Equivalent / indistinguish states:

Two states q_1 and q_2 are equivalent if both $\delta(q_1, x)$ and $\delta(q_2, x)$ are final states or non final states for all $x \in \Sigma^*$
it is denoted as $q_1 \equiv q_2$.

or

Two states q_1 and q_2 are K- equivalent $(K \geq 0)$ if both $\delta(q_1, x)$ and $\delta(q_2, x)$ for all string x of length K or less.

* any two final states are 0 equivalence.
* any two nonfinal states are 0 equivalence.
* equivalent states follow the Reflexive, symmetric and transitive Property.
* if q_1 and q_2 are K equivalent for all $K \geq 0$ then q_1 and q_2 are equivalent.
* The set of K equivalent sets of state denoted by π_K
* if $\pi_n = \pi_{n+1}$ Then the following result is the key to the construction of minimum state Automata.

Construction of minimum Automata: (by using SET method)

Step 0: Remove all inaccessible states from initial state.

Step 1: construction of π_0 (by defination of 0 equivalence)

$$\pi_0 = \{Q_1^0, Q_2^0\}$$ here Q^K show K equivalent states

where Q_1^0 = set of non final state.

Q_2^0 = set of final state.

Step 2: (construct π_{K+1} from π_K)

Let Q_i^K be any subset in π_K. if $q_1, q_2 \in Q_i^K$. then they are K+1 equivalent if $\delta(q_1, a)$ and $\delta(q_2, a)$ are in the same equivalence class in π_K, $\forall a \in \Sigma$.

in this way Q_i^K is further divided into K+1 equivalence classes. Repeat This Process for every Q_i^K in π_K to get π_{K+1}

Step 3:

construct π_{K+1} for $K = 0, 1, \ldots$ until $\pi_K = \pi_{K+1}$

Step 4: (construction of minimum Automata)

The state table is obtained by replacing a state q by the corresponding equivalence class.

Prob:

Construct a minimum state Automata equivalent to the Finite Automata described by ~~fig~~ given table.

State \ Σ	a=0	a=1
→ q_0	q_1	q_5
q_1	q_6	q_2
q_2	q_0	q_2
q_3	q_2	q_6
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	q_2

firstly, q_3 is unreachable state so remove it

Now $Q_1^0 = \{q_0, q_1, q_4, q_5, q_6, q_7\}$

$Q_2^0 = \{q_2\}$

$\pi_0 = \{\{q_0, q_1, q_4, q_5, q_6, q_7\}, \{q_2\}\}$

Now construct π_1

in Q_1^0, q_0 is equivalent to q_4, q_6

because $\delta(q_0/q_4/q_6, 0) \in Q_1^0$

and $\delta(q_0/q_4/q_6, 1) \in Q_1^0$

q_1 is one equivalent to q_7

and q_5 is alone so single state class

in Q_2^0 have only one state so it can not be further Partitioned.

So now $\pi_2 = \{\{q_0, q_4, q_6\}, \{q_1, q_7\}, \{q_5\}, \{q_2\}\}$

$Q_1' = \{q_0, q_4, q_6\}$, $Q_2' = \{q_1, q_7\}$, $Q_3' = \{q_5\}$, $Q_4' = \{q_2\}$

Now construct π_2

in Q_1' q_0 is 2 equivalent to q_4 & not 2 equivalent to q_6

because $\delta(q_0/q_4, 0) \in Q_2'$

$\delta(q_0/q_4, 1) \in Q_3'$

but $\delta(q_6, 0) \in Q_1'$

So Q_1' Partitioned into $\{q_0, q_4\}$ and $\{q_6\}$.

