7/12/23 = T.O.C (theory of computation

MTE

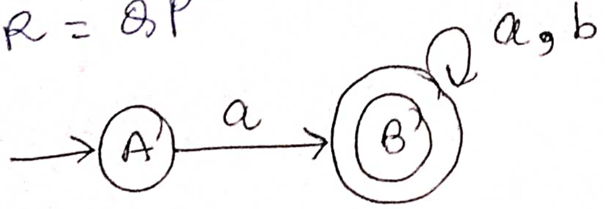
Arden's theorem

→ It state that if P and Q are two regular expression over Σ and if P is not ε, then the following equation in R is given by $R = Q + RP$ has unique solution

$$\boxed{R = QP^*}$$

$$R = Q + RP$$
$$R = Q + (Q + RP)P$$
$$R = Q + QP + RP^2$$
$$R = Q + QP + (Q + RP)P^2$$
$$R = Q + QP + QP^2 + RP^3 ----$$
$$R = Q(P^0 + P^1 + ---)$$
$$R = QP^*$$

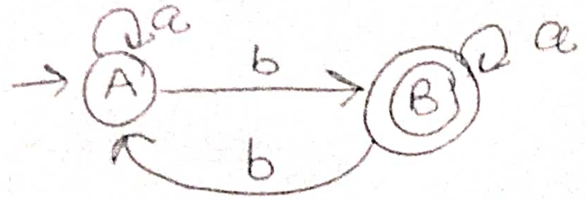
eg:-



$$A = \epsilon,$$

$$B = Aa + Ba + Bb$$
$$B = Aa + B(a+b)$$
$$B = a + B(a+b)$$
$$R = Q + RP$$
$$R = a(a+b)^*$$

Q)



$R = Q + RP$

$R = QP^*$

~~$R = Aa + Bb$~~

$A = \epsilon + Aa + Bb$ — (i)

$B = Ab + Ba$ — (ii)

$B - Ba = Ab$

$\frac{B(1-a)}{b}$

$B = (\epsilon + Aa + Bb)b + Ba$

$B = \epsilon b + Aab + Bb^2 + Ba$

$B = b + Aab + Bb^2 + Ba$

$B = \underbrace{(b + Aab)}_{Q} + \underset{R}{B}\underset{P}{(a+b^2)}$

$R =$

$R = (b + Aab)(a+b^2)^*$

$B = (b + Aab)(a+b^2)^*$

$B = \left(b + B\frac{(1-a)}{\cancel{b}} \times a\cancel{b}\right)(a+b^2)^*$

$B = (b + B(1-a)a)(a+b^2)^*$

$B = (b + Ba - Ba^2)(a+b^2)^*$

$A = \epsilon + Aa + Bb$ — (1)

$B = Ab + Ba$ — (2)

$R = Q + RP$

$R = QP^*$

$B = Aba^*$ — (3)

Put (3) in (1)

$A = \epsilon + Aa + Aba^*b$

$A = \epsilon + A(a + b^2a^*)$

$R = Q + RP$

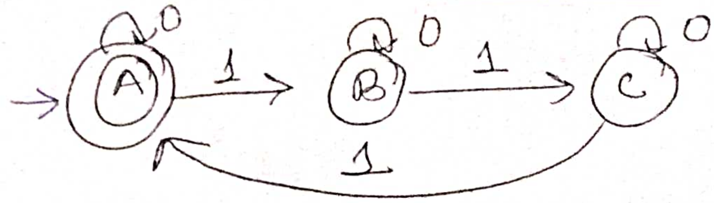
$R = QP^*$

$A = \epsilon(a + b^2a^*)^*$ — (4)

Put (4) in (3)

