# LAB 01 Report

1)

| n | x(n) | f(x(n)) |
|---|------|---------|
| 0 | -3.9269908e+00 | -6.874039e-01 |
| 1 | -3.5342917e+00 | -3.535040e-01 |
| 2 | -3.3379422e+00 | -1.595804e-01 |
| 3 | -3.2397674e+00 | -5.884414e-02 |
| 4 | -3.1906800e+00 | -7.923792e-03 |

solution = -3.239767e+00     iterations = 4

2)

| n | x(n) | f(x(n)) |
|---|------|---------|
| 0 | 1.0000000e+00 | 3.465573e-02 |
| 1 | 1.2500000e+00 | -1.409976e+00 |
| 2 | 1.1250000e+00 | -6.090797e-01 |
| 3 | 1.0625000e+00 | -2.669823e-01 |
| 4 | 1.0312500e+00 | -1.111478e-01 |
| 5 | 1.0156250e+00 | -3.700287e-02 |
| 6 | 1.0078125e+00 | -8.644252e-04 |

solution = 1     iterations = 6

| n | x(n) | f(x(n)) |
|---|------|---------|
| 0 | 5.0000000e-01 | -2.071068e-01 |
| 1 | 7.5000000e-01 | 1.553964e-01 |
| 2 | 6.2500000e-01 | -2.341978e-02 |
| 3 | 6.8750000e-01 | 6.657109e-02 |
| 4 | 6.5625000e-01 | 2.172452e-02 |
| 5 | 6.4062500e-01 | -8.100080e-04 |

solution = 6.250000e-01     iterations = 5

| n | x(n) | f(x(n)) |
|---|------|---------|
| 0 | 5.0000000e-01 | 8.987213e-01 |
| 1 | 2.5000000e-01 | -2.847458e-02 |
| 2 | 3.7500000e-01 | 4.393664e-01 |
| 3 | 3.1250000e-01 | 2.066817e-01 |
| 4 | 2.8125000e-01 | 8.943320e-02 |
| 5 | 2.6562500e-01 | 3.056423e-02 |

