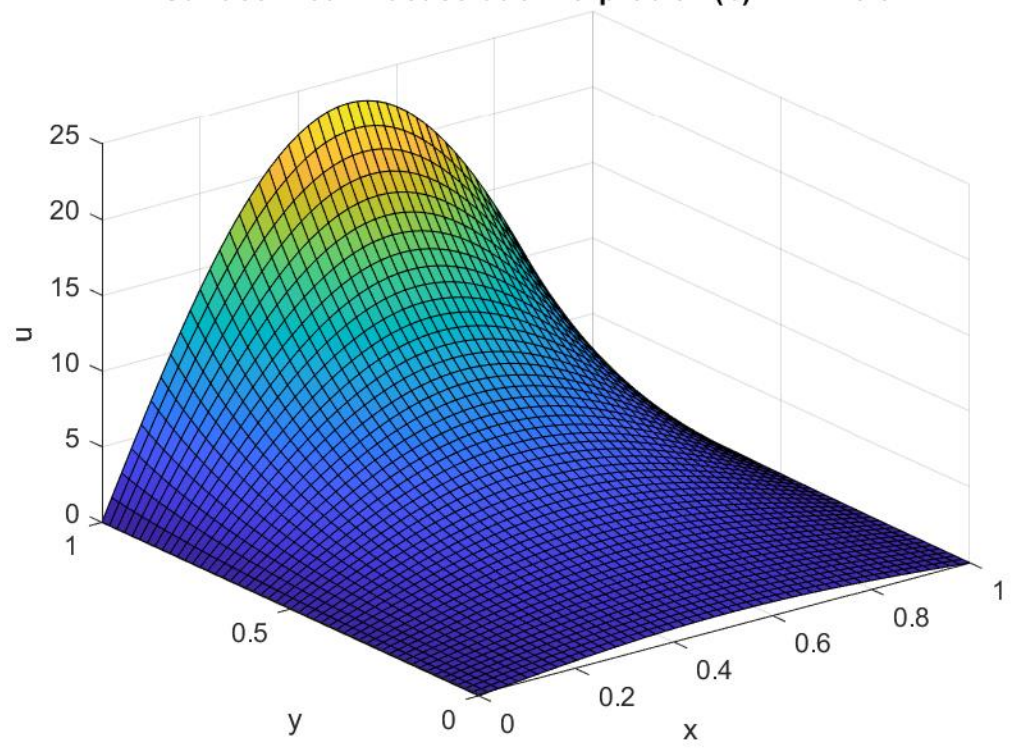

Error: 5-point stencil(Gauss Seidel) forproblem(a) h = k =0.02
×10^-15
Absolute Error
0
2
4
6
8
y
0
0.5
1
x
0
0.2
0.4
0.6
0.8
1

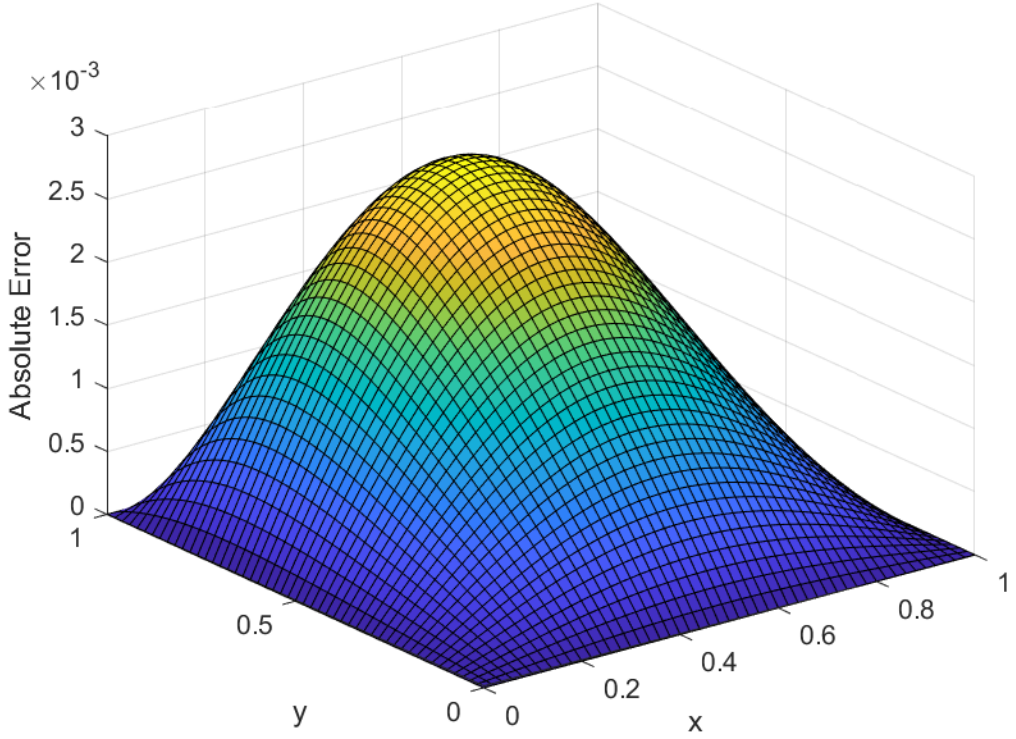

Error: 5-point stencil (Jacobi) forproblem(a) h = k =0.02
×10^-15
8
6
4
2
0
Absolute Error
1
0.5
y
0
0
0.2
0.4
0.6
0.8
1
x

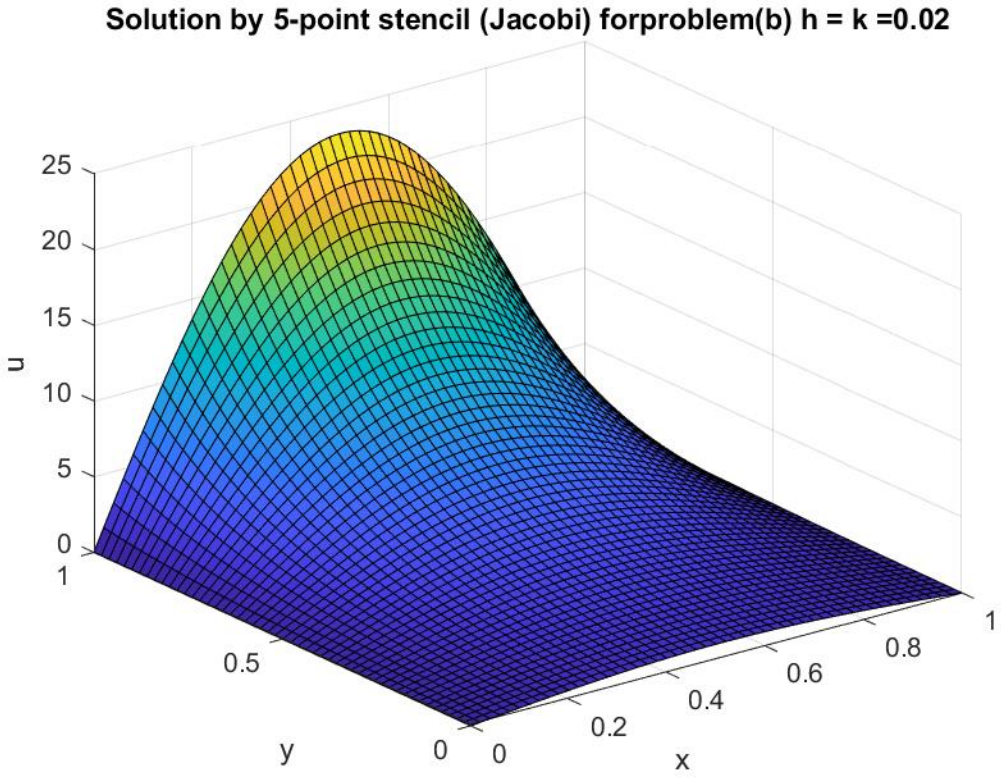

loglog plot: △ x(=△ y) vs max error, 5-point stencil (Gauss Siedel) for problem (
10^-14
10^-15
10^-16
Max Absolute Error
10^-2
10^-1
△ x(=△ y)

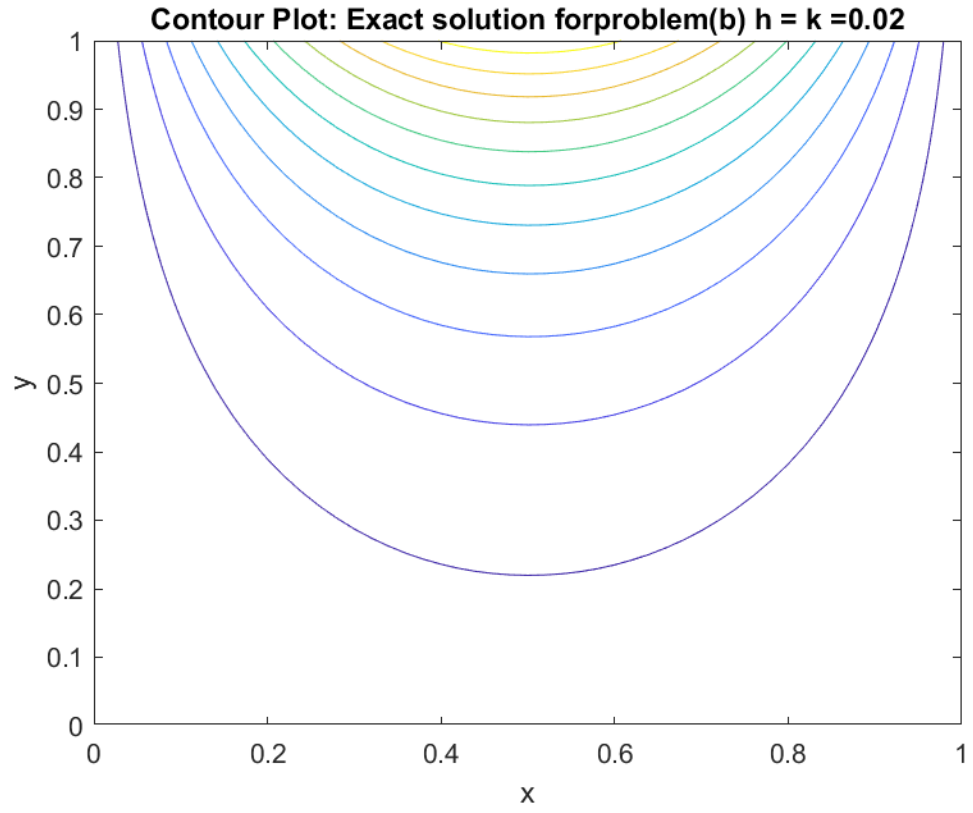

loglog plot: Δ x(=Δ y) vs max error, 5-point stencil (Jacobi) for problem (a)
Max Absolute Error
10^-14
10^-15
10^-16
10^-2
10^-1
Δ x(=Δ y)

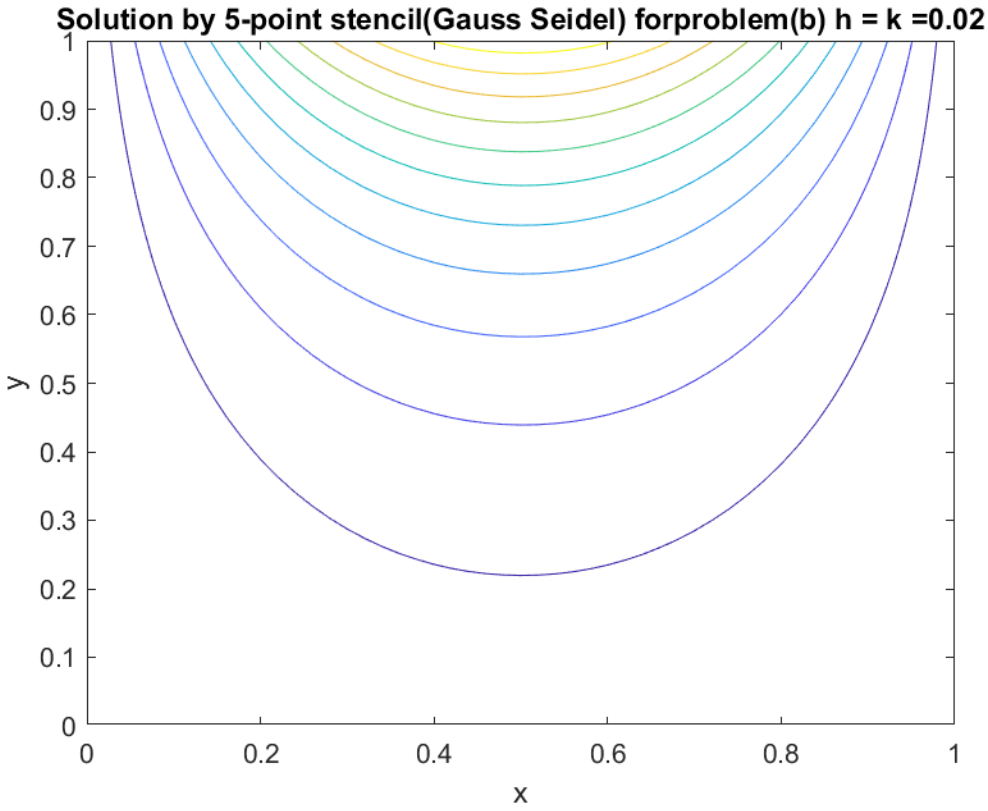
b)

