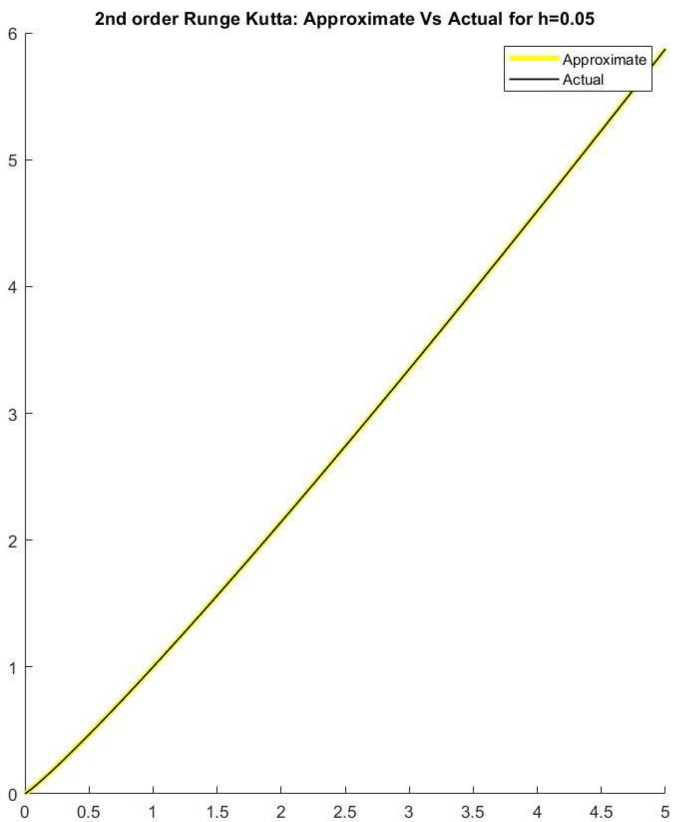
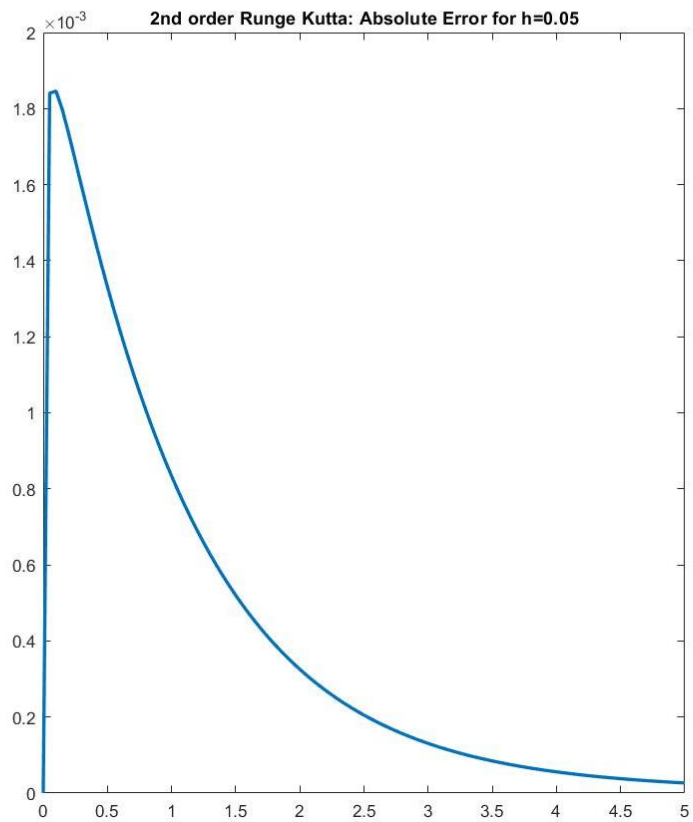
LAB 08

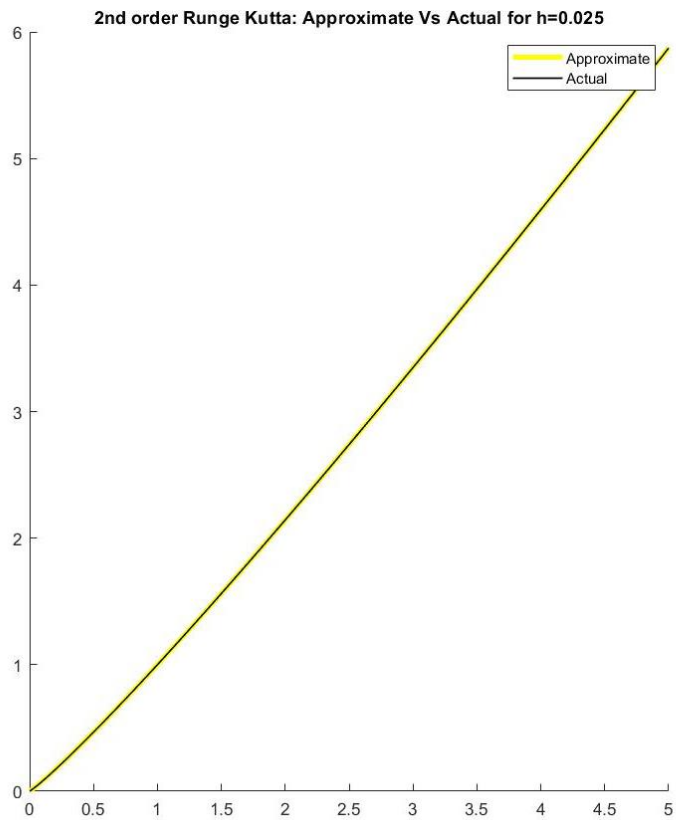
1)

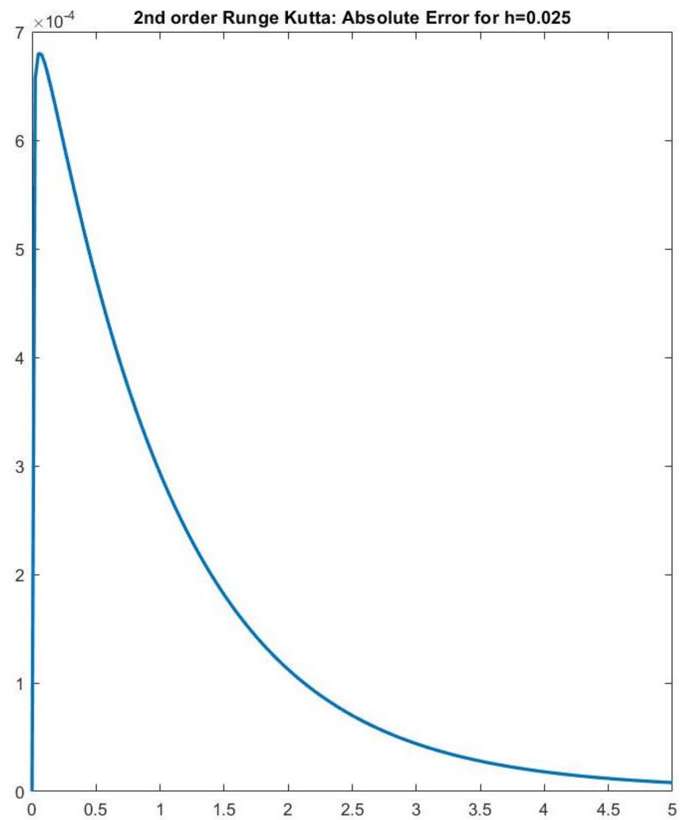




2nd order Runge Kutta: table for h =0.05

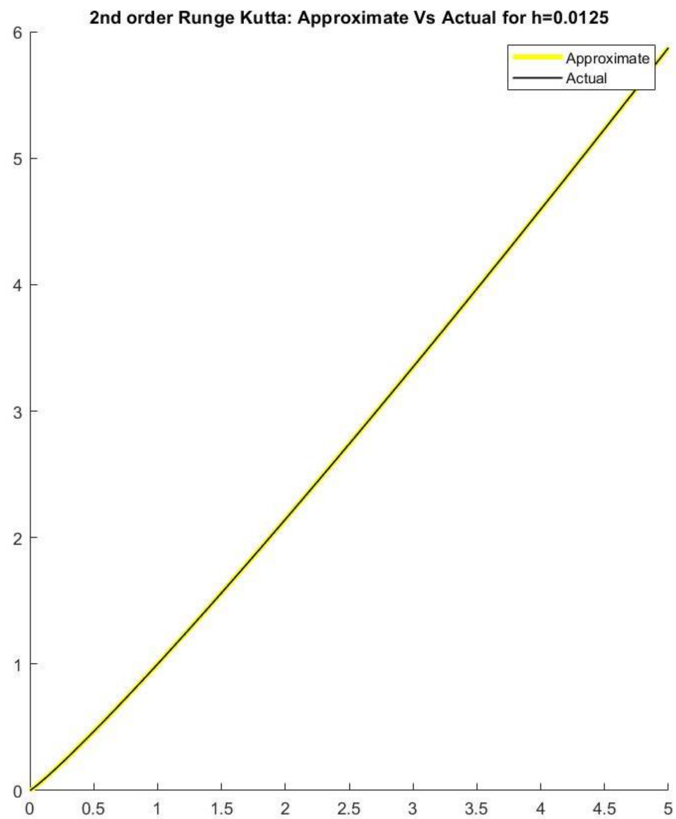
t	Approximate Y	Exact Y	Error
1	1.00083	1.00000	0.00083
2	2.14387	2.14355	0.00033
3	3.34850	3.34837	0.00013
4	4.59485	4.59479	0.00006
5	5.87312	5.87309	0.00003