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       6  2.5781250e-01           1.066368e-03
       7  2.5390625e-01          -1.369868e-02
       8  2.5585938e-01          -6.314807e-03
       9  2.5683594e-01          -2.623882e-03
solution = 2.568359e-01        iterations = 10
       n          x(n)            f(x(n))
       0  -2.5000000e+00         -3.668311e+00
       1  -2.2500000e+00         -6.139189e-01
       2  -2.1250000e+00          6.302468e-01
       3  -2.1875000e+00          3.807553e-02
       4  -2.2187500e+00         -2.808362e-01
       5  -2.2031250e+00         -1.195568e-01
       6  -2.1953125e+00         -4.027851e-02
       7  -2.1914062e+00         -9.851950e-04
solution = -2.195312e+00       iterations = 7
       n          x(n)            f(x(n))
       0  -5.0000000e-01         -7.903023e-01
       1  -7.5000000e-01         -1.686058e-01
       2  -8.7500000e-01          2.963056e-01
       3  -8.1250000e-01          5.288159e-02
       4  -7.8125000e-01         -6.081442e-02
       5  -7.9687500e-01         -4.680561e-03
       6  -8.0468750e-01          2.392518e-02
       7  -8.0078125e-01          9.578066e-03
       8  -7.9882812e-01          2.437641e-03
       9  -7.9785156e-01         -1.124244e-03
solution = -7.988281e-01       iterations = 10
       n          x(n)            f(x(n))
       0  2.5000000e-01          -5.077719e-01
       1  2.2500000e-01          -5.556713e-01
       2  2.1250000e-01          -5.797798e-01
       3  2.0625000e-01          -5.918713e-01
       4  2.0312500e-01          -5.979261e-01
       5  2.0156250e-01          -6.009556e-01
       6  2.0078125e-01          -6.024710e-01
```

solution = 2.000000e-01        iterations = 7

| n | x(n) | f(x(n)) |
|---|---|---|
| 0 | 1.2500000e+00 | 6.441530e-01 |
| 1 | 1.2250000e+00 | 6.402088e-01 |
| 2 | 1.2125000e+00 | 6.376986e-01 |
| 3 | 1.2062500e+00 | 6.363088e-01 |
| 4 | 1.2031250e+00 | 6.355803e-01 |
| 5 | 1.2015625e+00 | 6.352076e-01 |
| 6 | 1.2007812e+00 | 6.350192e-01 |

solution = 1.200000e+00        iterations = 7


3)

| n | x(n) | f(x(n)) |
|---|---|---|
| 0 | 2.9250000e+00 | 2.520312e-02 |
| 1 | 2.9125000e+00 | -2.942637e-01 |
| 2 | 2.9187500e+00 | -1.348723e-01 |
| 3 | 2.9218750e+00 | -5.492020e-02 |
| 4 | 2.9234375e+00 | -1.487995e-02 |
| 5 | 2.9242187e+00 | 5.156234e-03 |
| 6 | 2.9238281e+00 | -4.863195e-03 |
| 7 | 2.9240234e+00 | 1.461849e-04 |
| 8 | 2.9239258e+00 | -2.358589e-03 |

solution = 2.923926e+00        iterations = 9


4)
Rate of convergence c > d >  b > a
First Four iterations
a)

| n | x(n) |
|---|---|
| 1.000000e+00 | 3.430000e+02 |
| 3.430000e+02 | -2.253934e+25 |
| -2.253934e+25 | -3.383855e+253 |
| -3.383855e+253 | NaN |

b)

| n | x(n) |
|---|---|
| 1.000000e+00 | 7.000000e+00 |
| 7.000000e+00 | -3.358571e+02 |
| -3.358571e+02 | 3.788436e+07 |
| 3.788436e+07 | -5.437256e+22 |

c)

| n | x(n) |
|---|---|
| 1.000000e+00 | 2.200000e+00 |
| 2.200000e+00 | 1.819764e+00 |
| 1.819764e+00 | 1.583475e+00 |
| 1.583475e+00 | 1.489461e+00 |

d)

| n | x(n) |
|---|---|
| 1.000000e+00 | 1.500000e+00 |
| 1.500000e+00 | 1.450521e+00 |
| 1.450521e+00 | 1.498750e+00 |
| 1.498750e+00 | 1.451904e+00 |

5)
Using Newton's method to find roots of given equations (accurate to within $10^{-5}$).

a) $1 \le x \le 2$

| n | x(n) | f(x(n)) |
|---|---|---|
| 0 | 1.500000e+00 | -1.023283e+00 |
| 1 | 1.956490e+00 | 5.797014e-01 |
| 2 | 1.841533e+00 | 5.034095e-02 |
| 3 | 1.829506e+00 | 5.021213e-04 |

b)　$1.3 \leq x \leq 2$

| n | x(n) | f(x(n)) |
|---|------|---------|
| 0 | 1.650000e+00 | 3.653009e-01 |
| 1 | 1.258582e+00 | -3.857896e-01 |
| 2 | 1.365403e+00 | -7.277412e-02 |