in Q_2' q_1 is 2 equivalent to q_7 so no Partitioned

now $\pi_2 = \{\{q_0, q_4\}\{q_6\}, \{q_1, q_7\}, \{q_5\}, \{q_2\}\}$

$Q_1^2 = \{q_0, q_4\}$, $Q_2^2 = \{q_6\}$, $Q_3^2 = \{q_1, q_7\}$,

$Q_4^2 = \{q_5\}$, $Q_5^2 = \{q_2\}$

Now construct π_3

in Q_1^2 q_0 is 3 equivalent to q_4

in Q_3^2 q_1 is 3 equivalent to q_7

so $\pi_3 = \{\{q_0, q_4\}\{q_6\}, \{q_1, q_7\}, \{q_5\}, \{q_2\}\}$

$\pi_2 = \pi_3$ so π_2 gives us the equivalence classes

The minimum DFA is

$m' = \{Q', \{0,1\}, \delta', q_0', F'\}$

where

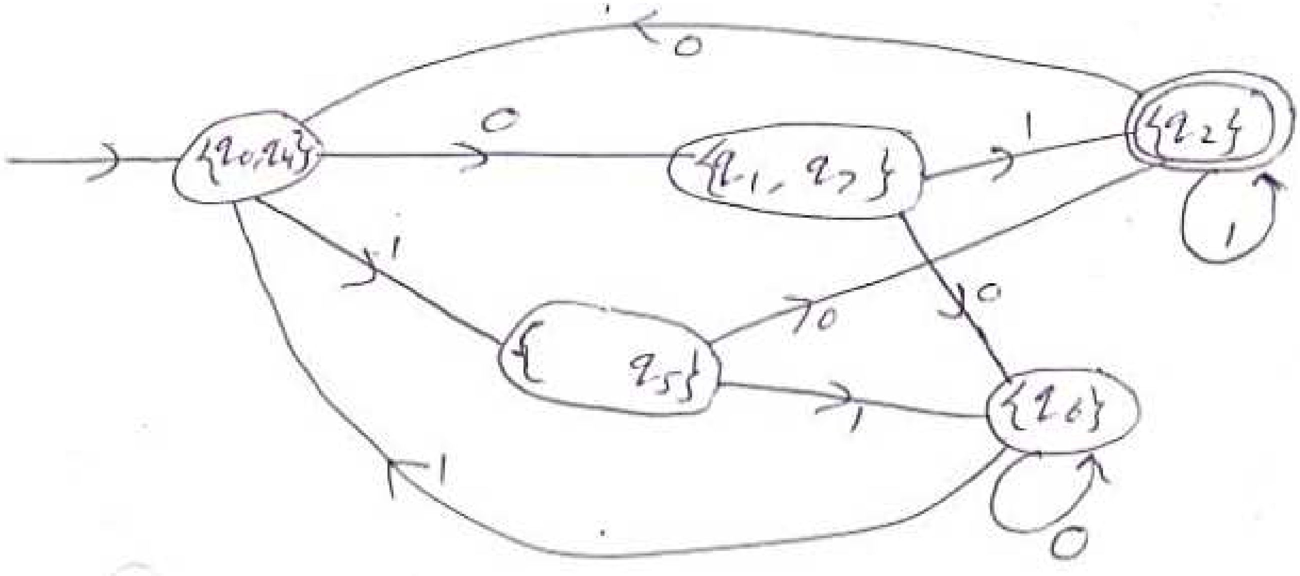
$Q' = \{ \{q_0, q_4\}, \{q_6\}, \{q_1, q_7\}, \{ \quad q_5\}, \{q_2\} \}$

$q_0' = \{q_0, q_4\}$

$F' = \{q_2\}$

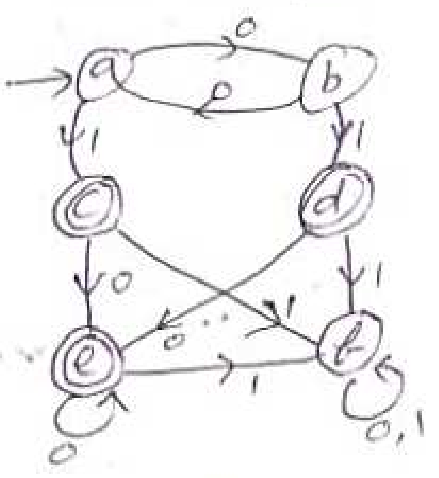
and δ' defined by table

$Q \backslash \Sigma$	0	1
$\{q_0, q_6\}$	$\{q_1, q_7\}$	$\{ \quad q_5\}$
$\{q_1, q_7\}$	$\{q_6\}$	$\{q_2\}$
$\{ \quad q_5\}$	$\{q_2\}$	$\{q_6\}$
$\{q_6\}$	$\{q_6\}$	$\{q_0, q_4\}$
$\{q_2\}$	$\{q_0, q_4\}$	$\{q_2\}$



Prob.

minimized the following DFA:



	0	1
a	b	c
b	a	d
c	e	f
d	e	f
e	e	f
f	f	f

$\pi_0 = \{\{a, b, f\}, \{c, d, e\}\}$

here $Q_1^0 = \{a, b, f\}$, $Q_2^0 = \{c, d, e\}$

now construct π_1

in Q_1^0 a is 1 equivalent to b but not f

in Q_2^0 c is 1 equivalent to d and e.

