QUESTION-1

"Imitation Game" which is wonderful and perfect. Stars played very well. I think stars lived this moment while they were playing. I impressed that the film told true story very well. As a matter of fact, it won Best Adapted Screenplay.
In my opinion, the film gives to us that Alan Turing is an incredible genius, he is a war hero and he abridged the time of Second World War.
Alan Turing's big secret is homosexual. Because the government was condemned homosexuals in that period of time. So, he kept a his secret.
He proposed to come to Bletchley. Because her parents didn't want to work in Bletchley.
If they had helped Peter's brother and other people there, Germans would have understood to break Enigma Code. Alan rescued 14 million people and shortened the time of Second World War that he decided to do this thing. I do same thing under same conditions too.

$$CSE\,321$$
$$HOMEWORK-2$$
$$QUESTION\ 2$$
$$Master\,Theorem$$

$$x(n)=ax\left(\frac{n}{b}\right)+f(n) \qquad Şartlar \rightarrow a\geq 1 \;\; b>1 \;\; ve \;\; f(n)\; asimptotik\;\; pozitif$$

$$Part-A \rightarrow x_1(n)=0.5x_1\left(\frac{n}{2}\right)+\frac{1}{n} \qquad f(n)=\frac{1}{n} \;\; a=0.5 \;\; b=2$$

$$a=0.5\;\; a\geq 1\;\; eşitsizliğini\;\; sağlamadığından\;\; dolayı\;\; Master\;\; Theorem\;\; bu\;\; denklemi\;\; çözemez.$$

$$Part-B \rightarrow x_2(n)=3x_2\left(\frac{n}{4}\right)+nlog(n) \qquad f(n)=nlogn\;(asimptotik\; pozitif)\; b=4 \;\; a=3$$

$$x=n^{(\log_b a)} \rightarrow n^{(\log_4 3)} \;\; bu\;\; fonksiyon\;\; ile\;\; f(n)'i\;\; karşılaştırırsak;$$

$$f(n)>x(n)\;\; olduğundan\;\; f(n)=\Omega(n^{(\log_b a+\varepsilon)})\;\; olur\;\; ve\, af\left(\frac{n}{b}\right)<cf(n)\;\; şartını\;\; sağlamalı$$

$$3f\left(\frac{n}{4}\right)<cf(n) \rightarrow 3\frac{n}{4}\log\left(\frac{n}{4}\right)<clogn$$

$$\frac{3}{4}n(logn-\log 4)<clogn \rightarrow \frac{3}{4}nlogn-\frac{(3nlog\,4)}{4}<cnlogn\;(Her\;\; iki\;\; tarafa\;\; da\;\; \frac{(3nlog\,4)}{4}\;\; eklersek)$$

$$\frac{3}{4}nlogn<cnlogn+\frac{(3\,nlog\,4)}{4} \quad Bu\;\; eşitsizliği\;\; sağlayan\;\; bir\;\; c\;\; sayısı\;\; buluruz\;\; c=\frac{3}{4}$$

$$Cevap \rightarrow x_2(n)\in\theta(nlogn)$$

$$Part-C \rightarrow x_3(n)=3x_3\left(\frac{n}{3}\right)+\frac{n}{2} \qquad f(n)=\frac{n}{2}\;(Asimptotik\,pozitif)\; a=3\;\; b=3$$

$$\frac{n}{2}\in\theta(n^1) \rightarrow d=1 \quad a=b^d \rightarrow 3=3^1 \rightarrow olduğu\;\; için\;\; Cevap \rightarrow \quad x_3(n)\in\theta(nlogn)$$

$$CSE\,321$$
$$HOMEWORK-2$$
$$QUESTION\,2$$

$$Part-D \rightarrow x_4(n)=6x_4(\frac{n}{3})+n^2 logn \quad f(n)n^2 logn\ (Asimptotik\ Pozitif)\ b=3\ a=6$$

$$x(n)=n^{(\log_3 6)}\ ile\ f(n)'i\ karşılaştırırsak\ f(n)>x(n)\ ve\ bu\ durumda;$$

$$f(n)=\Omega(n^{(\log_b(a+\varepsilon))})\ olur\ ve\ af(\frac{n}{b})<cf(n)\ ve\ c<1\ şartı\ sağlanmalıdır.$$