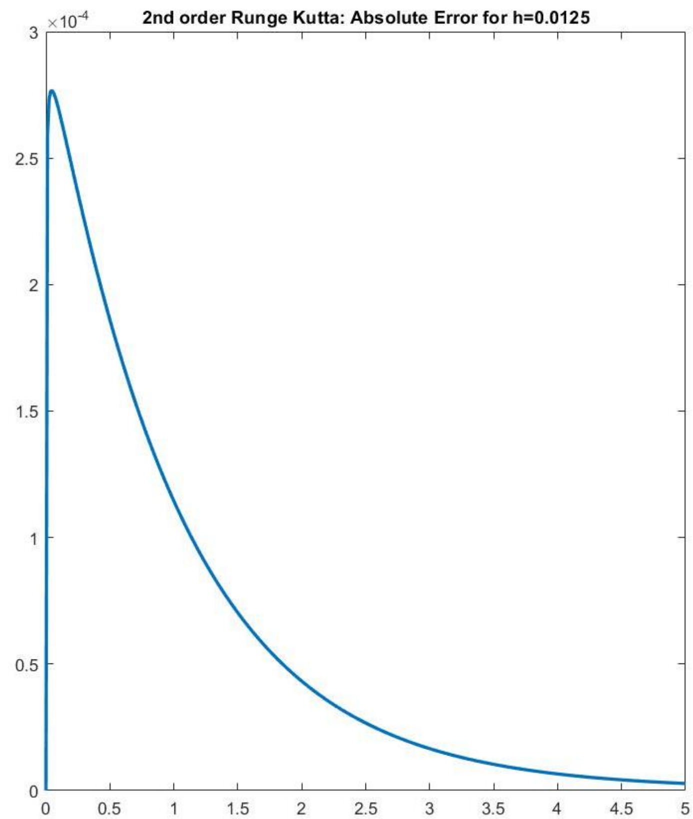




2nd order Runge Kutta: table for h =0.025

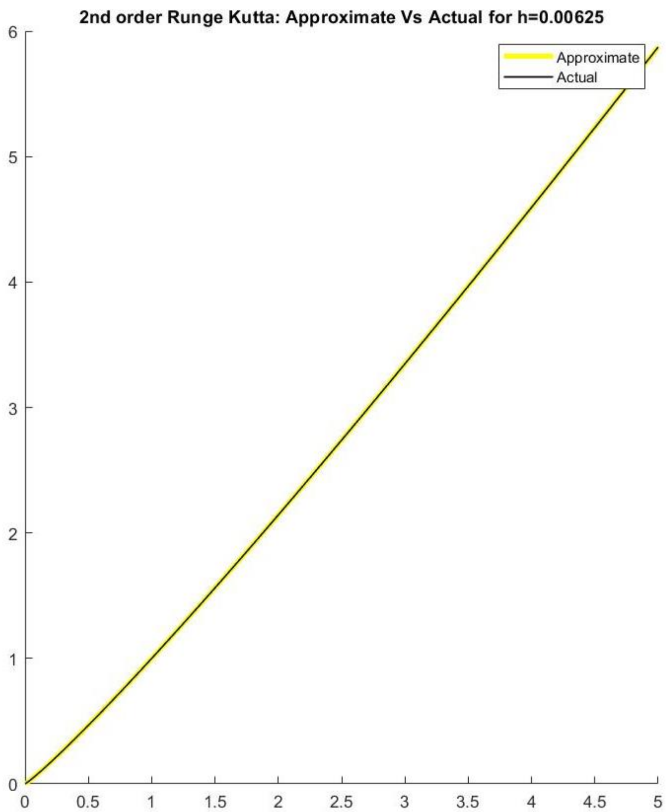
t	Approximate Y	Exact Y	Error
1	1.00029	1.00000	0.00029
2	2.14366	2.14355	0.00011
3	3.34841	3.34837	0.00004
4	4.59481	4.59479	0.00002
5	5.87310	5.87309	0.00001

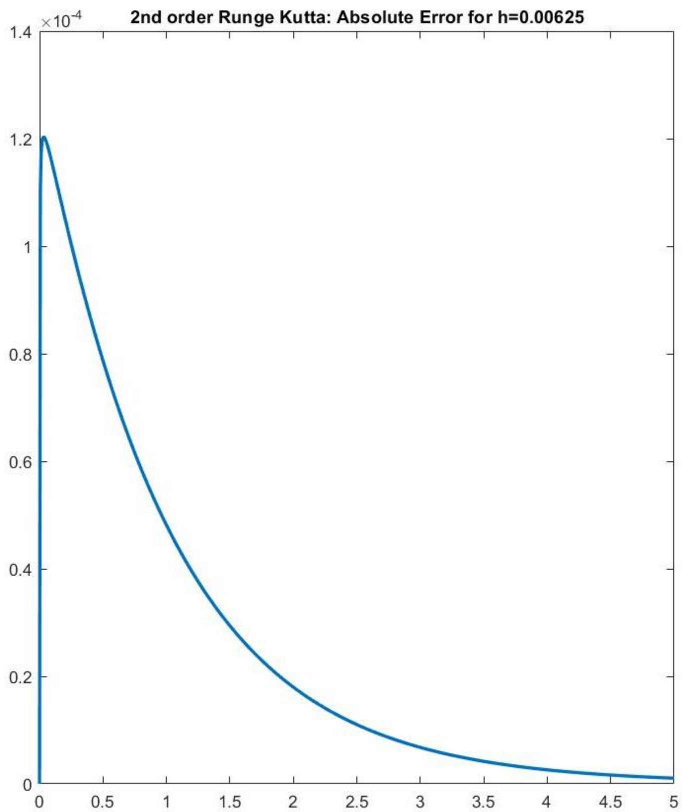




2nd order Runge Kutta: table for h =0.0125

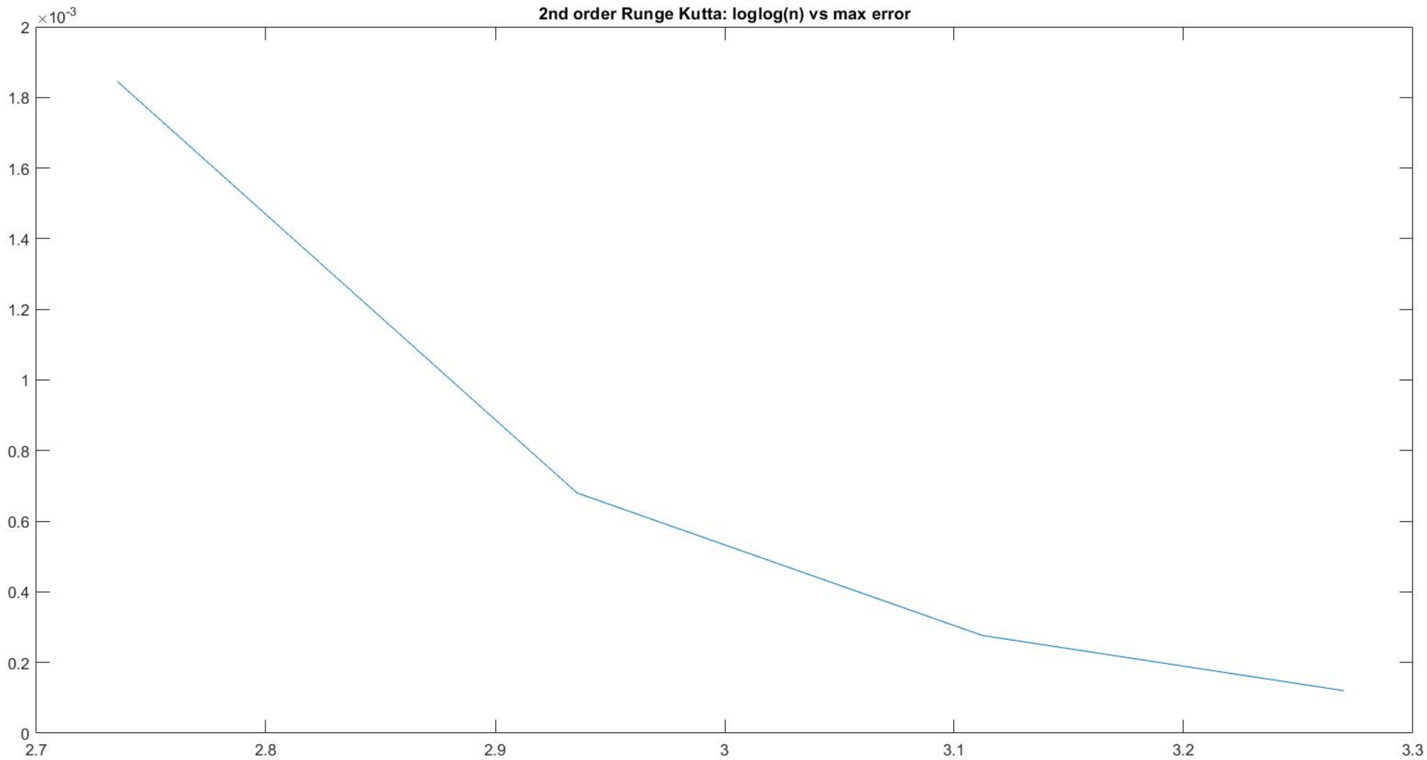
t	Approximate Y	Exact Y	Error
1	1.00011	1.00000	0.00011
2	2.14359	2.14355	0.00004
3	3.34839	3.34837	0.00002
4	4.59480	4.59479	0.00001
5	5.87310	5.87309	0.00000





2nd order Runge Kutta: table for h =0.00625

t	Approximate Y	Exact Y	Error
1	1.00005	1.00000	0.00005
2	2.14356	2.14355	0.00002
3	3.34838	3.34837	0.00001
4	4.59480	4.59479	0.00000
5	5.87310	5.87309	0.00000



2nd order Runge Kutta: Max error table

h	max error	log2(En/E2n)
5.000000e-02	0.00185	1.44083
2.500000e-02	0.00068	1.29734
1.250000e-02	0.00028	1.20144
6.250000e-03	0.00012	0.00000

How does this compare with the theoretical value of convergence of $O(h^2)$?

