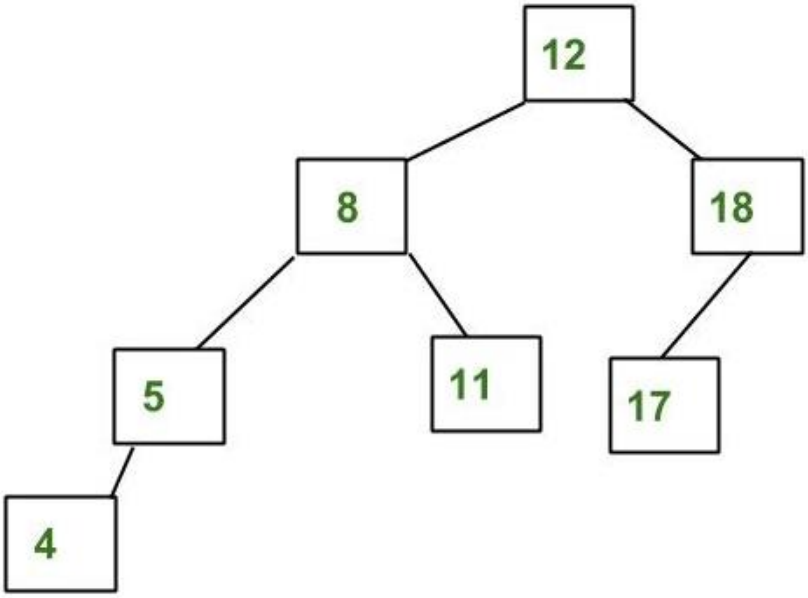
AVL Tree:

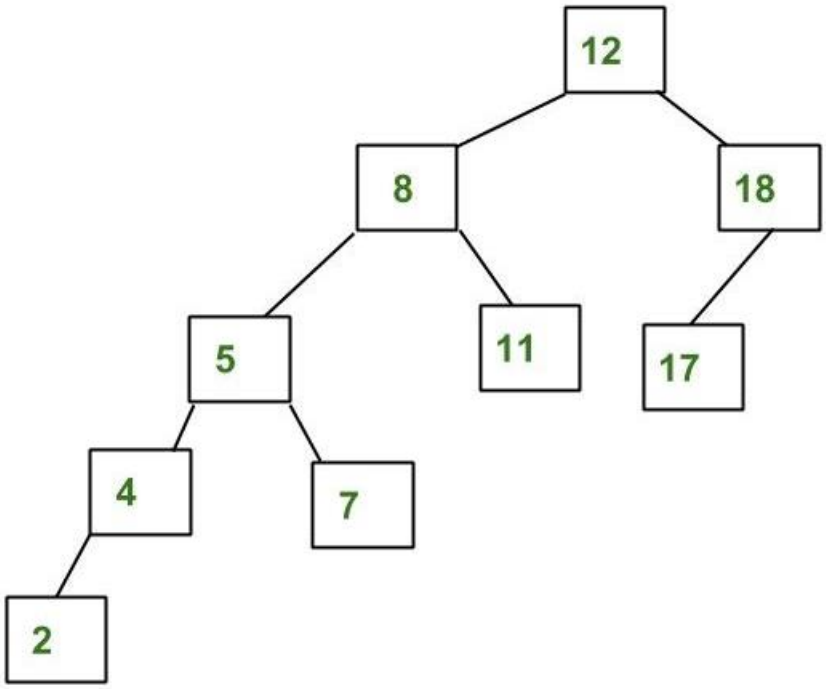
AVL tree is a self-balancing Binary Search Tree (**BST**) where the difference between heights of left and right subtrees cannot be more than **one** for all nodes.

Example of AVL Tree:



The above tree is AVL because the differences between the heights of left and right subtrees for every node are less than or equal to 1.

Example of a Tree that is NOT an AVL Tree:



The above tree is not AVL because the differences between the heights of the left and right subtrees for 8 and 12 are greater than 1.

KEY POINTS

- It is height balanced tree
- It is a binary search tree
- It is a binary tree in which the height difference between the left subtree and right subtree is almost one
- Height is the maximum depth from root to leaf

Characteristics of AVL Tree:

- It follows the general properties of a Binary Search Tree.
- Each subtree of the tree is balanced, i.e., the difference between the height of the left and right subtrees is at most 1.
- The tree balances itself when a new node is inserted. Therefore, the insertion operation is time-consuming

Application of AVL Tree:

- Most in-memory sets and dictionaries are stored using AVL trees.
- Database applications, where insertions and deletions are less common but frequent data lookups are necessary, also frequently employ AVL trees.
- In addition to database applications, it is employed in other applications that call for better searching.
- Most STL implementations of the ordered associative containers (sets, multisets, maps and multimaps) use red-black trees instead of AVL trees.

Advantages of AVL Tree:

- AVL trees can self-balance.
- It also provides faster search operations.
- AVL trees also have balancing capabilities with a different type of rotation
- Better searching time complexity than other trees, such as the binary Tree.
- Height must not be greater than log(N), where N is the total number of nodes in the Tree.

Disadvantages of AVL Tree:

- AVL trees are difficult to implement
- AVL trees have high constant factors for some operations.

Maximum & Minimum number of Nodes

Maximum number of nodes = $2^{H+1} - 1$
Minimum number of nodes of height H = min no of nodes of height (H-1) + min no of nodes of height(H-2) + 1
where H(0)=1

H(1)=2

Why AVL Trees?

*Most of the BST operations (e.g., search, max, min, insert, delete.. etc) take **O(h)** time where **h** is the height of the BST. The cost of these operations may become **O(n)** for a **skewed Binary tree**. If we make sure that the height of the tree remains **O(log(n))** after every insertion and deletion, then we can guarantee an upper bound of **O(log(n))** for all these operations. The height of an AVL tree is always **O(log(n))** where **n** is the number of nodes in the tree.*

Insertion in AVL Tree:

To make sure that the given tree remains AVL after every insertion, we must augment the standard BST insert operation to perform some re-balancing. Following are two basic operations that can be performed to balance a BST without violating the BST property (keys(left) < key(root) < keys(right)).