| 3 | 1.395989e+00 | -3.754240e-03 |
| 4 | 1.397743e+00 | -1.120807e-05 |

c)　$2 \leq x \leq 3$

| n | x(n) | f(x(n)) |
|---|------|---------|
| 0 | 2.500000e+00 | 1.168311e+00 |
| 1 | 2.372407e+00 | 1.513958e-02 |
| 2 | 2.370688e+00 | 7.986932e-06 |

$3 \leq x \leq 4$

| n | x(n) | f(x(n)) |
|---|------|---------|
| 0 | 3.500000e+00 | 3.027316e+00 |
| 1 | 3.783191e+00 | -1.033559e+00 |
| 2 | 3.724165e+00 | -3.350944e-02 |
| 3 | 3.722115e+00 | -4.441336e-05 |

d)　$1 \leq x \leq 2$

| n | x(n) | f(x(n)) |
|---|------|---------|
| 0 | 1.500000e+00 | -1.554651e-01 |
| 1 | 1.406721e+00 | 1.071863e-02 |
| 2 | 1.412370e+00 | 3.995300e-05 |

$e \leq x \leq 4$

| n | x(n) | f(x(n)) |
|---|------|---------|
| 0 | 3.000000e+00 | -9.861229e-02 |
| 1 | 3.059167e+00 | 3.692746e-03 |
| 2 | 3.057106e+00 | 4.476151e-06 |

e)　$0 \leq x \leq 1$

| n | x(n) | f(x(n)) |
|---|------|---------|

| 0 | 5.000000e-01 | 8.987213e-01 |
|---|---|---|
| 1 | 1.165089e+00 | -8.660907e-01 |
| 2 | 9.362269e-01 | -7.922199e-02 |
| 3 | 9.103967e-01 | -1.158090e-03 |

$3 \leq x \leq 5$

| n | x(n) | f(x(n)) |
|---|---|---|
| 0 | 4.000000e+00 | 6.598150e+00 |
| 1 | 3.784361e+00 | 1.043379e+00 |
| 2 | 3.735379e+00 | 4.474262e-02 |
| 3 | 3.733084e+00 | 9.450832e-05 |

f) $\quad 0 \leq x \leq 1$

| n | x(n) | f(x(n)) |
|---|---|---|
| 0 | 5.000000e-01 | -1.271051e-01 |
| 1 | 5.856438e-01 | -4.011277e-03 |
| 2 | 5.885294e-01 | -4.620254e-06 |

$3 \leq x \leq 4$

| n | x(n) | f(x(n)) |
|---|---|---|
| 0 | 3.500000e+00 | -3.809806e-01 |
| 1 | 3.079612e+00 | 1.596396e-02 |
| 2 | 3.096379e+00 | -1.434946e-05 |

$6 \leq x \leq 7$

| n | x(n) | f(x(n)) |
|---|---|---|
| 0 | 6.500000e+00 | 2.136165e-01 |
| 1 | 6.281599e+00 | -3.457208e-03 |

6)

For c = 1.500000

| n | x(n) | f(x(n)) |
|---|---|---|
| 0 | -1.000000e+00 | -3.467224e-01 |
| 1 | -3.624288e+00 | -1.717534e-01 |

|    |                  |                  |
|----|------------------|------------------|
| 2  | -7.521047e+00    | -6.084702e-02    |
| 3  | -1.113649e+01    | -1.874382e-02    |
| 4  | -1.348415e+01    | -3.227938e-03    |
| 5  | -1.407443e+01    | -1.345148e-04    |

For c = 1.700000

| n  | x(n)             | f(x(n))          |
|----|------------------|------------------|
| 0  | -1.000000e+00    | -5.467224e-01    |
| 1  | -5.138057e+00    | -3.155576e-01    |
| 2  | -1.544105e+01    | -1.938756e-01    |
| 3  | -6.186211e+01    | -1.453673e-01    |
| 4  | -6.183163e+02    | -1.308210e-01    |
| 5  | -5.063328e+04    | -1.292234e-01    |
| 6  | -3.313444e+08    | -1.292037e-01    |
| 7  | -1.418516e+16    | -1.292037e-01    |
| 8  | -2.599820e+31    | -1.292037e-01    |
| 9  | -8.732961e+61    | -1.292037e-01    |
| 10 | -9.853668e+122   | -1.292037e-01    |
| 11 | -1.254500e+245   | -1.292037e-01    |
| 12 | Inf              | Inf              |

For c = 1.200000

| n  | x(n)             | f(x(n))          |
|----|------------------|------------------|
| 0  | -1.000000e+00    | -4.672240e-02    |
| 1  | -1.353635e+00    | -7.167251e-03    |
| 2  | -1.429265e+00    | -2.167560e-04    |

7)
(a) Newton's method

[−1, 0]

| n  | x(n)             | f(x(n))          |
|----|------------------|------------------|
| 0  | -5.000000e-01    | 1.158750e+02     |
| 1  | 2.287736e-01     | -5.824239e+01    |
| 2  | -4.655754e-02    | 1.307989e+00     |

3    -4.065817e-02    -2.482166e-04

[0, 1]

| n | x(n) | f(x(n)) |
|---|---|---|
| 0 | 7.000000e-01 | -9.789300e+01 |
| 1 | 3.213170e+00 | 2.448768e+04 |
| 2 | 2.964466e+00 | 1.764678e+04 |
| 3 | 2.717169e+00 | 1.235507e+04 |
| 4 | 2.471905e+00 | 8.358849e+03 |
| 5 | 2.229638e+00 | 5.426652e+03 |
| 6 | 1.991879e+00 | 3.349347e+03 |
