

Optional Lab: Gradient Descent for Logistic Regression

Goals

In this lab, you will:

- update gradient descent for logistic regression.
- explore gradient descent on a familiar data set

```
In [1]: import copy, math
import numpy as np
%matplotlib widget
import matplotlib.pyplot as plt
from lab_utils.common import dlc, plot_data, plt_tumor_data, sigmoid, compute_cost_logistic
from plt_quad.logistic import plt_quad_logistic, plt_prob
plt.style.use('./deeplearning.mplstyle')
```

Data set

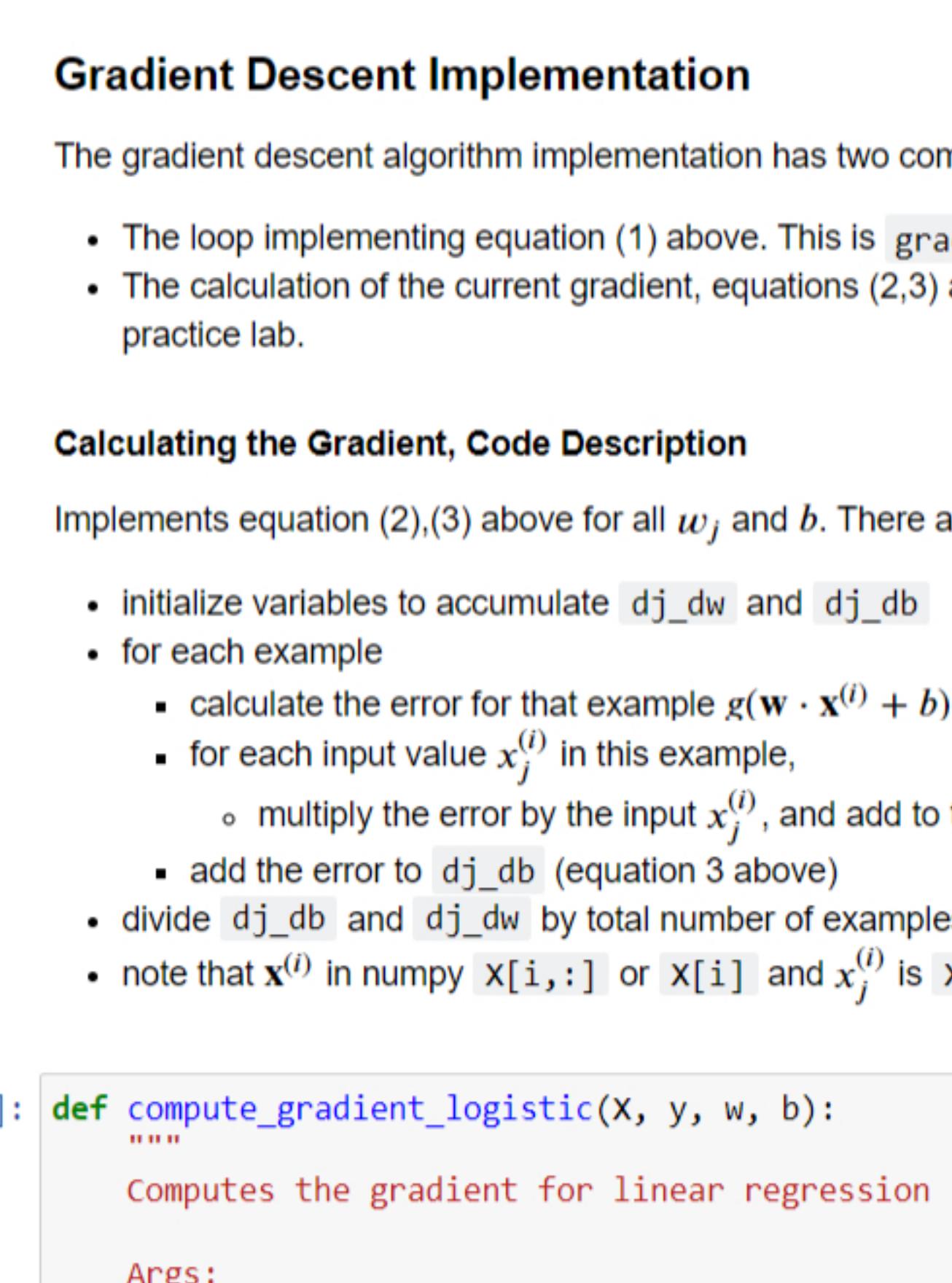
Let's start with the same two feature data set used in the decision boundary lab.

```
In [2]: X_train = np.array([[0.5, 1.5], [1.5, 0.5], [3, 0.5], [2, 2], [1, 2.5]])
y_train = np.array([0, 0, 1, 1, 1])
```

