

**DEPARTMENT OF MATHEMATICS
IIT GUWAHATI**

MA 332

Scientific Computing

Lab - III

1. Perform two iterations of Newton's method on these systems. a. Starting with $(1, 1)$ b. Starting with $(0, 0)$

$$\text{a. } \begin{cases} xy^2 + x^2y + x^4 &= 3, \\ x^3y^5 - 2x^5y - x^2 &= -2. \end{cases} \quad \text{b. } \begin{cases} \sin(4\pi x_1 x_2) - 2x_2 - x_1 &= 0, \\ \frac{4\pi-1}{4\pi}(e^{2x_1} - e) + 4ex_2^2 - 2ex_1 &= 0. \end{cases}$$

2. Use Newton's method to find a solution to the following nonlinear systems in the given domain. Iterate until $\|\mathbf{x}^{(k)} - \mathbf{x}^{(k-1)}\|_\infty < 10^{-6}$.

$$\begin{cases} 6x_1 - 2\cos(x_2 x_3) - 1 = 0, \\ 9x_2 + \sqrt{x_1^2 + \sin(x_3) + 1.06} + 0.9 = 0, \\ 60x_3 + 3e^{-x_1 x_2} + 10\pi - 3 = 0. \end{cases}$$

Use $\mathbf{x}^{(0)} = (0, 0, 0)^t$.

3. Solve this pair of simultaneous nonlinear equations by first eliminating y and then solving the resulting equation in x by Newton's method. Start with the initial value $x_0 = 1$.

$$\begin{cases} x^3 - 2xy + y^7 - 4x^3y &= 5, \\ y \sin x + 3x^2y + \tan x &= 4. \end{cases}$$

4. Use Newton's method and the modified Newton's method to find solutions accurate to within 10^{-5} to the following problems.

a) $\cos(x + \sqrt{2}) + x(x/2 + \sqrt{2}) = 0, \quad -2 \leq x \leq -1,$

b) $e^{6x} + 3(\ln 2)^2 e^{2x} - (\ln 8)e^{4x} - (\ln 2)^3 = 0, \quad -1 \leq x \leq 0.$

Are there any improvements in speed or accuracy after using the modified Newton's method?

5. Use modified Newton's method to find solutions of the following problems:

a) Use the value $p_0 = 1.3$ to find the quadruple root $p = 1$ of the equation $x^5 - 2x^4 - 2x^3 + 8x^2 - 7x + 2 = 0$

b) Find the double root $p = 1$ and triple root $p = 2$ of the polynomial $p(x) = x^5 - 8x^4 + 25x^3 - 38x^2 + 28x - 8$