ME351: 2024–25 - II
COURSE PROJECT REPORT
Reverse Engineering of a Mechanical Pencil

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Group Members: Contribution and Signature Sheet

Name	Contribution	Signature
Ayush Dwivedi	Creation of detailed CAD models for parts and complete assembly of the mechanical pencil.	
Raushan Kumar	Development of part and assembly drawings using Fusion 360 software.	
Rudradeep Datta	Analytical study of the spring including calculation of the factor of safety and the chance of fatigue failure.	
Navnit Patel	Theoretical calculations of buckling and surge avoidance in the spring.	
Sahil Kumar	Factor of safety analysis and stress assessment of the pencil sleeve under operational conditions.	
Soumyadeb Saha	Yield and fatigue strength analysis of the eraser holder component.	
Devraj Mathur	Precise measurement of mechanical pencil components using Vernier calipers and literature research.	
Keyansh Vaish	Brittle fracture analysis of the mechanical pencil lead using theoretical and empirical approaches.	

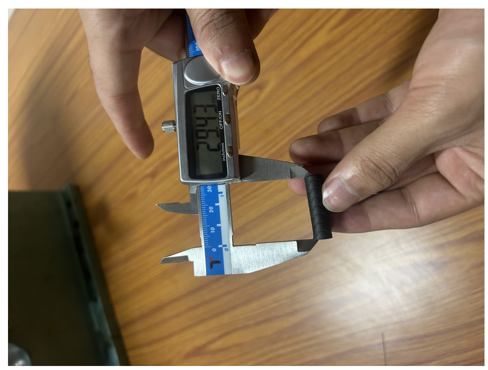
Contents

1 Measurement using Vernier Callipers

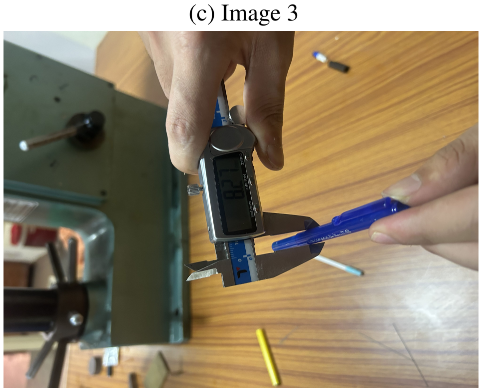
(a) Image 1(Assembled)

(b) Image 2(Disassembled)

(c) Image 3



(d) Image 4



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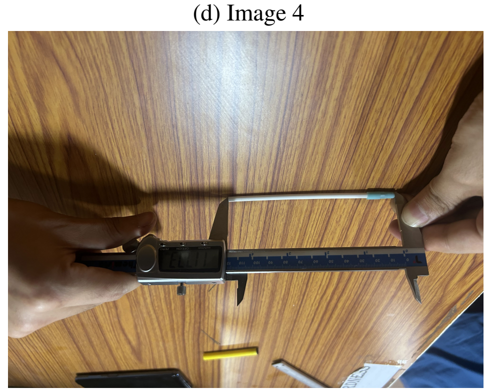

(f) Image 6

(a) Image 7

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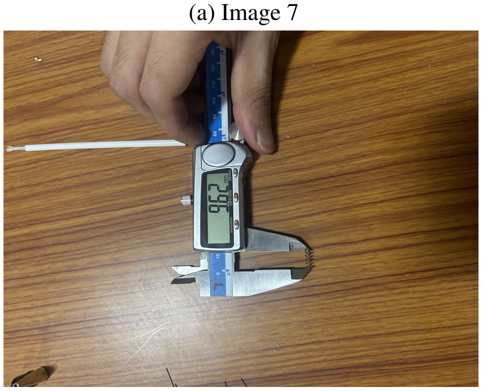

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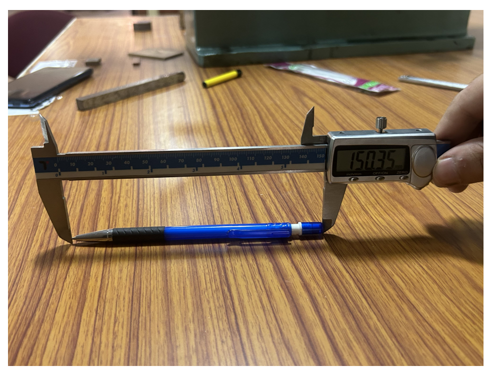
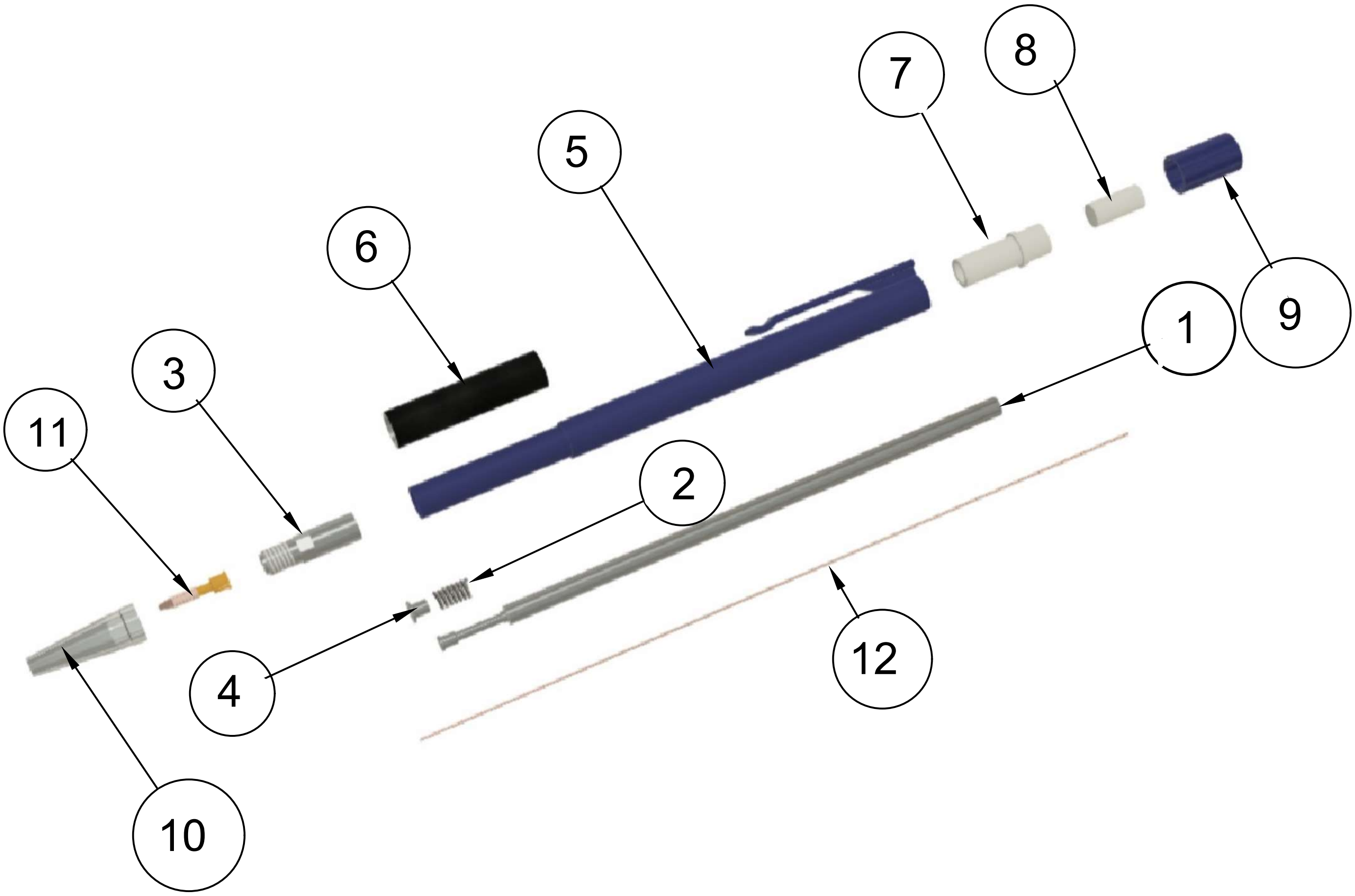