- Left Rotation
- Right Rotation

```
T1, T2 and T3 are subtrees of the tree, rooted with y (on the left
side) or x (on the right side)


     y                               x
    / \     Right Rotation          /  \
   x   T3   - - - - - - - >        T1   y
  / \       < - - - - - - -            / \
 T1  T2     Left Rotation            T2  T3


Keys in both of the above trees follow the following order

keys(T1) < key(x) < keys(T2) < key(y) < keys(T3)

So BST property is not violated anywhere.
```

Steps to follow for insertion:

Let the newly inserted node be **w**

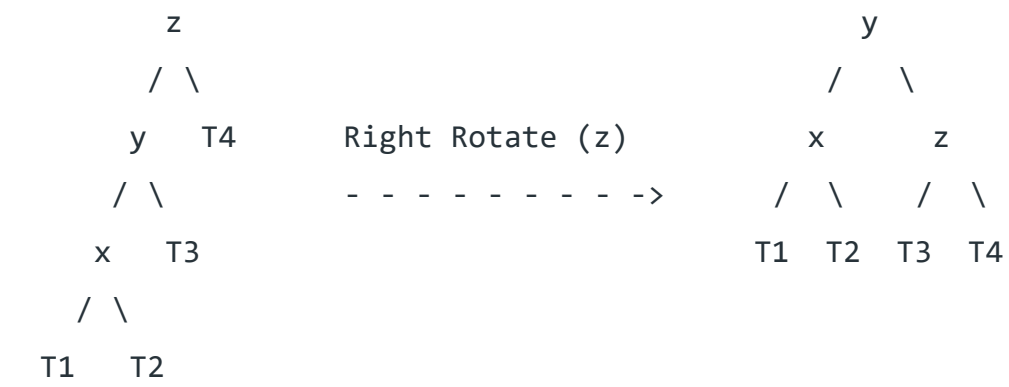
- Perform standard **BST** insert for **w**.

- Starting from **w**, travel up and find the first **unbalanced node**. Let **z** be the first unbalanced node, **y** be the **child** of **z** that comes on the path from **w** to **z** and **x** be the **grandchild** of **z** that comes on the path from **w** to **z**.
- Re-balance the tree by performing appropriate rotations on the subtree rooted with **z.** There can be 4 possible cases that need to be handled as **x, y** and **z** can be arranged in 4 ways.
- Following are the possible 4 arrangements:
    - y is the left child of z and x is the left child of y (Left Left Case)
    - y is the left child of z and x is the right child of y (Left Right Case)
    - y is the right child of z and x is the right child of y (Right Right Case)
    - y is the right child of z and x is the left child of y (Right Left Case)

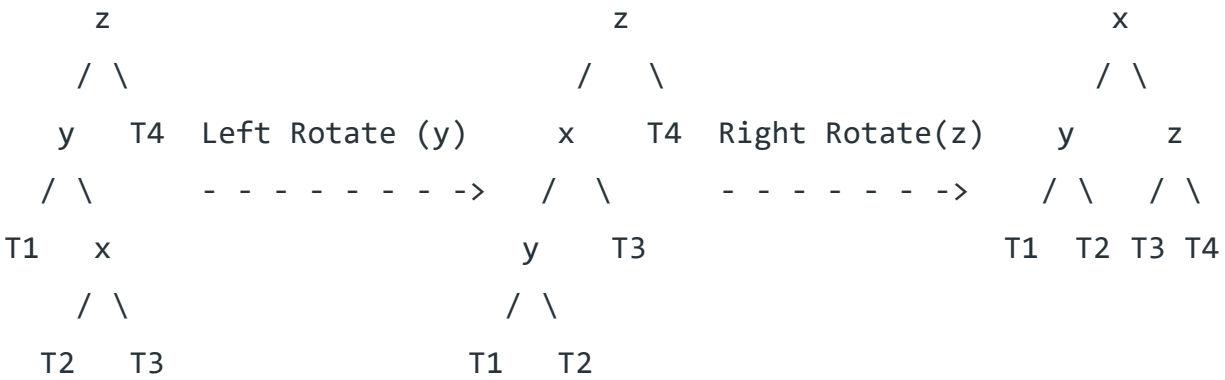
*Following are the operations to be performed in above mentioned 4 cases. In all of the cases, we only need to **re-balance** the subtree rooted with **z** and the complete tree becomes balanced as the height of the subtree (After appropriate rotations) rooted with **z** becomes the same as it was before insertion.*

1. Left Left Case

T1, T2, T3 and T4 are subtrees.