Numerically the max error i.e. error bound corresponding to step size is decreasing with decrease in step size but the results are not exactly quadratic as the theoretical argument suggests but are still better than linear. In case of h=0.05 and h=0.025 the step size is getting halved and the corresponding error bound is getting decreased by almost 1/3rd. And according to theory it should get reduced to 1/4th. One reason for this deviation is that max error does not exactly accounts for the error bound of the approximation and is a loose way of making an estimate. Another reason is as following:

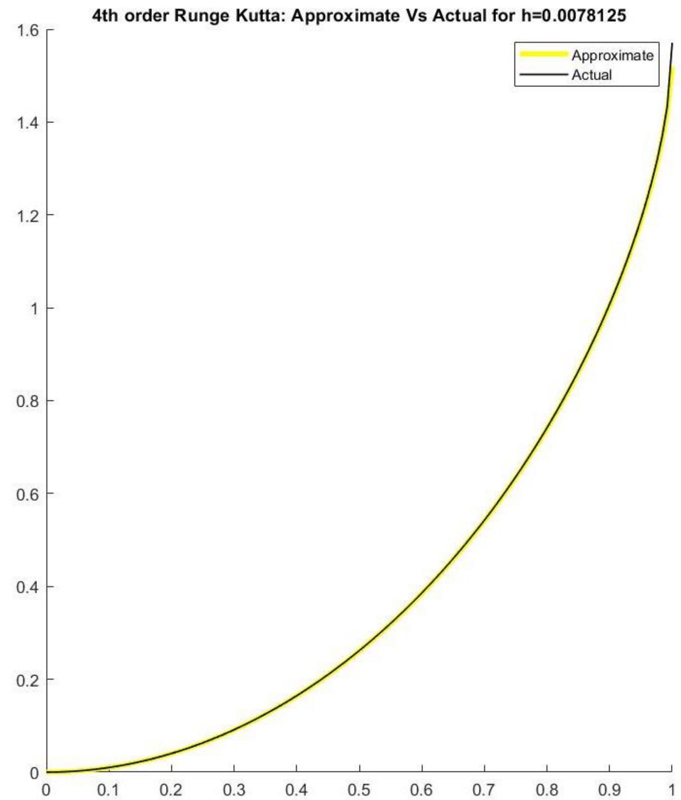
In general, Range Kutta methods of order 2 are of form:
$y(t+h) = y + w_1hf + w_2hf(t+\alpha h, y+\beta hf) + O(h^3)$
with constraints: $w_1 + w_2 = 1$, $w_2\alpha = ½$, $w_2\beta = ½$

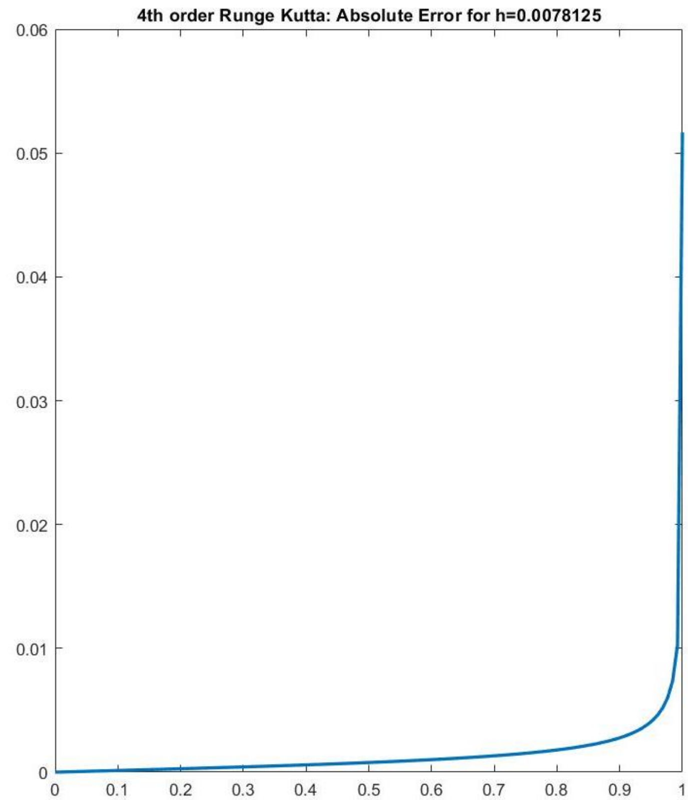
Convergence superiority is determined by appropriately choosing these weights for example:

i) One solution is (Huen's Method) : $w_1 = w_2 = ½$, $\alpha = \beta = 1$
ii) Another solution is (Modified Euler Method) : $w_1 = 0$, $w_2 = 1$, $\alpha = \beta = ½$ (This is the one I have used for this problem)

In general, Heun's method is superior to Modified Euler method even though theoretically both are of order 2.
The accuracy of the Heun's Method improves close to quadratically with the decrement of step size.

2)

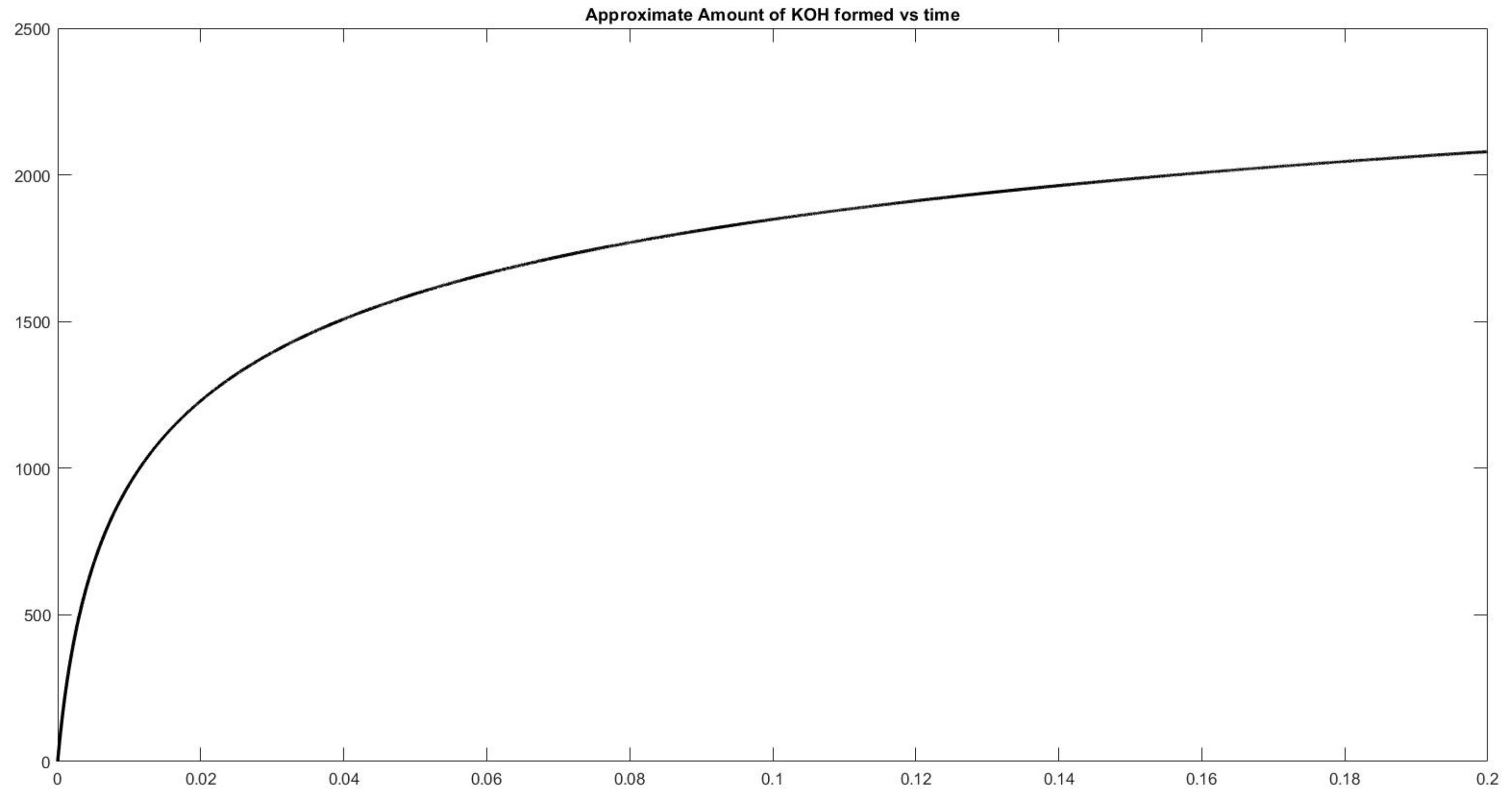




4th order Runge Kutta Table for h =0.0078125

t	Approximate Y	Exact Y	Error
0	0	0	0
1.02E-01	0.0102	0.01033	0.00014
2.03E-01	0.04127	0.04155	0.00028
3.05E-01	0.0939	0.09433	0.00043
4.06E-01	0.16935	0.16995	0.0006
5.08E-01	0.26969	0.27048	0.00079
6.09E-01	0.39827	0.39931	0.00103
7.11E-01	0.56087	0.56223	0.00136
8.13E-01	0.76872	0.7706	0.00187
9.14E-01	1.05106	1.05409	0.00302
1	1.51913	1.5708	0.05167

3)