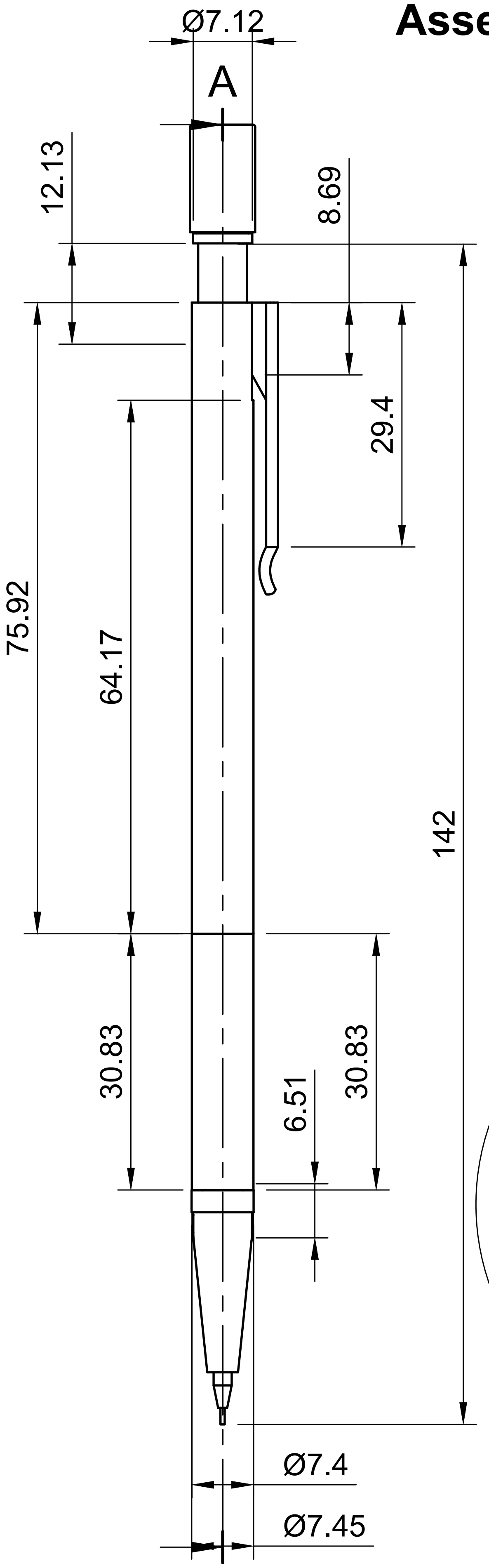
(f) Image 12



Parts List			
Item	Qty	Part Name	Material
1	1	Lead Dispenser	Polypropylene
2	1	Spring	Music Wire
3	1	Shaft Dispenser	Polypropylene
4	1	Dispenser Ring	Copper
5	1	Sleeve	Polypropylene
6	1	Grip	Thermoplastic
7	1	Eraser Holder	Polypropylene
8	1	Eraser	Rubber
9	1	Push Button	Polypropylene
10	1	Nose Cone	Stainless Steel
11	1	Lead Guide	Stainless Steel
12	1	Lead	Graphite

Assembly Drawing

All Dimensions are in mm



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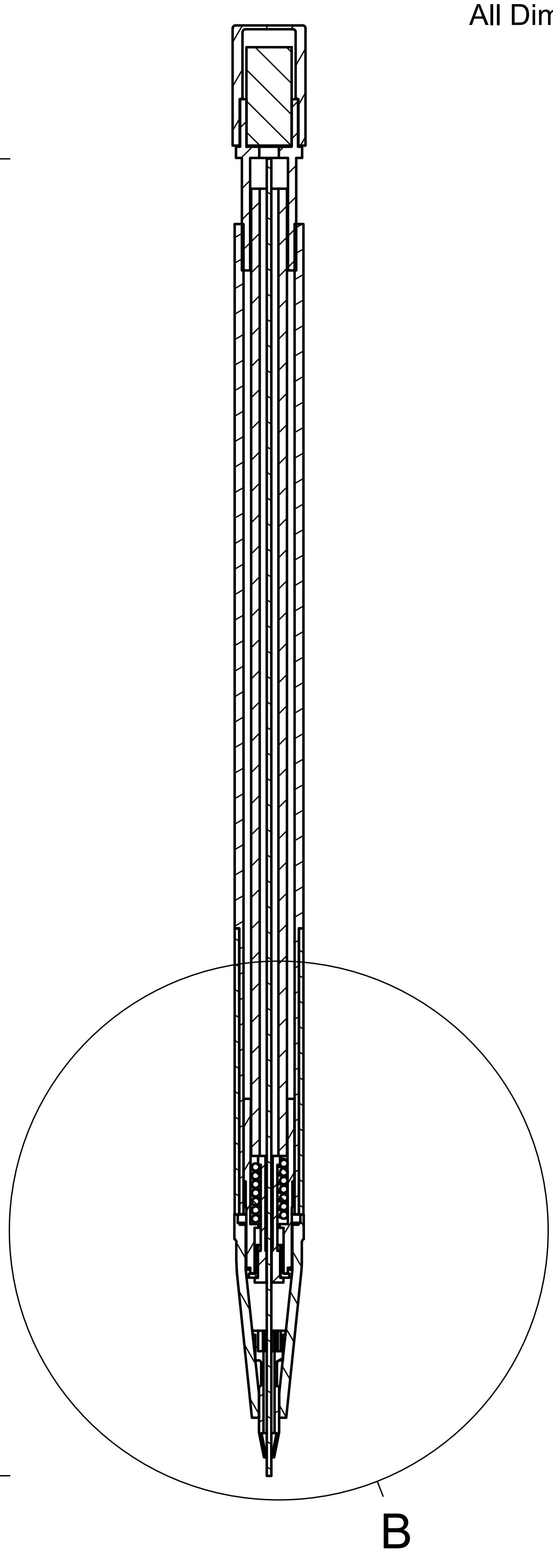

Section A-A

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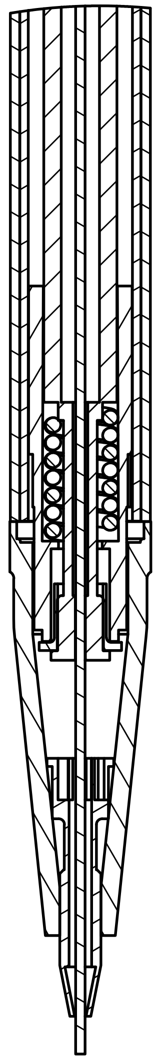