so $\pi_1 = \{\{a, b\}, \{f\}, \{c, d, e\}\}$

here $Q_1' = \{a, b\}$, $Q_2' = \{f\}$, $Q_3' = \{c, d, e\}$

now construct π_2

in Q_1' a is 2 equivalent to b

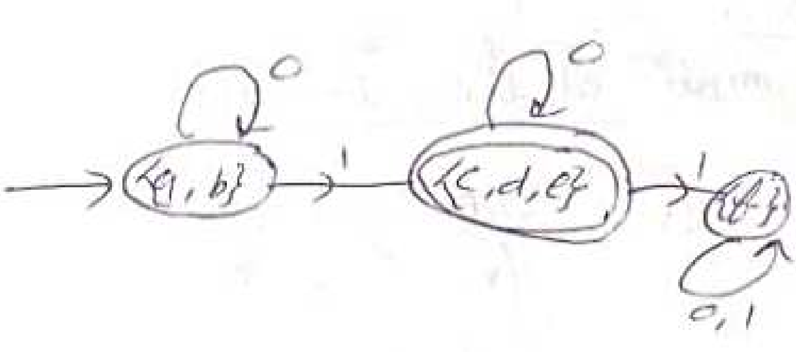
in Q_3' c is 2 equivalent to d and e

so $\pi_2 = \{\{a, b\}, \{f\}, \{c, d, e\}\}$

$\pi_1 = \pi_2$

so minimized DFA is

δ'	0	1
$\{a, b\}$	$\{a, b\}$	$\{c, d, e\}$
$\{c, d, e\}$	$\{c, d, e\}$	$\{f\}$
$\{f\}$	$\{f\}$	$\{f\}$



$q_0' = \{a, b\}$

$F' = \{c, d, e\}$

$\Sigma' = \{0, 1\}$

$Q' = \{\{a, b\}, \{f\}, \{c, d, e\}\}$

MYHILL NERODE Theorem

MyHill Nerode theorem is used to minimize a DFA.

Step 1: remove all unreachable states from DFA.

Step 2: Build a two dimensional $|Q| \times |Q|$ matrix labelling the right side $q_0, q_1, q_2, \ldots$ running down and Bottom as $q_0, q_1, q_2 \ldots$ running left to right

Put dashes (–) in the major diagonal and upper triangular part of the matrix.

* Now Perform operation in lower triangular *

Step 3: Put "X" between every final & non final states.

Step 4: (Now for every pair of states of empty positio
Let position of q_1 and q_2 is empty then put "x" between q_1 & q_2 if $\delta(q_1, a) = r_1$, $\delta(q_2, a) = r_2$ and r_1 and r_2 have "X" or "x". where $a \in \Sigma$

Step 5: Do the step 4 while put atleast one x in empty position.

Step 6: Now write o in every empty position.

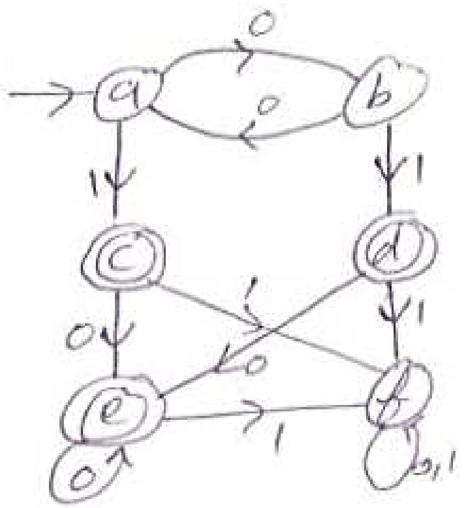
Step 7: To findout the states in minimised DFA check all the ~~rows~~ column of final matrix
if a column start with q_x is having o at q_y, q_z position then $\{q_x, q_y, q_z\}$ will a state in minimized machine.

if No "0" found in any column q_i then q_i is not equivalent to other states and write alone.

initial state/Final state will be that set which contains initial state / Final state.

For finding the transition between states of minimized machine we will adopt the same strategy as we adopt in minimization Process.

example:



$\Longrightarrow$

δ	0	1
a	b	c
b	a	d
c	e	f
d	e	f
e	e	f
f	f	f

solution

	a	b	c	d	e	f
a	–	–	–	–	–	–
b	O	–	–	–	–	–
c	X	X	–	–	–	–
d	X	X	O	–	–	–
e	X	X	O	O	–	–
f	x	x	X	X	X	–

set of Final state $F = \{c, d, e\}$

set of Non Final state $Q - F = \{a, b, f\}$

"X" the position of (a,c), (a,d), (a,e), (b,c), (b,d), (b,e), (f,c), (f,d), (f,e)

if (p,q) is dashes (-) then "X" (q,p)

Now for (a,f):

$\delta(a,0) = b$ & $\delta(f,0) = f$ & in Table (f,b) is empty.

