DEPARTMENT OF MATHEMATICS
IIT GUWAHATI

MA 332 Scientific Computing Lab - I

1. Use the Bisection method to find the root of the equation $e^x = \sin x$ closest to 0.

2. Use the Bisection method to find solution accurate to within 10^{-3} for the following problems.

   (a) $2 + \cos(e^x - 2) - e^x = 0 \quad \text{for} \quad 0.5 \le x \le 1.5$

   (b) $x - 2^{-x} = 0 \quad \text{for} \quad 0 \le x \le 1$

   (c) $e^x - x^2 + 3x - 2 = 0 \quad \text{for} \quad 0 \le x \le 1$

   (d) $2x\cos(2x) - (x+1)^2 = 0 \quad \text{for} \quad -3 \le x \le -2 \quad \text{and} \quad -1 \le x \le 0$

   (e) $x\cos(x) - 2x^2 + 3x - 1 = 0 \quad \text{for} \quad 0.2 \le x \le 0.3 \quad \text{and} \quad 1.2 \le x \le 1.3$

3. Find an approximation to $\sqrt[3]{25}$ correct to with in 10^{-4} using the Bisection algorithm.

4. The following four methods are proposed to compute $\sqrt[5]{7}$. Rank them in order based on their apparent speed of convergence, assuming $x_0 = 1$.

   (i) $x_n = x_{n-1}\left(1 + \dfrac{7 - x_{n-1}^5}{x_{n-1}^2}\right)^3$, (ii) $x_n = x_{n-1} - \dfrac{x_{n-1}^5 - 7}{x_{n-1}^2}$, (iii) $x_n = x_{n-1} - \dfrac{x_{n-1}^5 - 7}{5x_{n-1}^4}$,

   (iv) $x_n = x_{n-1} - \dfrac{x_{n-1}^5 - 7}{12}$. Based upon the first four iterations, which one do you think gives the best approximation to the solution?

5. Use Newton's method to find solution accurate to within 10^{-5} for the following problems.

   (a) $e^x + 2^{-x} + 2\cos(x) - 6 = 0 \quad \text{for} \quad 1 \le x \le 2$

   (b) $\ln(x-1) + \cos(x-1) = 0 \quad \text{for} \quad 1.3 \le x \le 2$

   (c) $2x\cos(2x) - (x-2)^2 = 0 \quad \text{for} \quad 2 \le x \le 3 \quad \text{and} \quad 3 \le x \le 4$

   (d) $(x-2)^2 - \ln x = 0 \quad \text{for} \quad 1 \le x \le 2 \quad \text{and} \quad e \le x \le 4$

   (e) $e^x - 3x^2 = 0 \quad \text{for} \quad 0 \le x \le 1 \quad \text{and} \quad 3 \le x \le 5$

   (f) $\sin x - e^{-x} = 0 \quad \text{for} \quad 0 \le x \le 1,\ 3 \le x \le 4 \quad \text{and} \quad 6 \le x \le 7$

6. Use Newton's method to find the negative zero of the function $f(x) = e^x - 1.5 - \tan^{-1} x$. Investigate the sensitivity of the root to perturbations in the constant term.

7. The fourth degree polynomial $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$, has two real zeros, one in $[-1, 0]$ and the other in $[0, 1]$, Attempt to approximate these zeros to within 10^{-6} using the

   (a) Newton's method

(b) Secant method

Use the midpoints of each interval as the initial approximation in (a) and the endpoints of each interval as the initial approximations in (b).

8. Use secant method to find the approximations to within 10^{-4} to all real zeros of the following functions.

   (a) $\sin(x/2) - 1$

   (b) $e^x - \tan x$

   (c) $x^3 - 12x^2 + 3x + 1$

   (d) $x^3 + 4.001x^2 + 4.002x + 1.101$

   (e) $x^6 - x^4 + 2x^3 - 3x^2 + x - 4$