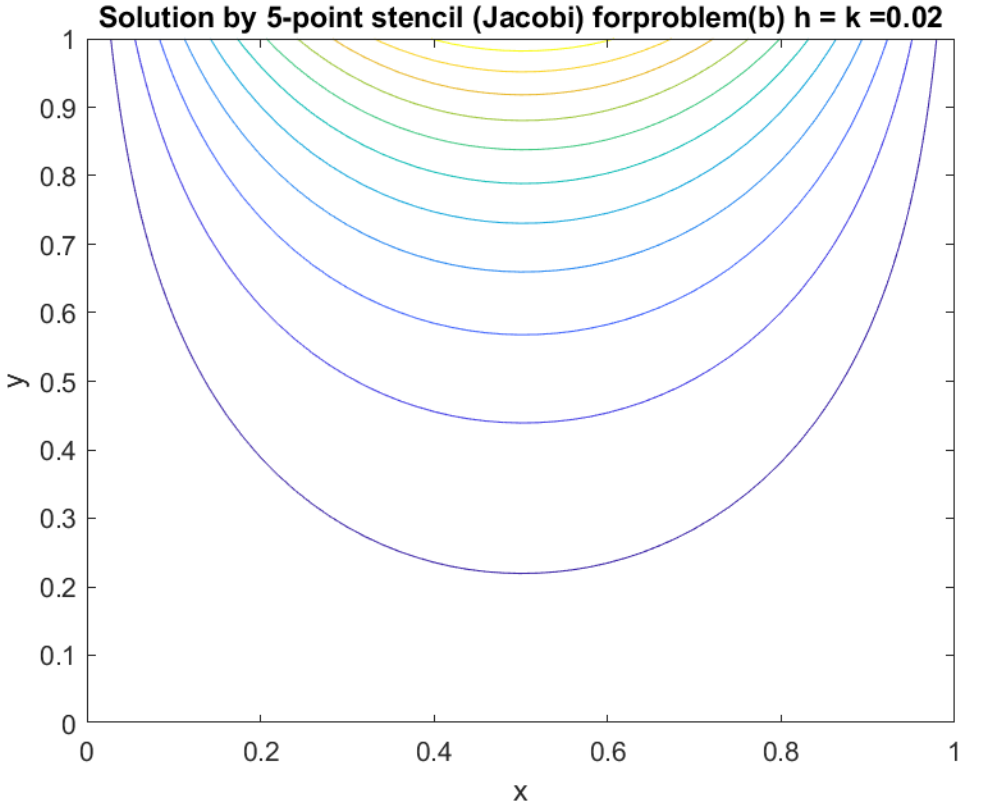

Surface Plot: Exact solution forproblem(b) h = k =0.02
u
25
20
15
10
5
0
1
0.5
y
0
0
0.2
0.4
0.6
0.8
1
x


Error: 5-point stencil(Gauss Seidel) forproblem(b) h = k =0.02
×10-3
Absolute Error
3
2.5
2
1.5
1
0.5
0
1
0.5
y
0
0
0.2
0.4
0.6
0.8
1
x

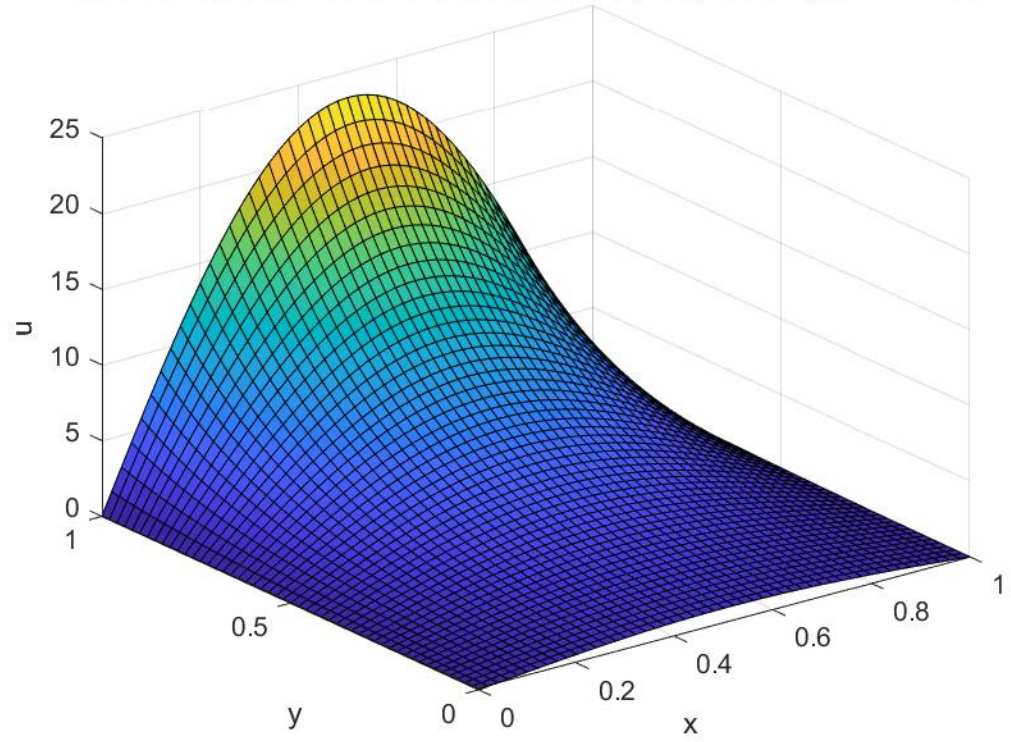

Solution by 5-point stencil (Jacobi) forproblem(b) h = k =0.02
25
20
15
10
5
0
u
1
0.5
y
0
0
0.2
0.4
0.6
0.8
1
x


Contour Plot: Exact solution forproblem(b) h = k =0.02
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
y
0
0.2
0.4
0.6
0.8
1
x

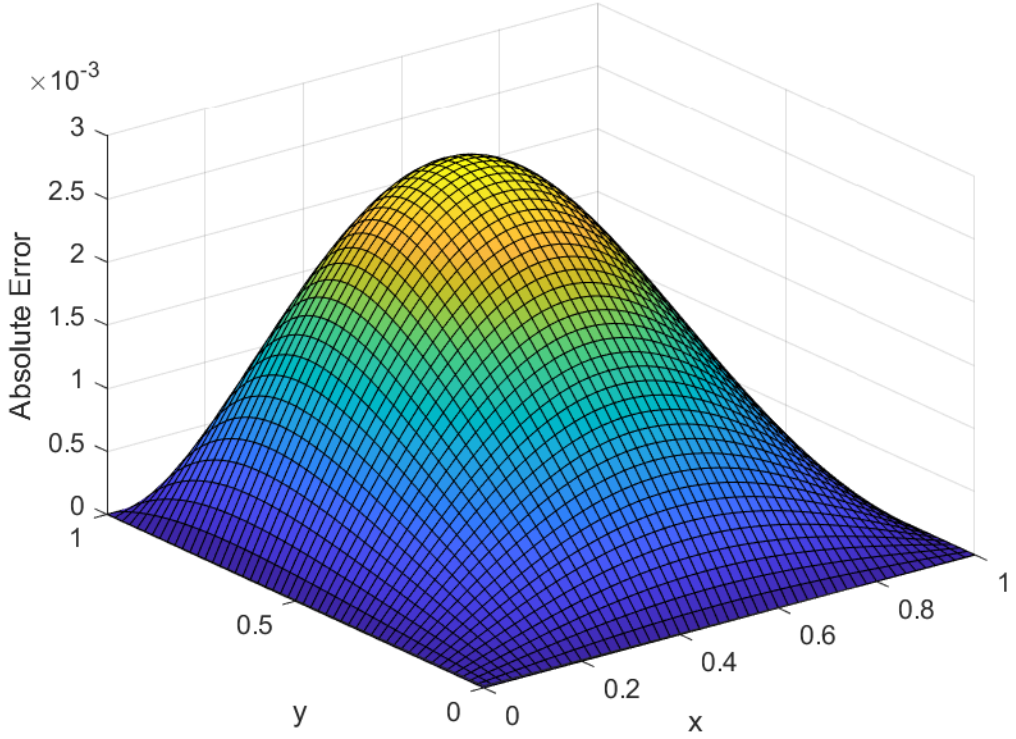

Solution by 5-point stencil(Gauss Seidel) forproblem(b) h = k =0.02
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1

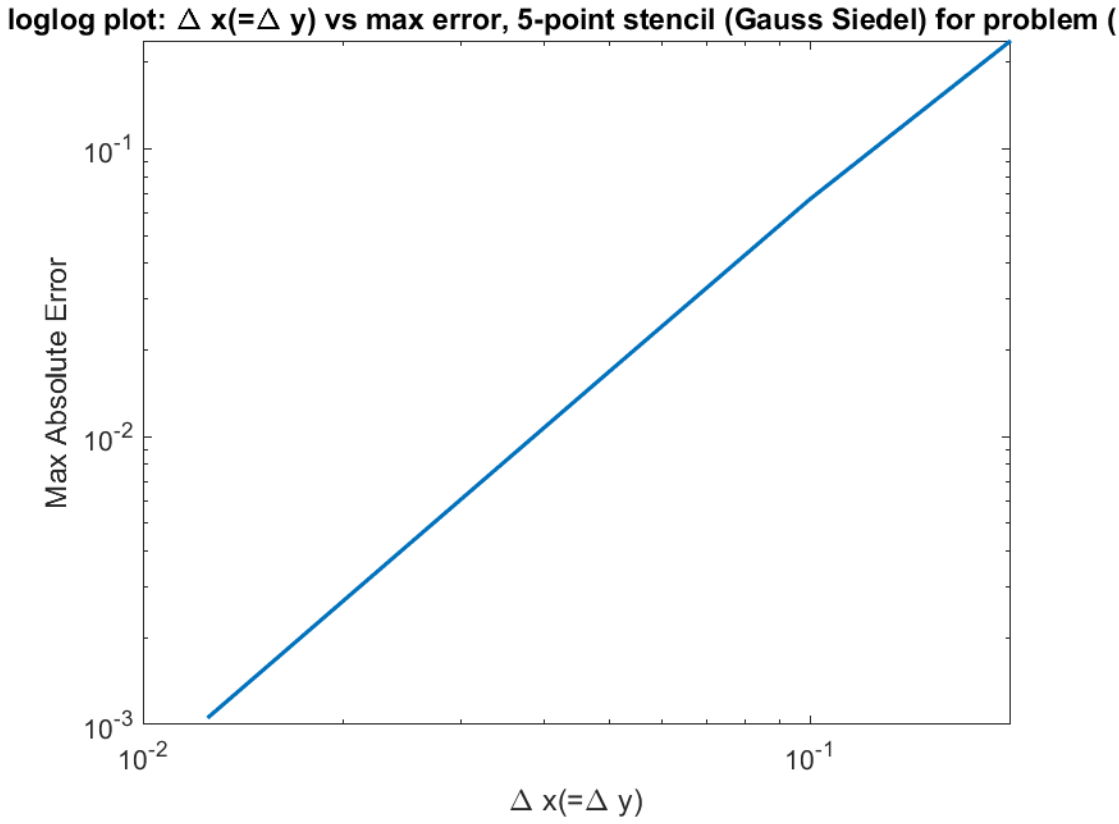

Solution by 5-point stencil (Jacobi) forproblem(b) h = k =0.02
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1