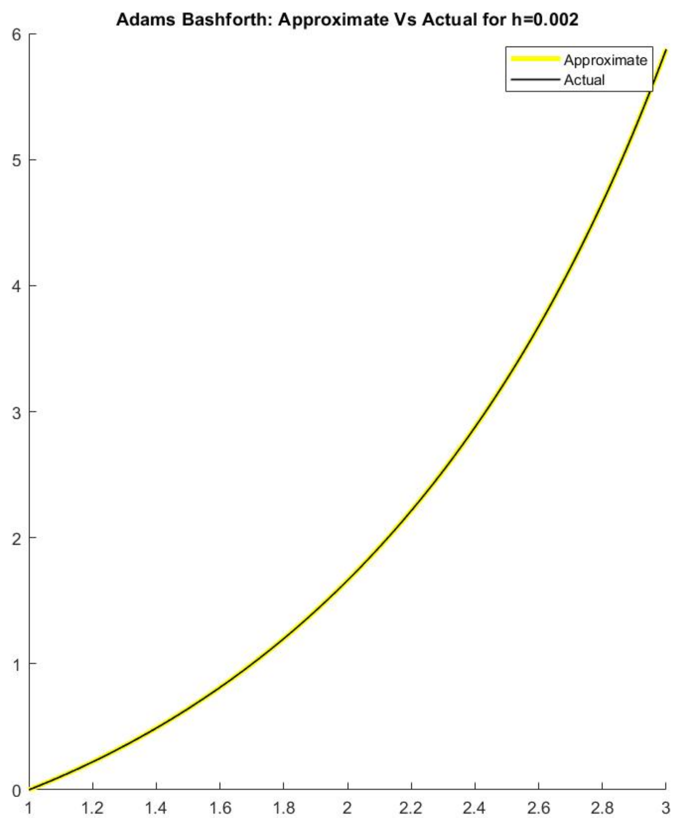


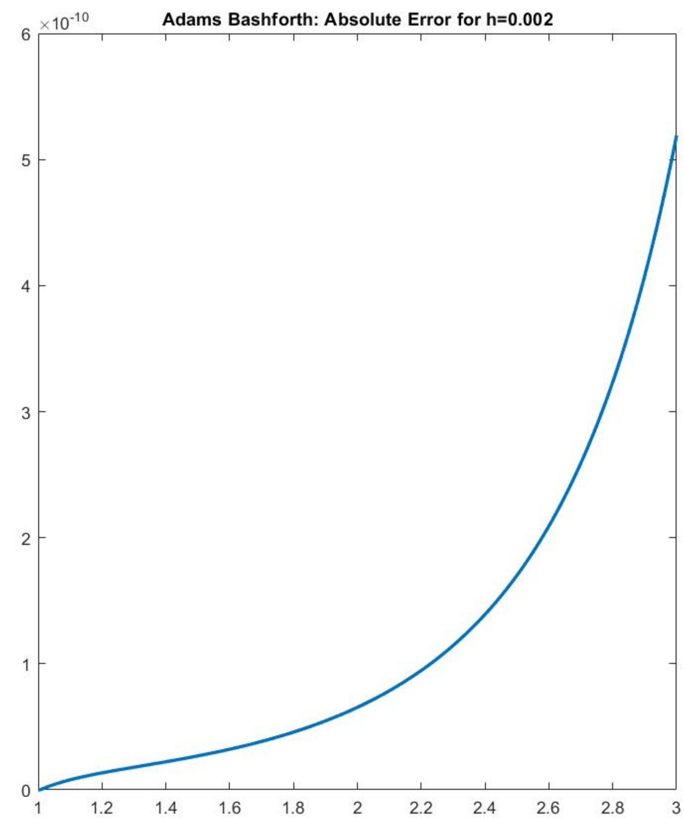
Approximate amount of KOH formed at 0.2 seconds: **2.0794e+03**

4)

Adams Bashforth: for problem (a)

i) using exact starting values



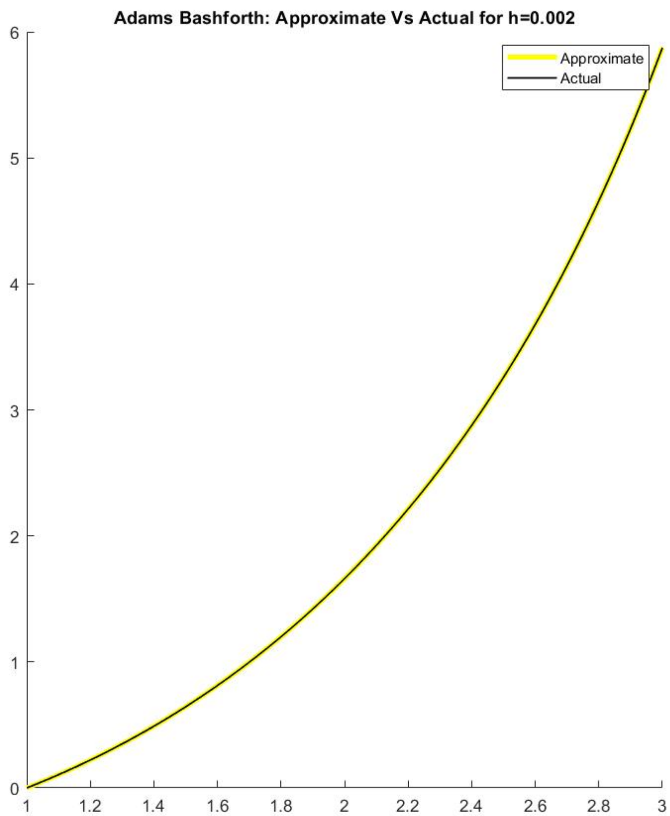


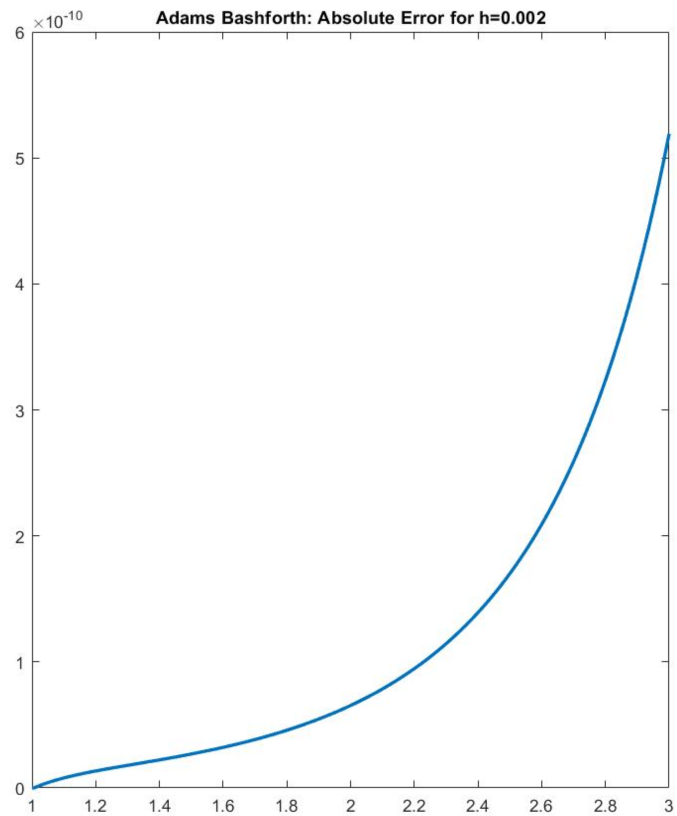
Adams Bashforth: table for problem (a) using exact starting values

t	Approximate Y	Exact Y	Error
1	0.00E+00	0	0.00E+00
1.10E+00	1.05E-01	0.10516	7.91E-12
1.20E+00	2.21E-01	0.22124	1.35E-11
1.30E+00	3.49E-01	0.34912	1.79E-11
1.40E+00	4.90E-01	0.48968	2.23E-11
1.50E+00	6.44E-01	0.64388	2.69E-11
1.60E+00	8.13E-01	0.81275	3.22E-11
1.70E+00	9.97E-01	0.99749	3.84E-11
1.80E+00	1.20E+00	1.19944	4.58E-11
1.90E+00	1.42E+00	1.42012	5.47E-11
2	1.66E+00	1.66128	6.54E-11
2.10E+00	1.92E+00	1.92496	7.86E-11

2.20E+00	2.21E+00	2.2135	9.46E-11
2.30E+00	2.53E+00	2.52963	1.15E-10
2.40E+00	2.88E+00	2.87655	1.39E-10
2.50E+00	3.26E+00	3.25802	1.70E-10
2.60E+00	3.68E+00	3.67848	2.09E-10
2.70E+00	4.14E+00	4.14324	2.59E-10
2.80E+00	4.66E+00	4.65867	3.23E-10
2.90E+00	5.23E+00	5.23248	4.08E-10
3	5.87E+00	5.8741	5.19E-10

ii) using starting values obtained from the Runge-Kutta method of order four





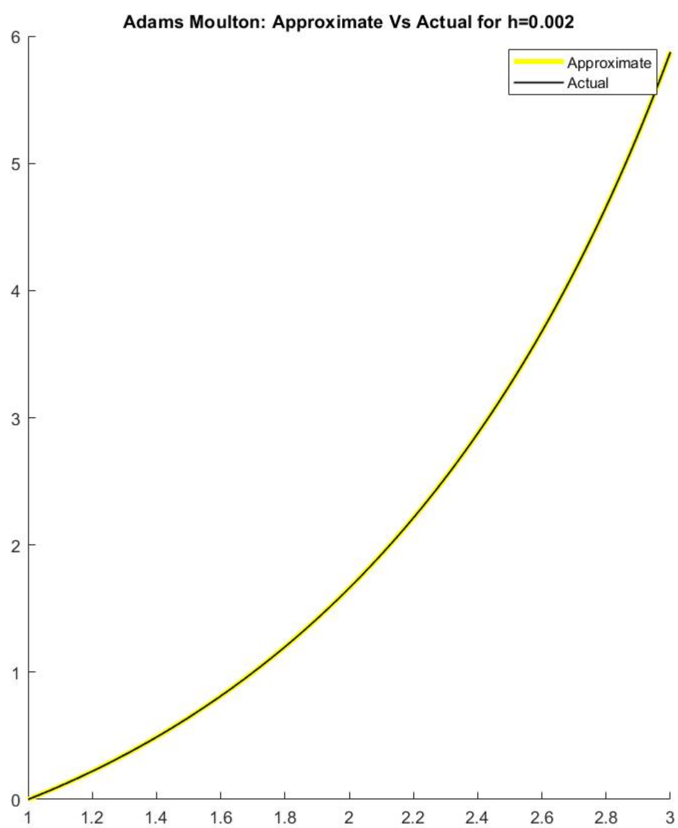
Adams Bashforth: table for problem (a) using starting values of 4th order Runge-Kutta method

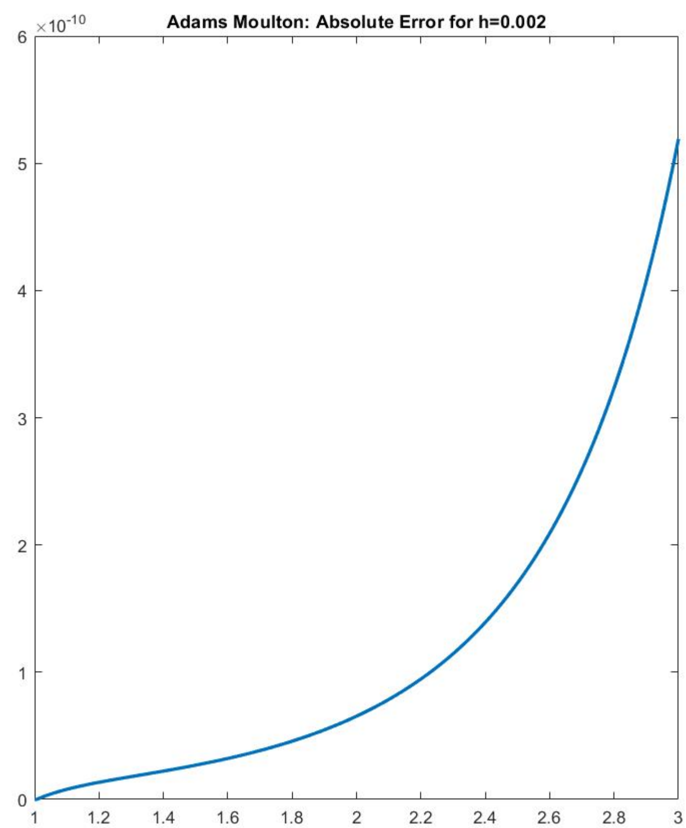
t	Approximate Y	Exact Y	Error
1	0.00E+00	0	0.00E+00
1.10E+00	1.05E-01	0.10516	7.91E-12
1.20E+00	2.21E-01	0.22124	1.34E-11
1.30E+00	3.49E-01	0.34912	1.79E-11
1.40E+00	4.90E-01	0.48968	2.23E-11
1.50E+00	6.44E-01	0.64388	2.69E-11
1.60E+00	8.13E-01	0.81275	3.22E-11