As before, we'll use a helper function to plot this data. The data points with label $y = 1$ are shown as red crosses, while the data points with label $y = 0$ are shown as blue circles.

```
In [3]: fig,ax = plt.subplots(1,1,figsize=(4,4))
plot_data(X_train, y_train, ax)

ax.axis([0, 4, 0, 3.5])
ax.set_ylabel('x1')
ax.set_xlabel('x0', fontsize=12)
plt.show()
```



Logistic Gradient Descent

Recall the gradient descent algorithm utilizes the gradient calculation:

$$\text{repeat until convergence: } \begin{cases} w_j = w_j - \alpha \frac{\partial J(w, b)}{\partial w_j} & \text{for } j := 0..n-1 \\ b = b - \alpha \frac{\partial J(w, b)}{\partial b} \end{cases} \quad (1)$$

Where each iteration performs simultaneous updates on w_j for all j , where

$$\frac{\partial J(w, b)}{\partial w_j} = \frac{1}{m} \sum_{i=0}^{m-1} (f_{w,b}(x^{(i)}) - y^{(i)}) x_j^{(i)} \quad (2)$$

$$\frac{\partial J(w, b)}{\partial b} = \frac{1}{m} \sum_{i=0}^{m-1} (f_{w,b}(x^{(i)}) - y^{(i)}) \quad (3)$$

- m is the number of training examples in the data set
- $f_{w,b}(x^{(i)})$ is the model's prediction, while $y^{(i)}$ is the target
- For a logistic regression model

$$z = w \cdot x + b$$

$$f_{w,b}(x) = g(z)$$

where $g(z)$ is the sigmoid function:

$$g(z) = \frac{1}{1+e^{-z}}$$

Gradient descent for logistic regression

$$\text{repeat } \left[\begin{array}{l} \text{loop like linear regression} \\ w_j = w_j - \alpha \left[\frac{1}{m} \sum_{i=0}^{m-1} (f_{w,b}(x^{(i)}) - y^{(i)}) x_j^{(i)} \right] \\ b = b - \alpha \left[\frac{1}{m} \sum_{i=0}^{m-1} (f_{w,b}(x^{(i)}) - y^{(i)}) \right] \end{array} \right] \quad (1)$$

Same concepts:

- Multiple gradient descent
- Learning curve
- Vectorized implementation
- Feature scaling

Linear regression $f_{w,b}(x) = w \cdot x + b$

Logistic regression $f_{w,b}(x) = \frac{1}{1+e^{-w \cdot x + b}}$

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Gradient Descent Implementation

The gradient descent algorithm implementation has two components:

- The loop implementing equation (1) above. This is `gradient_descent` below and is generally provided to you in optional and practice labs.
- The calculation of the current gradient, equations (2,3) above. This is `compute_gradient_logistic` below. You will be asked to implement this week's practice lab.

Calculating the Gradient, Code Description

Implements equation (2),(3) above for all w and b . There are many ways to implement this. Outlined below is:

- initialize variables to accumulate `dj_dw` and `dj_db`
- for each example
 - calculate the error for that example $g(w \cdot x^{(i)} + b) - y^{(i)}$
 - for each input value $x_j^{(i)}$ in this example,
 - multiply the error by the input $x_j^{(i)}$, and add to the corresponding element of `dj_dw`. (equation 2 above)
 - add the error to `dj_db` (equation 3 above)
- divide `dj_db` and `dj_dw` by total number of examples (m)
- note that $x^{(i)}$ in numpy $X[:,i]$ or $X[i,:]$ and $x_j^{(i)}$ is $X[:,i]$

```
In [6]: def compute_gradient_logistic(X, y, w, b):
    """
    Computes the gradient for linear regression

    Args:
        X (ndarray (m,n)): Data, m examples with n features
        y (ndarray (m,)) : target values
        w (ndarray (n,)) : model parameters
        b (scalar) : model parameter

    Returns:
        dj_dw (ndarray (n,)) : The gradient of the cost w.r.t. the parameters w.
        dj_db (scalar) : The gradient of the cost w.r.t. the parameter b.

    """
    m, n = X.shape
    dj_dw = np.zeros((n,))
    dj_db = 0.

    for i in range(m):
        f_wb_i = sigmoid(np.dot(X[i],w) + b)
        err_i = f_wb_i - y[i]
        for j in range(n):
            dj_dw[j] = dj_dw[j] + err_i * X[i,j]
        dj_db = dj_db + err_i
    dj_dw = dj_dw/m
    dj_db = dj_db/m

    return dj_db, dj_dw
```

Check the implementation of the gradient function using the cell below.

```
In [5]: X_tmp = np.array([[0.5, 1.5], [1,1], [1.5, 0.5], [3, 0.5], [2, 2], [1, 2.5]])
y_tmp = np.array([0, 0, 1, 1, 1])
w_tmp = np.array([2,-3])
b_tmp = 1.
dj_db_tmp, dj_dw_tmp = compute_gradient_logistic(X_tmp, y_tmp, w_tmp, b_tmp)
print("dJ_db: " + str(dj_db_tmp))
print("dJ_dw: " + str(dj_dw_tmp.tolist()))
```