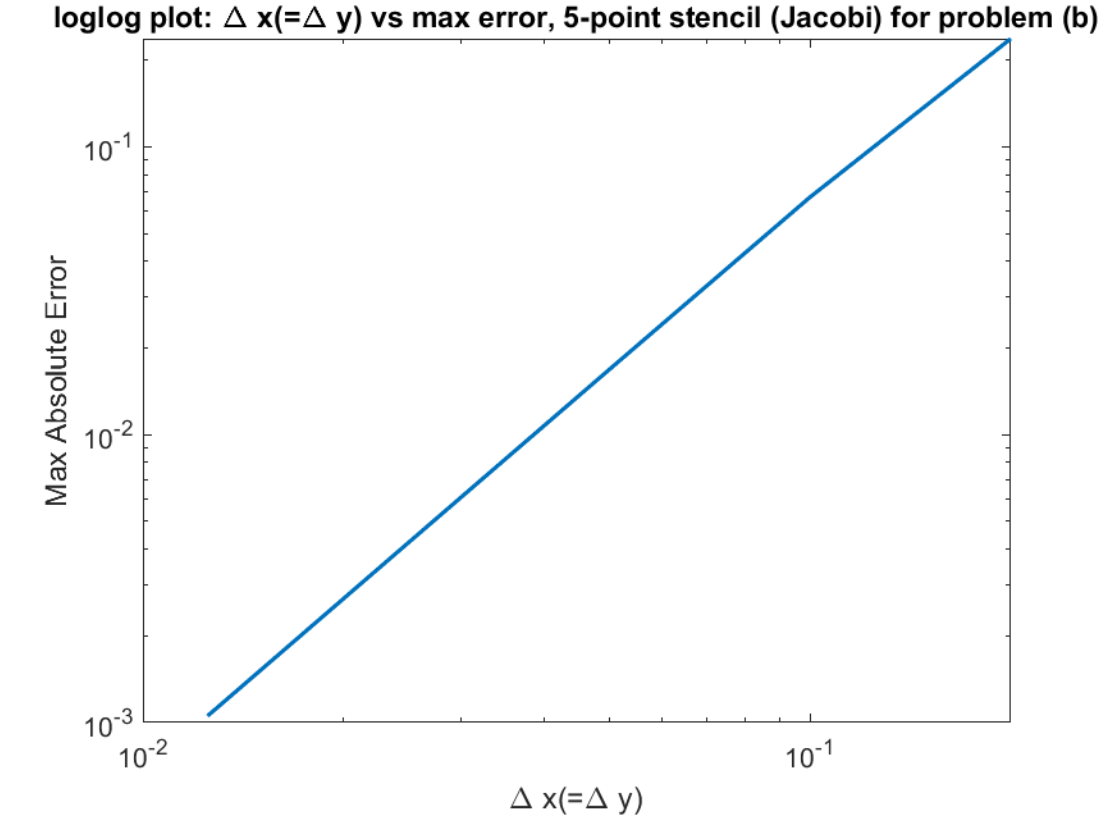
Solution by 5-point stencil(Gauss Seidel) forproblem(b) h = k =0.02



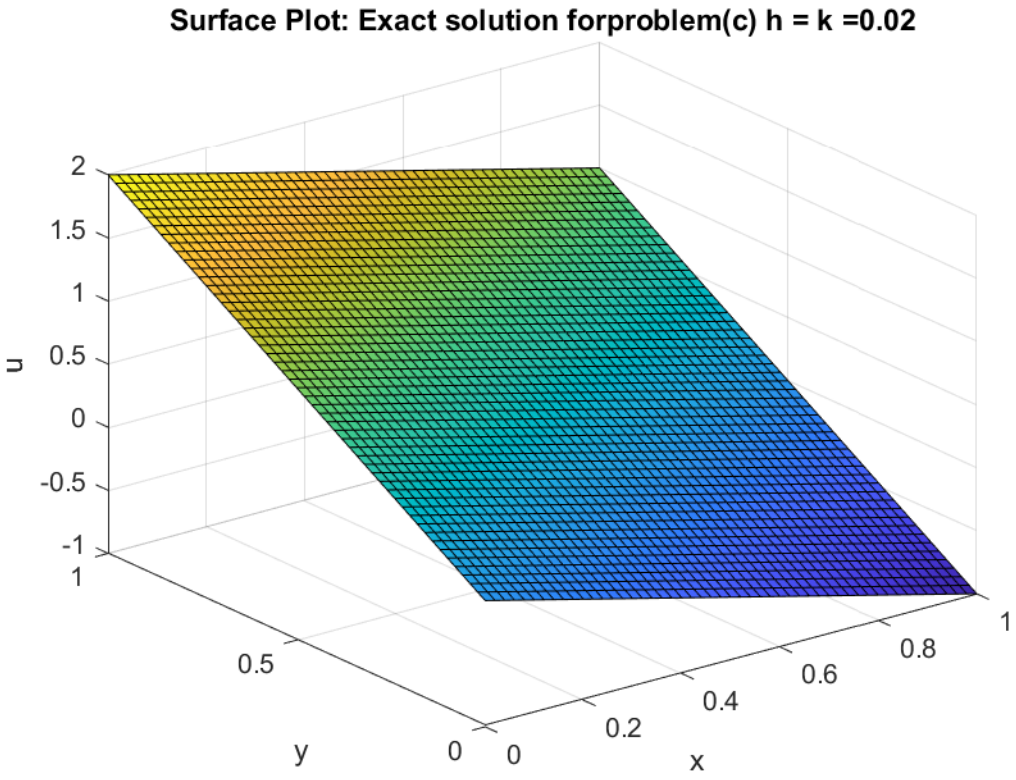
Error: 5-point stencil (Jacobi) forproblem(b) h = k =0.02

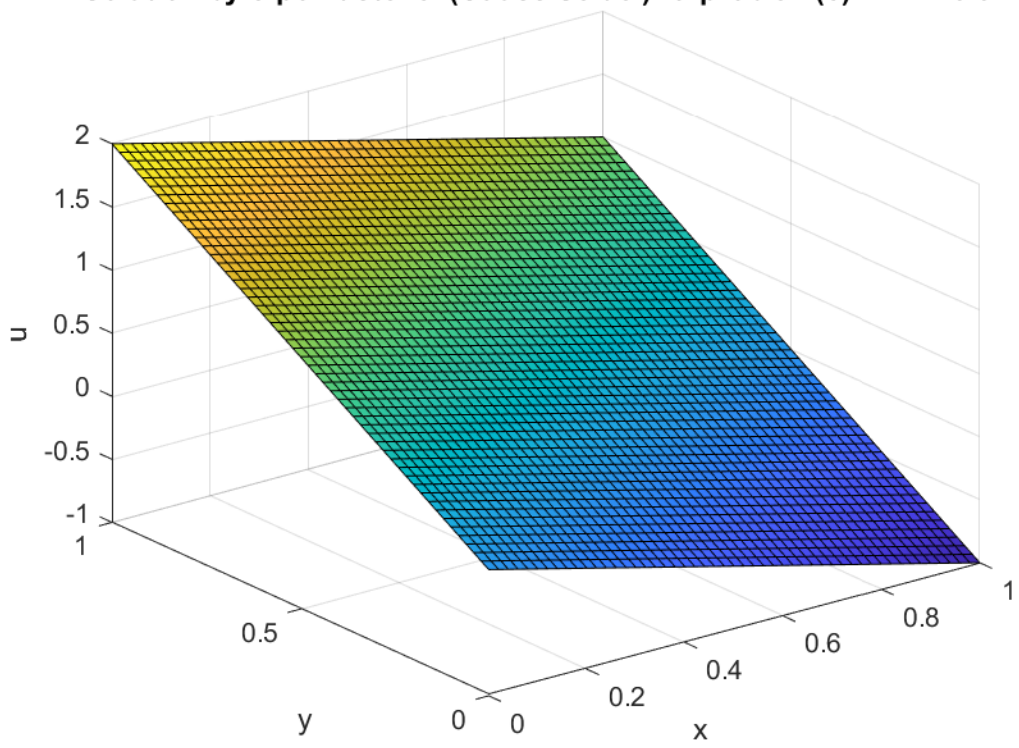


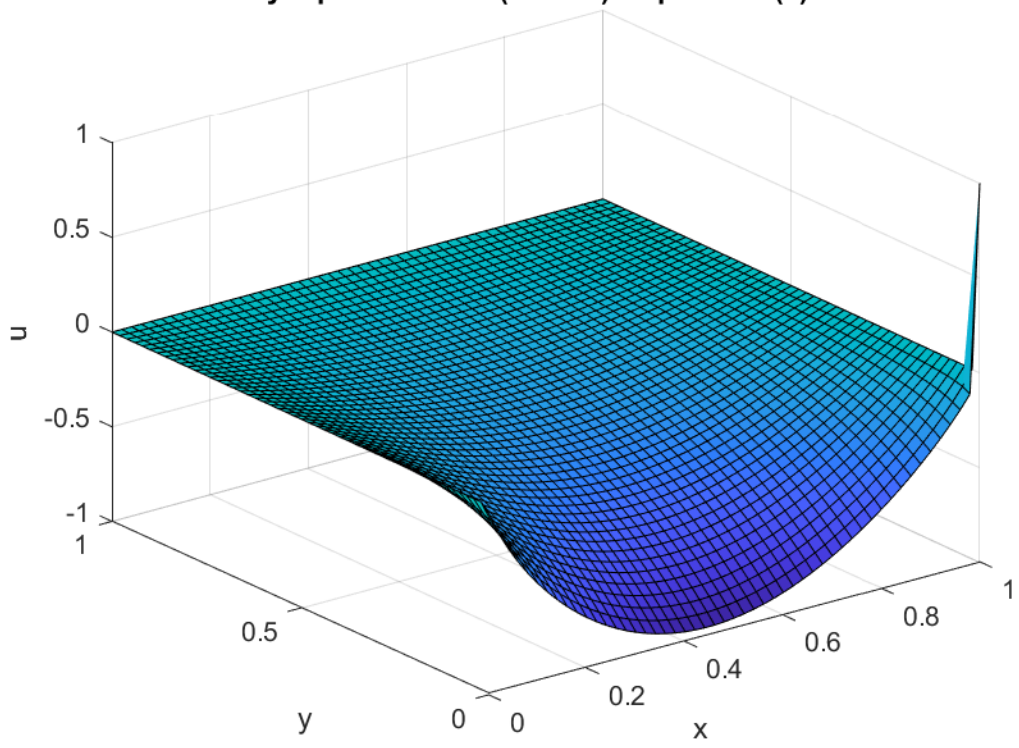

loglog plot: Δ x(=Δ y) vs max error, 5-point stencil (Gauss Siedel) for problem (
Max Absolute Error
10^-1
10^-2
10^-3
10^-2
10^-1
Δ x(=Δ y)

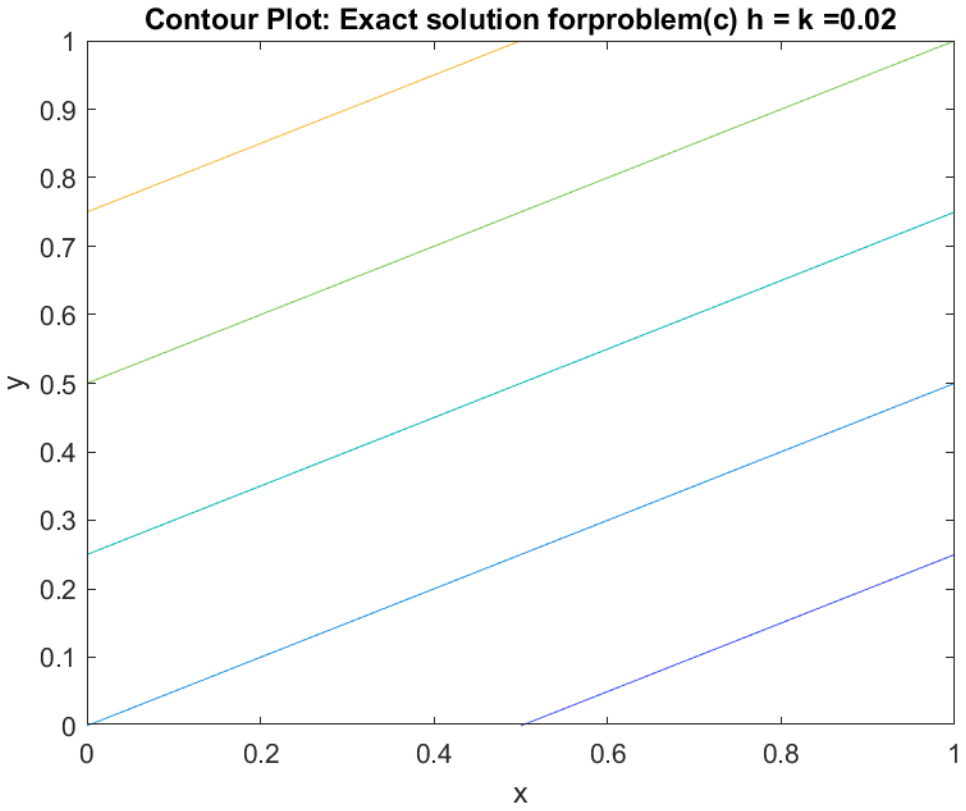

loglog plot: Δ x(=Δ y) vs max error, 5-point stencil (Jacobi) for problem (b)
Max Absolute Error
10^-1
10^-2
10^-3
10^-2
10^-1
Δ x(=Δ y)