$$6f(\frac{n}{3})<cf(n) \rightarrow 6\frac{n^2}{9}\log(\frac{n}{3})<cn^2 logn \rightarrow 6\frac{n^2}{9}(logn-\log 3)<cn^2 logn$$

$$\frac{(6n^2 logn)}{9}-\frac{(6n^2\log 3)}{9}<cn^2 logn\ (Her\ iki\ tarafa\ da\ \frac{(6n^2\log 3)}{9}\ eklersek)$$

$$\frac{(6n^2 logn)}{9}<cn^2 logn+\frac{(6n^2\log 3)}{9} \quad böylece\ bu\ eşitsizliği\ sağlayan\ bir\ c\ sayısı\ buluruz.\ c=\frac{6}{9}$$

$$Bu\ durumda\ Cevap \rightarrow x_4(n)\in\theta(n^2 logn)$$

$$Part-E \rightarrow x_5(n)=4x_5(\frac{n}{2})+\frac{n}{logn} \quad a=4\ b=2\ f(n)=\frac{n}{logn}\ (Asimptotik\ Pozitif\ değil)$$

$$Cevap \rightarrow Şartlar\ sağlanmadığından\ Master\ Theorem\ bu\ denklemi\ çözemez.$$

$$Part-F \rightarrow x_6(n)=2^n x_6(\frac{n}{2})+n^n \quad f(n)=n^n\ (Asimptotik\ Pozitif)\ b=2\ a=2^n\ (a\ sabit\ bir\ sayı\ değil)$$

$$Cevap \rightarrow Şartlar\ sağlanmadığından\ Master\ Theorem\ bu\ denklemi\ çözemez.$$

$$CSE\,321$$

$$HOMEWORK-3$$

$Part-A \rightarrow Soruda\ verilen\ algoritma\ verilen\ sayının\ karesini\ hesaplar.$

$Recurrence\ \ Relation$

$$T(n)=T(n-1)+2*n-1 \quad T(1)=1$$

$$\cancel{T(2)}=T(1)+2*2-1$$

$$\cancel{T(3)}=\cancel{T(2)}+2*3-1$$

$$\cancel{T(4)}=\cancel{T(3)}+2*4-1$$

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$$T(n)=\cancel{T(n-1)}+2*n-1$$

$$T(n)=T(1)+\sum_{i=2}^{n} 2i-1$$

$$T(n)=1+2\sum_{i=1}^{n-1} i+1-\sum_{i=1}^{n-1} 1$$

$$T(n)=2n\frac{(n-1)}{2}+2n-2-n+1+1$$

$$T(n)=n^2 \rightarrow Cevap \rightarrow T(n)\in\theta(n^2)$$

$$Part-B \rightarrow M(n)=0\ ,\ n=1 \quad veya \quad M(n)=M(n-1)+1\ ,\ otherwise$$

$$M(n)=M(n-1)+1$$

$$\cancel{M(2)}=M(1)+1$$

$$\cancel{M(3)}=\cancel{M(2)}+1$$

$$\cancel{M(4)}=\cancel{M(3)}+1$$

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$$M(n)=\cancel{M(n-1)}+1$$

$$M(n)=M(1)+\sum_{i=2}^{n} 1 \rightarrow M(n)=0+n-1 \rightarrow M(n)=n-1$$

$$Cevap \rightarrow M(n)\in\theta(n)$$

$$Part-C \rightarrow A(n)=0\ ,\ n=1 \quad veya \quad A(n)=A(n-1)+2\ ,\ otherwise$$

$$A(n)=A(n-1)+2$$

$$\cancel{A(2)}=A(1)+2$$

$$\cancel{A(3)}=\cancel{A(2)}+2$$

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$$A(n)=\cancel{A(n-1)}+2$$

$$A(n)=A(1)+\sum_{i=2}^{n} 2 \rightarrow A(n)=0+2n-2 \rightarrow A(n)=2n-2 \quad Cevap \rightarrow A(n)\in\theta(n)$$