2. Left Right Case



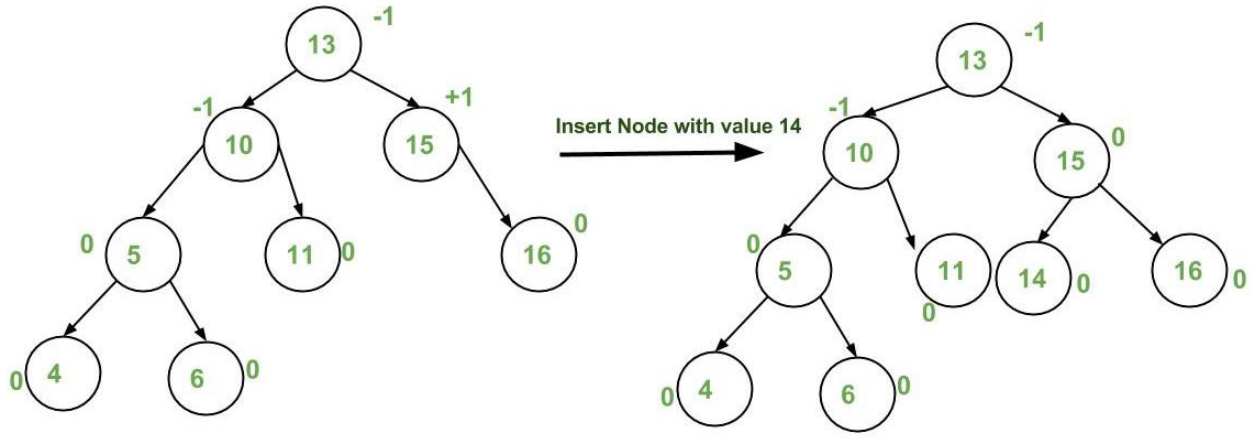
3. Right Right Case

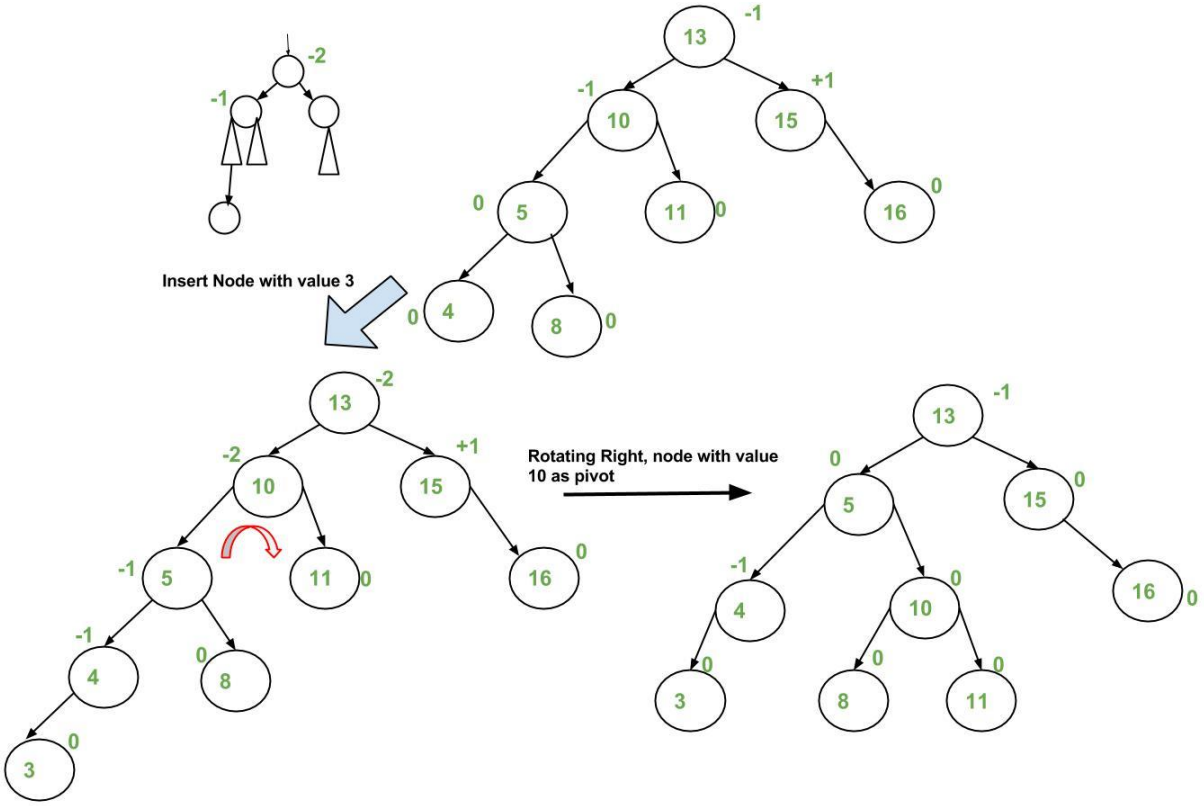
```
  z                                y
 /  \                            /   \
T1   y     Left Rotate(z)       z      x
    /  \   - - - - - - - ->    / \    / \
   T2   x                     T1  T2 T3  T4
       / \
     T3  T4
```