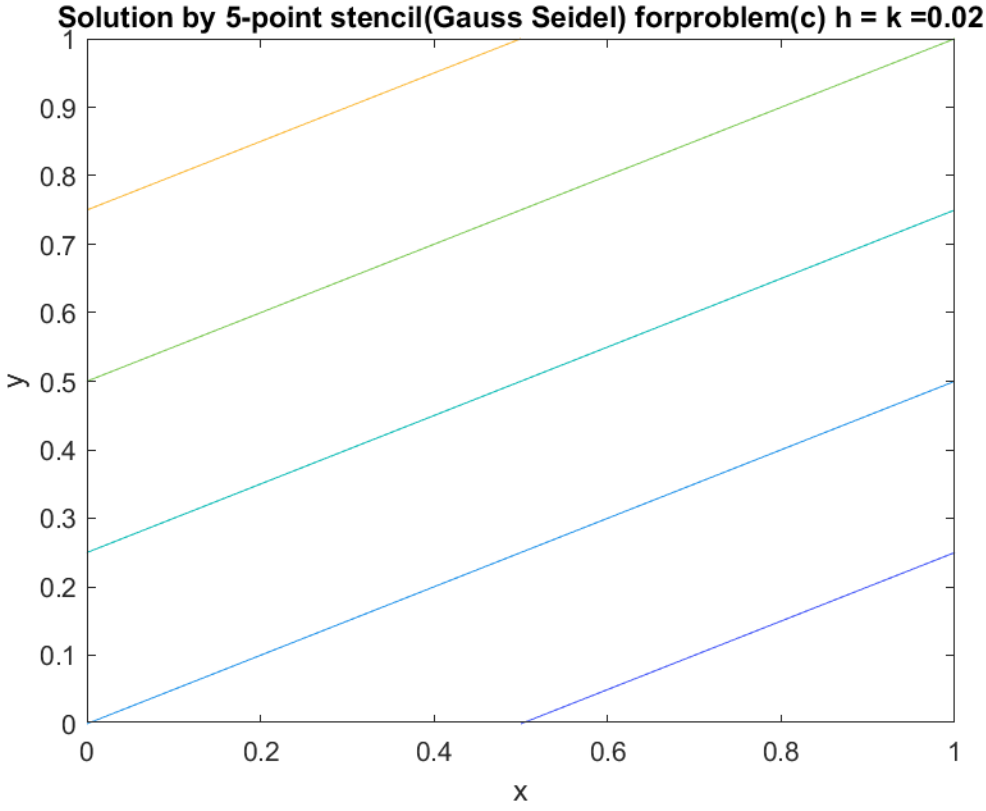
c)

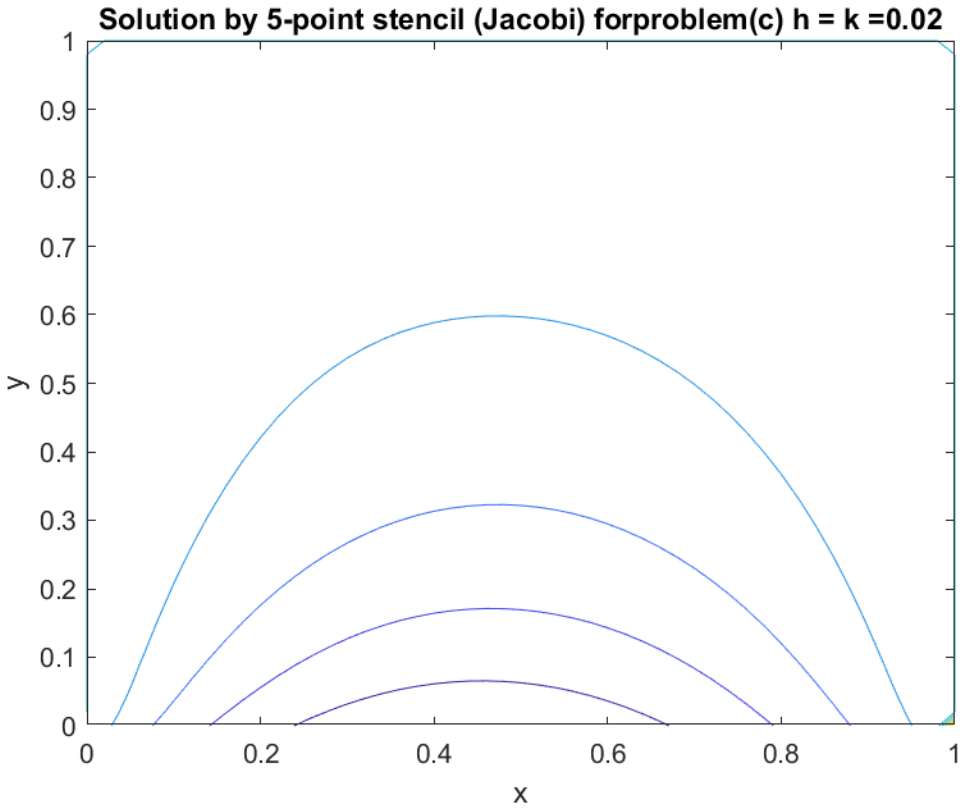

Surface Plot: Exact solution forproblem(c) h = k =0.02
u
2
1.5
1
0.5
0
-0.5
-1
y
1
0.5
0
x
0
0.2
0.4
0.6
0.8
1

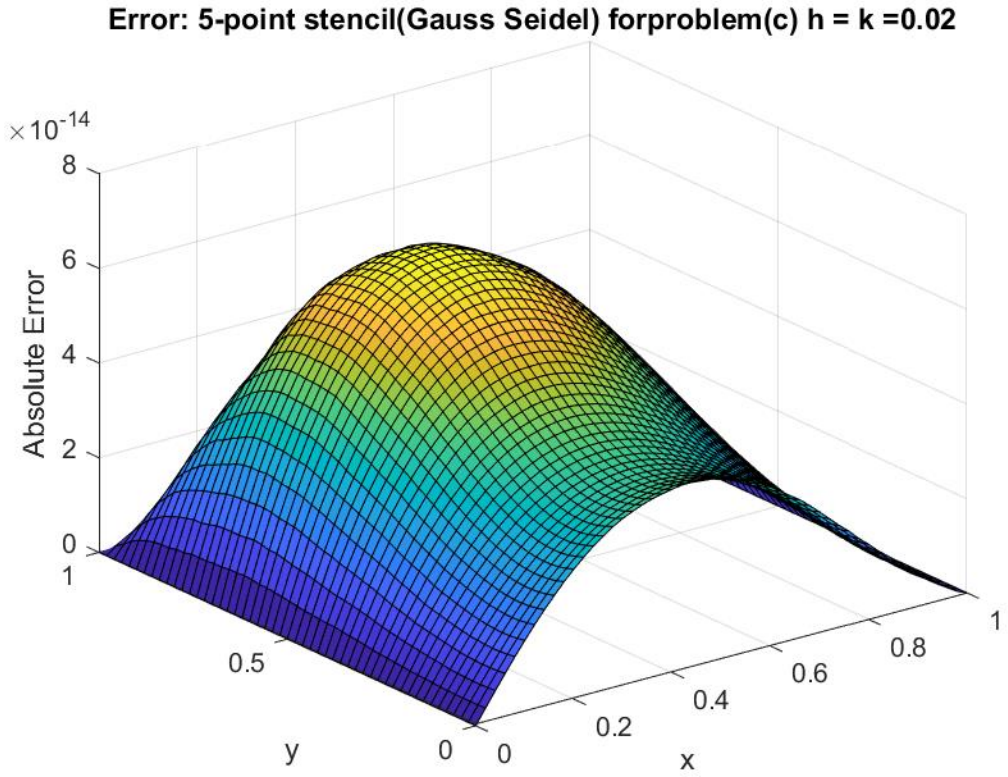

Solution by 5-point stencil(Gauss Seidel) forproblem(c) h = k =0.02
u
2
1.5
1
0.5
0
-0.5
-1
1
0.5
0
y
0
0.2
0.4
0.6
0.8
1
x

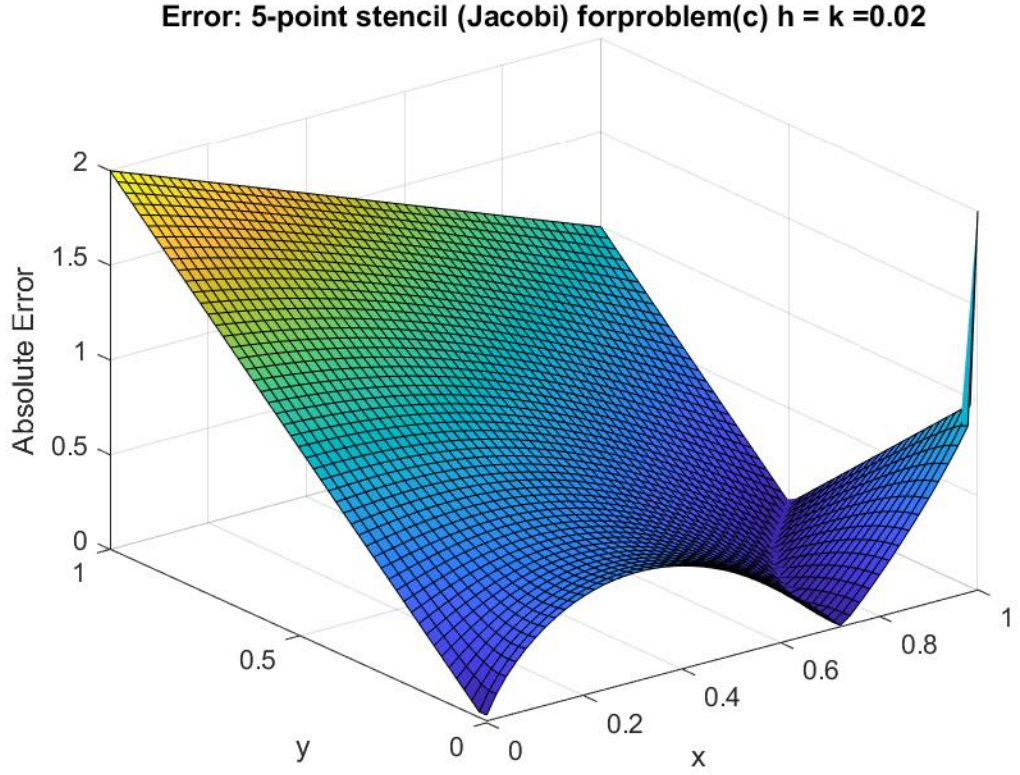

Solution by 5-point stencil (Jacobi) forproblem(c) h = k =0.02
u
1
0.5
0
-0.5
-1
1
0.5
0
y
0
0.2
0.4
0.6
0.8
1
x

