CSE 321

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HOMEWORK 1

1-) $T_1(n) = 3n^4 + 3n^3 + 1$
$T_2(n) = 3^n$
$T_3(n) = (n-2)!$
$T_4(n) = \ln^2 n$
$T_5(n) = 2^{2n}$
$T_6(n) = \sqrt[3]{n}$

Karmaşıklık Sırası

$T_4 < T_6 < T_1 < T_2 < T_5 < T_3$

$T_4 \in O(T_6)$ prove.

$\lim\limits_{n\to\infty} \frac{\ln^2 n}{\sqrt[3]{n}} = \frac{\infty}{\infty}$ Belirsizliği L'hospital $\lim\limits_{n\to\infty} \frac{2\ln n \cdot n^{-1}}{\frac{1}{3}\cdot n^{-2/3}} = \lim\limits_{n\to\infty} \frac{6\ln n}{\sqrt[3]{n}} = \frac{\infty}{\infty}$ Belirsizliği L'hospital

$= \lim\limits_{n\to\infty} \frac{6n^{-1}}{\frac{1}{3}n^{-2/3}} = \lim\limits_{n\to\infty} \frac{18}{\sqrt[3]{n}} = 0$ Buna göre $T_4 \in O(T_6)$

$T_6 \in O(T_1)$ prove.

$\lim\limits_{n\to\infty} \frac{\sqrt[3]{n}}{3n^4+3n^3+1} = \frac{\infty}{\infty}$ Belirsizliği L'hospital $\lim\limits_{n\to\infty} \frac{\frac{1}{3}n^{-2/3}}{12n^3+9n^2} = \lim\limits_{n\to\infty} \frac{\frac{1}{3}}{(12n^3+9n^2).n^{2/3}}$

$= \lim\limits_{n\to\infty} \frac{\overbrace{\frac{1}{3}}^{\text{sabit}}}{\underbrace{12n^{11/3}+9n^{8/3}}_{\text{sonsuz}}} = 0$ Buna göre $T_6 \in O(T_1)$

$T_1 \in O(T_2)$ prove.

$\lim\limits_{n\to\infty} \frac{3n^4+3n^3+1}{3^n} = \frac{\infty}{\infty}$ Belirsizliği L'hospital $\lim\limits_{n\to\infty} \frac{12n^3+9n^2}{3^n \ln 3} = \lim\limits_{n\to\infty} \frac{36n^2+18n}{3^n \ln^2 3} = \lim\limits_{n\to\infty} \frac{72n+18}{3^n \ln^3 3}$

$= \lim\limits_{n\to\infty} \frac{72}{3^n \ln^4 3} = 0$ Buna göre $T_1 \in O(T_2)$

$T_2 \in O(T_5)$ prove.

$= \lim\limits_{n\to\infty} \frac{3^n}{2^{2n}} = \lim\limits_{n\to\infty} \frac{3^n}{4^n} = \lim\limits_{n\to\infty} \left(\frac{3}{4}\right)^n \Rightarrow \frac{3}{4}$ sayısı 0 ile 1 arasında olduğundan dolayı $\lim\limits_{n\to\infty} \left(\frac{3}{4}\right)^n = 0$ Buna göre $T_2 \in O(T_5)$

$T_5 \in O(T_3)$ prove

$\lim\limits_{n\to\infty} \frac{4^n}{(n-2)!} = \lim\limits_{n\to\infty} \frac{4^n}{\frac{\sqrt{2\pi n}.\left(\frac{n}{e}\right)^n}{n^2-n}} = \lim\limits_{n\to\infty} \frac{4^n (n^2-n)}{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n} = \lim\limits_{n\to\infty} \frac{n^2-n}{\sqrt{2\pi n}.\left(\frac{4n}{e}\right)^n} = \lim\limits_{n\to\infty} \frac{n^2-n}{(2\pi n)^{1/2}\left(\frac{4n}{e}\right)^n}$

$n! = n(n-1).(n-2)!$

$(n-2)! = \frac{n!}{n(n-1)}$ → Striling's Formula

$(n-2)! = \frac{\sqrt{2\pi n}.\left(\frac{n}{e}\right)^n}{n^2-n}$

2 defa L'hospital uygularsak;

$\lim\limits_{n\to\infty} \frac{2}{\text{n'li terimden oluşan bir ifade}} = \frac{2}{\infty} = 0$ Buna göre $T_5 \in O(T_3)$

2-)a-) Verilen algoritma listenin en küçük ve en büyük elemanını bulur. Bunları toplar ve ikiye bölerek yeni bir sayı elde eder. Listenin içerisinde varsa bu sayıyı, yoksa bu sayıya en yakın sayıyı döndürür.

fruits $\longrightarrow$ Liste
fruit $\longrightarrow$ fruits listesinin elemanları
watermelon $\longrightarrow$ Listenin en büyük elemanı
plum $\longrightarrow$ Listenin en küçük elemanı
orange $\longrightarrow$ Return edilecek sayı
orangeTime $\longrightarrow$ While döngüsünü kontrol eden bir değişken

b-) Best Case

Listenin elemanları hiç shift edilmemisse meydana gelir.
Listenin elemanları hiç shift edilmemisse ve algoritmanın karmaşıklığı $\rightarrow \Theta(n)$'dir. $B(n) \in \Theta(n)$
Koddaki for loop bir kere çalışır ve algoritmanın karmaşıklığı $\rightarrow \Theta(n)$'dir.

Worst Case

Eğer liste, en küçük eleman sonda olacak şekilde shift edilmişse meydana gelir.
Fruits döngüsünde, döngü $\rightarrow 1+2+3+4+\dots+n$ defa çalışır
$$\sum_{i=1}^{n} i = \frac{n.(n+1)}{2} = \frac{n^2+n}{2} = \Theta(n^2)$$
Alttaki döngü $\rightarrow \Theta(n)$ $\quad W(n) = \Theta(n^2) + \Theta(n)$
$W(n) \in \Theta(n^2)$

Average Case

Eğer liste, en küçük eleman sonda olmayacak şekilde shift edilmişse meydana gelir.
$P \rightarrow 0 \leq P \leq 1$
$P_i = L[x] = \frac{P}{n}$

$[\,|\,|\,|\,\dots\,|\,|\,]$ (1 2 3 … n-1 n)

$$A(n) = \sum_{i=1}^{n} i \cdot P_i \Rightarrow A(n) = \sum_{i=1}^{n} i \cdot \frac{P}{n} = \frac{n.(n+1)}{2} \cdot \frac{P}{n}$$
$$= \frac{pn+p}{2} \in O(n) \qquad A(n) \in O(n)$$

3-)a-) $\sum_{i=0}^{n-1} (i^2+1)^2$ $\quad i^2+1 \rightarrow$ Nondecreasing Function

$$\sum_{i=1}^{n} ((i-1)^2+1)^2 \rightarrow \sum_{i=1}^{n} \underbrace{(i^2-2i+2)^2}_{g(i)}$$

$$\underbrace{\int_0^n g(x)dx}_{\text{Lower Bound}} \leq \sum_{i=1}^{n} g(i) \leq \underbrace{\int_1^{n+1} g(x)dx}_{\text{Upper Bound}}$$

$$\int_0^n (x^2-2x+2)^2 dx$$
$$\int_0^n (x^2-2(x-1))^2 dx \qquad u = x-1, \quad x^2 = (u+1)^2$$
$$\int ((u+1)^2-2u)^2 du = \int (u^2+\cancel{2u}+1-\cancel{2u})^2 du = \int (u^2+1)^2 du = \int (u^4+2u^2+1) du = \int u^4 du + \int 2u^2 du + \int du$$
$$= \frac{u^5}{5} + \frac{2u^3}{3} + u = \frac{(x-1)^5}{5}\Big|_0^n + \frac{2(x-1)^3}{3}\Big|_0^n + x-1\Big|_0^n$$
$$= \frac{(n-1)^5}{5} - \left(\frac{-1}{5}\right)^5 + \frac{2(n-1)^3}{3} - \left(\frac{-2}{3}\right)^3 + n - \cancel{1} + \cancel{1}$$
$$= \frac{(n-1)^5}{5} + \left(\frac{1}{5}\right)^5 + \frac{2(n-1)^3}{3} + \left(\frac{2}{3}\right)^3 + n \Big\} \text{ Lower Bound}$$