B-B

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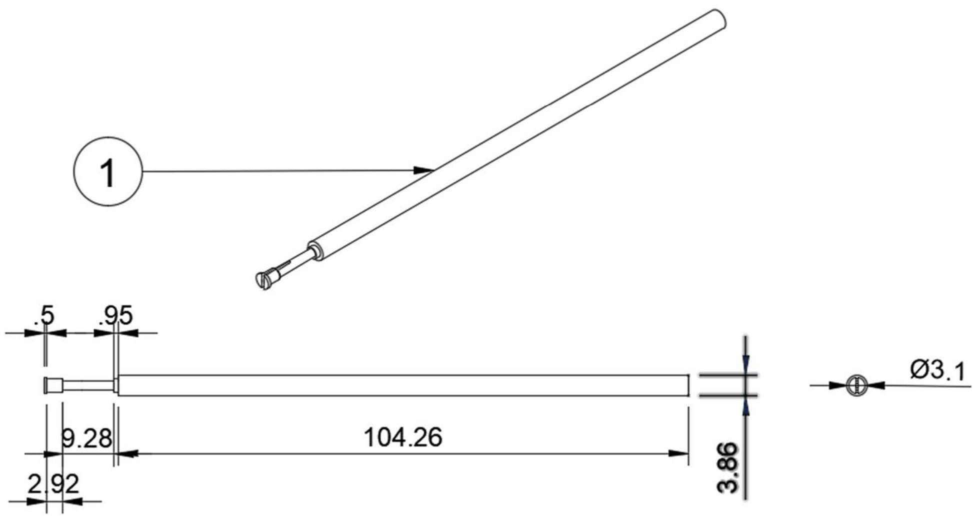
Part Drawing

All Dimensions are in mm



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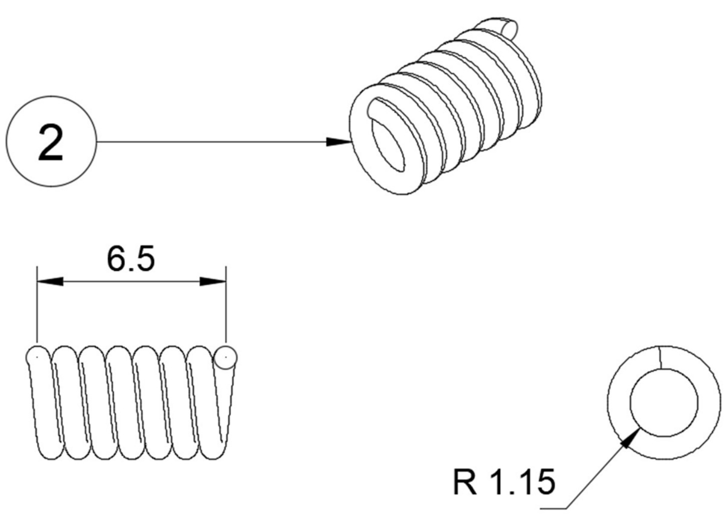

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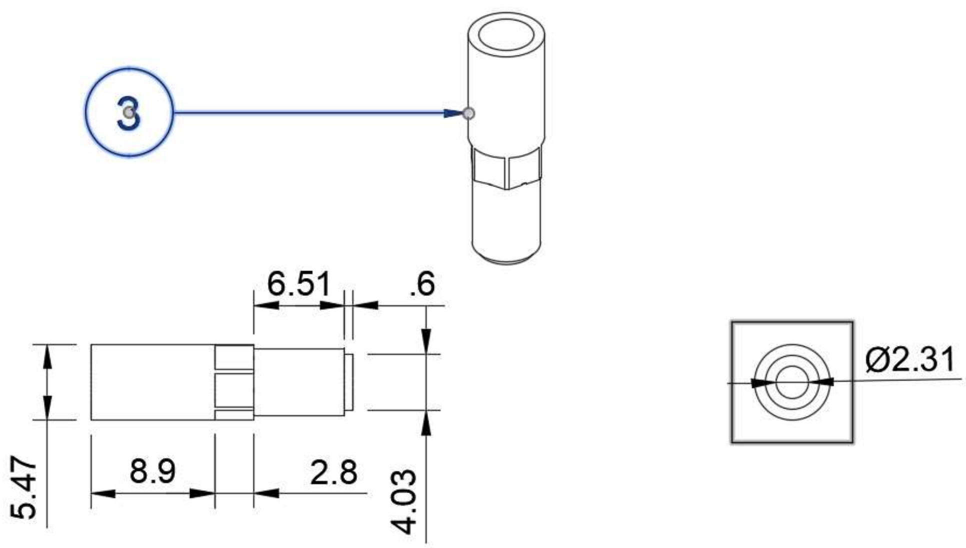
Part Drawing



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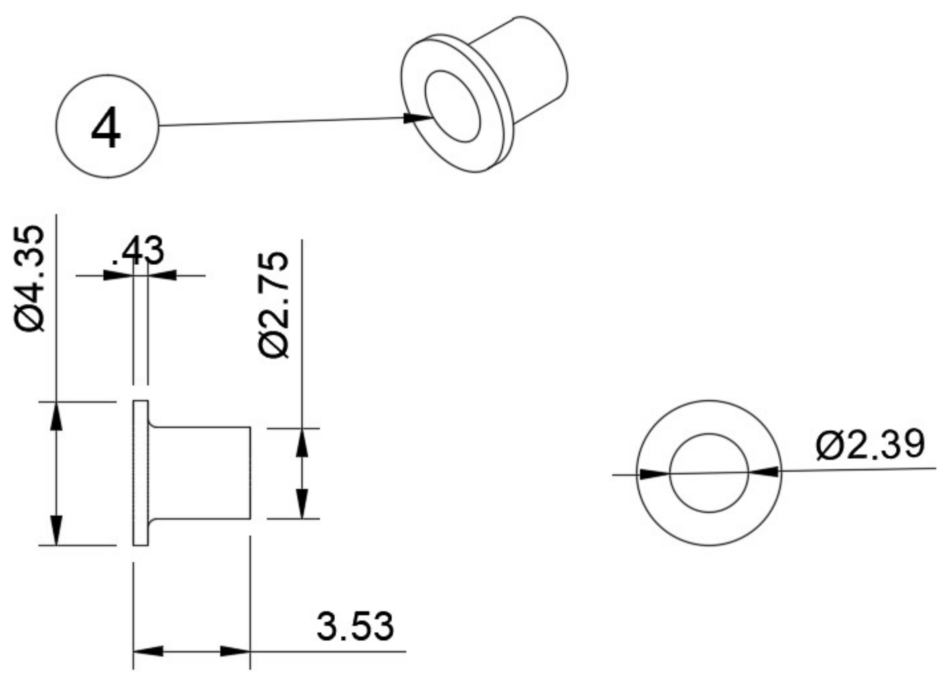