| 7 | 1.761072e+00 | 1.940287e+03 |
| 8 | 1.541264e+00 | 1.035549e+03 |
| 9 | 1.339268e+00 | 4.943452e+02 |
| 10 | 1.166404e+00 | 1.997524e+02 |
| 11 | 1.039684e+00 | 5.992828e+01 |
| 12 | 9.753179e-01 | 8.835149e+00 |
| 13 | 9.623327e-01 | -4.381786e-02 |
| 14 | 9.624016e-01 | 2.136299e-03 |
| 15 | 9.623983e-01 | -1.036215e-04 |
| 16 | 9.623984e-01 | 5.027435e-06 |
| 17 | 9.623984e-01 | -2.439146e-07 |

(b) Secant method

[−1, 0]

| n | x(n) | f(x(n)) |
|---|---|---|
| 0 | 0.000000e+00 | -9.000000e+00 |
| 1 | -1.000000e+00 | 4.330000e+02 |
| 2 | -2.036199e-02 | -4.496381e+00 |
| 3 | -3.043025e-02 | -2.266891e+00 |
| 4 | -4.066741e-02 | 1.800163e-03 |
| 5 | -4.065928e-02 | -7.404578e-07 |
| 7 | -4.065929e-02 | -2.451372e-13 |

[0, 1]

| n | x(n) | f(x(n)) |
|---|---|---|
| 0 | 2.000000e+00 | 3.409000e+03 |
| 1 | 1.000000e+00 | 2.700000e+01 |
| 2 | 9.920166e-01 | 2.093626e+01 |
| 3 | 9.644521e-01 | 1.374285e+00 |
| 4 | 9.625157e-01 | 7.815526e-02 |
| 5 | 9.623989e-01 | 3.215332e-04 |
| 7 | 9.623984e-01 | 7.574502e-08 |

8)

a)

| n | x(n) | f(x(n)) |
|---|---|---|
| 0 | 0.000000e+00 | -1.000000e+00 |
| 1 | 4.712389e+00 | -2.928932e-01 |
| 2 | 6.664324e+00 | -1.189418e+00 |
| 3 | 4.074695e+00 | -1.068751e-01 |
| 4 | 3.819031e+00 | -5.681902e-02 |
| 5 | 3.528826e+00 | -1.868518e-02 |
| 6 | 3.386628e+00 | -7.495891e-03 |
| 7 | 3.291367e+00 | -2.802738e-03 |
| 8 | 3.234478e+00 | -1.078260e-03 |
| 9 | 3.198907e+00 | -4.105830e-04 |
| 10 | 3.177032e+00 | -1.569931e-04 |
| 11 | 3.163491e+00 | -5.993890e-05 |

b)

| n | x(n) | f(x(n)) |
|---|---|---|
| 0 | -1.500000e+00 | 1.432455e+01 |
| 1 | 1.500000e+00 | -9.619731e+00 |
| 2 | 2.947355e-01 | 1.039194e+00 |
| 3 | 4.122430e-01 | 1.072901e+00 |
| 4 | -3.328043e+00 | 2.245051e-01 |
| 5 | -4.317809e+00 | 2.414760e+00 |

| 6 | -3.226591e+00 | 1.248959e-01 |
| 7 | -3.167072e+00 | 6.761201e-02 |
| 8 | -3.096823e+00 | 3.931112e-04 |
| 9 | -3.096412e+00 | 1.613459e-08 |

c)

| n | x(n) | f(x(n)) |
|---|------|---------|
| 0 | -1.000000e+00 | -1.500000e+01 |
| 1 | 1.145000e-01 | 1.187678e+00 |
| 2 | 3.272995e-02 | 1.085370e+00 |
| 3 | -8.347542e-01 | -1.044771e+01 |
| 4 | -4.890839e-02 | 8.244535e-01 |
| 5 | -1.063857e-01 | 5.438239e-01 |
| 6 | -2.177693e-01 | -2.327167e-01 |
| 7 | -1.843894e-01 | 3.256917e-02 |
| 8 | -1.884875e-01 | 1.510856e-03 |
| 9 | -1.886868e-01 | -1.076148e-05 |

| n | x(n) | f(x(n)) |
|---|------|---------|
| 0 | 1.145000e-01 | 1.187678e+00 |
| 1 | 1.000000e+00 | -7.000000e+00 |
| 2 | 2.429478e-01 | 1.034900e+00 |
| 3 | 3.404565e-01 | 6.699044e-01 |
| 4 | 5.194220e-01 | -5.391851e-01 |
| 5 | 4.396136e-01 | 8.467922e-02 |
| 6 | 4.504463e-01 | 7.913146e-03 |
| 7 | 4.515629e-01 | -1.422163e-04 |
| 8 | 4.515432e-01 | 2.302310e-07 |

d)

| n | x(n) | f(x(n)) |
|---|------|---------|
| 0 | -1.000000e+00 | 1.000000e-01 |
| 1 | 1.000000e+00 | 1.010400e+01 |
| 2 | -1.019992e+00 | 1.203841e-01 |
| 3 | -1.044349e+00 | 1.462310e-01 |

| 4 | -9.065455e-01 | 1.610415e-02 |
| 5 | -8.894912e-01 | 3.065170e-03 |
| 6 | -8.854821e-01 | 1.114226e-04 |
| 7 | -8.853309e-01 | 8.435256e-07 |

e)

| n | x(n) | f(x(n)) |
|---|------|---------|
| 0 | -1.000000e+00 | -1.000000e+01 |
| 1 | 1.000000e+00 | -4.000000e+00 |
| 2 | 2.333333e+00 | 1.391495e+02 |
| 3 | 1.037257e+00 | -3.870613e+00 |
| 4 | 1.072333e+00 | -3.713007e+00 |
| 5 | 1.898687e+00 | 3.462820e+01 |
| 6 | 1.152358e+00 | -3.192659e+00 |
| 7 | 1.215360e+00 | -2.584577e+00 |
| 8 | 1.483140e+00 | 3.213978e+00 |
| 9 | 1.334717e+00 | -7.740795e-01 |
| 10 | 1.363526e+00 | -1.739914e-01 |
| 11 | 1.371879e+00 | 1.394562e-02 |
| 12 | 1.371259e+00 | -2.235544e-04 |
| 13 | 1.371269e+00 | -2.804559e-07 |