$$\int_1^{n+1} (x^2-2x+2)^2 dx$$
$$= \frac{(x-1)^5}{5}\Big|_1^{n+1} + \frac{2(x-1)^3}{3}\Big|_1^{n+1} + x-1\Big|_1^{n+1}$$
$$= \frac{n^5}{5} + \frac{2n^3}{3} + n \Big\} \text{ Upper Bound}$$
$$\frac{(n-1)^5}{5} + \left(\frac{1}{5}\right)^5 + \frac{2(n-1)^3}{3} + \left(\frac{2}{3}\right)^3 + n \leq g(n) \leq \frac{n^5}{5} + \frac{2n^3}{3} + n$$
$$g(n) \in \Theta(n^5)$$

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b-) $\sum_{i=2}^{n-1} \log i^2$ $\quad \log(\ldots) \rightarrow$ Nondecreasing Function

$$\sum_{i=1}^{n-2} \log(i+1)^2 \qquad \int_0^{n-2} \log(x+1)^2 dx = (x+1).(\log(x+1)^2) - 2 \Big|_0^{n-2}$$

$$= n-1(\log(n-1)^2) \cancel{-2+2}$$

$$= \underbrace{(n-1).(\log(n-1)^2)}_{\text{Lower Bound}}$$

$$\int_1^{n-1} \log(x+1)^2 dx = (x+1).(\log(x+1)^2) - 2 \Big|_1^{n-1} = n.(\log n^2) \cancel{-2} - [2(\log 4) \cancel{-2}]$$

$$= \underbrace{n.(\log n^2) - 2\log 4}_{\text{Upper Bound}} \qquad (n-1).(\log(n-1)^2) \leq g(n) \leq (n.\log n^2) - 2\log 4$$

$$g(n) \in \Theta(n \log n)$$

c-) $\sum_{i=1}^{n} (i+1).2^{i-1}$

$$\int_0^n (x+1)2^{x-1} dx = \frac{1}{2} \int_0^n \underbrace{(x+1)}_{u}.\underbrace{2^x dx}_{dv} \qquad dx = du \quad \int 2^x dx = \frac{2^x}{\ln 2} = v$$

$$\frac{1}{2}(x+1)\frac{2^x}{\ln 2} - \int \frac{2^x}{\ln 2} dx = \frac{1}{2}(x+1)\frac{2^x}{\ln 2} - \frac{1}{\ln 2}\int 2^x dx$$

$$= \frac{1}{2}(x+1).\frac{2^x}{\ln 2} - \frac{1}{\ln 2}.\frac{2^x}{\ln 2} = \frac{1}{2}\frac{(x+1).2^x}{\ln 2}\Big|_0^n - \frac{2^x}{\ln^2 2}\Big|_0^n$$

$$= \frac{(n+1).2^n}{2\ln 2} - \frac{1}{\ln 2} - \left[\frac{2^n}{\ln^2 2} - \frac{1}{\ln^2 2}\right] = \underbrace{\frac{(n+1)2^n}{2\ln 2} - \frac{2^n}{\ln^2 2} + \frac{1-\ln 2}{\ln^2 2}}_{\text{Lower Bound}}$$

$$\int_1^{n+1} (x+1)2^{x-1} dx = \frac{(x+1)2^x}{2\ln 2}\Big|_1^{n+1} - \frac{2^x}{\ln^2 2}\Big|_1^{n+1}$$