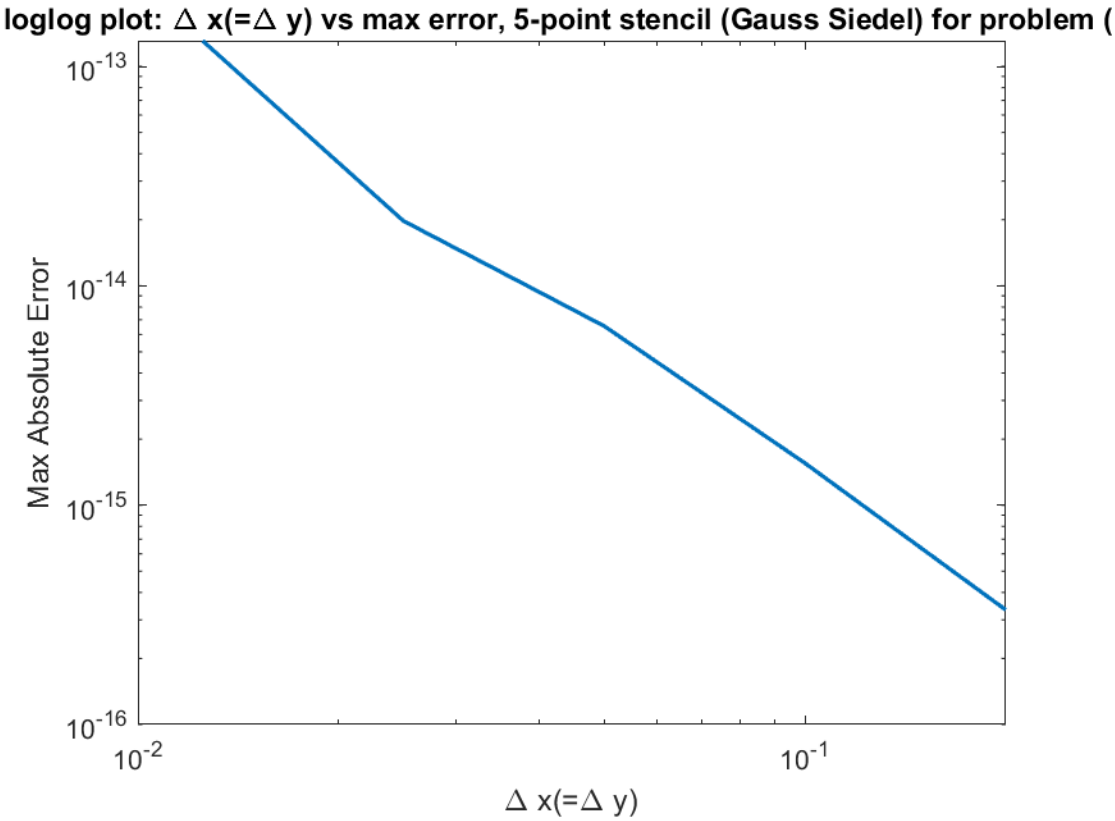

Contour Plot: Exact solution forproblem(c) h = k =0.02
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1

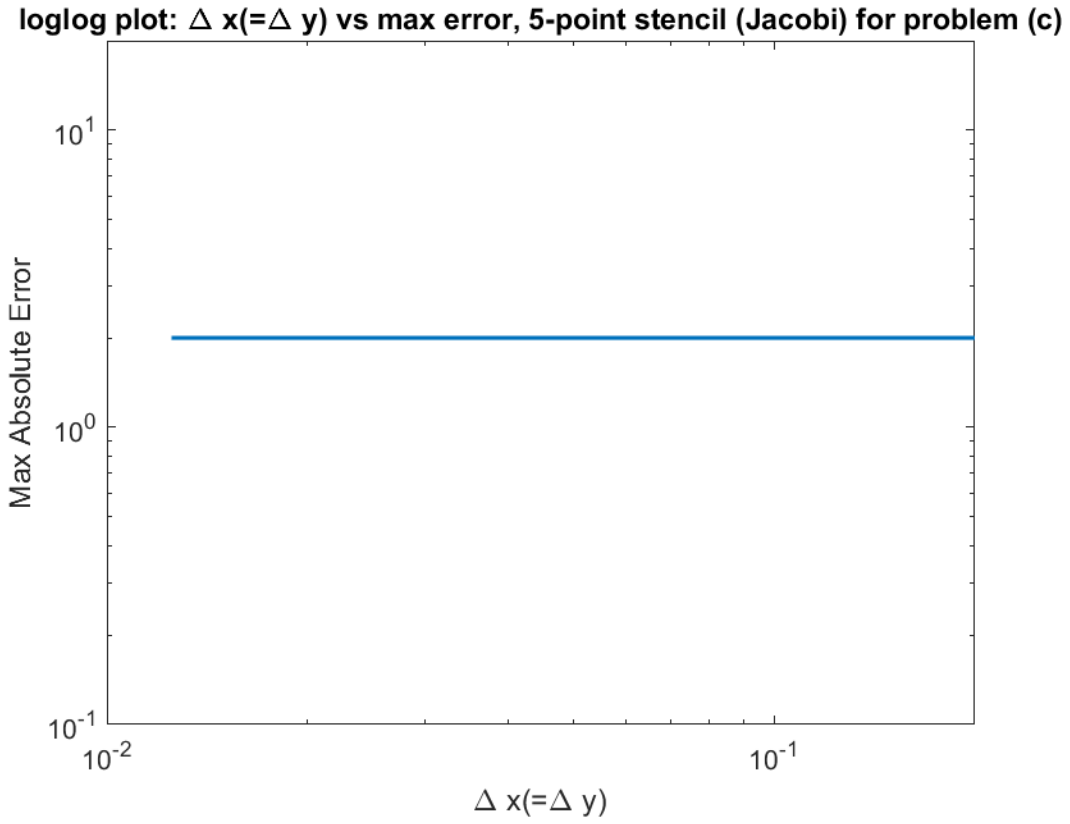

Solution by 5-point stencil(Gauss Seidel) forproblem(c) h = k =0.02
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1


Solution by 5-point stencil (Jacobi) forproblem(c) h = k =0.02
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1

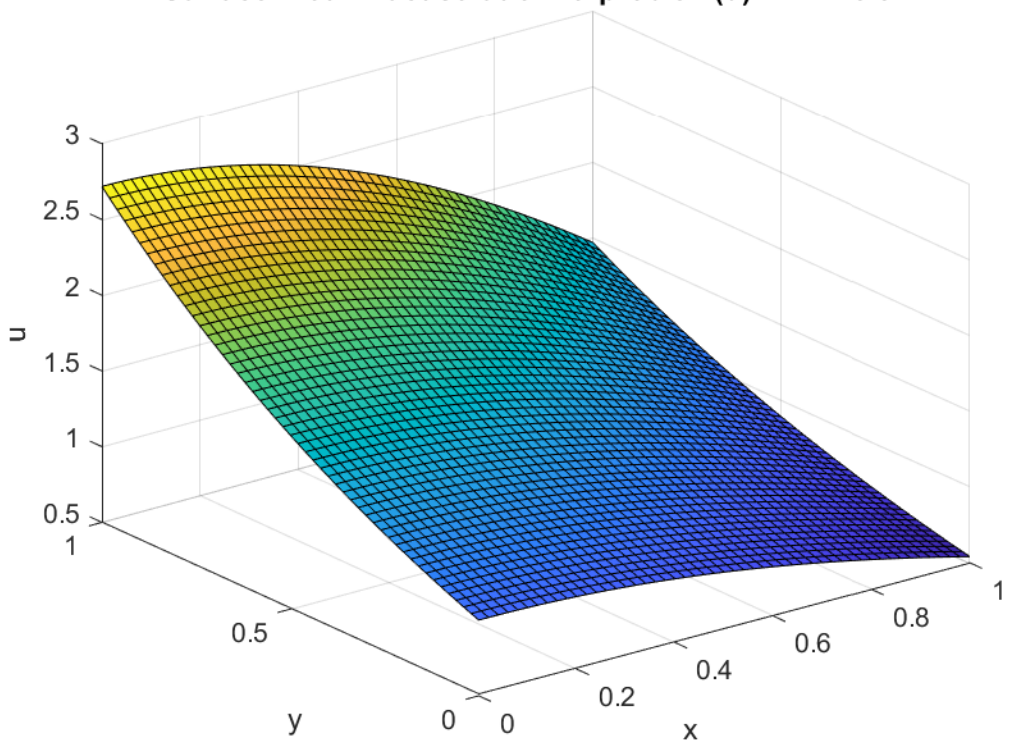

Error: 5-point stencil(Gauss Seidel) forproblem(c) h = k =0.02
×10^-14
Absolute Error
0
2
4
6
8
y
0
0.5
1
x
0
0.2
0.4
0.6
0.8
1

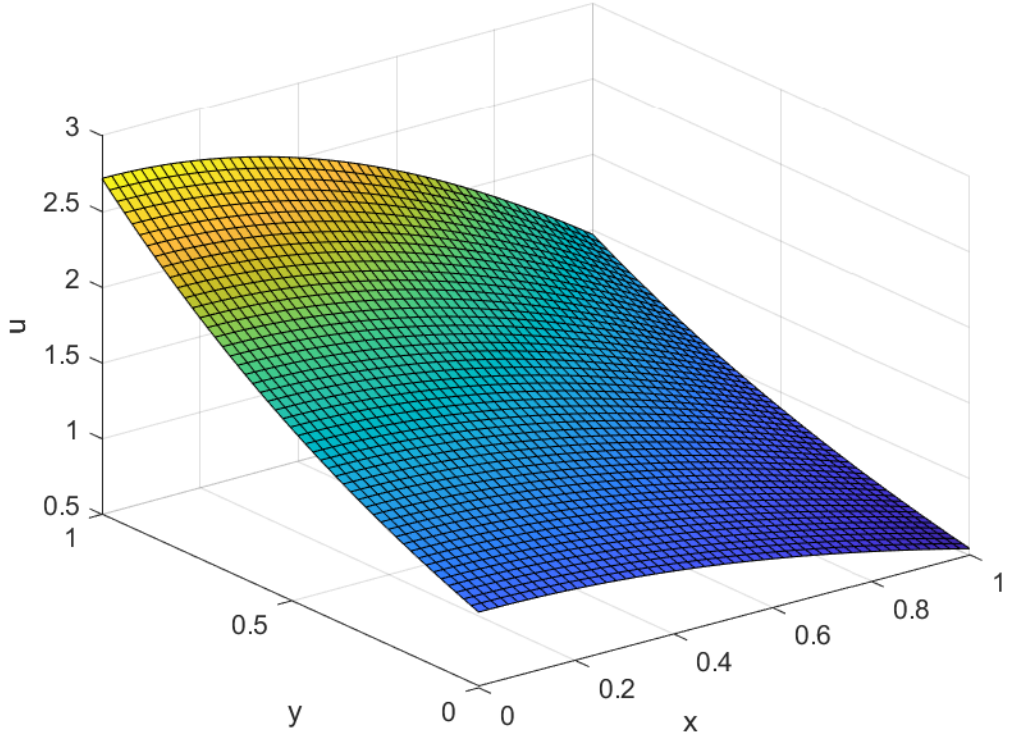

Error: 5-point stencil (Jacobi) forproblem(c) h = k =0.02
Absolute Error
2
1.5
1
0.5
0
1
0.5
0
y
0
0.2
0.4
0.6
0.8
1
x

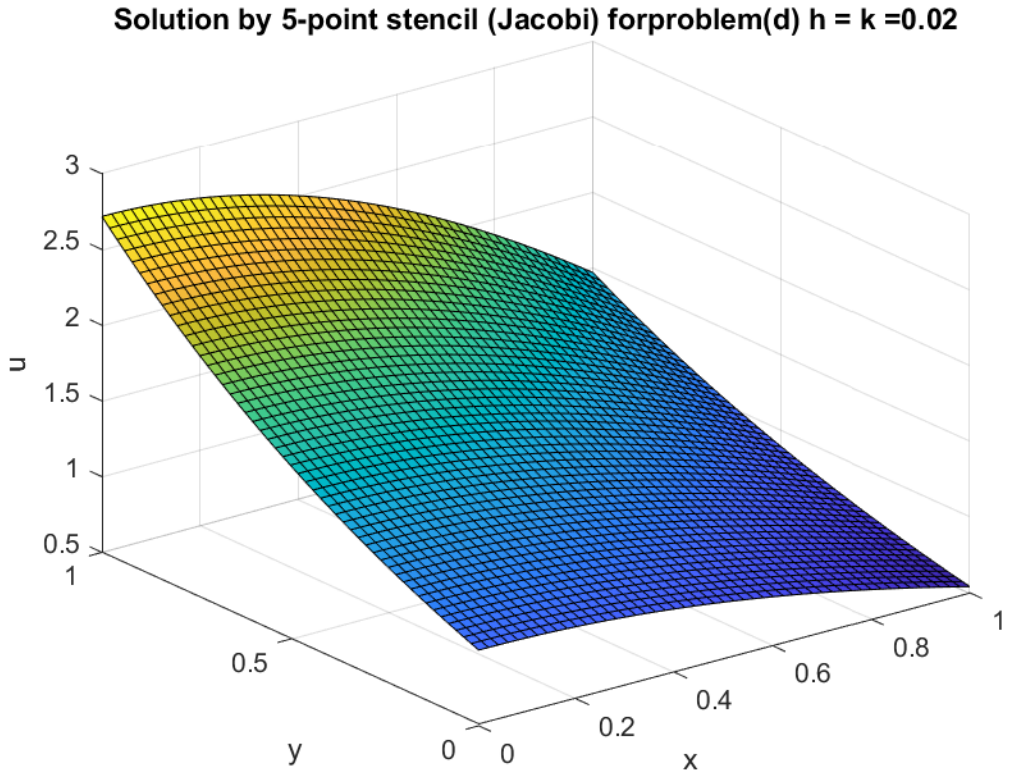

loglog plot: Δ x(=Δ y) vs max error, 5-point stencil (Gauss Siedel) for problem (
Max Absolute Error
10^-13
10^-14
10^-15
10^-16
10^-2
10^-1
Δ x(=Δ y)