Expected output

`dj_db: 0.49861806546328574`

`dj_dw: [0.49833393278696, 0.49883942983996693]`

Gradient Descent Code

The code implementing equation (1) above is implemented below. Take a moment to locate and compare the functions in the routine to the equations above.

```
In [8]: def gradient_descent(X, y, w_in, b_in, alpha, num_iters):
    """
    Performs batch gradient descent

    Args:
        X (ndarray (m,n)) : Data, m examples with n features
        y (ndarray (m,)) : target values
        w_in (ndarray (n,)) : Initial values of model parameters
        b_in (scalar) : Initial values of model parameter
        alpha (float) : Learning rate
        num_iters (scalar) : number of iterations to run gradient descent

    Returns:
        w (ndarray (n,)) : Updated values of parameters
        b (scalar) : Updated value of parameter
    """
    # An array to store cost J and w's at each iteration primarily for graphing later
    J_history = []
    w = copy.deepcopy(w_in) # avoid modifying global w within function
    b = b_in

    for i in range(num_iters):
        # calculate the gradient and update the parameters
        dj_db, dj_dw = compute_gradient_logistic(X, y, w, b)

        # Update Parameters using w, b, alpha and gradient
        w = w - alpha * dj_dw
        b = b - alpha * dj_db

        # Save cost J at each iteration
        if i<100000: # prevent resource exhaustion
            J_history.append( compute_cost_logistic(X, y, w, b) )

        # Print cost every at intervals 10 times or as many iterations if < 10
        if i % math.ceil(num_iters / 10) == 0:
            print(f"iteration {i:4d}: cost {J_history[-1]}")

    return w, b, J_history
```

Let's run gradient descent on our data set.

```
In [9]: w_tmp = np.zeros_like(X_train[0])
b_tmp = 0.
alph = 0.1
iters = 10000
```

`w_out, b_out, _ = gradient_descent(X_train, y_train, w_tmp, b_tmp, alph, iters)`

`print("updated parameters: w:[%s], b:[%s]" % (w_out, b_out))`

`Iteration 0: Cost: 0.684610468560574`

`Iteration 1000: Cost: 0.1509977666876081`

`Iteration 2000: Cost: 0.05705327279402531`

`Iteration 3000: Cost: 0.0242907594216820076`

`Iteration 4000: Cost: 0.01338847298845684`

`Iteration 5000: Cost: 0.0068603798822120097`

`Iteration 6000: Cost: 0.003458159608793`

`Iteration 7000: Cost: 0.001724237033259295`

`Iteration 8000: Cost: 0.00085031712410914`

`Iteration 9000: Cost: 0.000419031712410914`

`updated parameters: w:[5.28 5.08], b:-14.222409982019837`

Let's plot the results of gradient descent:

```
In [10]: fig,ax = plt.subplots(1,1,figsize=(5,4))
# plot the probability
plt_prob(ax, w_out, b_out)
```

`# Plot the original data`

`ax.set_ylabel('x1')`

`ax.set_xlabel('x0')`

`ax.axis([0, 4, 0, 3.5])`

`ax.plot(X_train,y_train,ax)`

`# Plot the decision boundary`

`x0 = -b_out/w_out[0]`

`x1 = -b_out/w_out[1]`

`ax.plot([x0,x1],[x1,x1],c='blue',lw=1)`

`plt.show()`

In the plot above:

- the shading reflects the probability $y=1$ (result prior to decision boundary)
- the decision boundary is the line at which the probability = 0.5

Another Data set

Let's return to a one-variable data set. With just two parameters, w , b , it is possible to plot the cost function using a contour plot to get a better idea of what gradient descent is doing.

```
In [11]: x_train = np.array([0., 1, 2, 3, 4, 5])
y_train = np.array([0, 0, 0, 1, 1, 1])
```

As before, we'll use a helper function to plot this data. The data points with label $y = 1$ are shown as red crosses, while the data points with label $y = 0$ are shown as blue circles.

```
In [12]: fig,ax = plt.subplots(1,1,figsize=(4,3))
plt_tumor_data(x_train, y_train, ax)
plt.show()
```


In the plot below, try:

- changing w and b by clicking within the contour plot on the upper right.
 - changes may take a second or two
 - note the changing value of cost on the upper left plot.
 - note the cost is accumulated by a loss on each example (vertical dotted lines)
- run gradient descent by clicking the orange button.
 - note the steadily decreasing cost (contour and cost plot are in log(cost))
 - clicking in the contour plot will reset the model for a new run
- to reset the plot, re-run the cell

```
In [13]: w_range = np.arange([-1, 7])
b_range = np.arange([-1,-14])
quad = plt_quad_logistic(x_train, y_train, w_range, b_range)
```


In []:

In []: