Regular Language / Reguler set : A set L is called Regular set if we design a DFA or NFA or Regular expression for this set.

Non Regular Language / Non Regular set :

A set L is called non Regular set if we can't desgin a DFA or NFA or R.E. for this set.

NOTE : (i) For proof the set is regular, design a DFA / NFA / R.E.

(ii) For Proof the set is non regular use the concept of pumping Lema.

(iii) Every finite set is Always regular set.

(iv) Every infinite set must contain cycle (loop) or any set contain cycle then it is infinite set.

(v) any infinite set may regular or nonregular.

Arden's Theorem:

Let P and Q be two regular expression over Σ. if P does not contain ε, then the following equition in R, namely

$$R = Q + RP \qquad \text{---} \quad (1).$$

has a unique salution given by $R = QP^*$.

Proof: in eq. (1) replac R by $Q + RP$ on the R.H.S.

we get the equation

$$R = Q + RP = Q + (Q + RP)P$$
$$= Q + QP + RP^2$$

now again replace R by $Q + RP$ on the R.H.S.

$$R = Q + QP + QP^2 + RP^3$$

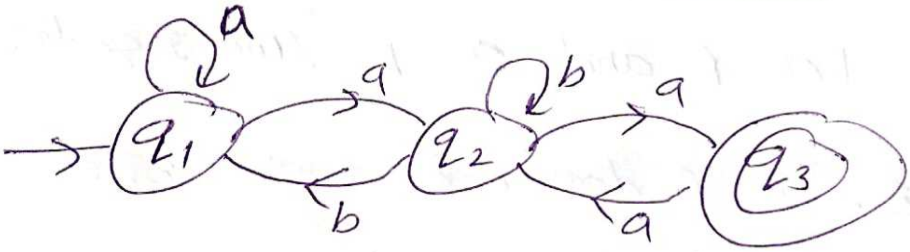
$$R = Q + QP + QP^2 + \cdots QP^i + RP^{i+1}$$
$$= Q(\varepsilon + P + P^2 + \cdots P^i) + RP^{i+1} \quad \text{for } i \geq 0$$
$$\text{---} \quad (2)$$

Now suppose any salution of R satisfies eq (1) then it must satisfy eq. (2).

Let w be a a string of length i in the set R then w must belongs to the set $Q(\varepsilon + P + P^2 + \cdots P^i) + RP^{i+1}$.

As P does not contain ε, RP^{i+1} has no string of length less than $i+1$ so w is not in the set RP^{i+1} This means that w belongs to the set $Q(\varepsilon + P + P^2 + P^3 + \cdots P^i)$. and it is equivalent to QP^*.

NFA without ε move or
DFA to Regular expression:



$$q_1 = q_1 a + q_2 b + \varepsilon \quad \text{— (I)}$$

$$q_2 = q_1 a + q_2 b + q_3 a \quad \text{— (II)}$$

$$q_3 = q_2 a \quad \text{— (III)}$$

by using eq. (II) & (III)

$$q_2 = q_1 a + q_2 b + (q_2 a) a$$

$$= q_1 a + q_2 (b + aa)$$

$$q_2 = q_1 a (b + aa)^* \quad \text{— (IV)}$$

by using eq. I & IV

$$q_1 = q_1 a + q_1 a (b+aa)^* b + \varepsilon$$

$$q_1 = q_1 (a + a(b+aa)^* b) + \varepsilon$$

$$q_1 = \varepsilon \cdot (a + a(b+aa)^* b)^* \quad \text{— (V)}$$

(by using Arden Th.)

by using eq. IV & V

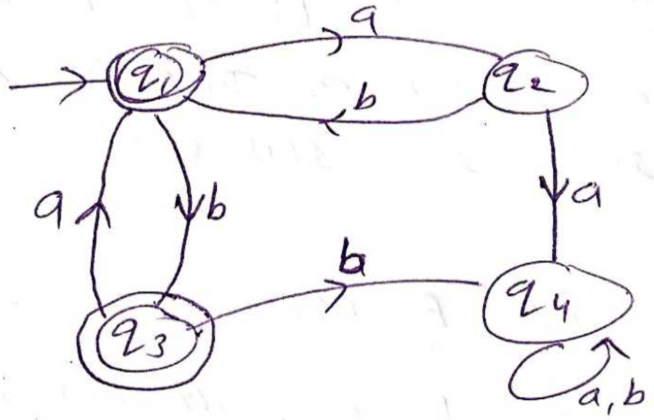
$$q_2 = (a + a(b+aa)^* b)^* a (b+aa)^* \quad \text{— (VI)}$$

by using eq. III & (VI)

$$q_3 = (a + a(b+aa)^* b)^* a (b+aa)^* a$$

since q_3 is final state, so the R.E. equivalent to FA is $(a + a(b+aa)^* b)^* a (b+aa)^* a$.

Q convert to R.E.



$$q_1 = q_2 b + q_3 a + \varepsilon \quad \text{— (I)}$$
$$q_2 = q_1 a \quad \text{— (II)}$$
$$q_3 = q_1 b \quad \text{— (III)}$$
$$q_4 = q_2 a + q_3 b + q_4 a + q_4 b \quad \text{— (IV)}$$

here q_1 and q_3 are final states so we find R.E. for both equation I & III and then union b/w them perform

by using eq. I, II, III

$$q_1 = q_1 ab + q_1 ba + \varepsilon$$
$$q_1 = q_1 (ab + ba) + \varepsilon$$
$$q_1 = \varepsilon \cdot (ab + ba)^*$$ (by using Arden Theorem)
$$q_1 = (ab + ba)^* \quad \text{— (V)}$$

by using eq. (III) & (V)

$$q_3 = (ab + ba)^* b \quad \text{— (VI)}$$

So R.E. for this DFA is

$$= (ab + ba)^* + (ab + ba)^* b$$
$$= (ab + ba)^* (\varepsilon + b)$$