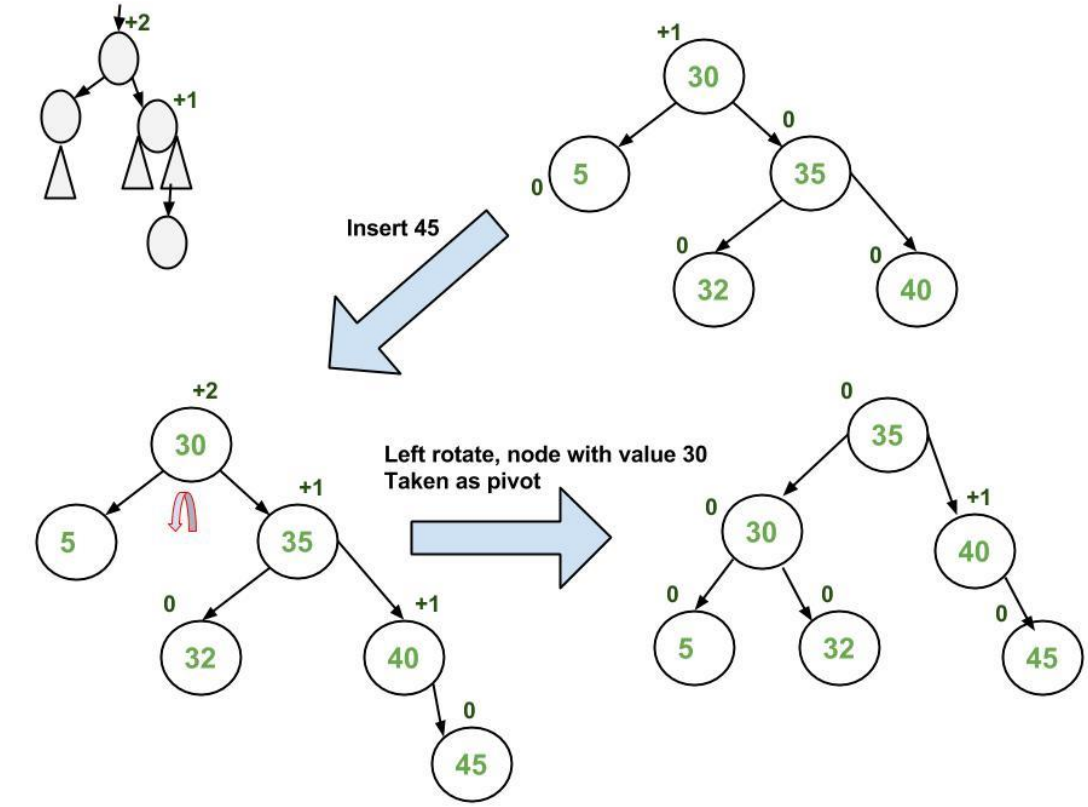
4. Right Left Case

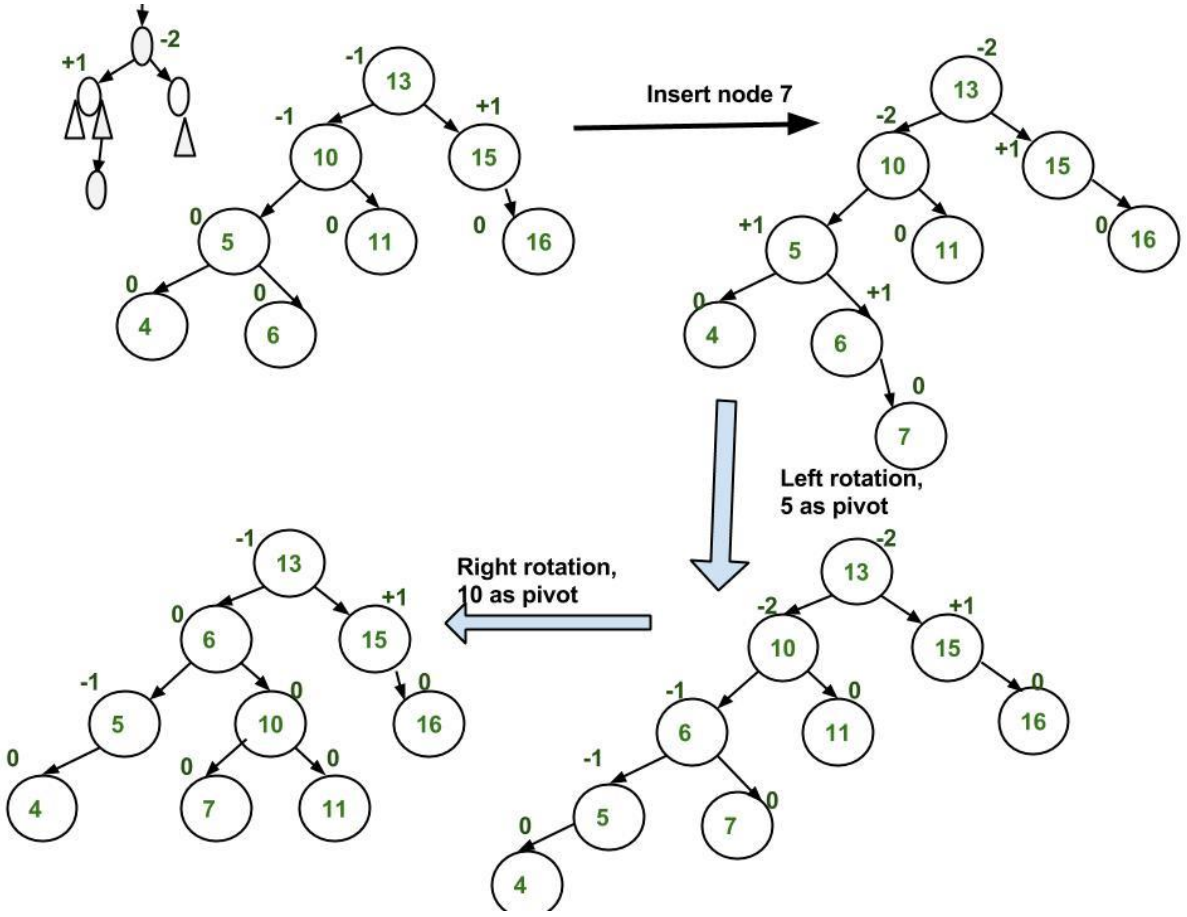
```
   z                            z                           x
  / \                          / \                        /  \
T1   y   Right Rotate (y)    T1   x   Left Rotate(z)   z      y
    / \  - - - - - - - ->        /  \  - - - - - ->   / \    / \
   x   T4                      T2   y                T1  T2 T3  T4
  / \                              /  \
T2   T3                           T3   T4
```