1.70E+00	9.97E-01	0.99749	3.84E-11
1.80E+00	1.20E+00	1.19944	4.58E-11
1.90E+00	1.42E+00	1.42012	5.47E-11
2	1.66E+00	1.66128	6.54E-11
2.10E+00	1.92E+00	1.92496	7.86E-11
2.20E+00	2.21E+00	2.2135	9.46E-11
2.30E+00	2.53E+00	2.52963	1.14E-10
2.40E+00	2.88E+00	2.87655	1.39E-10
2.50E+00	3.26E+00	3.25802	1.70E-10
2.60E+00	3.68E+00	3.67848	2.09E-10
2.70E+00	4.14E+00	4.14324	2.59E-10
2.80E+00	4.66E+00	4.65867	3.23E-10
2.90E+00	5.23E+00	5.23248	4.08E-10
3	5.87E+00	5.8741	5.19E-10

Adams Moulton: for problem (a)

i) using exact starting values

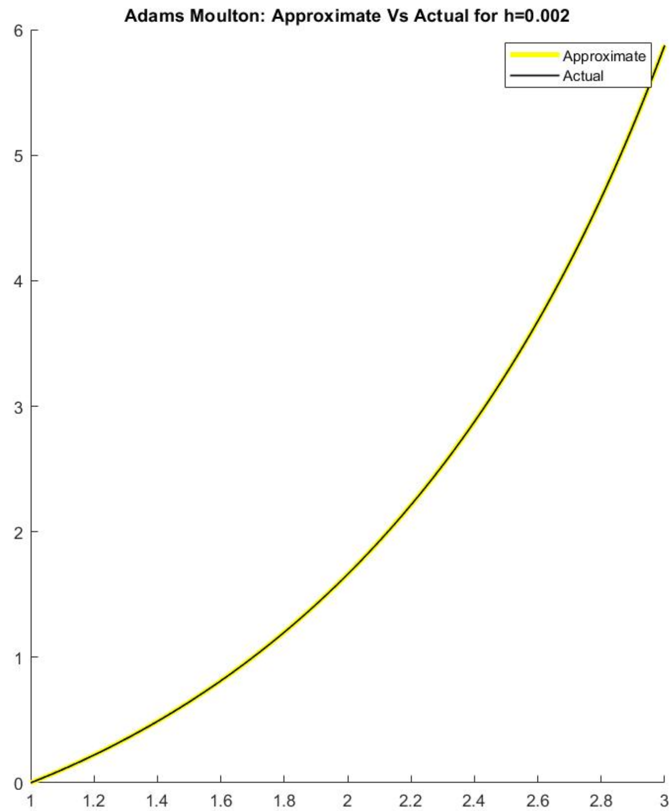


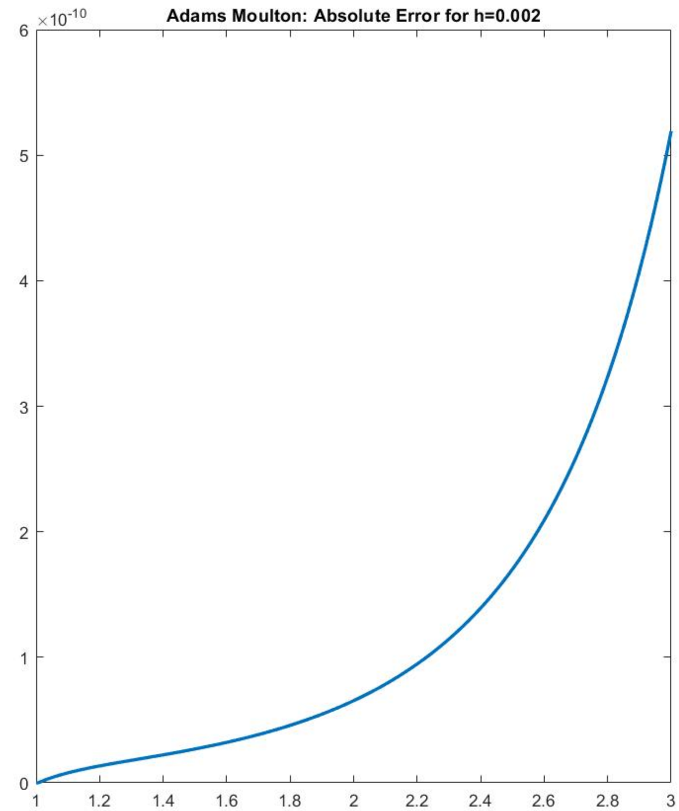


Adams Moulton: table for problem (a) using exact starting values

t	Approximate Y	Exact Y	Error
1	0.00E+00	0	0.00E+00
1.10E+00	1.05E-01	0.10516	7.91E-12
1.20E+00	2.21E-01	0.22124	1.35E-11
1.30E+00	3.49E-01	0.34912	1.79E-11
1.40E+00	4.90E-01	0.48968	2.23E-11
1.50E+00	6.44E-01	0.64388	2.69E-11
1.60E+00	8.13E-01	0.81275	3.22E-11
1.70E+00	9.97E-01	0.99749	3.84E-11
1.80E+00	1.20E+00	1.19944	4.58E-11
1.90E+00	1.42E+00	1.42012	5.47E-11
2	1.66E+00	1.66128	6.54E-11
2.10E+00	1.92E+00	1.92496	7.86E-11
2.20E+00	2.21E+00	2.2135	9.46E-11
2.30E+00	2.53E+00	2.52963	1.15E-10
2.40E+00	2.88E+00	2.87655	1.39E-10
2.50E+00	3.26E+00	3.25802	1.70E-10
2.60E+00	3.68E+00	3.67848	2.09E-10
2.70E+00	4.14E+00	4.14324	2.59E-10
2.80E+00	4.66E+00	4.65867	3.23E-10
2.90E+00	5.23E+00	5.23248	4.08E-10
3	5.87E+00	5.8741	5.19E-10

ii) using starting values obtained from the Runge-Kutta method of order four





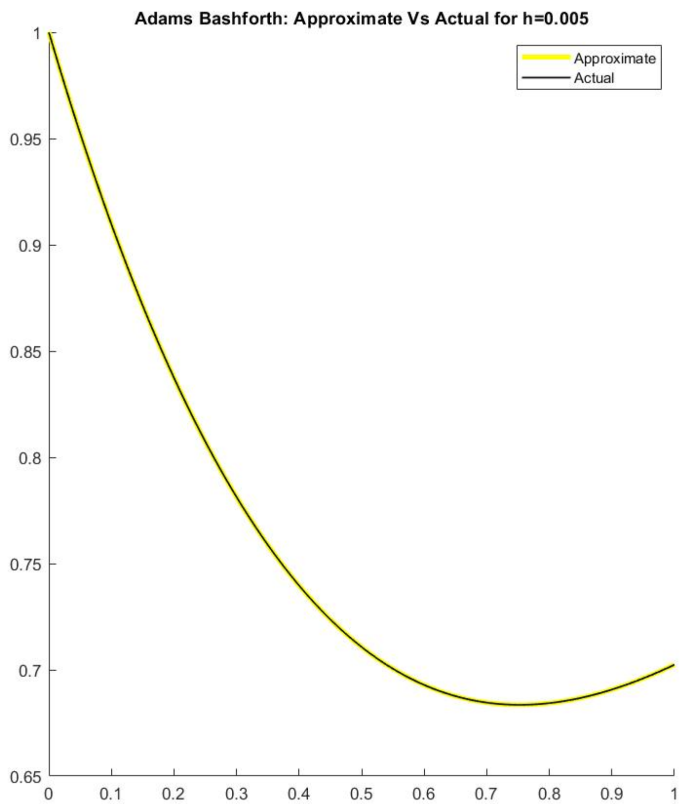
Adams Moulton: table for problem (a) using starting values of 4th order Runge-Kutta method

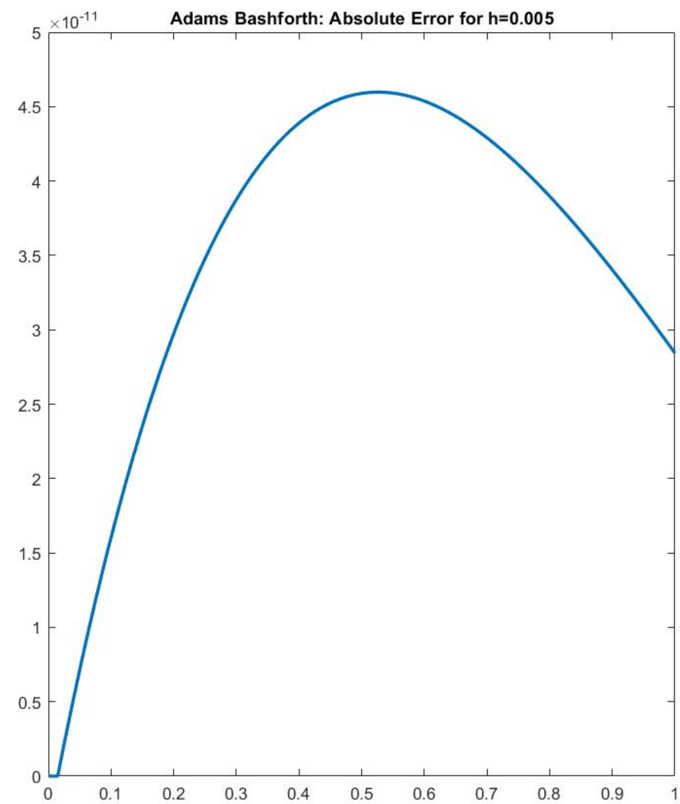
t	Approximate Y	Exact Y	Error
1	0.00E+00	0	0.00E+00
1.10E+00	1.05E-01	0.10516	7.91E-12
1.20E+00	2.21E-01	0.22124	1.34E-11
1.30E+00	3.49E-01	0.34912	1.79E-11
1.40E+00	4.90E-01	0.48968	2.23E-11
1.50E+00	6.44E-01	0.64388	2.69E-11
1.60E+00	8.13E-01	0.81275	3.22E-11
1.70E+00	9.97E-01	0.99749	3.84E-11
1.80E+00	1.20E+00	1.19944	4.58E-11
1.90E+00	1.42E+00	1.42012	5.47E-11
2	1.66E+00	1.66128	6.54E-11
2.10E+00	1.92E+00	1.92496	7.86E-11
2.20E+00	2.21E+00	2.2135	9.46E-11
2.30E+00	2.53E+00	2.52963	1.14E-10
2.40E+00	2.88E+00	2.87655	1.39E-10
2.50E+00	3.26E+00	3.25802	1.70E-10
2.60E+00	3.68E+00	3.67848	2.09E-10