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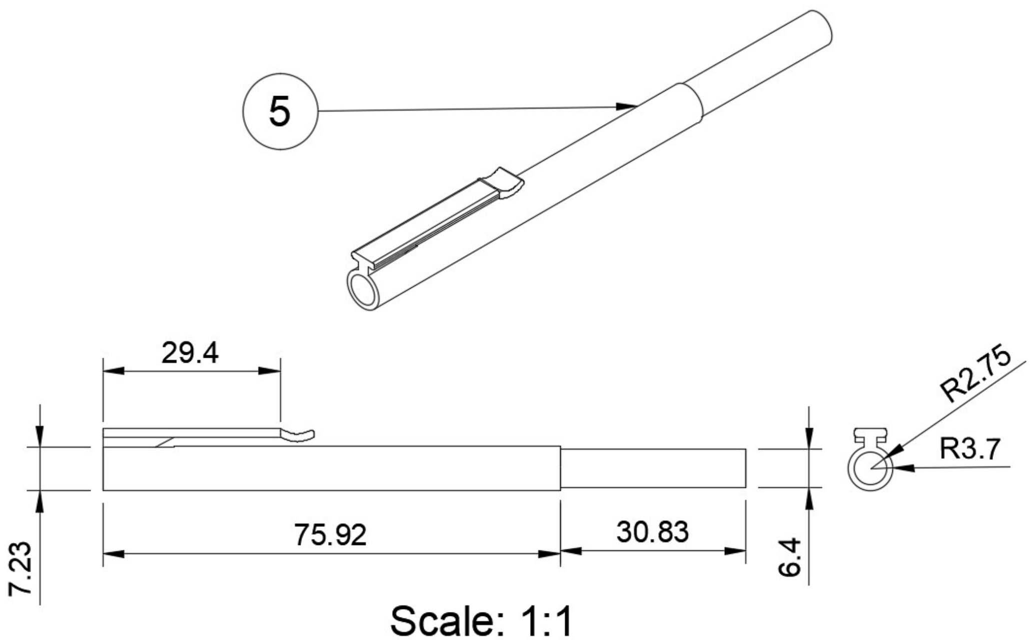
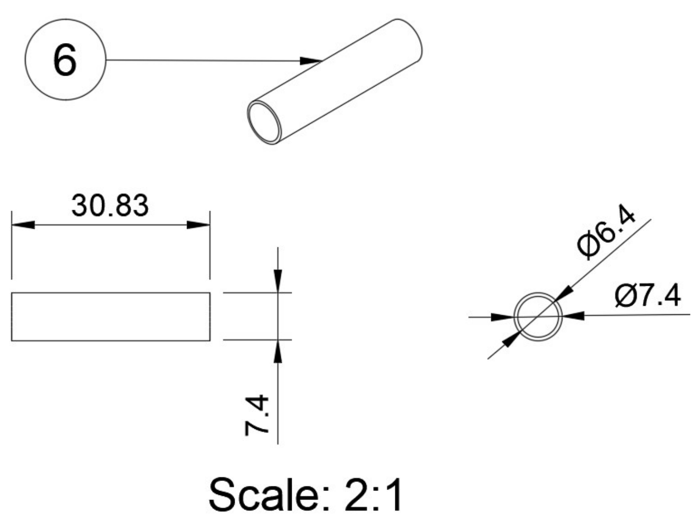
Part Drawing





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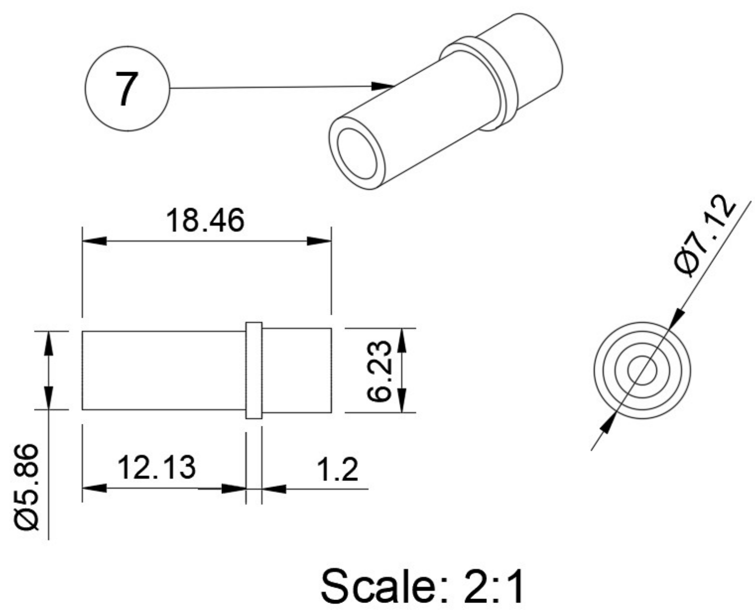
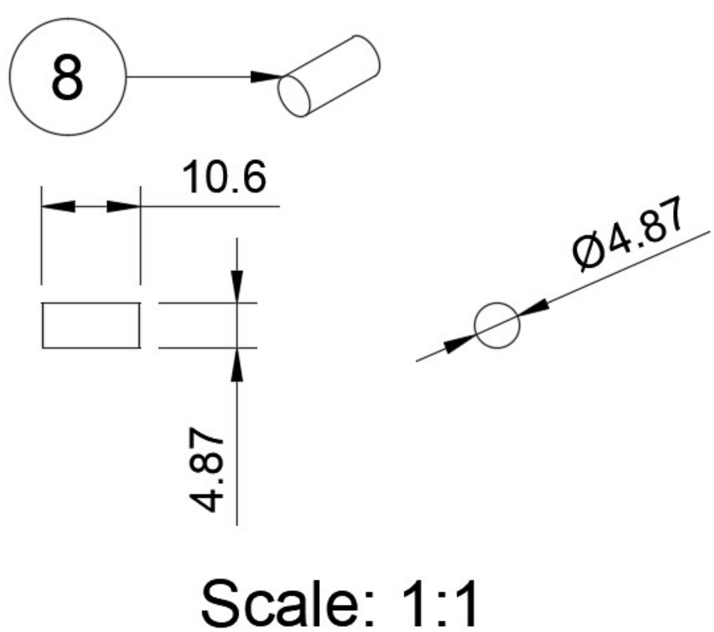
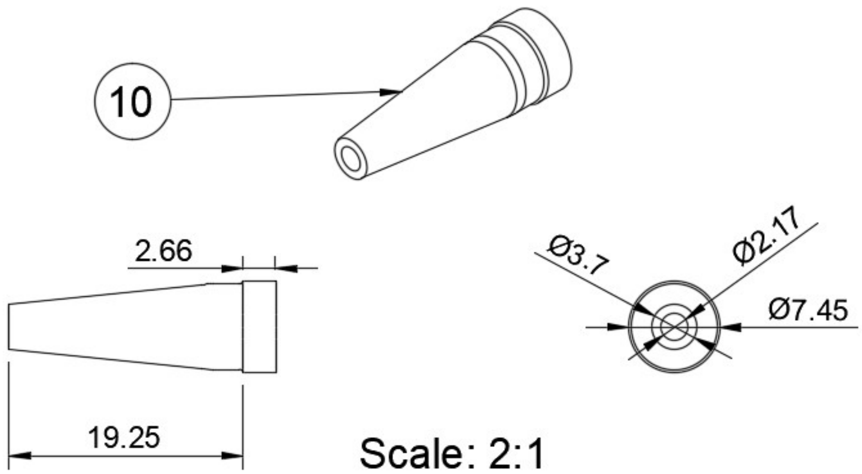




Part Drawing



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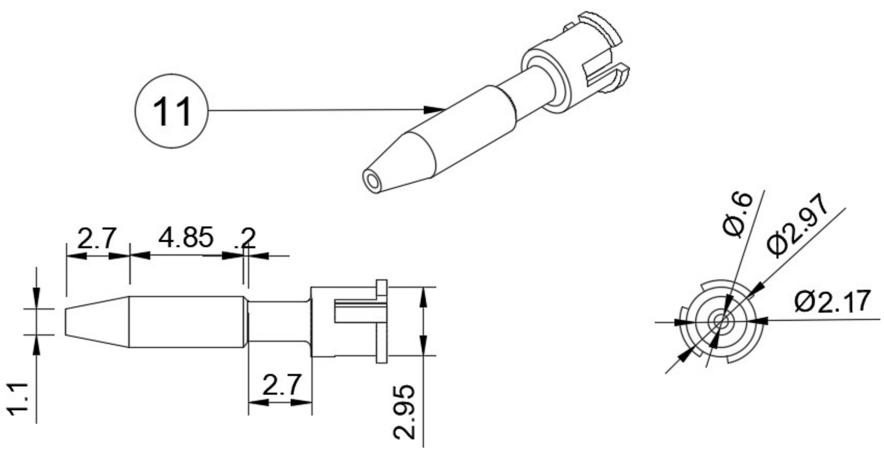

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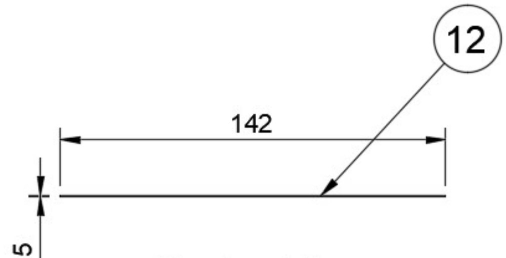

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3 Functional Analysis and descriptions of Mechanical Pencil Components

Nose Cone

The nose cone, a conical part made from SUS 304 stainless steel, is threaded to attach securely to the front of the pencil barrel. Its main function is to keep the lead sleeve aligned coaxially, ensuring precise contact with the paper without wobbling.