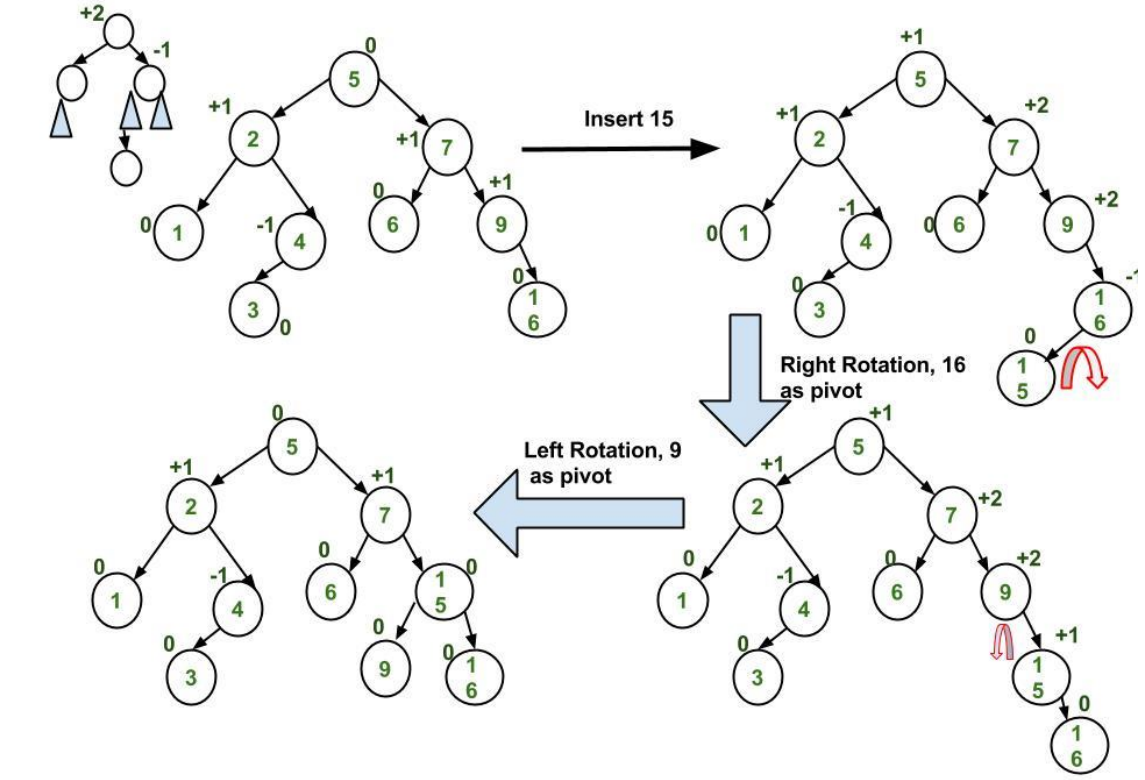
Illustration of Insertion at AVL Tree




-2
-1
13
-1
+1
10
15
0
5
11
0
16
0
Insert Node with value 3
0
4
8
0
13
-2
-2
+1
10
15
-1
5
11
0
16
0
-1
4
0
8
3
0
Rotating Right, node with value
10 as pivot
13
-1
0
5
15
0
-1
4
0
10
0
16
0
0
3
0
8
0
11


+2
+1
+1
30
0
0
5
35
0
32
0
40
Insert 45
+2
30
5
+1
35
0
32
+1
40
0
45
Left rotate, node with value 30
Taken as pivot
0
35
0
30
+1
40
0
5
0
32
0
45


Insert node 7
Left rotation,
5 as pivot
Right rotation,
10 as pivot


Insert 15
Right Rotation, 16
as pivot
Left Rotation, 9
as pivot

Approach:

The idea is to use recursive BST insert, after insertion, we get pointers to all ancestors one by one in a bottom-up manner. So we don't need a parent pointer to travel up. The recursive code itself travels up and visits all the ancestors of the newly inserted node.

Follow the steps mentioned below to implement the idea:

- Perform the normal BST insertion.
- The current node must be one of the ancestors of the newly inserted node. Update the **height** of the current node.
- Get the balance factor **(left subtree height – right subtree height)** of the current node.
- If the balance factor is greater than **1,** then the current node is unbalanced and we are either in the **Left Left case** or **left Right case**. To check whether it is **left left case** or not, compare the newly inserted key with the key in the **left subtree root**.
- If the balance factor is less than **-1**, then the current node is unbalanced and we are either in the Right Right case or Right-Left case. To check whether it is the Right Right case or not, compare the newly inserted key with the key in the right subtree root.

Below is the implementation of the above approach:

```
// Java program for insertion in AVL Tree

class Node {

    int key, height;

    Node left, right;



    Node(int d) {

        key = d;

        height = 1;

    }

}
```

```
class AVLTree {

    Node root;

    // A utility function to get the height of the tree
    int height(Node N) {
        if (N == null)
            return 0;

        return N.height;
    }

    // A utility function to get maximum of two integers
    int max(int a, int b) {
        return (a > b) ? a : b;
    }

    // A utility function to right rotate subtree rooted with y
    // See the diagram given above.
    Node rightRotate(Node y) {
```

```
    Node x = y.left;

    Node T2 = x.right;



    // Perform rotation

    x.right = y;

    y.left = T2;



    // Update heights

    y.height = max(height(y.left), height(y.right)) + 1;

    x.height = max(height(x.left), height(x.right)) + 1;



    // Return new root

    return x;

}



// A utility function to left rotate subtree rooted with x

// See the diagram given above.

Node leftRotate(Node x) {

    Node y = x.right;

    Node T2 = y.left;
```

```
    // Perform rotation
    y.left = x;
    x.right = T2;

    //  Update heights
    x.height = max(height(x.left), height(x.right)) + 1;
    y.height = max(height(y.left), height(y.right)) + 1;

    // Return new root
    return y;
}

// Get Balance factor of node N
int getBalance(Node N) {
    if (N == null)
        return 0;

    return height(N.left) - height(N.right);
}

Node insert(Node node, int key) {
```

```
/* 1.  Perform the normal BST insertion */
if (node == null)
    return (new Node(key));

if (key < node.key)
    node.left = insert(node.left, key);
else if (key > node.key)
    node.right = insert(node.right, key);
else // Duplicate keys not allowed
    return node;

/* 2. Update height of this ancestor node */
node.height = 1 + max(height(node.left),
                      height(node.right));

/* 3. Get the balance factor of this ancestor
      node to check whether this node became
      unbalanced */
int balance = getBalance(node);
```

```
        // If this node becomes unbalanced, then there

        // are 4 cases Left Left Case

        if (balance > 1 && key < node.left.key)

            return rightRotate(node);



        // Right Right Case

        if (balance < -1 && key > node.right.key)

            return leftRotate(node);



        // Left Right Case

        if (balance > 1 && key > node.left.key) {

            node.left = leftRotate(node.left);

            return rightRotate(node);

        }



        // Right Left Case

        if (balance < -1 && key < node.right.key) {

            node.right = rightRotate(node.right);

            return leftRotate(node);

        }



        /* return the (unchanged) node pointer */
```

```
        return node;
    }


    // A utility function to print preorder traversal
    // of the tree.
    // The function also prints height of every node
    void preOrder(Node node) {
        if (node != null) {
            System.out.print(node.key + " ");
            preOrder(node.left);
            preOrder(node.right);
        }
    }


    public static void main(String[] args) {
        AVLTree tree = new AVLTree();


        /* Constructing tree given in the above figure */
        tree.root = tree.insert(tree.root, 10);
        tree.root = tree.insert(tree.root, 20);
        tree.root = tree.insert(tree.root, 30);
        tree.root = tree.insert(tree.root, 40);
```

```
        tree.root = tree.insert(tree.root, 50);

        tree.root = tree.insert(tree.root, 25);



        /* The constructed AVL Tree would be

             30

            /  \

          20   40

         /  \     \

        10  25    50

        */

        System.out.println("Preorder traversal" +

                           " of constructed tree is : ");

        tree.preOrder(tree.root);

    }

}
```