2.70E+00	4.14E+00	4.14324	2.59E-10
2.80E+00	4.66E+00	4.65867	3.23E-10
2.90E+00	5.23E+00	5.23248	4.08E-10
3	5.87E+00	5.8741	5.19E-10

Adams Bashforth: for problem (b)

i) using exact starting values

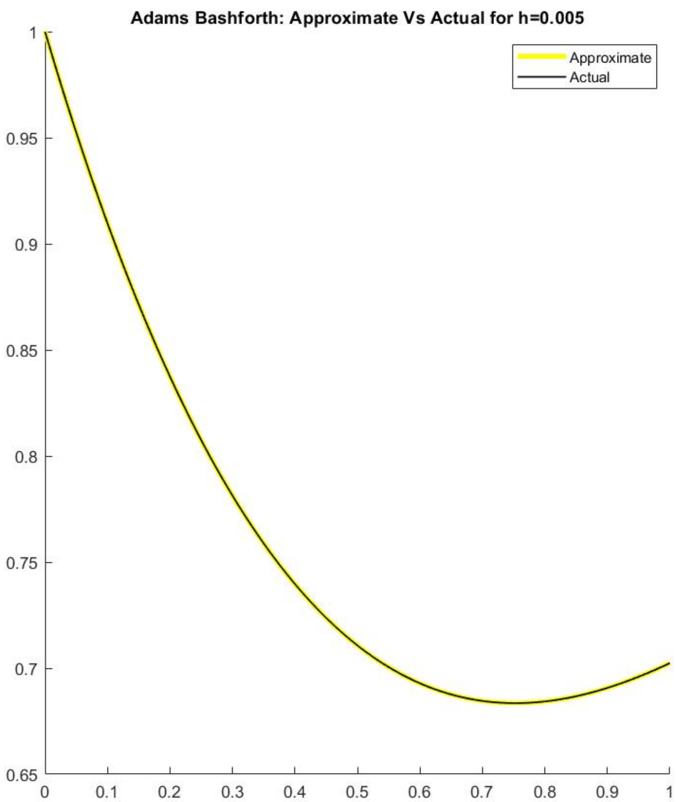


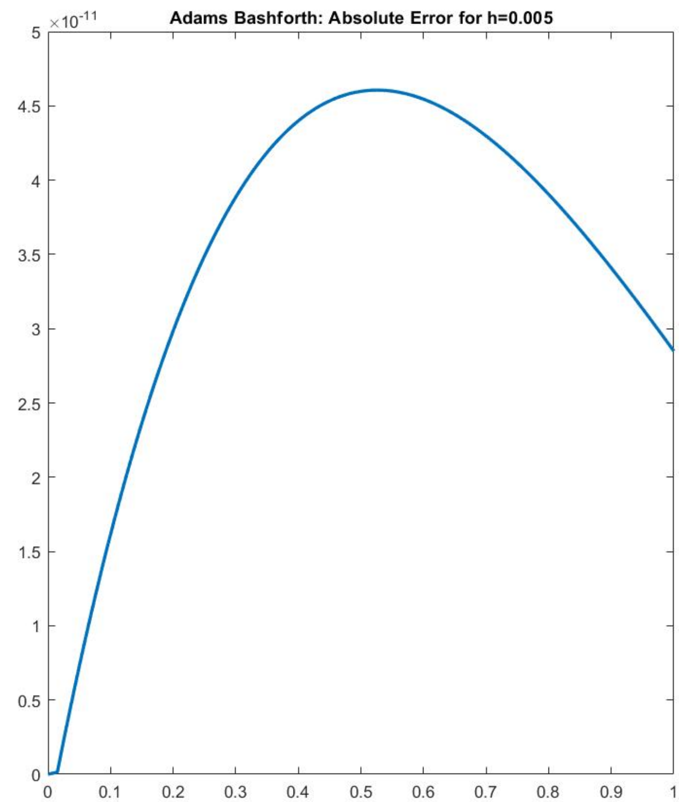


Adams Bashforth: table for problem (b) using exact starting values

t	Approximate Y	Exact Y	Error
0	1.00E+00	1	0.00E+00
2.50E-01	8.07E-01	0.80745	3.48E-11
5.00E-01	7.11E-01	0.71072	4.59E-11
7.50E-01	6.84E-01	0.68352	4.11E-11
1	7.02E-01	0.7024	2.84E-11

ii) using starting values obtained from the Runge-Kutta method of order four



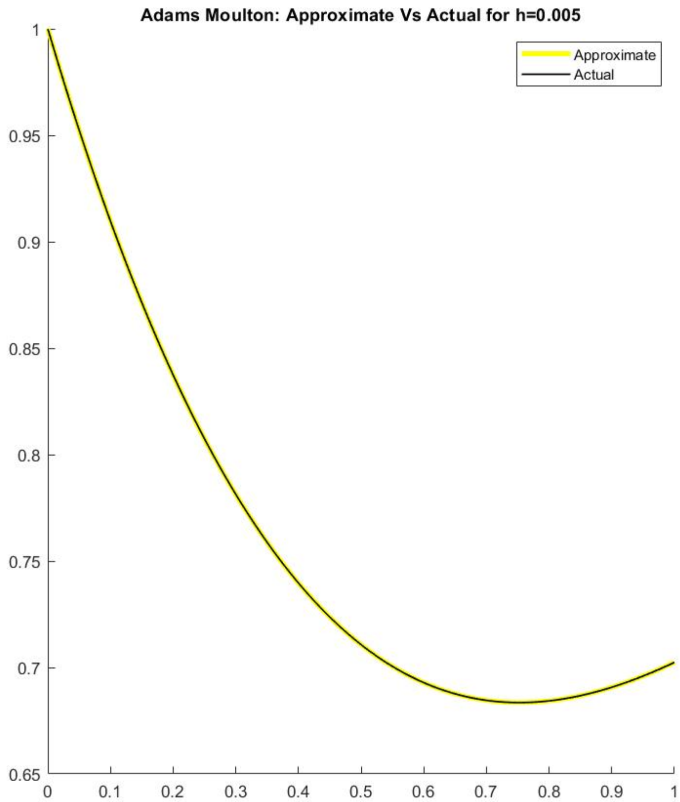


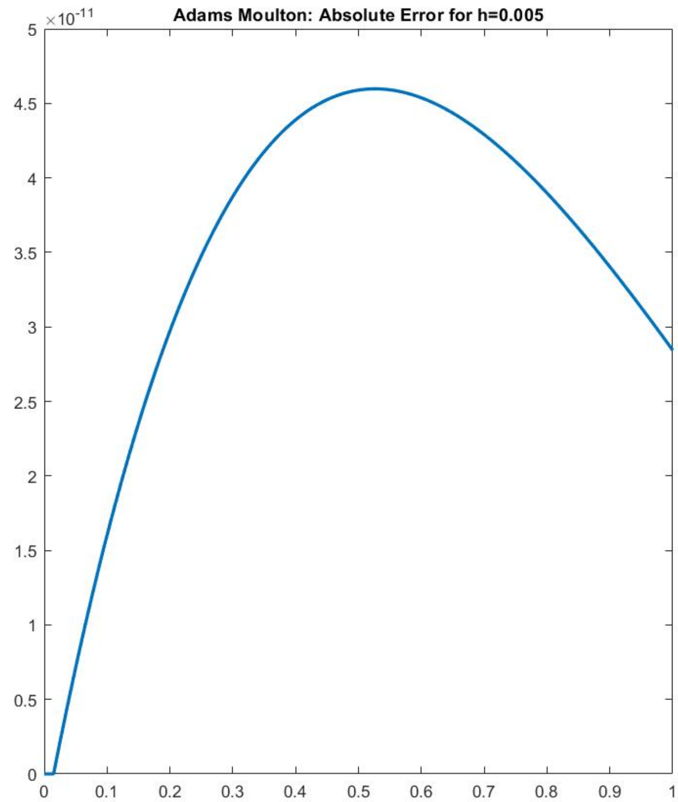
Adams Bashforth: table for problem (b) using starting values of 4th order Runge-Kutta method

t	Approximate Y	Exact Y	Error
0	1.00E+00	1	0.00E+00
2.50E-01	8.07E-01	0.80745	3.49E-11
5.00E-01	7.11E-01	0.71072	4.60E-11
7.50E-01	6.84E-01	0.68352	4.12E-11
1	7.02E-01	0.7024	2.85E-11