$B = (a + b^2a^*)^* ba^*$

Q)



$$A = \epsilon + A\cdot 0 + C\cdot 1 \quad \text{—} \ (i)$$

$$B = A\cdot 1 + B\cdot 0 \quad \text{—} \ (ii)$$

$$\underbrace{C}_{R} = \underbrace{C\cdot 0}_{RP} + \underbrace{B\cdot 1}_{Q} \quad \text{—} \ (iii)$$

$$\boxed{R = Q + RP}$$

$$R = QP^*$$

$$C = B\cdot 1 0^* \quad \text{—} \ (iv)$$

$$B = \underline{A\cdot 1} + B\cdot 0$$

$$R = Q + RP$$

$$R = QP^*$$

$$B = A\cdot 1 0^* \quad \text{—} \ (v)$$

$$C = A\cdot 1 0^* 1 0^* \quad \text{—} \ (vi)$$

Put (vi) in (I)

$$A = \epsilon + A\cdot 0 + A 1 0^* 1 0^* \cdot 1$$

$$A = \epsilon + A(0 + 1 0^* 1 0^* 1)$$

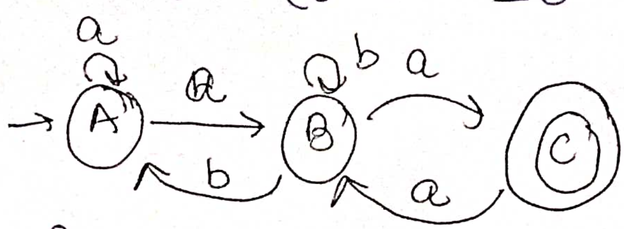
$$R = Q + RP$$

$$R = QP^*$$

$$A = \epsilon(0 + 1 0^* 1 0^* 1)^*$$

$$A = (0 + 1 0^* 1 0^* 1)^*$$

Q)



convert FA to R.E

for state (A) :—

$$A = \epsilon + Aa + Bb \quad \text{—} \ (1)$$

for state (B) :—

$$B = Aa + Bb + Ca \quad \text{—} \ (2)$$

for state C :—

$$C = Ba \quad \text{—} \ (3)$$

from eq. 2

$B = Aa + Bb + Ca$

$B = a(A+C) + Bb$

$\cancel{R = Q + RP}$ (Arden's theorem)

$\cancel{R = QP^*}$

$B = a(A+C)b^*$ —— (5)

$B = Aa + Bb + Baa$

$B = Aa + B(b + a \cdot a)$

$R = Q + RP$

$R = QP^*$

$B = Aa(b+a^2)^*$

from eq (1)

$A = \epsilon + Aa + Bb$

$A = \epsilon + Aa + Aa(b+a^2)^* b$

$A = \epsilon + A(a + a(b+a^2)^* b)$

$R = Q + RP$

$R = QP^*$

$A = \epsilon(a + (a(b+a^2)^* b)$

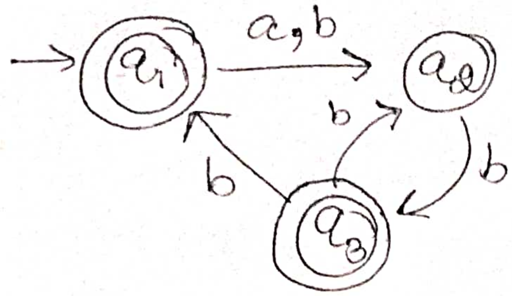
$B = [a + (a(b+a^2)^* b] a(b+a^2)^*$

$C = Ba$

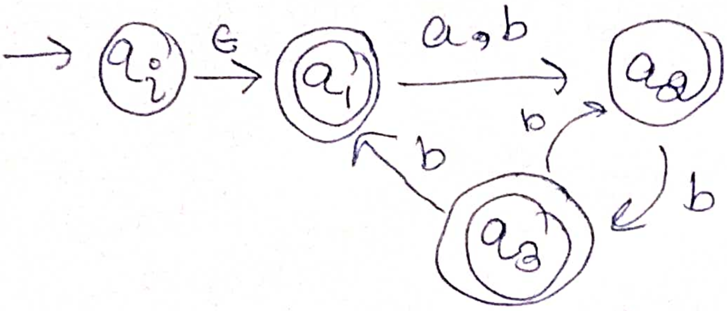
$C = (a + (a(b+a^2)^* b) a(b+a \cdot a)^* a$

★ FA → R.E (state elimination)