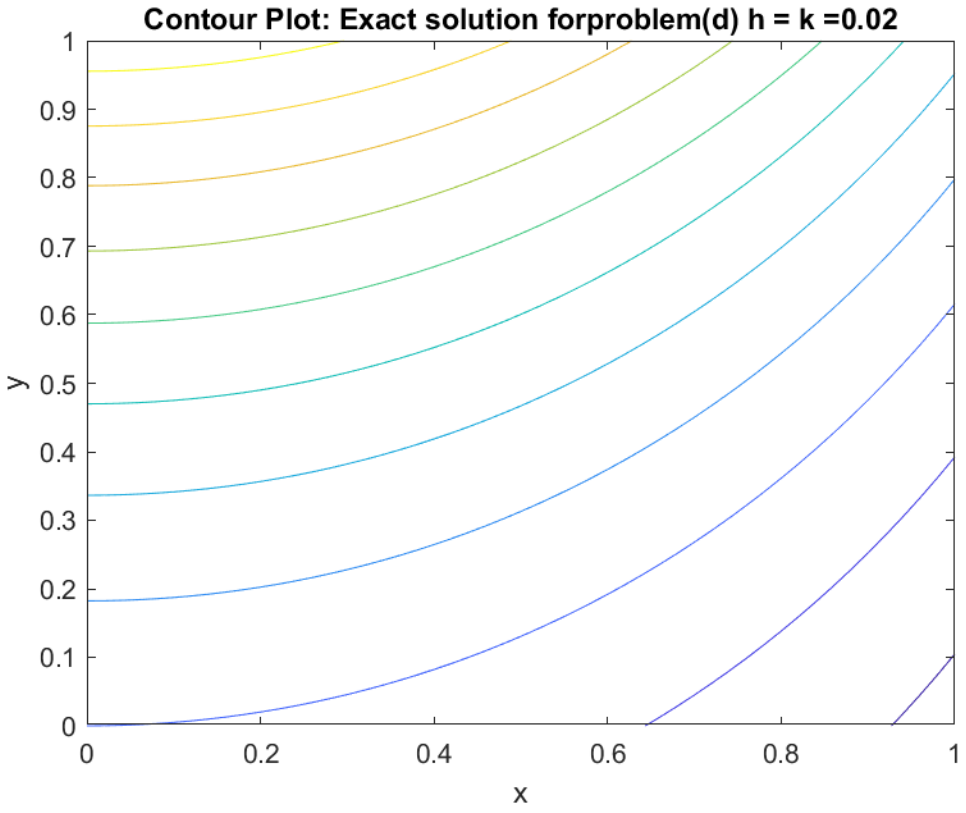

loglog plot: Δ x(=Δ y) vs max error, 5-point stencil (Jacobi) for problem (c)
Max Absolute Error
10^1
10^0
10^-1
10^-2
10^-1
Δ x(=Δ y)

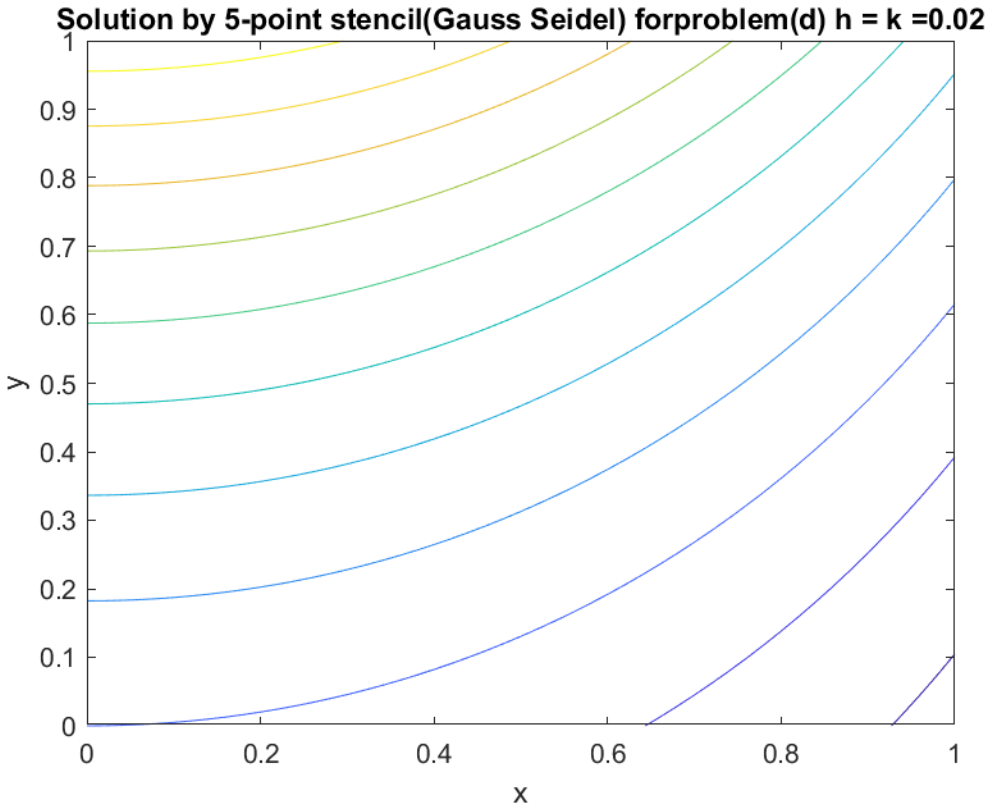
d)

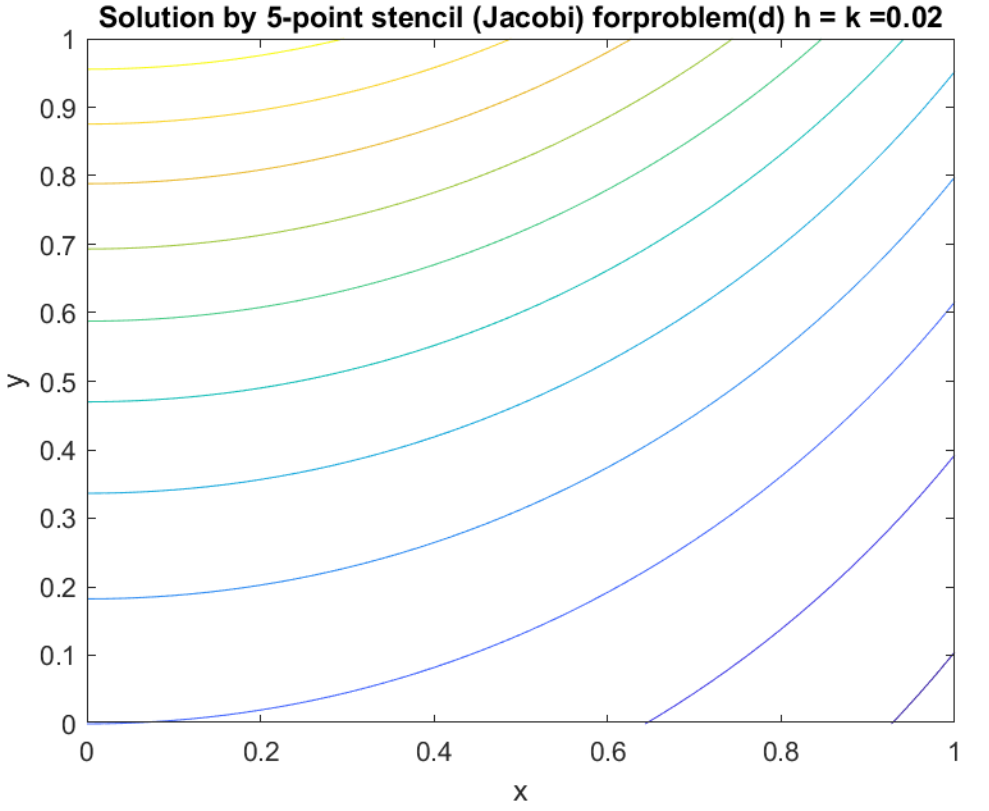

Surface Plot: Exact solution forproblem(d) h = k =0.02
u
3
2.5
2
1.5
1
0.5
1
0.5
0
y
0
0.2
0.4
0.6
0.8
1
x

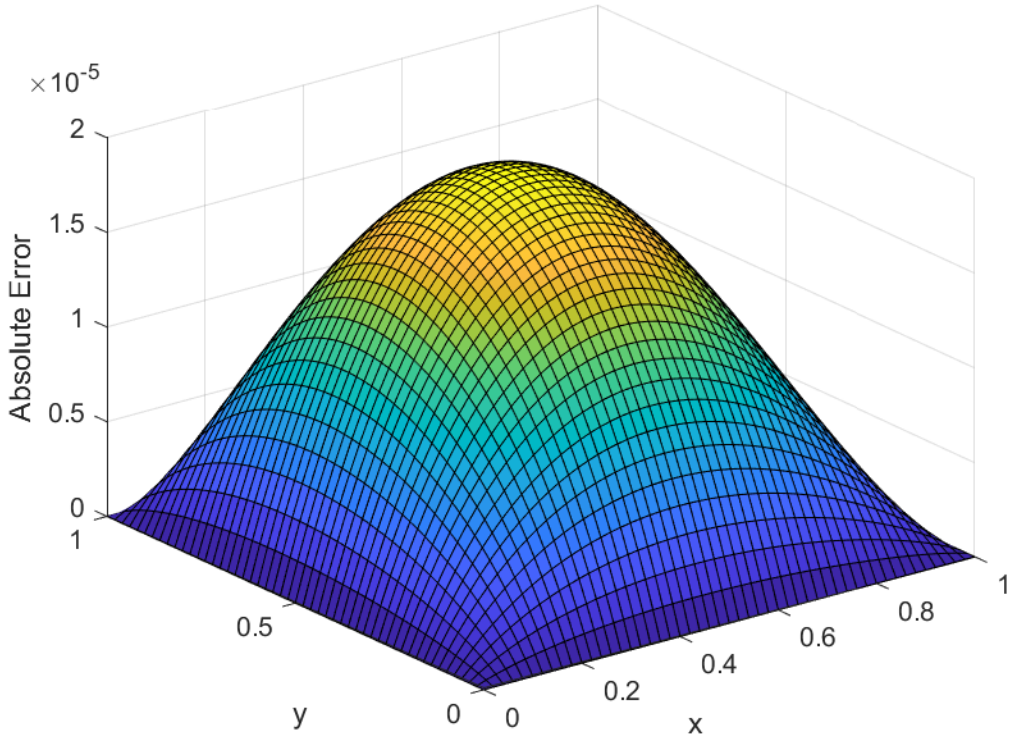

Solution by 5-point stencil(Gauss Seidel) forproblem(d) h = k =0.02
u
3
2.5
2
1.5
1
0.5
1
0.5
0
y
0
0.2
0.4
0.6
0.8
1
x