Adams Moulton: for problem (b)

i) using exact starting values

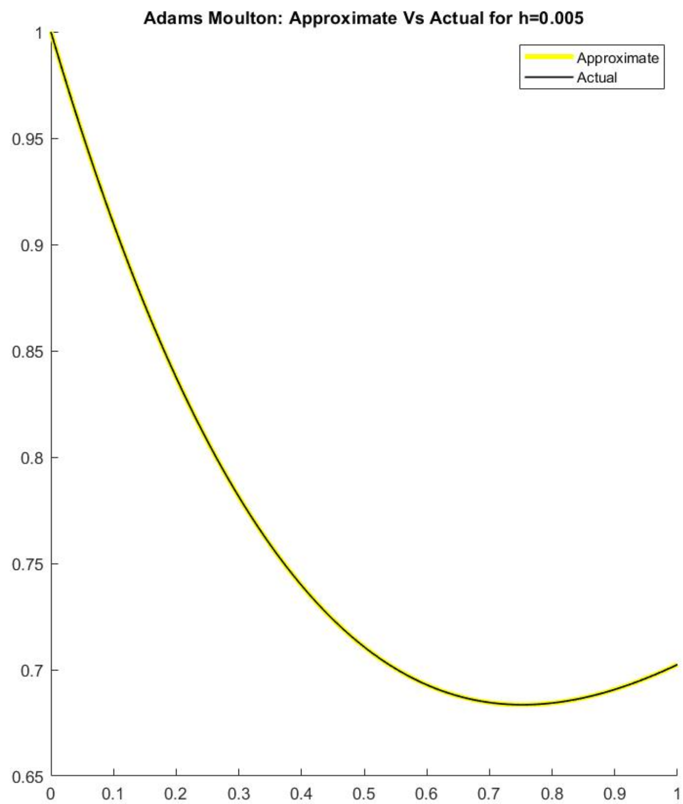


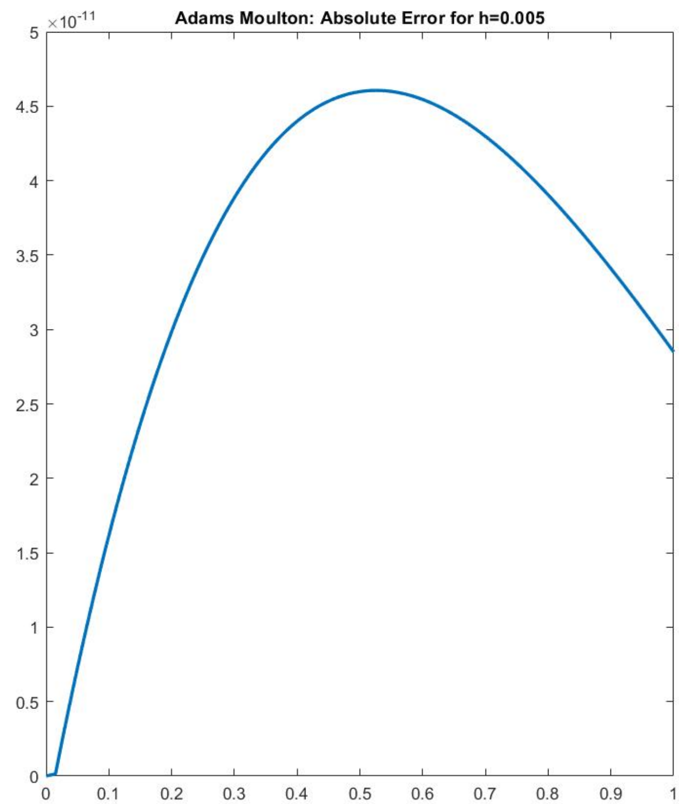


Adams Moulton: table for problem (b) using exact starting values

t	Approximate Y	Exact Y	Error
0	1.00E+00	1	0.00E+00
2.50E-01	8.07E-01	0.80745	3.48E-11
5.00E-01	7.11E-01	0.71072	4.59E-11
7.50E-01	6.84E-01	0.68352	4.11E-11
1	7.02E-01	0.7024	2.84E-11

ii) using starting values obtained from the Runge-Kutta method of order four



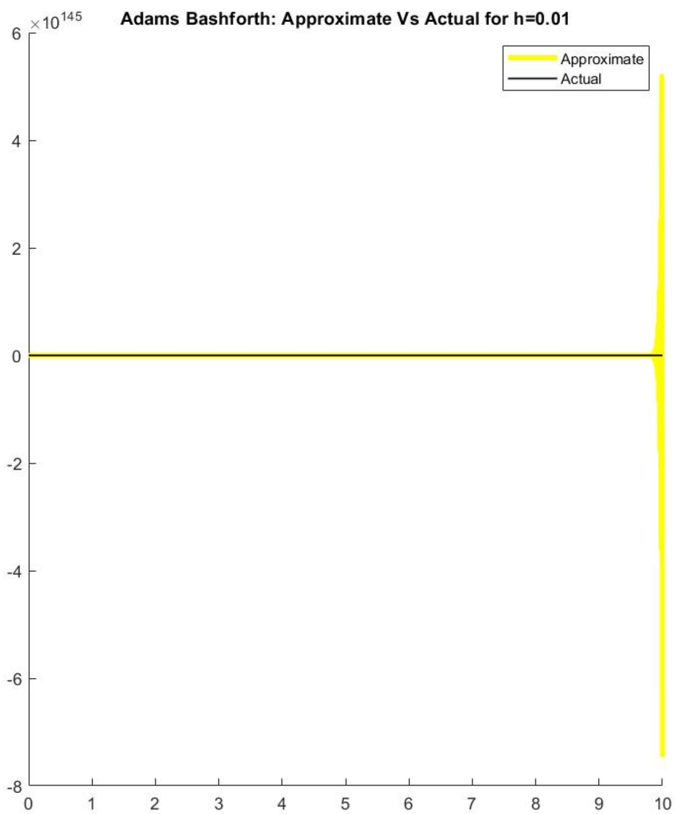


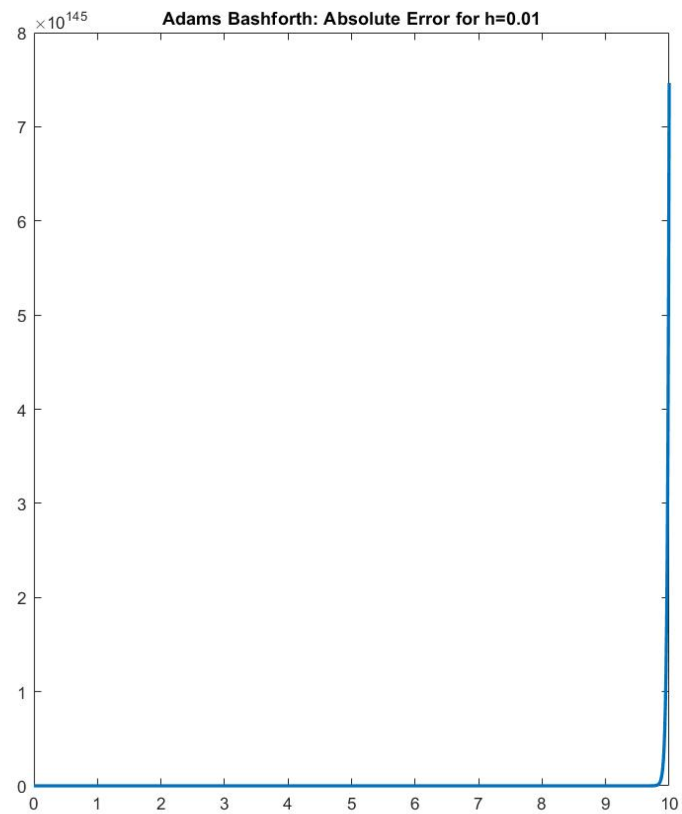
Adams Moulton: table for problem (b) using starting values of 4th order Runge-Kutta method

t	Approximate Y	Exact Y	Error
0	1.00E+00	1	0.00E+00
2.50E-01	8.07E-01	0.80745	3.49E-11
5.00E-01	7.11E-01	0.71072	4.60E-11
7.50E-01	6.84E-01	0.68352	4.12E-11
1	7.02E-01	0.7024	2.85E-11

5)

Adams Bashforth for h =0.01

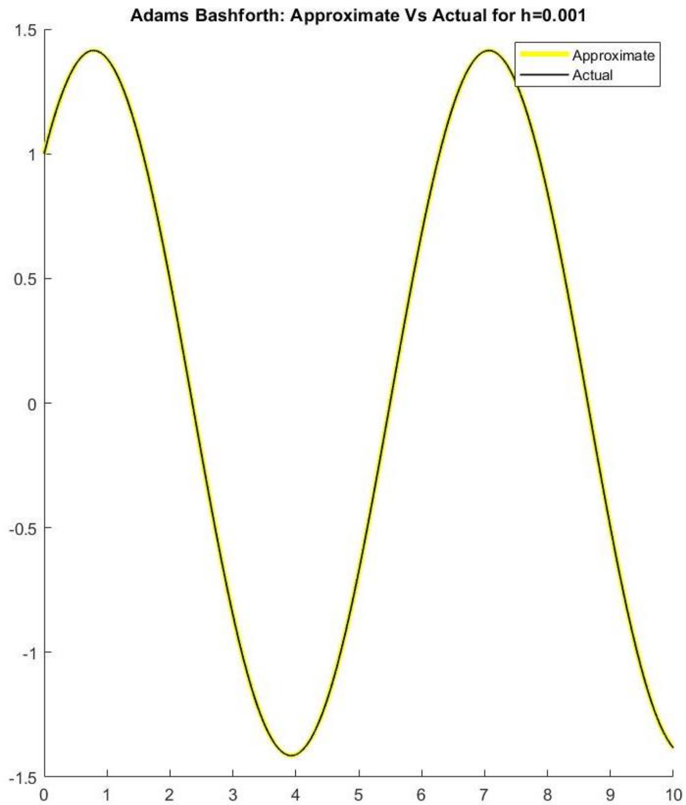


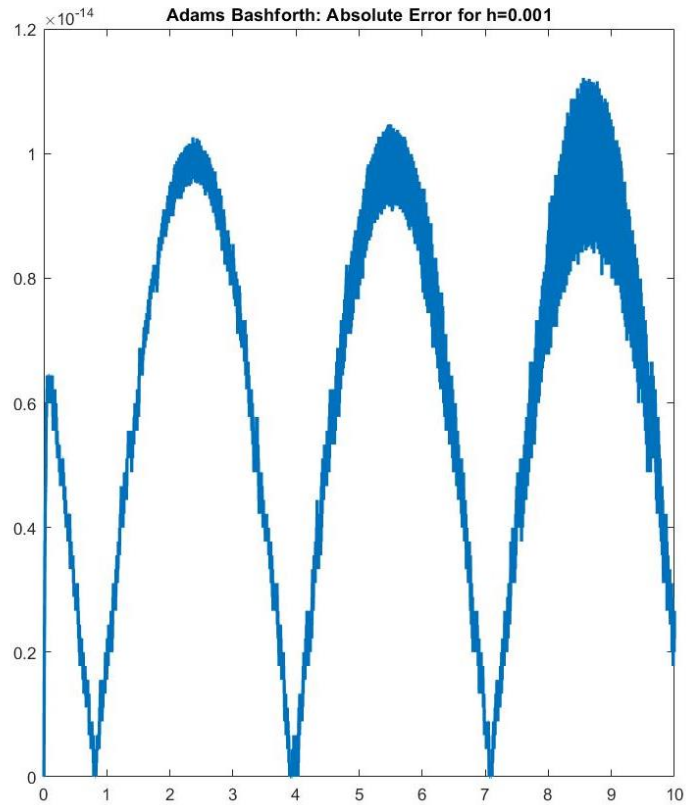


Adams Bashforth: table for h =0.01

t	Approximate Y	Exact Y	Error
1	-1.20059e+04	1.38177	1.200732213e+04
2	-6.82792e+19	0.49315	6.827915449e+19
3	-3.88267e+35	-0.84887	3.882666667e+35
4	-2.20786e+51	-1.41045	2.207862789e+51
5	-1.25549e+67	-0.67526	1.255492298e+67
6	-7.13931e+82	0.68075	7.139306472e+82
7	-4.05974e+98	1.41089	4.059737920e+98
8	-2.30855e+114	0.84386	2.308553645e+114
9	-1.31275e+130	-0.49901	1.312749748e+130
10	-7.46490e+145	-1.38309	7.464898662e+145

