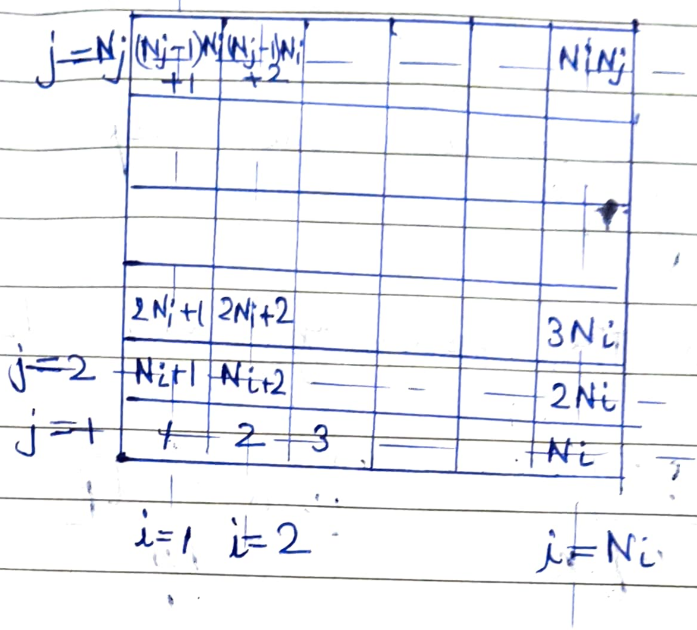
Grid Detail :



Given parameters : $N_i = 128$

$N_j = 128$

Length of domain $L_i = 1$

Width of domain $L_j = 1$

Since, we follow Lexicographic ordering of control volumes, the control volumes are numbered as : $1, 2, 3, \dots (N_j - 1)N_i + 1, \dots N_i N_j$

Hence, we represent the points on the grid using a 1D array of size $N_i N_j \times 1$.

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Discretized Equation detail:

Given equation: $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$

This can be written as $\nabla \cdot (\nabla T) = 0$.

Integrating this equation over the control volume we can write,

$$\int_{CV} \nabla \cdot (\nabla T) = 0$$

Using Gauss-divergence theorem we can rewrite this as:

$$\oint_{CS} \nabla T \cdot \hat{n}\, dA = 0$$

We now split the integral into the individual boundaries,

$$\int_e \nabla T \cdot \hat{i}\, A_e + \int_n \nabla T \cdot \hat{j}\, A_n + \int_w \nabla T \cdot (-\hat{i})\, A_w$$
$$+ \int_s \nabla T \cdot (-\hat{j})\, A_s = 0$$

Here $\nabla T = \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j}$

$\therefore \quad \nabla T \cdot \hat{i} = \frac{\partial T}{\partial x} \quad \& \quad \nabla T \cdot \hat{j} = \frac{\partial T}{\partial y}$

Therefore, our equation can be written as –

$$\frac{\partial T}{\partial x}\Big|_e A_e - \frac{\partial T}{\partial x}\Big|_w A_w + \frac{\partial T}{\partial y}\Big|_n A_n - \frac{\partial T}{\partial y}\Big|_s A_s = 0$$

Replacing the derivatives with their central difference approximations, we can write,

$$\frac{(T_E - T_P)}{\Delta x} A_e - \frac{(T_P - T_W)}{\Delta x} A_w + \frac{(T_N - T_P)}{\Delta y} A_n - \frac{(T_P - T_S)}{\Delta y} A_s = 0$$

Rearranging this equation we can write

$$T_P\left(\frac{A_e}{\Delta x} + \frac{A_w}{\Delta x} + \frac{A_n}{\Delta y} + \frac{A_s}{\Delta y}\right) = \frac{A_e}{\Delta x} T_E + \frac{A_W}{\Delta x} T_W + \frac{A_N}{\Delta y} T_N + \frac{A_S}{\Delta y} T_S$$

let $a_E = \frac{A_e}{\Delta x}$ $a_W = \frac{A_w}{\Delta x}$ $a_N = \frac{A_n}{\Delta y}$ $a_S = \frac{A_s}{\Delta y}$

$$a_P = a_E + a_W + a_N + a_S$$

$$\Rightarrow \boxed{a_P T_P = a_E T_E + a_W T_W + a_S T_S + a_N T_N}$$

Boundary Condition Detail:

Bottom wall: Given $T_{Bottom} = 0$

In the eq^n, $\left.\frac{\partial T}{\partial x}\right|_e A_e - \left.\frac{\partial T}{\partial x}\right|_w A_w + \left.\frac{\partial T}{\partial y}\right|_n A_n$

$$- \left.\frac{\partial T}{\partial y}\right|_s A_s = 0$$

The term term $\left.\frac{\partial T}{\partial y}\right|_s$ at first row (cell nos - 1,2 ... Ni) can be written as:

$$\left.\frac{\partial T}{\partial y}\right|_s = \frac{T_p - T_{wall}}{\Delta y/2} = 2\left(\frac{T_p - 0}{\Delta y}\right)$$

$$= \frac{2T_p}{\Delta y}$$

The other terms can be discretized in the earlier manner, we get

$$\frac{(T_E - T_p)A_e}{\Delta x} \frac{(T_p - T_w)A_w}{\Delta x} + \frac{(T_N - T_p)A_N}{\Delta y} - \frac{2T_p}{\Delta y} A_s = 0$$

Rearranging the terms we can write,

$$T_p\left(\frac{A_e}{\Delta x} + \frac{A_w}{\Delta x} + \frac{A_N}{\Delta y} + \frac{2A_s}{\Delta y}\right) = \frac{A_e}{\Delta x}T_e + \frac{A_w}{\Delta x}T_w + \frac{A_N}{\Delta y}T_N$$

$$\Rightarrow T_p(a_E + a_W + a_N + S_p) = a_E + a_W + a_N$$

$$S_p = \frac{2A_s}{\Delta y} \quad \& \quad a_S = 0$$

Since the Left and Right walls also have a similar boundary conditions, $T_{wall} = 0$

we can proceed in a similar manner and write

For left wall:

$$\frac{(T_E - T_P)A_E}{\Delta x} - \frac{2T_P}{\Delta x}A_W + \frac{(T_N - T_P)A_N}{\Delta y} - \frac{(T_P - T_S)}{\Delta y}A_S = 0$$

Rearranging the terms,

$$T_P\left(\frac{A_E}{\Delta x} + \frac{A_N}{\Delta y} + \frac{A_S}{\Delta y} + \frac{2A_W}{\Delta x}\right) = \frac{A_E}{\Delta x}T_E + \frac{A_N}{\Delta y}T_N + \frac{A_S}{\Delta y}T_S$$

$$T_P(a_E + a_N + a_S + S_P) = a_E T_E + a_N T_N + a_S T_S$$

$$S_P = \frac{2A_W}{\Delta x} \quad \& \quad a_W = 0$$

For Right wall:

Using a similar approach, we get

$$T_P\left(\frac{A_W}{\Delta x} + \frac{A_N}{\Delta y} + \frac{A_S}{\Delta y} + \frac{2A_E}{\Delta x}\right) = \frac{A_W}{\Delta x}T_W + \frac{A_N}{\Delta y}T_N + \frac{A_S}{\Delta y}T_S$$

$$T_P(a_W + a_N + a_S + S_P) = a_W T_W + a_N T_N + a_S T_S$$

$$S_P = \frac{2A_E}{\Delta x} \quad \& \quad a_E = 0$$

For Top Wall : Given $T_{top} = 1.0$

The term $\left.\frac{\partial T}{\partial y}\right|_n$ at the top wall can be replaced with the following finite difference approx^n:

$$\left.\frac{\partial T}{\partial y}\right|_n = \frac{T_{wall} - T_P}{\frac{\Delta y}{2}} = \frac{2(1 - T_P)}{\Delta y}$$

$$= \frac{2 - 2T_P}{\Delta y}$$

Substituting this value in the earlier equation, we can write,

$$\frac{2(T_E - T_P)A_e}{\Delta x} - \frac{(T_P - T_W)A_w}{\Delta x} + \frac{(2 - 2T_P)A_N}{\Delta y} - \frac{(T_P - T_S)A_S}{\Delta y} = 0$$

Rearranging the terms,

$$T_P\left(\frac{A_e}{\Delta x} + \frac{A_w}{\Delta y} + \frac{2A_N}{\Delta y} + \frac{A_S}{\Delta y}\right) = \frac{A_e}{\Delta x}T_E + \frac{A_W}{\Delta x}T_W + \frac{2A_N}{\Delta y} + \frac{A_S}{\Delta y}T_S$$

$$T_P(a_E + a_W + a_S + S_P) = a_E T_E + a_W T_W + a_S T_S + S_U$$

$$S_P = \frac{2A_N}{\Delta y} \qquad S_U = \frac{2A_N}{\Delta y} \qquad a_N = 0$$