$\delta(a,1) = c$ & $\delta(f,1) = f$ & in Table (f,c) have "X"

~~the~~ so Put "x" at position (a,f)

Now for (a,b):

$\delta(a,0) = b$ & $\delta(b,0) = a$ & (b,a) is empty

$\delta(a,1) = c$ & $\delta(b,1) = d$ & (c,d) is empty

So remain ~~left~~ empty Position of (a,b)

Now for (b,f):

$\delta(b,0) = a$ & $\delta(f,0) = f$ & (a,f) have "x"

so put "x" ~~at~~ Position (b,f)

Now for (c,e): $\delta(c,0) = e$, $\delta(e,0) = e$ and (e,e) have -

{- is equivalent to empty}

$\delta(c,1) = f$, $\delta(e,1) = f$ & (f,f) have -

So remain empty ~~at~~ Position (c,e)

Now for (c,d): $\delta(c,0) = e$, $\delta(d,0) = e$ & e,e have -

$\delta(c,1) = f$, $\delta(d,f) = f$ & f,f have -

So remain empty ~~at~~ Position (c,d)

Now for (d, e):

$\delta(d, 0) = e$, $\delta(e, 0) = e$, (e, e) have –
$\delta(d, 1) = f$, $\delta(e, 1) = f$, (f, f) have –
so remain empty the position of (d, e)

Now again check for (a, b)

$\delta(a, 0) = b$, $\delta(b, 0) = a$ & (a, b) not x ar n
$\delta(a, 1) = c$, $\delta(b, 1) = d$ & (c, d) " "
so remain empty the position of (a, b)

Now for (c, e):

$\delta(c, 0) = e$, $\delta(e, 0) = e$ & (e, e) have –
$\delta(c, 1) = f$, $\delta(e, 1) = f$ & (f, f) have –

so remain empty.

Now for (c, d)

$\delta(c, 0) = e$, $\delta(d, 0) = e$ & (e, e) have –
$\delta(c, 1) = f$, $\delta(d, 1) = f$ & (f, f) have –

so No change

Now for (d, e)

$\delta(d, 0) = e$, $\delta(e, 0) = e$, (e, e) have –
$\delta(d, 1) = f$, $\delta(f, 0) = f$, (f, f) have –

so No change

~~So~~ Now put 0 at empty position.

equivalent states are ~~[illegible]~~

(a, b), (c, d, e), (f)

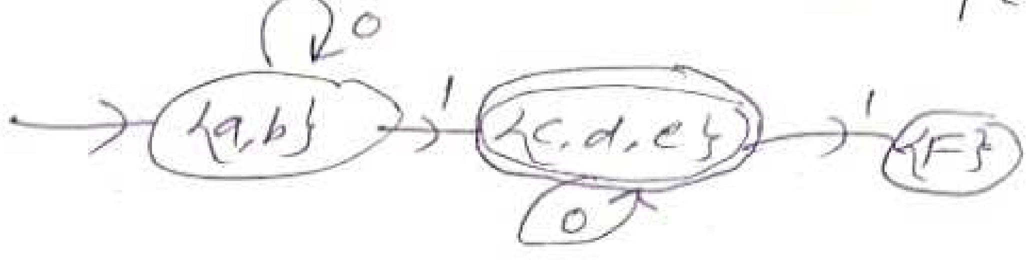
$Q = \{\{a,b\}, \{c,d,e\}, \{f\}\}$

$q_0 = \{a,b\}$

$F = \{c,d,e\}$

$\Sigma = \{0,1\}$

δ	0	1
$\{a,b\}$	$\{a,b\}$	$\{c,d,e\}$
$\{c,d,e\}$	$\{c,d,e\}$	$\{f\}$
$\{f\}$	$\{f\}$	$\{f\}$



Prob: minimized by myhill Nerode Theorem

δ	a	b
$\rightarrow q_0$	q_1	q_4
q_1	q_2	q_3
q_2	q_7	q_8
q_3	q_8	q_7
q_4	q_5	q_6
q_5	q_7	q_8
q_6	q_7	q_8
q_7	q_7	q_7
q_8	q_8	q_8

	q_0	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8
q_0	-	x	-	-	-	-	-	-	-
q_1	x	-	-	-	-	-	-	-	-
q_2	X	X	-	-	-	-	-	-	-
q_3	x	x	O	-	-	-	-	-	-
q_4	x	O	X	X	-	-	-	-	-
q_5	X	X	O	O	X	-	-	-	-
q_6	X	X	O	O	X	O	-	-	-
q_7	x	x	X	X	x	X	X	-	-
q_8	x	x	X	X	x	X	X	O	-

$Q' = \{\{q_0\}, \{q_1, q_4\}, \{q_2, q_3, q_5, q_6\} \{q_7, q_8\}\}$

δ	a	b
$\rightarrow \{q_0\}$	$\{q_1, q_4\}$	$\{q_1, q_4\}$
$\{q_1, q_4\}$	$\{q_2, q_3, q_5, q_6\}$	$\{q_2, q_3, q_5, q_6\}$
$\{q_2 q_3 q_5 q_6\}$	$\{q_7, q_8\}$	$\{q_7, q_8\}$
$\{q_7 q_8\}$	$\{q_7, q_8\}$	$\{q_7, q_8\}$