$$= \frac{(n+2)2^{n+1}}{2\ln 2} - \frac{4}{\ln 2} - \left[\frac{2^{n+1}}{\ln^2 2} - \frac{2}{\ln^2 2}\right] = \underbrace{\frac{(n+2).2^{n+1}}{2\ln 2} - \frac{2^{n+1}}{\ln^2 2} + \frac{2-4\ln^2 2}{\ln^2 2}}_{\text{Upper Bound}}$$

$$\frac{(n+1)2^n}{2\ln 2} - \frac{2^n}{\ln^2 2} + \frac{1-\ln 2}{\ln^2 2} \leq g(n) \leq \frac{(n+2).2^{n+1}}{2\ln 2} - \frac{2^{n+1}}{\ln^2 2} + \frac{2-4\ln^2 2}{\ln^2 2}$$

$$g(n) \in \Theta(n.2^n)$$

d-) $\sum_{i=0}^{n-1}\sum_{J=0}^{i-1}(i+J)$

$$\sum_{J=0}^{i-1} i + \sum_{J=0}^{i-1} J \qquad \sum_{J=1}^{i} i-1 + \sum_{J=1}^{i} J \qquad \frac{(i-1).i}{2} = \frac{i^2-i}{2} + i = \frac{i^2+i}{2}$$

$$\sum_{i=0}^{n-1} \frac{i^2+i}{2} = \frac{1}{2}\sum_{i=1}^{n} (i-1)^2 + (i-1) = \frac{1}{2}\sum_{i=1}^{n} i^2 - 2i + 1 + i - 1$$

$$= \sum_{i=1}^{n} \frac{i^2-i}{2} \} f(i)$$

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$$\int_0^n \frac{x^2-x}{2}\,dx \le f(x) \le \int_1^{n+1} \frac{x^2-x}{2}\,dx$$

$$\int_0^n \frac{x^2}{2}\,dx - \int_0^n \frac{dx}{2} = \frac{x^3}{6}\Big|_0^n - \frac{x^2}{4}\Big|_0^n = \frac{n^3}{6} - \frac{n^2}{4} \Big\} \text{ Lower Bound}$$

$$\int_1^{n+1} \frac{x^2-x}{2}\,dx = \int_1^{n+1} \frac{x^2\,dx}{2} - \int_1^{n+1} \frac{dx}{2} = \frac{x^3}{6}\Big|_1^{n+1} - \frac{x^2}{4}\Big|_1^{n+1} = \frac{(n+1)^3}{6} - \frac{1}{6} - \left[\frac{(n+1)^2}{4} - \frac{1}{4}\right]$$

$$= \underbrace{\frac{(n+1)^3}{6} - \frac{(n+1)^2}{4} + \frac{1}{12}}_{\text{Upper Band}}$$

$$\frac{n^3}{6} - \frac{n^2}{4} \le f(n) \le \frac{(n+1)^3}{6} - \frac{(n+1)^2}{4} + \frac{1}{12}$$

$f(n) \in \Theta(n^3)$

C Code for option a

```
int toplama(int n){
  int i=0;
  int sum = 0;
  int temp = 0;
  for(i=0; i<=n-1; i++){
      temp = (i*i)+1;
      sum = sum + (temp * temp);
  }
  return sum;
}
```

C Code for option d

```
int toplama(int n){
  int i = 0;
  int j = 0;
  int sum = 0;
  for(i=0; i<=n-1; i++){
     for(j=0; j<=i-1; j++){
         sum = sum + i + j;
     }
  }
  return sum;
}
```

4-)
```
int fun(int n){
 int count = 0;
 for(int i=n; i>0; i/=2){      } O(logn)  } O(n logn)
  for(int J=0; J<i; J++){      } O(2n)
     count += 1;
  }
 }
return count;
}
```

içerideki döngü $\rightarrow n + \frac{n}{2} + \frac{n}{4} + \frac{n}{8} + \cdots$

$n.\underbrace{\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots\right)}_{\cong 1} = 2n$

Dışarıdaki döngü, i'nin değeri her defasında yarı yarıya azalır.