It also protects the internal chuck mechanism—absorbing impact if excess pressure is applied while writing, preventing direct force on the chuck jaws.

The cone is screwed into the barrel's internal threads, with torque carefully controlled to ensure a firm fit while allowing easy removal for maintenance or replacement.

SUS 304 steel provides a yield strength (Syt) of 205 MPa, ultimate tensile strength (Sut) of 515 MPa, Young's modulus of 193 GPa, and shear modulus of 86 GPa.

Lead Guide

The lead guide is a small, precision-engineered component made of steel with a chrome finish. It is located at the front of the pencil, just behind the nose cone. Its primary function is to keep the graphite lead centered within the shaft and properly aligned with the writing surface. The chrome coating minimizes friction, allowing the lead to move smoothly while reducing wear and preventing jamming.

This component plays a critical role in supporting the lead during writing to prevent breakage. It works in coordination with the cone and chuck mechanism to stabilize the lead and guide its advancement through the pencil.

Barrel (Sleeve)

The pencil barrel, whether cylindrical or faceted, is ergonomically designed to suit grip preferences. It is typically manufactured from polypropylene using injection molding techniques. This thermoplastic material is lightweight, cost-effective, and can be molded with textures or scale markings for added usability.

Internally, the barrel contains a chamber about 30–40 mm deep, with integrated ribs that help in aligning spare leads and reducing internal rattling. Additionally, the molding includes features such as bosses, which facilitate the attachment of the nose cone and the installation of the pocket clip.

Mechanically, the polypropylene barrel exhibits a yield strength (S_{yt}) 40 MPa, and a Young's modulus of approximately 1.3 GPa.

Grip

The grip, made from molded thermoplastic elastomer (TPE), is typically 20–30 mm long and features ribbing or knurled textures for better hold. Located just above the lead sleeve, it distributes finger pressure evenly, offering comfort and stability during extended use.

With a Shore A hardness of 50 to 70, the grip absorbs micro-vibrations and reduces hand fatigue. Its rubberized surface increases friction, improving control.

Overmolded with a bond strength resisting over 5 N of pull, the grip stays firmly attached. TPE material properties include a tensile strength of 5–30 MPa, yield strength of 3–20 MPa, and a Young's modulus between 5 and 50 MPa.

Pocket Clip

The pocket clip is either metallic or plastic and is attached externally to the barrel. Its primary function is to secure the pencil to clothing, notebooks, or folders, allowing for portability. It is engineered to endure over 5,000 insertion and removal cycles without experiencing permanent deformation.

Eraser Cap (Push Button)

The eraser cap serves two functions: it acts as a protective cover for the eraser and operates as the actuator for the lead advancement mechanism. When pressed, it activates the internal system responsible for propelling the lead forward. The typical stroke length is between 1.5 mm and 2 mm. The cap is often designed with fluting or knurling to provide better grip during use.

Eraser and Eraser Holder

The eraser is a cylindrical rod made of synthetic rubber (such as vinyl or PVC) with a diameter of approximately 4 mm and a length ranging from 10 to 12 mm. It is optimized for effective removal of graphite without damaging paper surfaces.

The eraser holder, a thin metallic sleeve, grips the eraser and may include a small set screw or a plastically deformed collar to keep it firmly in place.

Lead Dispenser

The lead dispenser comprises the shaft and the chuck mechanism. The shaft, an internal rod, links the eraser cap or push button to the chuck, transmitting force during pressing to advance the lead.

The chuck, made of polypropylene (Syt = 40 MPa, E = 1.3 GPa), consists of two spring-loaded jaws near the pencil tip. At rest, the jaws grip the lead; pressing the button lifts the chuck, causing the jaws to open and advance the lead. Releasing the button allows the jaws to close and hold the new lead segment in place.

Spring

The spring is a coiled component that surrounds the shaft and plays a crucial role in resetting the mechanism after each click. It is made from music wire and functions by returning the eraser cap or button to its original position after being pressed. It also provides the necessary tension to drive the clutch mechanism consistently.

The mechanical properties of music wire include an ultimate tensile strength of

$$S_{ut} = \frac{A}{d^m} = 2596.60\,\mathrm{MPa},$$

where A and m are material-specific constants and d is the wire diameter. The Young's modulus is $207\,\mathrm{GPa}$, and the shear modulus is $80\,\mathrm{GPa}$.

Dispenser Ring

The dispenser ring is a thin annular collar located just behind the chuck assembly. It is designed to fit around the pencil's internal shaft and usually sits in a groove formed into the shaft. This component assists in the guidance and locking of the chuck mechanism during the push-release cycle, ensuring smooth and consistent lead advancement.

Graphite Lead

The writing core of the mechanical pencil consists of a rod made from a composite of compressed graphite and clay, sometimes supplemented with wax or resin to improve writing smoothness. These leads are available in various diameters, such as 0.5 mm, and come in multiple hardness grades.

As the user writes, the lead gradually wears down and deposits graphite onto the paper. Mechanically, this graphite-clay composite is relatively brittle, with a Young's modulus of about 10 GPa and a yield/ultimate tensile strength in the range of 20–40 MPa.

Summary of Material Properties

Component(s)	Material	Mechanical Properties
Nose Cone	SUS 304 Stainless Steel	Yield Strength: 205 MPa Ultimate Tensile Strength: 515 MPa Young's Modulus: 193 GPa Shear Modulus: 86 GPa
Spring	Music Wire	Ultimate Tensile Strength: 2596.60 MPa Young's Modulus: 207 GPa Shear Modulus: 80 GPa
Barrel, Chuck	Polypropylene (PP)	Yield Strength: 40 MPa Young's Modulus: 1.3 GPa
Grip	Thermoplastic Elastomer (TPE)	Tensile Strength: 5–30 MPa Yield Strength: 3–20 MPa Young's Modulus: 5–50 MPa Shore A Hardness: 50–70
Graphite Lead	Graphite-Clay Composite	Young's Modulus: 10 GPa Yield/Ultimate Strength: 20–40 MPa

Table 2: Summary of Mechanical Pencil Components and Their Material Properties

Component	During Writing	While Advancing Lead	During Erasing
Lead	Experiences friction and contact force while pressing	Encounters friction in the dispenser and guidance system	No significant effect
Lead Guide	Provides support at the pencil tip	Prevents bending; ensures straight lead advancement	Not involved
Collet	Transfers force from lead guide	Receives force from lead dispenser, especially where screw is present	No role
Spring	No effect	Under axial force, causing a twisting effect	No role
Grip Area	Subject to friction and pressure from fingers	Not impacted	Not impacted
Sleeve	Feels reaction force from hand holding the pencil's weight	Compressed by finger pressure	Normal force from fingers; typically higher than writing
Eraser Holder	Not active	Compressed when eraser is pushed	Repeated loading due to rubbing motion
Lead Dispenser	Provides counter force to the lead	Transmits force from eraser holder; friction helps push lead out	Not active

Table 3: Functional roles of mechanical pencil components under different usage scenarios

4 Brittle Fracture Analysis of Mechanical Pencil Lead

Objective: To estimate the stress components and assess the brittle fracture behavior of a mechanical pencil lead subjected to combined axial and bending forces, considering the lead as a circular brittle shaft.