Output

```
Preorder traversal of the constructed AVL tree is

30 20 10 25 40 50
```

Complexity Analysis

Time Complexity: O(log(n)), For Insertion

Auxiliary Space: O(1)

The rotation operations (left and right rotate) take constant time as only a few pointers are being changed there. Updating the height and getting the balance factor also takes constant time. So the time complexity of the AVL insert remains the same as the BST insert which is O(h) where h is the height of the tree. Since the AVL tree is balanced, the height is O(Logn). So time complexity of AVL insert is O(Logn).

Deletion in an AVL Tree

To make sure that the given tree remains AVL after every deletion, we must augment the standard BST delete operation to perform some re-balancing. Following are two basic operations that can be performed to re-balance a BST without violating the BST property (keys(left) < key(root) < keys(right)).

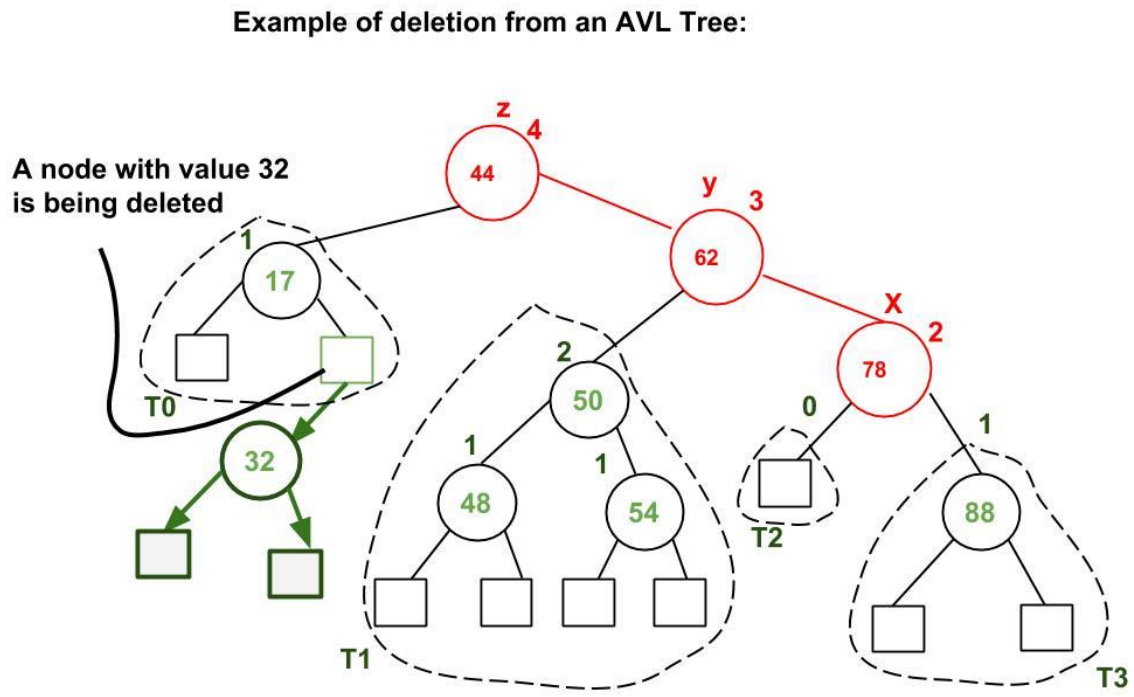
1. Left Rotation
2. Right Rotation

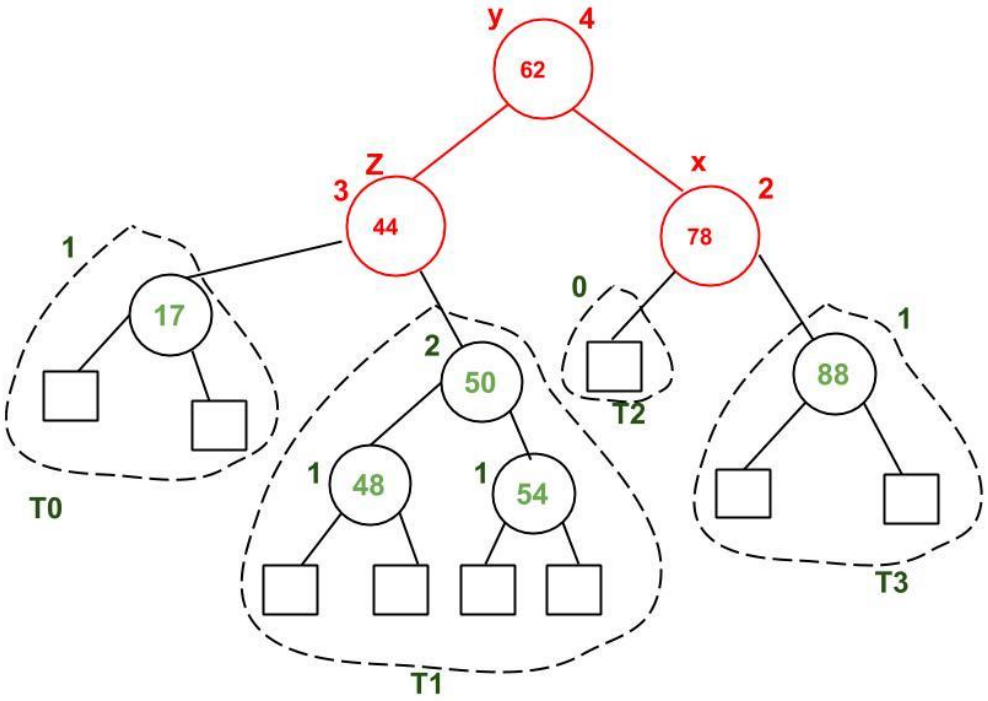
Let w be the node to be deleted

1. Perform standard BST delete for w.
2. Starting from w, travel up and find the first unbalanced node. Let z be the first unbalanced node, y be the larger height child of z, and x be the larger height child of y. Note that the definitions of x and y are different from insertion here.
3. Re-balance the tree by performing appropriate rotations on the subtree rooted with z. There can be 4 possible cases that needs to be handled as x, y and z can be arranged in 4 ways. Following are the possible 4 arrangements:
   1. y is left child of z and x is left child of y (Left Left Case)
   2. y is left child of z and x is right child of y (Left Right Case)
   3. y is right child of z and x is right child of y (Right Right Case)
   4. y is right child of z and x is left child of y (Right Left Case)

Like insertion, the four operations to be performed in above mentioned 4 cases. Note that, unlike insertion, fixing the node z won't fix the complete AVL tree. After fixing z, we may have to fix ancestors of z as well.

Example:





A node with value 32 is being deleted. After deleting 32, we travel up and find the first unbalanced node which is 44. We mark it as z, its higher height child as y which is 62, and y's higher height child as x which could be either 78 or 50 as both are of same height. We have considered 78. Now the case is Right Right, so we perform left rotation.

Java implementation

Following is the java implementation for AVL Tree Deletion. The following Java implementation uses the recursive BST delete as basis. In the recursive BST delete, after deletion, we get pointers to all ancestors one by one in bottom up manner. So we don't need parent pointer to travel up. The recursive code itself travels up and visits all the ancestors of the deleted node.