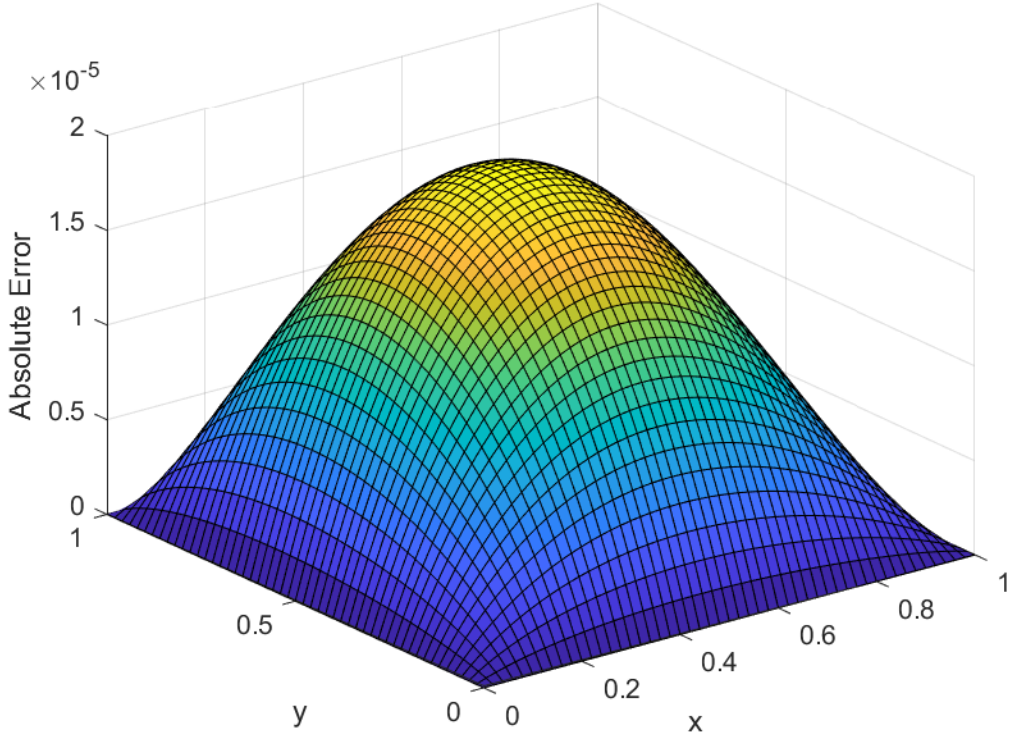

Solution by 5-point stencil (Jacobi) forproblem(d) h = k =0.02
u
3
2.5
2
1.5
1
0.5
1
0.5
0
y
0
0.2
0.4
0.6
0.8
1
x

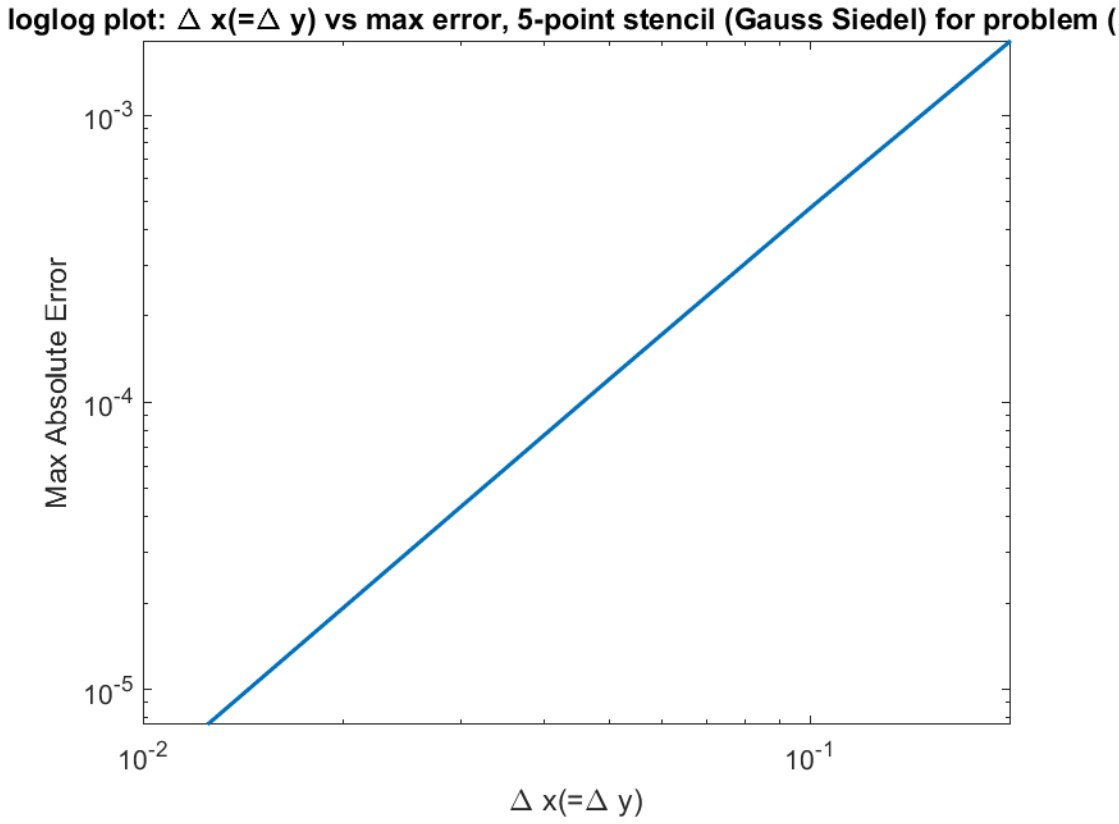

Contour Plot: Exact solution forproblem(d) h = k =0.02
y
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
x

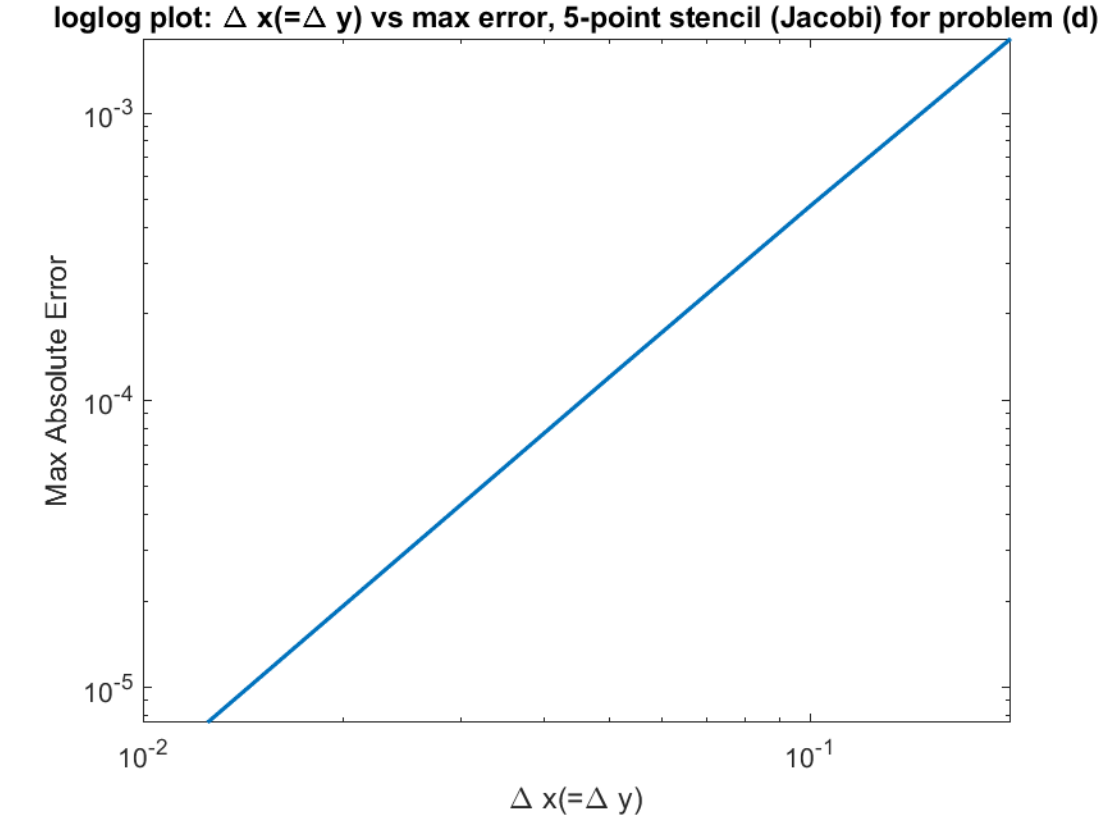

Solution by 5-point stencil(Gauss Seidel) forproblem(d) h = k =0.02
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1

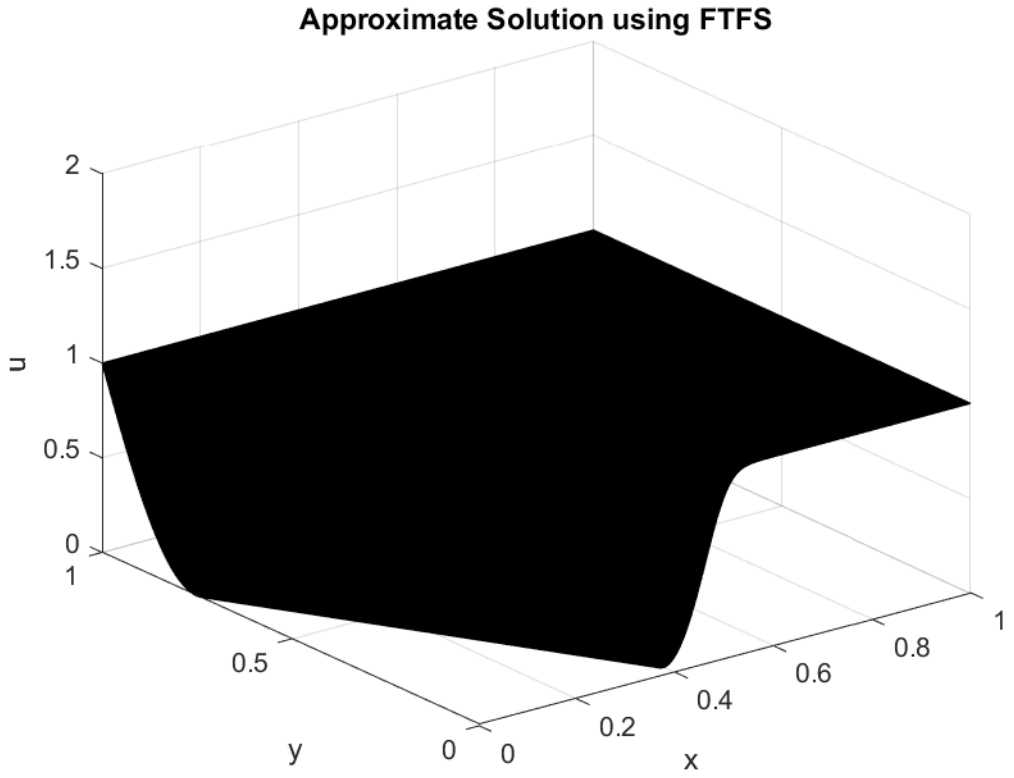

Solution by 5-point stencil (Jacobi) forproblem(d) h = k =0.02
y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1


Error: 5-point stencil(Gauss Seidel) forproblem(d) h = k =0.02
×10^-5
2
1.5
1
0.5
0
Absolute Error
1
0.5
0
y
0
0.2
0.4
0.6
0.8
1
x

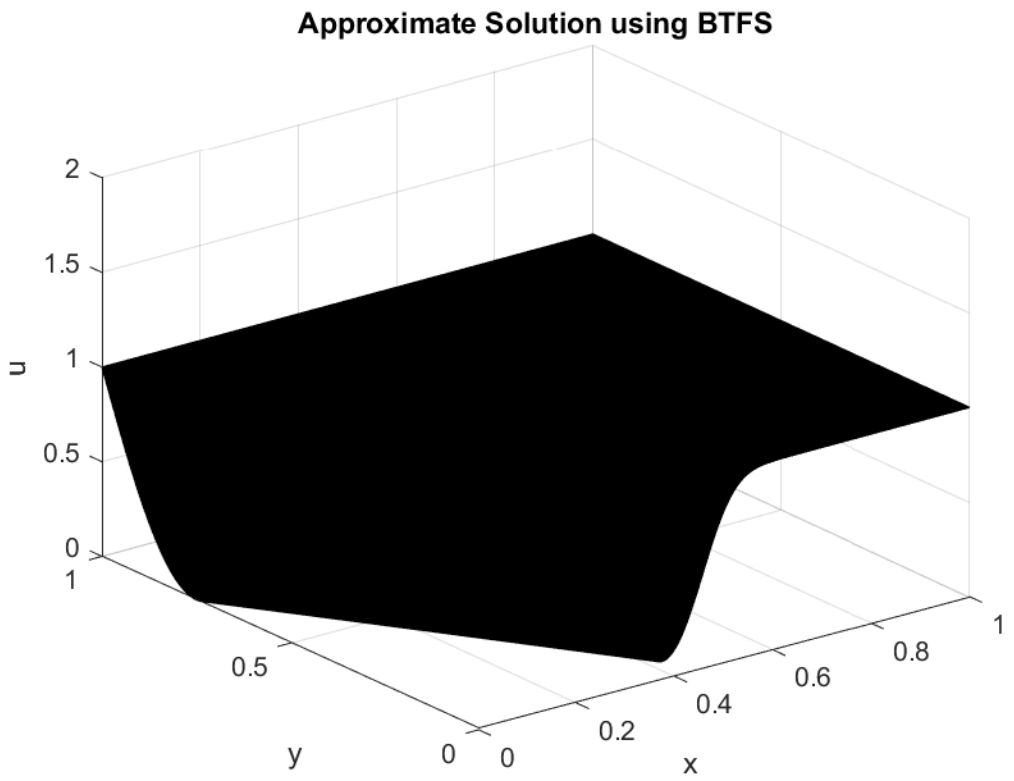

Error: 5-point stencil (Jacobi) forproblem(d) h = k =0.02
×10^-5
2
1.5
1
0.5
0
Absolute Error
1
0.5
0
y
0
0.2
0.4
0.6
0.8
1
x


loglog plot: Δ x(=Δ y) vs max error, 5-point stencil (Gauss Siedel) for problem (
Max Absolute Error
10^-3
10^-4
10^-5
10^-2
10^-1
Δ x(=Δ y)