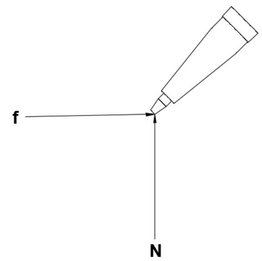

Figure 3: N is normal reaction and f represents friction force

Equilibrium and Stress Components

To determine internal forces, we begin with equilibrium in the horizontal plane. Assuming contact friction μ and a contact angle θ, the net force components are:

$$F_{xx} = -N(\sin\theta + \mu\cos\theta)$$
$$F_{xy} = N(\cos\theta - \mu\sin\theta)$$
$$F_{xz} = \mu N$$

Here, N is the normal reaction force. As analysis is confined to the x-y plane, F_{xz} is not used further.

Stress Due to Bending and Axial Load

Assuming failure initiates at point A (maximum stress location), we evaluate normal stress σ_{xx} due to axial load and bending moment M_{xy}:

$$\sigma_{xx} = \frac{F_{xx}}{\pi r^2} + M_{xy}\left(\frac{R}{I_{xx}}\right)$$

Where: - $M_{xy} = F_{xy} \cdot l$ and $I_{xx} = \frac{\pi d^4}{64}$, the moment of inertia for a circular cross-section

Shear stress is evaluated at the same point using:

$$\tau_{xy} = \frac{F_{xy}}{\pi r^2} + \frac{VQ}{Ib} \quad \text{but as } Q = 0, \quad \tau_{xy} = \frac{F_{xy}}{A}$$

Total Stress Components

Total stresses are derived from internal forces, considering geometry and load conditions:

$$\sigma_{xx} = -\left(\frac{N\sin\theta + \mu N\cos\theta}{\pi r^2}\right) - \frac{(N\cos\theta - \mu N\sin\theta)\cdot l \cdot R}{\frac{\pi d^4}{64}}$$

An additional torsional shear stress expression is introduced:

$$T_{xy} = -N(\cos\theta - \mu\sin\theta)\cdot\frac{1}{\pi r^4}$$

Assumptions and Given Parameters

For simplification and numerical estimation:

$$\theta \approx 45^\circ, \quad \cos\theta \approx \frac{1}{\sqrt{2}}, \quad l = 2\,\text{mm}, \quad \frac{d}{2} = r = 0.25\,\text{mm}, \quad \mu = 0.15$$

Stress Substitution and Simplification

Substituting values:

$$\sigma_{xx} = -\left(\frac{N\sin\theta + \mu N\cos\theta}{\pi r^2}\right) - \frac{(N\cos\theta - \mu N\sin\theta)\cdot l \cdot r}{\frac{\pi d^4}{64}}$$

$$= -\left(\frac{N\left(\frac{1}{\sqrt{2}} + 0.15\cdot\frac{1}{\sqrt{2}}\right)}{\pi(0.25\times10^{-3})^2}\right) - \frac{N\left(\frac{1}{\sqrt{2}} - 0.15\cdot\frac{1}{\sqrt{2}}\right)\cdot(2\times10^{-3})\cdot(0.25\times10^{-3})}{\frac{\pi(0.5\times10^{-3})^4}{64}}$$

$$= -\left(\frac{N\cdot\left(\frac{1+0.15}{\sqrt{2}}\right)}{\pi\cdot(0.25\times10^{-3})^2}\right) - \left(\frac{N\cdot\left(\frac{1-0.15}{\sqrt{2}}\right)\cdot(2\times10^{-3})\cdot(0.25\times10^{-3})}{\frac{\pi\cdot(0.5\times10^{-3})^4}{64}}\right)$$

$$= -\left(\frac{1.15N}{\sqrt{2}\cdot\pi\cdot(0.25\times10^{-3})^2}\right) - \left(\frac{0.85N\cdot(0.5\times10^{-6})}{\sqrt{2}\cdot\left(\frac{\pi\cdot(0.5\times10^{-3})^4}{64}\right)}\right)$$
$$= N106.16MPa$$

Similarly, $\tau_{xy} = N\cdot 3.10MPa$

Principal Stress Calculation

Normal and shear stresses are used to compute principal stress:

$$D_{xn} = -N_x(102.16\times10^6), \quad \tau_{xy} = -N\left[\frac{1}{2} - \frac{0.15}{52}\right] = N(3.11)\times10^6$$

$$\sigma_{1,2} = \frac{\sigma_{xx}}{2} \pm \sqrt{\left(\frac{\sigma_{xx}}{2}\right)^2 + (\tau_{xy})^2}$$

$$\sigma_{1,2} = \frac{-102.16}{2} \pm \sqrt{\left(\frac{102.16}{2}\right)^2 + (3.06)^2}$$
$$= -51.08 \pm 51.168$$

$$\begin{aligned} \sigma_1 &= 0.08\,\text{N} \\ \sigma_3 &= -102.24\,\text{N} \\ S_{ut} &= 30\,\text{MPa} \\ S_{uc} &= 150\,\text{MPa} \\ N &= 1 \end{aligned}$$

Using the modified Goodman relation:

$$\frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} = \frac{1}{n_f}$$

Substituting values:

$$\frac{0.08}{30} + \frac{102.24}{150} = \frac{1}{n_f}$$

$$0.00267 + 0.6816 = \frac{1}{n_f}$$

$$0.68427 = \frac{1}{n_f}$$

$$n_f = \frac{1}{0.68427} \approx 1.461$$

Factor of Safety, $n_f \approx 1.46$

5 Factor of Saftey Analysis of Spring

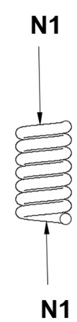

Figure 4: N1 is the applied axial force

5.1 Fatigue in Spring

In this study, we aim to evaluate the fatigue performance of a helical compression spring from a mechanical pencil. The primary goal is to compute the **factor of safety against fatigue failure**, based on the cyclic stresses developed during use. We utilize geometric measurements of the spring, estimate applied loading conditions, and apply fatigue failure theories such as the Goodman criterion.

Spring Parameters

The spring was measured using vernier calipers to have:

- Wire diameter, $d = 0.33\,\mathrm{mm}$
- Mean coil diameter, $D = 3.3\,\mathrm{mm}$

The spring index C, which represents the ratio of coil diameter to wire diameter, is given by:

$$C = \frac{D}{d} = \frac{3.3}{0.33} = 10$$

Shear Stress Calculation

The shear stress τ in the spring due to an axial force F is given by the following expression:

$$\tau = \frac{8FD}{\pi d^3} \cdot K_B$$

where:

- F: Axial load on the spring
- D: Mean coil diameter
- d: Wire diameter
- K_B: Bergsträsser correction factor for curvature

The Bergsträsser factor K_B accounts for the stress concentration due to the curvature of the spring wire and is defined as:

$$K_B = \frac{4C + 2}{4C - 3}$$

Substituting $C = 10$, we get:

$$K_B = \frac{4(10) + 2}{4(10) - 3} = \frac{42}{37} \approx 1.135$$

Hence, the expression for shear stress becomes:

$$\tau = \frac{8FD}{\pi d^3} \times 1.135$$

This expression will be used in the subsequent sections for fatigue analysis under cyclic loading.

Load Conditions and Stress Values

In practical usage, the spring is subjected to a cyclic load varying between a maximum and a minimum force:

- Estimated maximum load: $F_{\text{max}} = 1\,\text{N}$, approximated as the force required to actuate a mechanical pencil.
- Estimated minimum load (preload): $F_{\text{min}} = 0.05\,\text{N}$

Using the shear stress formula:

$$\tau = \frac{8FD}{\pi d^3} \cdot K_B$$

and substituting $F_{\text{max}} = 1\,\text{N}$, $F_{\text{min}} = 0.05\,\text{N}$, $D = 3.3\,\text{mm}$, $d = 0.33\,\text{mm}$, and $K_B = 1.135$, we compute:

$$\tau_{\text{max}} = 265.404\,\text{MPa}$$
$$\tau_{\text{min}} = 13.27\,\text{MPa}$$

These values represent the maximum and minimum shear stresses during the load cycle.