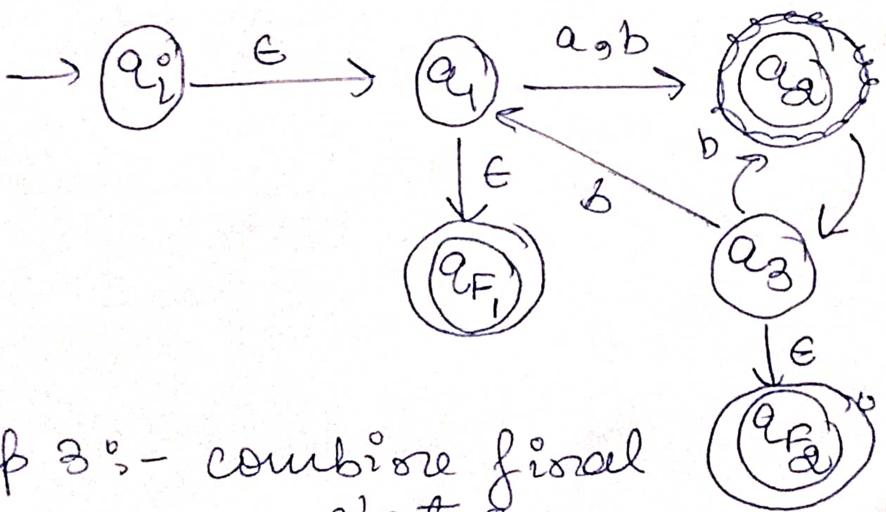
Q)



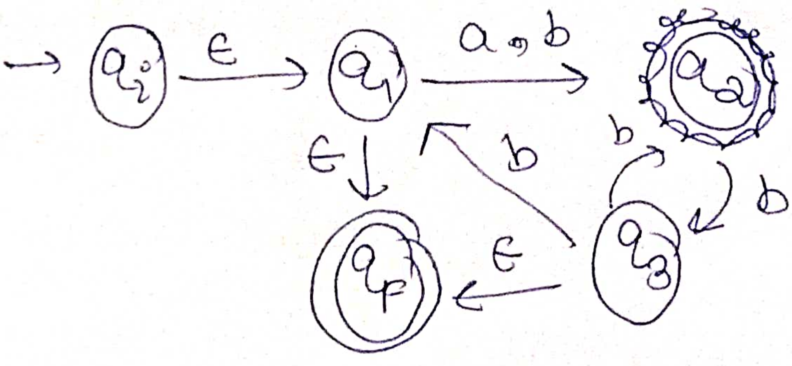
step 1:- create new initial state because incoming edge is present



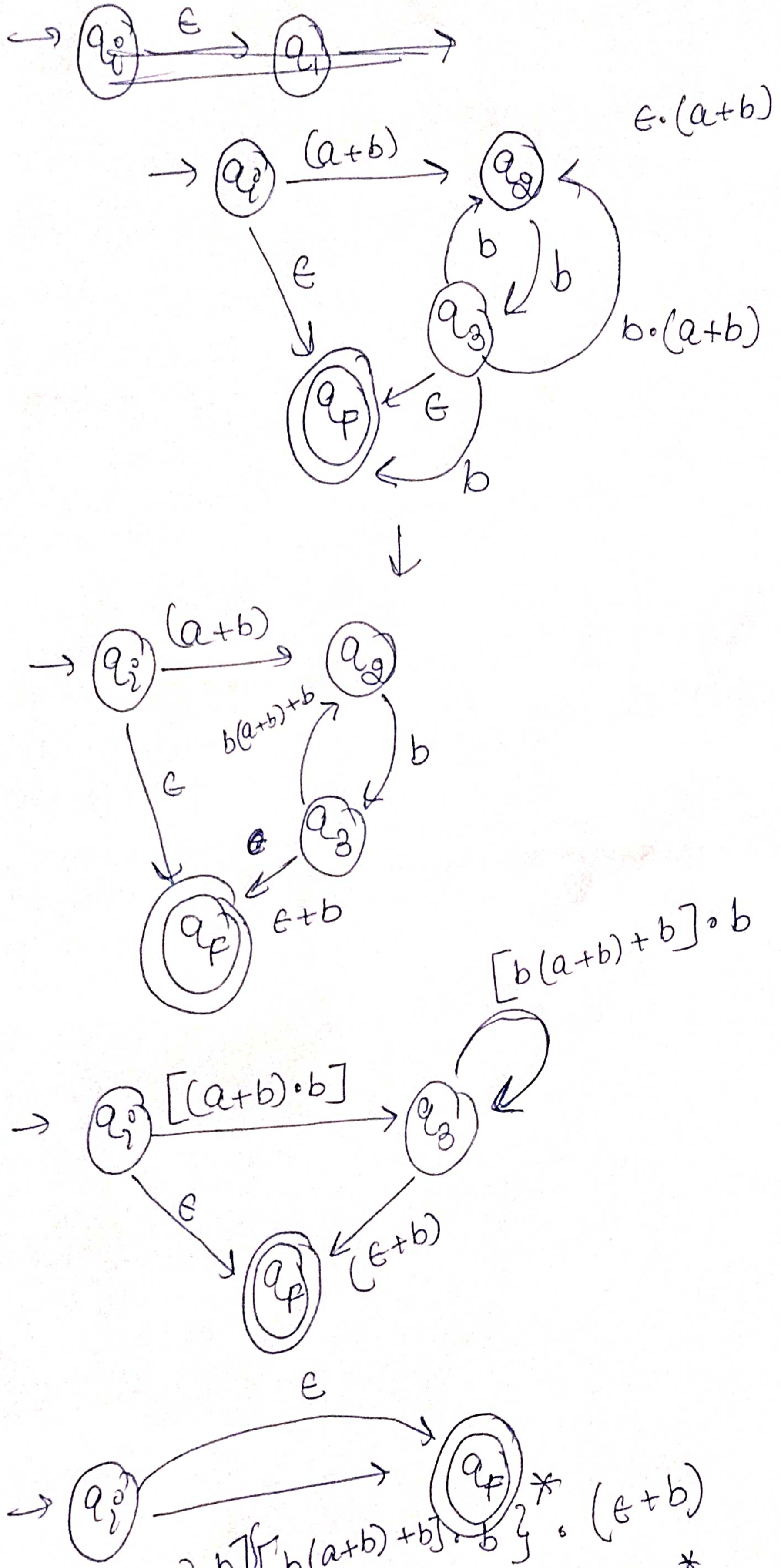
step 2:- check final state has ~~out~~going transition if yes, create new final state :-