$\frac{n}{2^i} \le 1 \quad n \le 2^i$ (Her iki tarafın logaritması)

$\log_2 n \le i = \log n \le i$

içerideki döngü $\rightarrow$ 2n

Dışarıdaki döngü $\rightarrow$ logn

Algoritmanın Karmaşıklığı

$O(2n.\log n) = O(n\log n)$

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5-) a.) $n^3 \in O(3^{2n})$ prove.

$\lim_{n\to\infty} \frac{n^3}{3^{2n}} = \frac{\infty}{\infty}$ Belirsizliği L'hospital

$\lim_{n\to\infty} \frac{3n^2}{2\,3^{2n}\ln 3} = \frac{\infty}{\infty}$ Belirsizliği L'hospital

$\lim_{n\to\infty} \frac{6n}{4\ln^2 3 . 3^{2n}} = \frac{\infty}{\infty}$ Belirsizliği L'hospital

$\lim_{n\to\infty} \frac{6}{8\ln^3 3 . 3^{2n}} = 0$ $\quad \lim_{n\to\infty} \frac{n^3}{3^{2n}} = 0$ olduğundan dolayı $n^3 \in O(3^{2n})$

b-) $n \in o(\log\log n)$ prove

$\lim_{n\to\infty} \frac{n}{\log(\log n)} = \frac{\infty}{\infty}$ Belirsizliği L'hospital

$\lim_{n\to\infty} \frac{1}{\frac{\frac{1}{n\ln 10}}{\log n \ln 10}} = \lim_{n\to\infty} \frac{1}{\frac{1}{\ln^2 10 \log n . n}} = \lim_{n\to\infty} \ln^2 10 \log n . n = \infty$

$\lim_{n\to\infty} \frac{n}{\log\log n} = \infty$ olduğundan $n \in \omega(\log\log n)$

c-) $n^2 \log^2 n \in O(n!)$ prove

$\lim_{n\to\infty} \frac{n^2\log^2 n}{(n!)} \longrightarrow$ Stirling's Formula $\quad n! = \sqrt{2\pi n}.\left(\frac{n}{e}\right)^n$

$\lim_{n\to\infty} \frac{n^2\log^2 n}{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n} = \frac{\infty}{\infty}$ Belirsizliği L'hospital

$\lim_{n\to\infty} \frac{n^{3/2}\log^2 n}{\sqrt{2\pi}\left(\frac{n}{e}\right)^n} = \frac{\frac{3}{2}n \log^2 n + \frac{n^{3/2}\,2\log n}{\ln 10}}{\sqrt{2\pi}.\frac{1}{e}\left(\frac{n}{e}\right)^n . \ln\left(\frac{n}{e}\right)}$ Payda paya göre daha hızlı büyüyeceğinden dolayı sonuç 0 çıkar.

Buna göre $\lim_{n\to\infty} \frac{n^2\log^2 n}{n!} = 0$, $n^2\log^2 n \in O(n!)$

d-) $\sqrt{10n^2+7n+3} \in \Theta(n)$ prove.

$\lim_{n\to\infty} \sqrt{ax^2+bx+c} = \lim_{n\to\infty} \sqrt{a}\left|x+\frac{b}{2a}\right|$ özelliğinden;

$\lim_{n\to\infty} \frac{\sqrt{10n^2+7n+3}}{n}$

$\lim_{n\to\infty} \frac{\sqrt{10}\left|n+\frac{7}{20}\right|}{n} = \lim_{n\to\infty} \frac{\sqrt{10}n + \frac{7\sqrt{10}}{20}}{n} = \lim_{n\to\infty} \frac{20\sqrt{10}n + 7\sqrt{10}}{20n} = \frac{\infty}{\infty}$ Belirsizliği L'hospital

$\lim_{n\to\infty} \frac{\cancel{20}\sqrt{10}}{\cancel{20}} = \sqrt{10} \Rightarrow \lim_{n\to\infty} \frac{\sqrt{10n^2+7n+3}}{n} = \sqrt{10}$ olduğundan $\sqrt{10n^2+7n+3} \in \Theta(n)$