Mean and Alternating Shear Stress

From the maximum and minimum shear stresses, we calculate the mean stress τ_m and the alternating (or amplitude) stress τ_a as follows:

$$\tau_m = \frac{\tau_{\text{max}} + \tau_{\text{min}}}{2} = \frac{265.404 + 13.27}{2} = 139.337\,\text{MPa}$$
$$\tau_a = \frac{\tau_{\text{max}} - \tau_{\text{min}}}{2} = \frac{265.404 - 13.27}{2} = 126.067\,\text{MPa}$$

These values of τ_m and τ_a represent the stress conditions under cyclic loading, which are essential for evaluating fatigue life using fatigue failure criteria such as the Goodman approach.

Calculating ultimate shear strength

Since the wire diameter $d = 0.33\,\mathrm{mm} < 10\,\mathrm{mm}$, Zimmerli's fatigue data is applicable for estimating fatigue limits. Assuming the spring is unpeened, the endurance limits for shear are taken as:

$$S_{sa}^{z} = 241\,\mathrm{MPa}, \quad S_{sm}^{z} = 379\,\mathrm{MPa}$$

Assuming the spring is made from music wire, the ultimate tensile strength S_{ut} is estimated using the empirical relation:

$$S_{ut} = \frac{A}{d^m}$$

where:

- $A = 2211\,\mathrm{MPa\,mm}^m$ is a material constant specific to music wire,
- $m = 0.145$ is the material exponent,
- $d = 0.33\,\mathrm{mm}$ is the wire diameter.

Substituting the values:

$$S_{ut} = \frac{2211}{(0.33)^{0.145}} = 2596.60\,\mathrm{MPa}$$

The ultimate shear strength S_{su} is approximated using:

$$S_{su} = 0.67 \times S_{ut}$$

$$S_{su} = 0.67 \times 2596.60\,\mathrm{MPa} = 1739.72\,\mathrm{MPa}$$

Endurance Limit Estimation Using Goodman Relation

To determine the endurance limit in shear (S_{se}), we use the linear Goodman relation of the form:

$$\frac{y}{x - S_{su}} = \frac{S_{sa}^{z}}{S_{sm}^{z} - S_{su}}$$

This simplifies to a linear equation:

$$y = -0.177x + 308.125$$

To find S_{se}, we evaluate this line at $x = 0$:

$$S_{se} = -0.177 \times 0 + 308.125 = 308.125\,\mathrm{MPa}$$

Factor of Safety for infinite life

The factor of safety n_f is calculated using the modified Goodman criterion for shear as:

$$n_f = \frac{1}{\left(\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}}\right)}$$

Substituting the given values:

$$n_f = \frac{1}{\left(\frac{126.067}{308.125} + \frac{139.337}{1739.72}\right)} = 2.044$$

Conclusion: The fatigue analysis of the mechanical pencil spring, based on measured geometry and estimated loading, indicates a **factor of safety of 2.044** under cyclic shear loading. This suggests that the spring is well within safe operating limits for infinite life, ensuring reliable performance during repeated use without risk of fatigue failure.

5.2 Buckling of Spring

A compression spring is susceptible to buckling when its deflection becomes excessive, particularly in slender configurations. To ensure the stability of the spring under loading, we evaluate the buckling criteria using geometric and material parameters.

The spring is made of music wire, with the following properties:

- Modulus of elasticity: $E = 207\,\mathrm{GPa}$
- Modulus of rigidity: $G = 80\,\mathrm{GPa}$
- Mean coil diameter: $D = 3.3\,\mathrm{mm}$
- Free length: $L_0 = 9.62\,\mathrm{mm}$

Assuming the spring is fixed at both ends, the end condition factor is taken as:

$$\alpha = 0.5$$

The constant C_2', which is used in determining the buckling susceptibility of the spring, is calculated as:

$$C_2' = 2\pi^2 \cdot \frac{E-G}{2G+E} = 2\pi^2 \times \frac{207-80}{2(80)+207} = 6.831$$

The effective slenderness ratio λ_{eff} is given by:

$$\lambda_{\text{eff}} = \frac{\alpha L_0}{D} = \frac{0.5 \times 9.62}{3.3} = 1.457$$

Next, we evaluate the critical stability factor:

$$\frac{C_2'}{\lambda_{\text{eff}}^2} = \frac{6.831}{(1.457)^2} = 3.218 > 1$$

Since this value is greater than 1, the spring satisfies the first condition for buckling stability.

To further confirm stability, we compare the critical buckling length L_{cr} with the free length L_0. The critical buckling length is given by:

$$L_{\text{cr}} = \frac{\pi D \sqrt{2(E-G)/(2G+E)}}{\alpha}$$

Substituting the known values:

$$L_{\text{cr}} = \frac{\pi \times 3.3 \times \sqrt{2(207-80)/(2 \times 80 + 207)}}{0.5} = 17.249\,\mathrm{mm}$$

Since $L_{\text{cr}} = 17.249\,\mathrm{mm} > L_0 = 9.62\,\mathrm{mm}$, the second buckling criterion is also satisfied.

Conclusion: Based on both conditions, the spring in the mechanical pencil is *absolutely stable* with respect to buckling during operation.

5.3 Avoiding Spring Surge

Spring surge is a dynamic instability that can occur when the natural frequency of the spring matches the frequency of external cyclic loading, resulting in resonance and potentially damaging oscillations. To avoid spring surge, the natural frequency of the spring must be sufficiently higher than the operational excitation frequency.

The natural frequency f of the spring (in Hz), when both ends are in contact with rigid plates (i.e., fixed-fixed boundary condition), is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

where:

- k: Axial stiffness (spring constant) of the spring in $\mathrm{N\,m^{-1}}$
- m: Equivalent mass of the spring in kg

The stiffness k of a helical compression spring is defined as:

$$k = \frac{Gd^4}{8D^3N_a}$$

where:

- $G = 80 \times 10^9\,\mathrm{Pa}$: Modulus of rigidity of music wire
- $d = 0.33\,\mathrm{mm}$: Wire diameter
- $D = 3.3\,\mathrm{mm}$: Mean coil diameter
- $N_a = 5$: Number of active coils

The number of active coils N_a is obtained from the total number of coils N_t, adjusted for squared and ground ends:

$$N_a = N_t - 2 = 7 - 2 = 5$$

The mass m of the spring, assuming a uniform distribution and density ρ of steel, is calculated using:

$$m = \frac{\pi^2 d^2 D N_a \rho}{4}$$

where:

- $\rho = 7850\,\mathrm{kg\,m^{-3}}$: Density of music wire

Substituting the values, we compute:

$$k = 660\,\mathrm{N\,m^{-1}}, \quad m = 3.480\,\mathrm{kg}$$

Hence, the natural frequency of the spring is:

$$f = \frac{1}{2}\sqrt{\frac{660}{3.480}} = 5.565\,\mathrm{Hz}$$

Conclusion: Since the operational excitation frequency of the spring in a mechanical pencil is expected to be significantly lower than $5.565\,\mathrm{Hz}$, the spring is not susceptible to surge. Therefore, spring surge is not a concern in this application.