step 3:- combine final state :-



Step 4:- we have to eliminate all state except initial and final:-

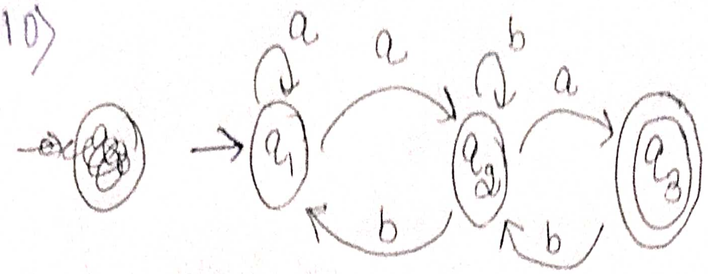


$[(a+b).b][\{b(a+b)+b\}.b]^*.(\epsilon+b)$

R.E = $\epsilon + [(a+b).b][\{ b(a+b)+b$

Q Convert DFA → R.E using Arden's theorem

10)



18

for state q_1

$q_1 = \epsilon + q_1 a + q_2 b$ —— (1)

for state q_2

$q_2 = q_2 b + q_1 a + q_3 b$ —— (2)

for state q_3

$q_3 = q_2 a$ —— (3)

$R = Q + RP$

Put (3) in eq. (2)

$q_2 = q_2 b + q_1 a + q_2 ab$

$q_2 = q_1 a + q_2 (b + ab)$

$R = Q + R P$

$R = QP^*$

$q_2 = q_1 a (b+ab)^*$ —— (4)

Put eq^n (4) in eq. (1)

$q_1 = \epsilon + q_1 a + q_1 a (b+ab)^*$

$q_1 = \epsilon + q_1 (a + a(b+ab)^*)$

$R = Q + RP$

$R = QP^*$

$$q_1 = \epsilon[a+a(b+ab)^*]^* \quad \text{——— (5)}$$

Put q_1 eq^n (5) in eq^n (4)

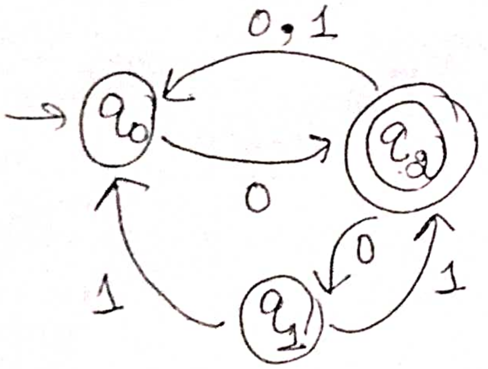
$$q_2 = [a+a(b+ab)^*]^*[a+(b+ab)^*] \quad \text{— (6)}$$

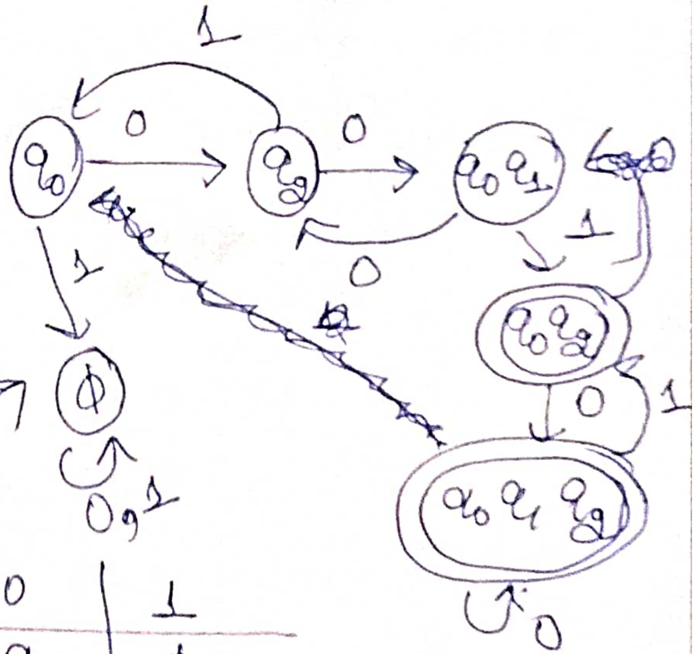
Put eq^n (6) in eq^n (3)

$$q_3 = q_2 a$$

R.E. $\boxed{q_3 = [a+a(b+ab)^*]^*[a+(b+ab)^*]a}$

Q.11) convert NFA to DFA





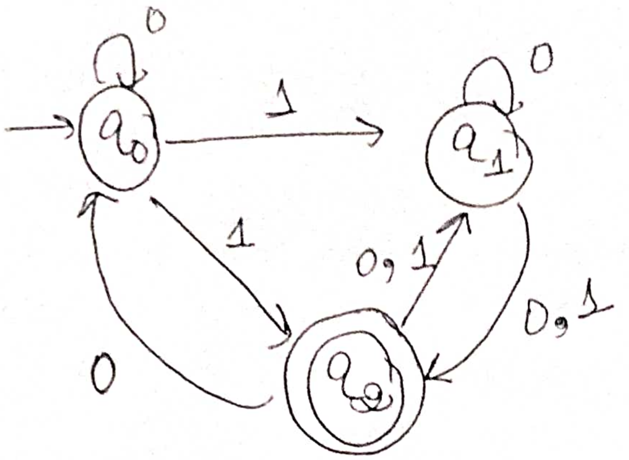
Transition table :-

	0	1
q_0	q_2	ϕ
q_1	ϕ	q_0, q_2
q_2	q_0, q_1	q_0

	0	1
q_0	q_2	ϕ
q_2	$\{q_0, q_1\}$	q_0
$\{q_0, q_1\}$	q_2	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$	q_0
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2\}$
ϕ	ϕ	ϕ

Q) convert NFA to DFA

12)

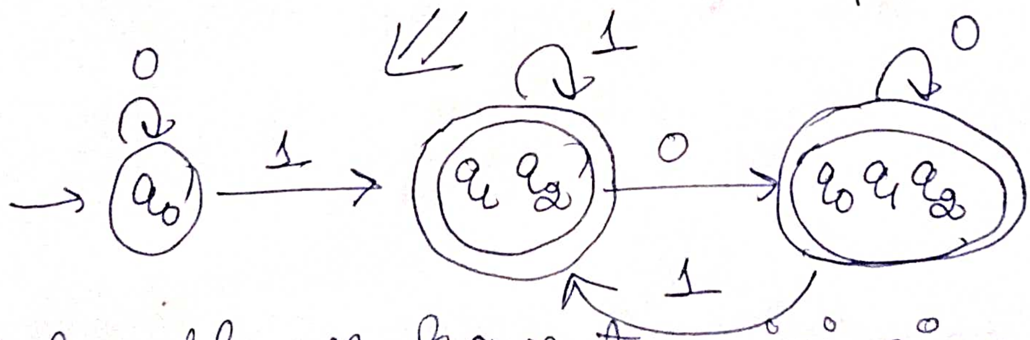


Transition table :-

Q	0	1
q_0	q_0	q_1, q_2
q_1	q_1, q_2	q_2
q_2	q_0, q_1	q_1

DFA transition table

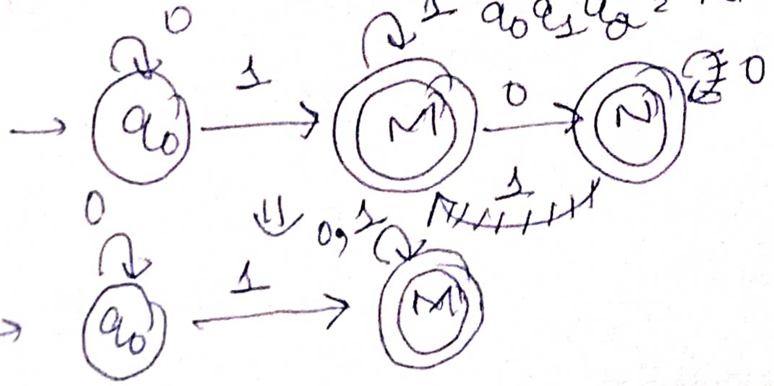
Q	0	1
q_0	q_0	$\{q_1, q_2\}$
$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$

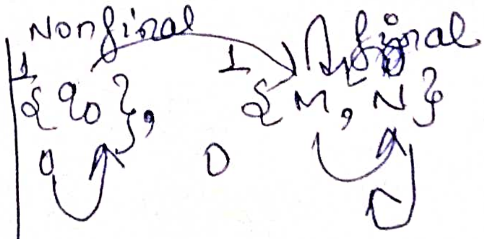


→ If possible we have to minimize the DFA

Let assume $q_1 q_2 = M$

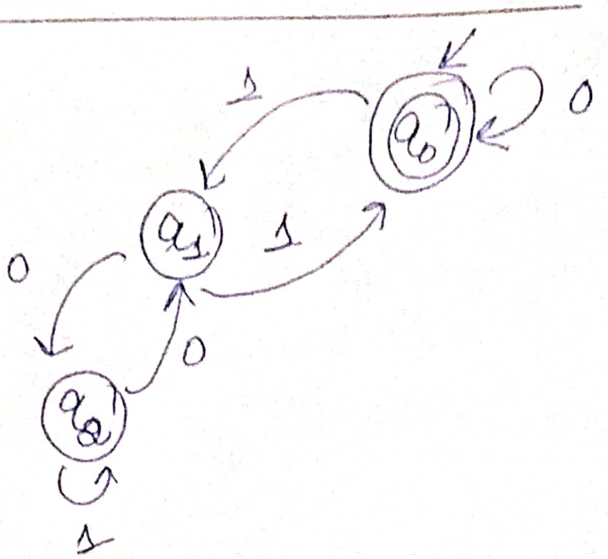
$q_0 q_1 q_2 = N$





M and N are equal states

Q14) convert DFA to R.E



Here we have two methods to convert DFA to R.E but we use Arden's theorem

for state q_0 :-

$q_0 = \epsilon + q_0 \cdot 0 + q_1 \cdot 1$ — (1)

for state q_1 :-

$q_1 = q_2 \cdot 0 + q_0 \cdot 1$ — (2)

for state q_2 :-

$q_2 = q_2 \cdot 1 + q_1 \cdot 0$ — (3)

$R = Q + RP$

$R = QP^*$

~~Put eq^n (2) in eq^n (1)~~

$\cancel{q_0 = \epsilon + q_0 \cdot 0 + q_2 \cdot 0 + q_0 \cdot 1}$

put eq (3) in eq (2)

$q_1 = (q_2 \cdot 1 + q_1 \cdot 0) \cdot 0 + q_0 \cdot 1$

$q_1 = q_2 \cdot 1 \cdot 0 + q_1 \cdot 0 \cdot 0 + q_0 \cdot 1$

Put eq 4 in eq 3

$$q_2 = q_2 \cdot 1 + (q_2 \cdot 0 + q_0 \cdot 1) \cdot 0$$

$$q_2 = q_2 \cdot 1 + q_2 \cdot 0 \cdot 0 + q_0 \cdot 1 \cdot 0$$

$$q_2 = q_2(1 + 0 \cdot 0) + q_0 \cdot 1 \cdot 0$$

$$q_2 = q_0 \cdot 1 \cdot 0 + q_2(1 + 0 \cdot 0)$$

$$R = Q + R\,P$$

$$R = QP^*$$

$$q_2 = q_0 \cdot 1 \cdot 0\,(1 + 0 \cdot 0)^*$$

we have:-

$$q_0 = \epsilon + q_0 \cdot 0 + q_2 \cdot 0 + q_0 \cdot 1$$

$$q_0 = \epsilon + q_0 \cdot 0 + q_0 \cdot 1 \cdot 0\,(1 + 0 \cdot 0)^* \cdot 0 + q_0 \cdot 1$$

$$q_0 = \epsilon + q_0(0 + \cancel{q_0}\, 1 \cdot 0 \cdot (1 + 0 \cdot 0)^* \cdot 0 + 1)$$

$$R = Q + RP$$

$$R = QP^*$$

$$q_0 = \epsilon\,[\,\cancel{q_0}\,(0 + 1 \cdot 0\,(1 + 0 \cdot 0)^* \cdot 0 + 1]^*$$

$$\boxed{q_0 = [\,(0 + 1 \cdot 0\,(1 + 0 \cdot 0)^* \cdot 0 + 1]^*}$$

R.E

Minimization of DFA

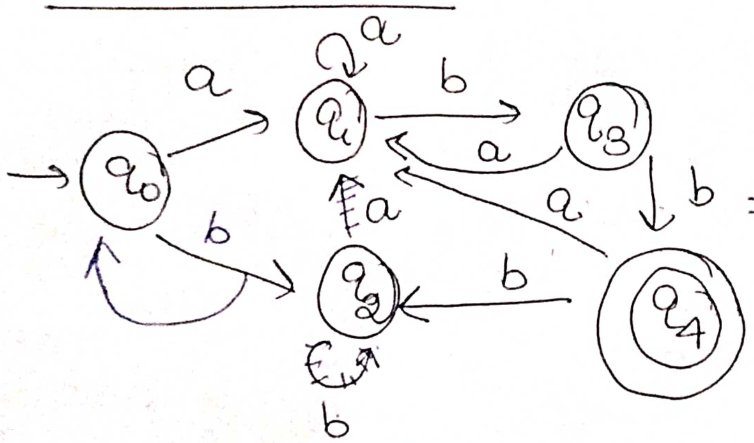
1) Remove all unreachable state. {जहाँ पहुँच नहीं सकते Initial state से}
2)

[Set of Non-final] [Set of final state]

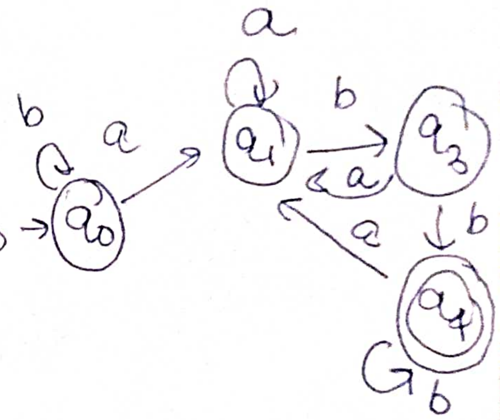
$\Pi_0 = [q_0, q_1]\ [q_3, q_5]$

$\Pi_1 = [q_0, q_1]$

Q18) Minimize DFA



⇒



step 1 :- there is no any unreachable state :-

step 2 :-

Transition table

Q	a	b
q_0	q_1	q_2
q_1	q_1	q_3
q_2	q_1	q_2
q_3	q_1	q_4
q_4	q_1	q_2

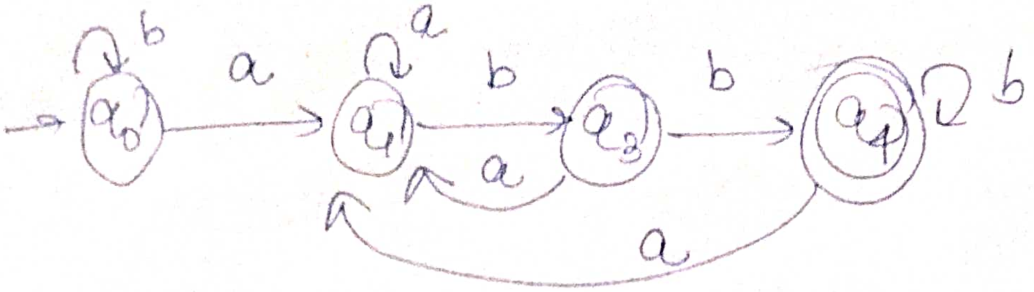
$\Pi_0 = [q_0, q_1, q_2, q_3]\ [q_4]$

$\Pi_1 = [q_0, q_1, q_2]\ [q_3]\ [q_4]$

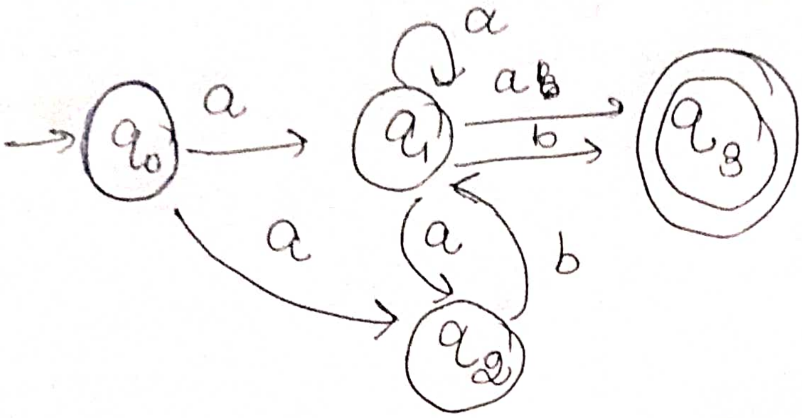
$\Pi_2 = [q_0, q_2]\ [q_1]\ [q_3]\ [q_4]$

$\cancel{\Pi_3 =}$

Here q_0 state is equal to q_2.



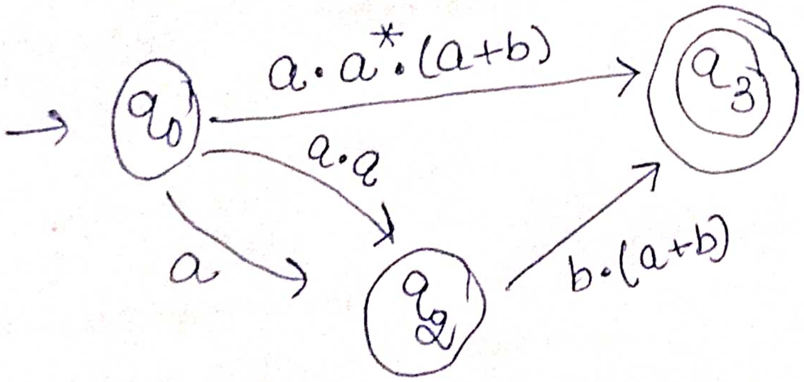
Q) convert DFA to RE

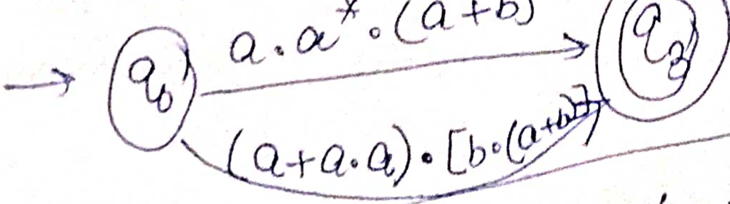


step 1:- If there is any incoming transition to initial state create a new initial state

step 2:- create new final state

step 3:- Reduce no. of states.





$$\boxed{RE = a.a^*.(a+b) + (a+a.a).[b.(a+b)]}$$