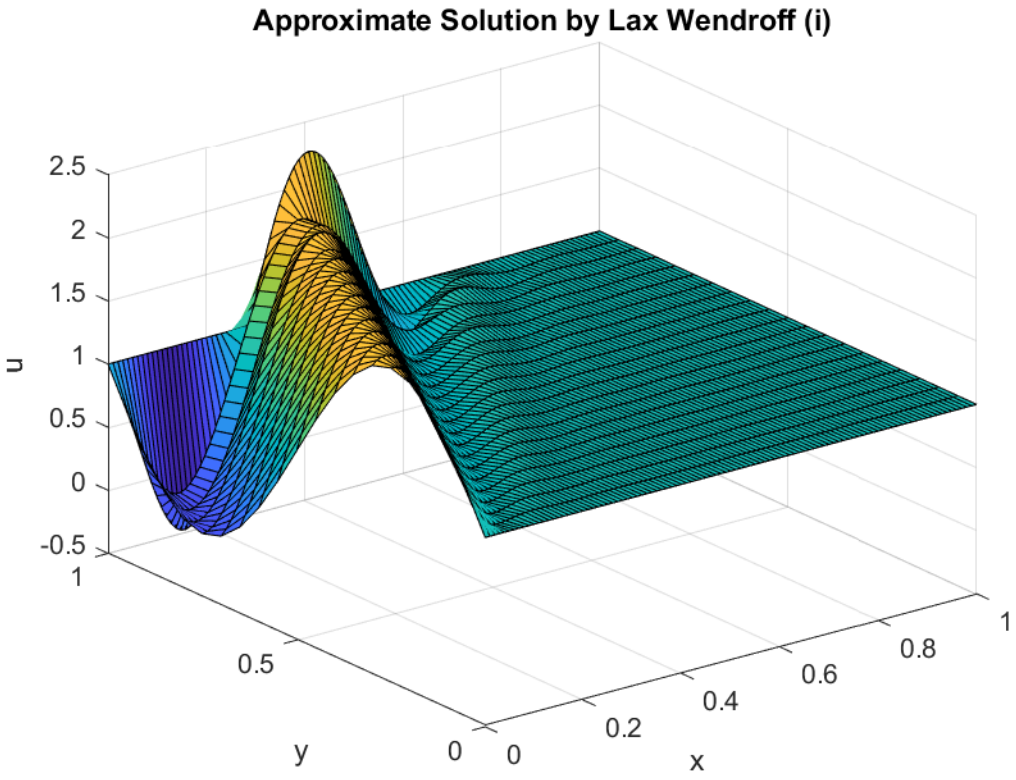

loglog plot: Δ x(=Δ y) vs max error, 5-point stencil (Jacobi) for problem (d)
Max Absolute Error
10^-3
10^-4
10^-5
10^-2
10^-1
Δ x(=Δ y)

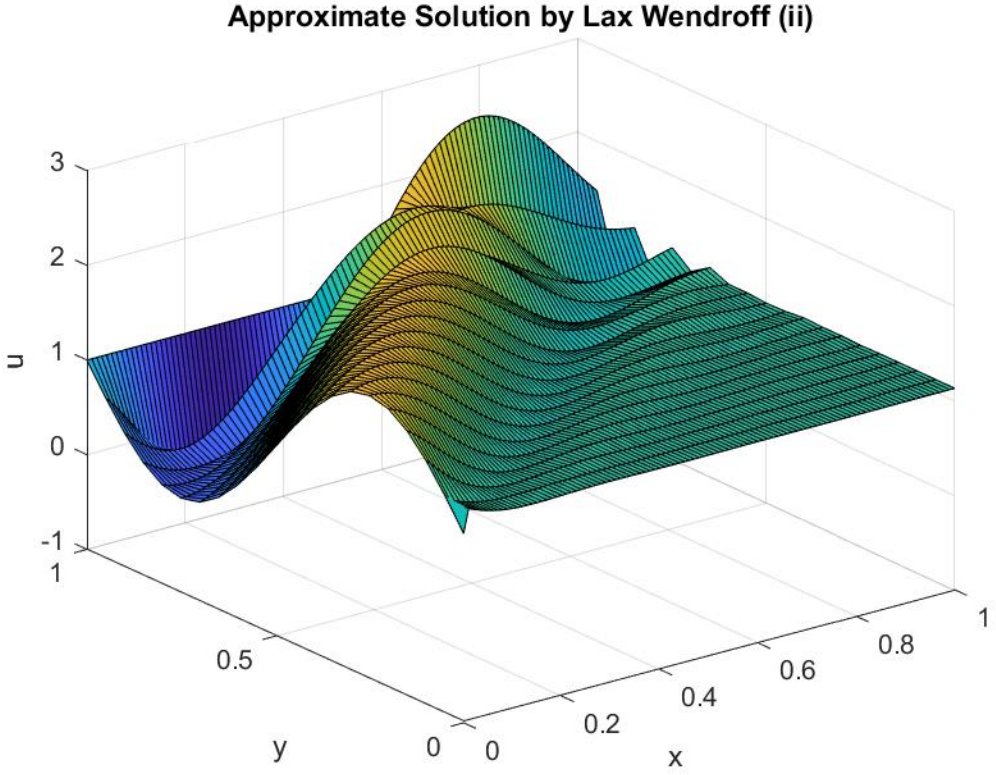

Approximate Solution using FTFS
u
2
1.5
1
0.5
0
1
0.5
0
y
0
0.2
0.4
0.6
0.8
1
x

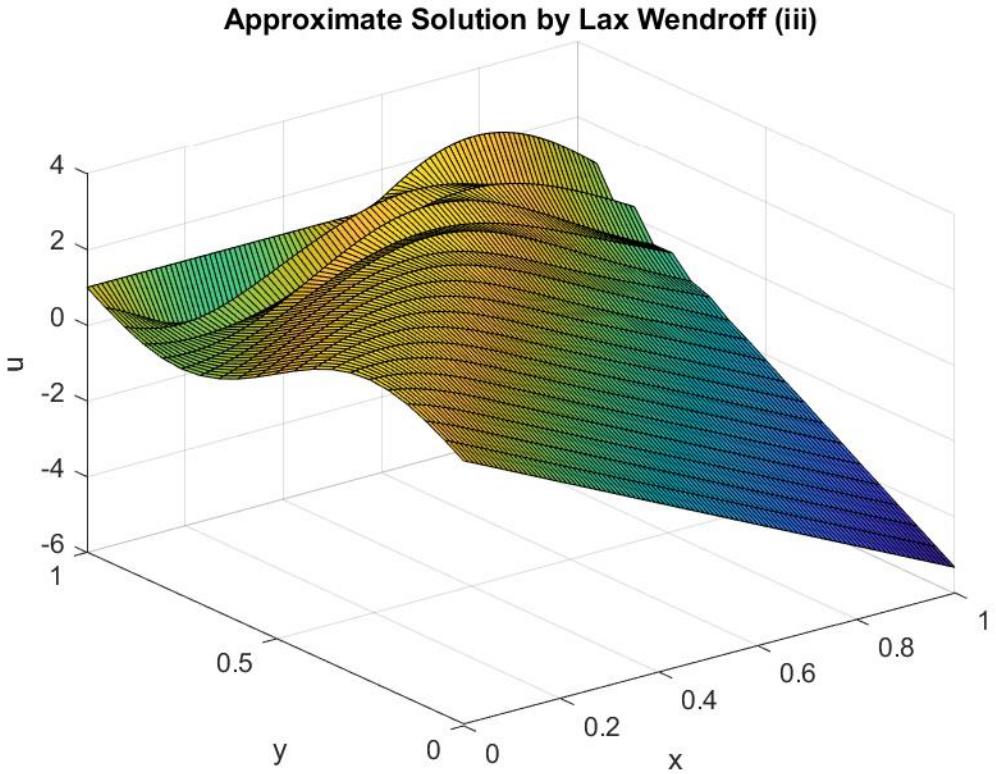
3)

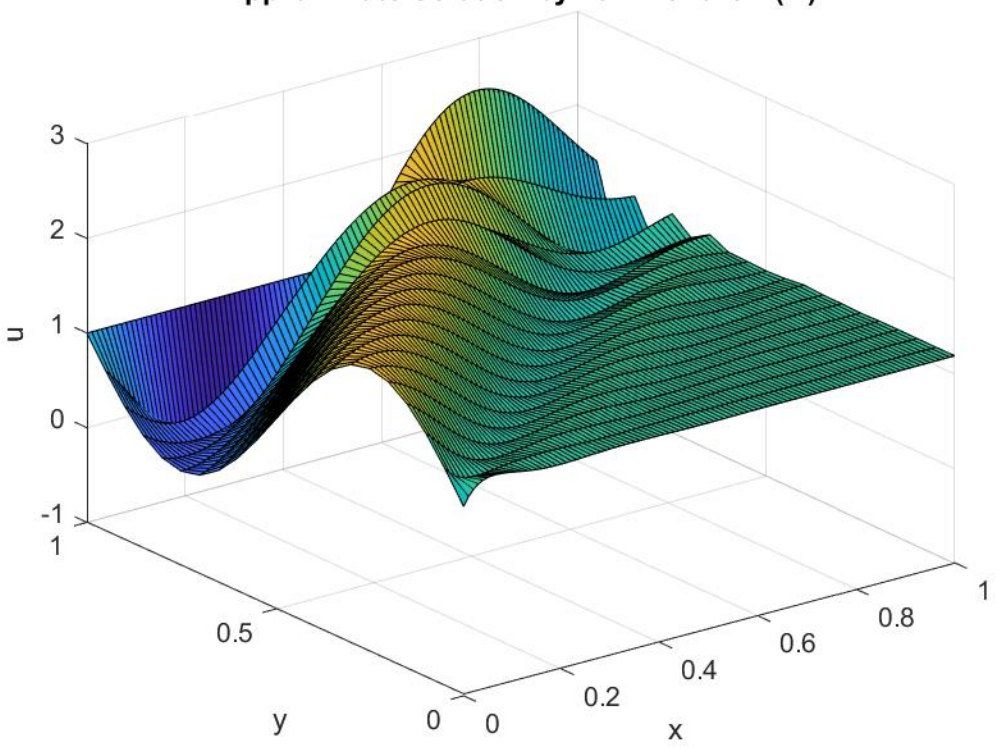

Approximate Solution using BTFS
u
2
1.5
1
0.5
0
1
0.5
0
y
0
0.2
0.4
0.6
0.8
1
x

4)


Approximate Solution by Lax Wendroff (i)
u
2.5
2
1.5
1
0.5
0
-0.5
y
1
0.5
0
x
0
0.2
0.4
0.6
0.8
1


Approximate Solution by Lax Wendroff (ii)
u
3
2
1
0
-1
y
1
0.5
0
x
0
0.2
0.4
0.6
0.8
1


Approximate Solution by Lax Wendroff (iii)
u
4
2
0
-2
-4
-6
y
1
0.5
0
x
0
0.2
0.4
0.6
0.8
1


Approximate Solution by Lax Wendroff (iv)
u
3
2
1
0
-1
1
0.5
0
y
0
0.2
0.4
0.6
0.8
1
x