1. Perform the normal BST deletion.
2. The current node must be one of the ancestors of the deleted node. Update the height of the current node.
3. Get the balance factor (left subtree height – right subtree height) of the current node.
4. If balance factor is greater than 1, then the current node is unbalanced and we are either in Left Left case or Left Right case. To check whether it is Left Left case or Left Right case, get the balance factor of left subtree. If balance factor of the left subtree is greater than or equal to 0, then it is Left Left case, else Left Right case.
5. If balance factor is less than -1, then the current node is unbalanced and we are either in Right Right case or Right Left case. To check whether it is Right Right case or Right Left case, get the balance factor of right subtree. If the balance factor of the right subtree is smaller than or equal to 0, then it is Right Right case, else Right Left case.

```
// Java program for deletion in AVL Tree

class Node

{

    int key, height;

    Node left, right;



    Node(int d)

    {

        key = d;

        height = 1;

    }

}



class AVLTree

{

    Node root;



    // A utility function to get height of the tree

    int height(Node N)

    {

        if (N == null)
```

```
        return 0;

    return N.height;

}



// A utility function to get maximum of two integers

int max(int a, int b)

{

    return (a > b) ? a : b;

}



// A utility function to right rotate subtree rooted with y

// See the diagram given above.

Node rightRotate(Node y)

{

    Node x = y.left;

    Node T2 = x.right;



    // Perform rotation

    x.right = y;

    y.left = T2;



    // Update heights
```

```
    y.height = max(height(y.left), height(y.right)) + 1;

    x.height = max(height(x.left), height(x.right)) + 1;



    // Return new root

    return x;

}



// A utility function to left rotate subtree rooted with x

// See the diagram given above.

Node leftRotate(Node x)

{

    Node y = x.right;

    Node T2 = y.left;



    // Perform rotation

    y.left = x;

    x.right = T2;



    // Update heights

    x.height = max(height(x.left), height(x.right)) + 1;

    y.height = max(height(y.left), height(y.right)) + 1;
```

```
    // Return new root
    return y;
}

// Get Balance factor of node N
int getBalance(Node N)
{
    if (N == null)
        return 0;
    return height(N.left) - height(N.right);
}

Node insert(Node node, int key)
{
    /* 1. Perform the normal BST rotation */
    if (node == null)
        return (new Node(key));

    if (key < node.key)
        node.left = insert(node.left, key);
    else if (key > node.key)
```

```
        node.right = insert(node.right, key);
    else // Equal keys not allowed
        return node;

    /* 2. Update height of this ancestor node */
    node.height = 1 + max(height(node.left),
                          height(node.right));

    /* 3. Get the balance factor of this ancestor
    node to check whether this node became
    Wunbalanced */
    int balance = getBalance(node);

    // If this node becomes unbalanced, then
    // there are 4 cases Left Left Case
    if (balance > 1 && key < node.left.key)
        return rightRotate(node);

    // Right Right Case
    if (balance < -1 && key > node.right.key)
        return leftRotate(node);
```

```
    // Left Right Case
    if (balance > 1 && key > node.left.key)
    {
        node.left = leftRotate(node.left);
        return rightRotate(node);
    }

    // Right Left Case
    if (balance < -1 && key < node.right.key)
    {
        node.right = rightRotate(node.right);
        return leftRotate(node);
    }

    /* return the (unchanged) node pointer */
    return node;
}

/* Given a non-empty binary search tree, return the
node with minimum key value found in that tree.
Note that the entire tree does not need to be
```

```
searched. */
Node minValueNode(Node node)
{
    Node current = node;

    /* loop down to find the leftmost leaf */
    while (current.left != null)
    current = current.left;

    return current;
}

Node deleteNode(Node root, int key)
{
    // STEP 1: PERFORM STANDARD BST DELETE
    if (root == null)
        return root;

    // If the key to be deleted is smaller than
    // the root's key, then it lies in left subtree
    if (key < root.key)
        root.left = deleteNode(root.left, key);
```

```
// If the key to be deleted is greater than the

// root's key, then it lies in right subtree

else if (key > root.key)

    root.right = deleteNode(root.right, key);