6 Factor of Safety Analysis of Sleeve

Geometry and Material Properties

Let the outer barrel of the mechanical pencil be modeled as a thin-walled hollow cylinder with:

$$\begin{aligned} d_o &= \text{outer diameter}, \\ t &= \text{wall thickness}, \\ d_i &= d_o - 2\,t = \text{inner diameter}, \\ A &= \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \text{cross-sectional area}, \\ I &= \frac{\pi}{64}\left(d_o^4 - d_i^4\right) = \text{second moment of area}, \\ c &= \frac{d_o}{2} = \text{distance to outer fiber}, \\ S &= \frac{I}{c} = \text{section modulus}, \\ S_{\text{yt}} &= \text{material yield strength}. \end{aligned}$$

Load Identification

We consider two primary loading modes:

- **Axial (compressive) load:**

$$F = \text{writing pressure (N)}.$$

- **Bending load:**

$$M = F \times L, \quad L = \text{lever arm (distance from grip to tip)}.$$

Stress Computation

Axial Stress

$$\sigma_{\text{ax}} = \frac{F}{A}.$$

Bending Stress

$$\sigma_{\text{bend}} = \frac{M\,c}{I} = \frac{M}{S}.$$

Combined Stress

For ductile materials, the worst-case normal stress at the outer fiber is the sum:

$$\sigma_{\text{tot}} = \left|\sigma_{\text{ax}}\right| + \left|\sigma_{\text{bend}}\right|.$$

Factor of Safety

Define the factor of safety (FoS) as

$$\text{FoS} = \frac{S_{\text{yt}}}{\sigma_{\text{tot}}}.$$

Calculation

a. Observed data

$$\begin{array}{ll} \text{Material: Polypropylene}, & S_{\text{yt}} = 40\ \text{MPa}, \\ d_o = 7.10\ \text{mm}, & t = 1.04\ \text{mm} \ \Rightarrow\ d_i = 5.02\ \text{mm}, \\ F = 10\ \text{N}, & L = 43.590\ \text{mm}. \end{array}$$

b. Cross-Sectional Properties

$$A = \frac{\pi}{4}\left(7.10^2 - 5.02^2\right)\,\mathrm{mm}^2 = \frac{\pi}{4} \times 25.210 \approx 19.790\,\mathrm{mm}^2,$$
$$I = \frac{\pi}{64}\left(7.10^4 - 5.02^4\right)\,\mathrm{mm}^4 = \frac{\pi}{64} \times 1906.108 \approx 93.518\,\mathrm{mm}^4,$$
$$c = \frac{7.10}{2} = 3.55\,\mathrm{mm}, \quad S = \frac{I}{c} = \frac{93.518}{3.55} \approx 26.343\,\mathrm{mm}^3.$$

c. Stress Calculations

$$\sigma_{\mathrm{ax}} = \frac{10\,\mathrm{N}}{19.790 \times 10^{-6}\,\mathrm{m}^2} \approx 0.505\,\mathrm{MPa},$$
$$M = 10\,\mathrm{N} \times 43.590\,\mathrm{mm} = 435.9\,\mathrm{N}\cdot\mathrm{mm},$$
$$\sigma_{\mathrm{bend}} = \frac{M}{S} = \frac{435.9\,\mathrm{N}\cdot\mathrm{mm}}{36.343\,\mathrm{mm}^3} \approx 11.994\,\mathrm{MPa},$$
$$\sigma_{\mathrm{tot}} = 0.505 + 11.994 = 12.499\,\mathrm{MPa}.$$

d. Factor of Safety

$$\mathrm{FoS} = \frac{40\,\mathrm{MPa}}{12.499\,\mathrm{MPa}} \approx 3.20.$$

Interpretation: The barrel's factor of safety is about 3.20 for the assumed 10 N writing force and bending moment.

7 Yield and Fatigue Strength Analysis of Pencil Eraser Holder

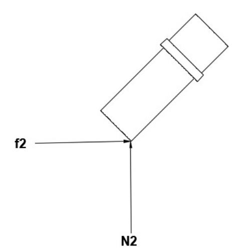

Figure 5: N2 is normal reaction and f2 represents friction force

Material Properties

$$S_{ut} = 30\,\mathrm{MPa}$$
$$S_{uc} = 40\,\mathrm{MPa}$$
$$S_e = 13\,\mathrm{MPa}$$

Stress Calculations

Given geometry:

$$r = 0.0036\,\mathrm{m}, \quad l = 0.02406\,\mathrm{m}, \quad a = 0.0026\,\mathrm{m}, \quad \mu = 0.15$$

Axial stress and Shear stress:

$$\sigma_{xx} = -\left[\frac{N\sin\theta - \mu N\cos\theta}{\pi r^2}\right] - \left[\frac{N\cos\theta + \mu N\sin\theta}{\pi r^4}\right] \cdot 4l(r)$$

$$\tau_{xy} = \frac{N\cos\theta + \mu N\sin\theta}{\pi r^2}$$

$$\sigma_{xx} = -N\left(\frac{\frac{1}{\sqrt{2}} - \frac{0.15}{\sqrt{2}}}{\pi(0.0036)^2}\right) - N\left(\frac{(\frac{1}{\sqrt{2}} + \frac{0.15}{\sqrt{2}}) \cdot (0.0206)(0.0036)}{\frac{\pi(0.0036)^4}{4}}\right)$$
$$= -N(0.54) \quad \text{(approx)}$$

$$\tau_{xy} = \frac{N\left(\frac{1}{\sqrt{2}} + \frac{0.15}{\sqrt{2}}\right)}{\pi(0.0036)^2}$$
$$= N(0.04)$$

Principal Stresses

$$\sigma_1 = \frac{-0.54 + \sqrt{(0.54)^2 + (0.039)^2}}{2} = 0.1138N$$
$$\sigma_2 = \frac{-0.54 - \sqrt{(0.54)^2 + (0.039)^2}}{2} = -0.654N$$

Yield Strength - Modified Mohr Coulomb

$$\frac{\sigma_1}{S_{ut}} - \frac{\sigma_2}{S_{uc}} = \frac{1}{n_d} \Rightarrow \frac{0.1138N}{30} + \frac{0.654N}{40} = \frac{1}{n_f}$$

Taking N = 10, we get n$_f \approx 4.96$

Fatigue Analysis - Goodman Criterion

$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = \frac{1}{n_d}$$

Where:

$$S_m = \frac{\tau_{xy}}{2} = \frac{0.039}{2} = 0.02$$
$$S_a = \frac{\tau_{xy}}{2} = 0.02$$

$$\frac{0.02}{13} + \frac{0.02}{30} = \frac{1}{n_f} \Rightarrow n_f \approx 45.35$$

8 Conclusion

A comprehensive reverse engineering and mechanical analysis of a mechanical pencil has led to several important findings regarding the safety and durability of its key components:

- **Spring (Music Wire)**
  Subjected to cyclic shear loading.
  Demonstrated a high factor of safety (FoS = 2.04).
  Proven stable against buckling and free from spring surge.
  Indicates reliable performance over long-term, repeated use.
- **Graphite Lead**
  Characterized by brittleness and low tensile strength.
  Exhibited a relatively low FoS of approximately 1.46.
  Most likely to fail due to brittle fracture under combined axial and bending stresses.
  Identified as the most vulnerable component in the system.
- **Sleeve (Polypropylene, Hollow Cylinder)**
  Subjected to combined axial and bending loads.
  Achieved a FoS of about 3.20.
  Confirms strong structural integrity during typical writing conditions.
- **Eraser Holder**
  Evaluated for both yield and fatigue failure.
  High safety margins: FoS 4.96 (yield), FoS 45.35 (fatigue).
  Very unlikely to fail, even under frequent erasing cycles.

Overall Summary: The analysis confirms that the majority of the mechanical pencil's components are structurally sound and perform reliably under normal usage conditions. The spring, sleeve, and eraser holder each exhibit high factors of safety, indicating strong resistance to failure. However, the graphite lead—due to its material properties—remains the most failure-prone part and should be considered the primary limitation in terms of overall durability.

9 References

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