Adams Bashforth for h =0.001

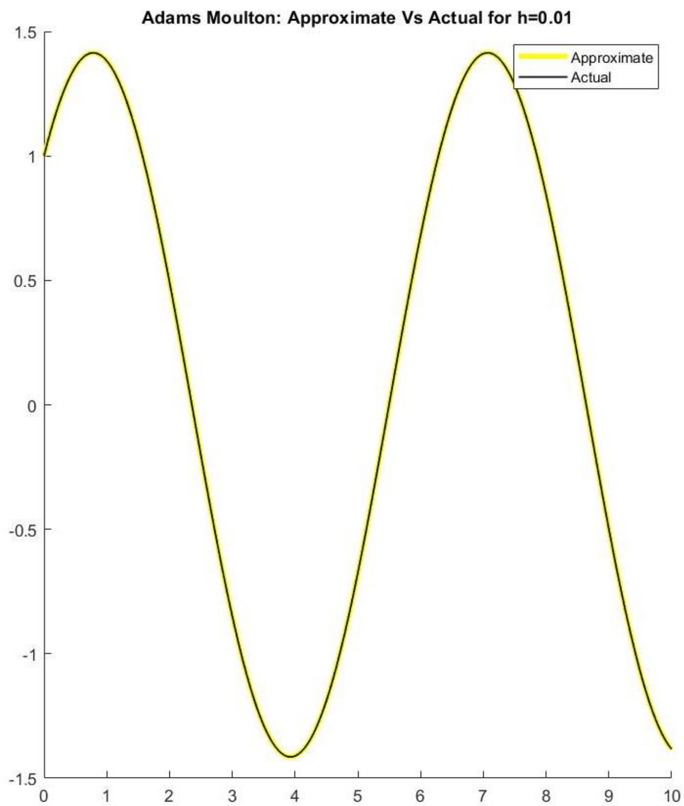


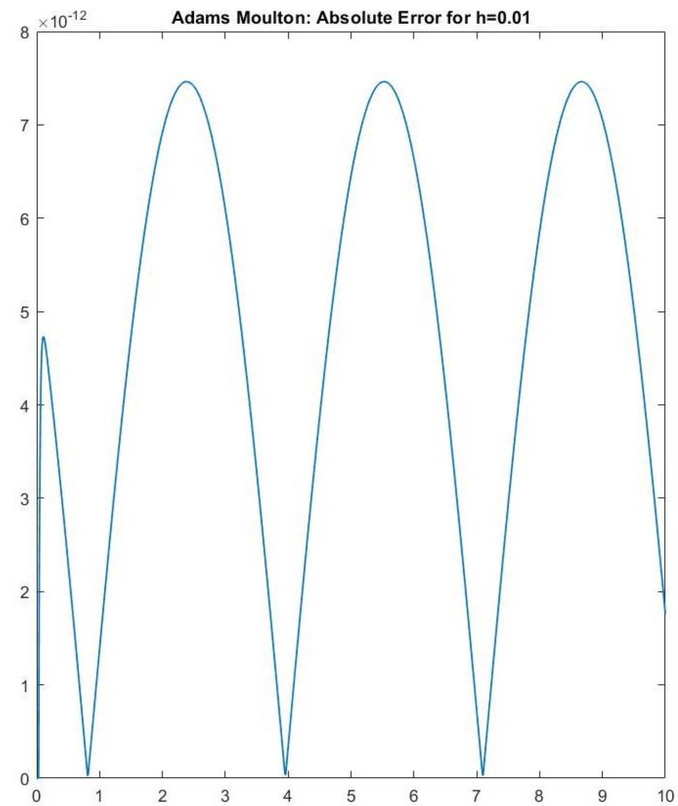


Adams Bashforth: table for h =0.001

t	Approximate Y	Exact Y	Error
1	1.38177e+00	1.38177	1.776356839e-15
2	4.93151e-01	0.49315	9.214851104e-15
3	-8.48872e-01	-0.84887	8.104628080e-15
4	-1.41045e+00	-1.41045	2.220446049e-16
5	-6.75262e-01	-0.67526	8.548717290e-15
6	6.80755e-01	0.68075	8.881784197e-15
7	1.41089e+00	1.41089	8.881784197e-16
8	8.43858e-01	0.84386	7.882583475e-15
9	-4.99012e-01	-0.49901	9.159339953e-15
10	-1.38309e+00	-1.38309	2.220446049e-15

Adams Moulton for h =0.01

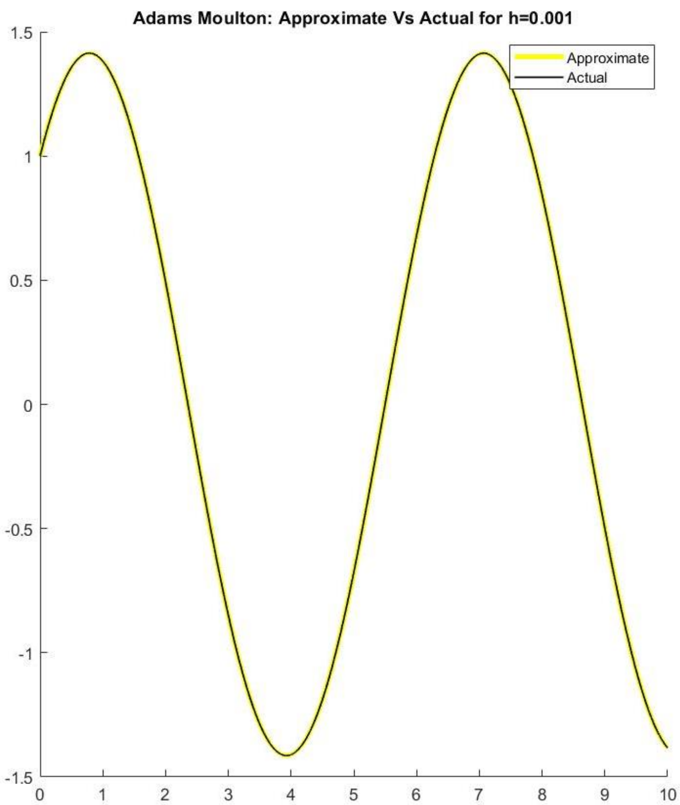


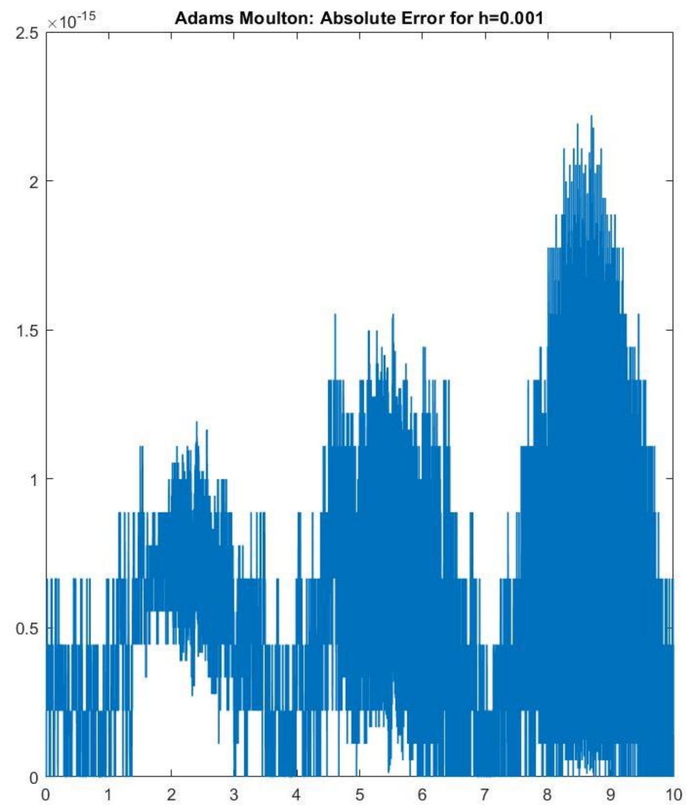


Adams Moulton: table for h =0.01

t	Approximate Y	Exact Y	Error
1	1.38177e+00	1.38177	1.385558335e-12
2	4.93151e-01	0.49315	6.918743356e-12
3	-8.48872e-01	-0.84887	6.091016580e-12
4	-1.41045e+00	-1.41045	3.368416657e-13
5	-6.75262e-01	-0.67526	6.455058710e-12
6	6.80755e-01	0.68075	6.638245509e-12
7	1.41089e+00	1.41089	7.185363415e-13
8	8.43858e-01	0.84386	5.862088592e-12
9	-4.99012e-01	-0.49901	7.053413409e-12
10	-1.38309e+00	-1.38309	1.759703494e-12

Adams Moulton for h =0.001





Adams Moulton: table for h =0.001

t	Approximate Y	Exact Y	Error
1	1.38177e+00	1.38177	4.440892099e-16
2	4.93151e-01	0.49315	7.771561172e-16
3	-8.48872e-01	-0.84887	4.440892099e-16
4	-1.41045e+00	-1.41045	4.440892099e-16
5	-6.75262e-01	-0.67526	6.661338148e-16
6	6.80755e-01	0.68075	4.440892099e-16
7	1.41089e+00	1.41089	2.220446049e-16
8	8.43858e-01	0.84386	7.771561172e-16
9	-4.99012e-01	-0.49901	7.771561172e-16
10	-1.38309e+00	-1.38309	0.000000000e+00