// if key is same as root's key, then this is the node

// to be deleted

else

{



    // node with only one child or no child

    if ((root.left == null) || (root.right == null))

    {

        Node temp = null;

        if (temp == root.left)

            temp = root.right;

        else

            temp = root.left;



        // No child case

        if (temp == null)
```

```
            {
                temp = root;
                root = null;
            }
            else // One child case
                root = temp; // Copy the contents of
                             // the non-empty child
        }
        else
        {
             // node with two children: Get the inorder
            // successor (smallest in the right subtree)
            Node temp = minValueNode(root.right);

            // Copy the inorder successor's data to this node
            root.key = temp.key;

            // Delete the inorder successor
            root.right = deleteNode(root.right, temp.key);
        }
    }
```

```
// If the tree had only one node then return
if (root == null)
    return root;

// STEP 2: UPDATE HEIGHT OF THE CURRENT NODE
root.height = max(height(root.left), height(root.right)) + 1;

// STEP 3: GET THE BALANCE FACTOR OF THIS NODE (to check whether
// this node became unbalanced)
int balance = getBalance(root);

// If this node becomes unbalanced, then there are 4 cases
// Left Left Case
if (balance > 1 && getBalance(root.left) >= 0)
    return rightRotate(root);

// Left Right Case
if (balance > 1 && getBalance(root.left) < 0)
{
    root.left = leftRotate(root.left);
    return rightRotate(root);
}
```

```
    // Right Right Case
    if (balance < -1 && getBalance(root.right) <= 0)
        return leftRotate(root);

    // Right Left Case
    if (balance < -1 && getBalance(root.right) > 0)
    {
        root.right = rightRotate(root.right);
        return leftRotate(root);
    }

    return root;
}

// A utility function to print preorder traversal of
// the tree. The function also prints height of every
// node
void preOrder(Node node)
{
    if (node != null)
    {
```

```
        System.out.print(node.key + " ");
        preOrder(node.left);
        preOrder(node.right);
    }
}
public static void main(String[] args)
{
    AVLTree tree = new AVLTree();
    /* Constructing tree given in the above figure */
    tree.root = tree.insert(tree.root, 9);
    tree.root = tree.insert(tree.root, 5);
    tree.root = tree.insert(tree.root, 10);
    tree.root = tree.insert(tree.root, 0);
    tree.root = tree.insert(tree.root, 6);
    tree.root = tree.insert(tree.root, 11);
    tree.root = tree.insert(tree.root, -1);
    tree.root = tree.insert(tree.root, 1);
    tree.root = tree.insert(tree.root, 2);
    /* The constructed AVL Tree would be
    9
    / \
    1 10
```

```
/ \ \

0 5 11

/ / \

-1 2 6

*/

System.out.println("Preorder traversal of "+

                   "constructed tree is : ");

tree.preOrder(tree.root);

 tree.root = tree.deleteNode(tree.root, 10);

 /* The AVL Tree after deletion of 10

1

/ \

0 9

/     / \

-1 5 11

/ \

2 6

*/

System.out.println("");

System.out.println("Preorder traversal after "+

                "deletion of 10 :");

tree.preOrder(tree.root);
```

```
    }

}

// This code has been contributed by Mayank Jaiswal
```


Output:

```
Preorder traversal of the constructed AVL tree is

9 1 0 -1 5 2 6 10 11

Preorder traversal after deletion of 10

1 0 -1 9 5 2 6 11
```

Time Complexity: The rotation operations (left and right rotate) take constant time as only few pointers are being changed there. Updating the height and getting the balance factor also take constant time. So the time complexity of AVL delete remains same as BST delete which is O(h) where h is height of the tree. Since AVL tree is balanced, the height is O(Logn). So time complexity of AVL delete is O(Log n).

Auxiliary Space: O(1), since no extra space is used.

Advantages Of AVL Trees

- It is always height balanced
- Height Never Goes Beyond LogN, where N is the number of nodes
- It give better search than compared to binary search tree
- It has self balancing capabilities

Summary of AVL Trees

- These are self-balancing binary search trees.
- Balancing Factor ranges -1, 0, and +1.
- When balancing factor goes beyond the range require rotations to be performed
- Insert, delete, and search time is O(log N).
- AVL tree are mostly used where search is more frequent compared to insert and delete operation.