Tasarladığım algoritma,verilen bir listedeki çürük olan cevizin indexini bulur ve onu return eder. Çalışma mantığı,ilk önce listeyi ikiye böler sonra compareScales fonksiyonunu kullanarak çürük olan cevizin listenin solunda mı yoksa sağında mı olduğuna karar verir ve karar verdikten sonra tekrar ikiye böler. Listede 1 eleman kalana kadar bu işlem recursive bir şekilde devam eder. Recursive'de en son dönecek olan fonksiyonun içerisinde, recursive olarak bulunan sonuçtan global olarak tuttuğum, listenin size'ini çıkararak çürük olan cevizin listedeki yerini bulur.

$$CSE\,321$$
$$HOMEWORK-2$$
$$QUESTION\,5$$
$$Best\ Case \rightarrow Eğer\ listede\ bir\ eleman\ varsa\ bu\ durum\ meydana\ gelir.$$
$$B(n)\in\theta(1)$$
$$Worst\ Case \rightarrow T(n)=1,\ n=1$$
$$T(n)=T(\frac{n}{2})+4,\ otherwise$$

$$T(n)=T(\frac{n}{2})+4$$
$$T(\frac{n}{2})=T(\frac{n}{2^2})+4$$
$$T(\frac{n}{4})=T(\frac{n}{2^3})+4$$
$$T(\frac{n}{8})=T(\frac{n}{2^4})+4$$
$$\begin{matrix}. & . & . & .\\ . & . & . & .\\ . & . & . & .\end{matrix}$$
$$T(\frac{n}{2^k})=T(\frac{n}{2^{(k+1)}})+4$$
$$T(n)=T(\frac{n}{2^{(k+1)}})+\sum_{i=1}^{k+1}4$$

$\frac{n}{2^{(k+1)}}=0$ $olabilmesi\ için$ $2^{(k+1)}>n$ $olduğunda\ integer\ bölmesi\ yaparsak\ sayı\ sıfır\ olur.$

$$Bu\ durumda;$$
$$2^{(k+1)}>n\ (Her\ iki\ tarafın\ logn'i\ alınırsa)$$
$$k+1>logn \rightarrow k>logn-1$$
$$T(n)=0+4k+4 \rightarrow T(n)=0+4logn-4+4 \rightarrow T(n)=4logn$$
$$Cevap \rightarrow W(n)\in\theta(logn)$$

$$CSE\,321$$
$$HOMEWORK-2$$
$$QUESTION\,6$$

$$Part-A \rightarrow T_1(n)=3T_1(n-1)\ for\ \ n>1\ \ T_1(1)=4$$
$$T_1(2)=3T_1(1)$$
$$T_1(3)=3^2T_1(1)$$
$$T_1(4)=3^3T_1(1)$$

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$$T_1(n)=3^{(n+1)}T_1(1) \rightarrow T_1(n)=3^n 3*4$$
$$T_1(n)=12*3^n \quad Forward\ \ Subsitution\ 3^n=3*3^n-1 \rightarrow 3^n=3^n$$
$$\textcolor{red}{Cevap} \rightarrow T_1(n)\in\theta(3^n)$$
$$Part-B \rightarrow T_2(n)=T_2(n-1)+n\ for\ \ n>1\ ,\ T_2(0)=0$$
$$\cancel{T_2(1)}=T_2(0)+1$$
$$\cancel{T_2(2)}=\cancel{T_2(1)}+2$$
$$\cancel{T_2(3)}=\cancel{T_2(2)}+3$$

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$$T_2(n)=\cancel{T_2(n-1)}+n$$
$$T_2(n)=T_2(0)+\sum_{i=1}^{n} i \rightarrow T_2(n)=n\frac{(n+1)}{2}=\frac{(n^2+n)}{2}$$
$$Forward\ \ Subsitution\ \ \frac{(n^2+n)}{2}=\frac{(n-1)^2}{2}+\frac{(n-1)}{2}+n \rightarrow \frac{(n^2+n)}{2}=\frac{(n^2-2n+1+n-1+2n)}{2}$$
$$\frac{(n^2+n)}{2}=\frac{(n^2+n)}{2}\ \ \textcolor{red}{Cevap} \rightarrow T_2(n)\in\theta(n^2)$$

$$CSE\,321$$
$$HOMEWORK-2$$
$$QUESTION\,6$$

$$Part-C \rightarrow T_3(n)=T\left(\frac{n}{2}\right)+n \quad for\ n>1\ ,\ T_3(1)=0 \quad (solve\ for\ n=2^k)$$

$$\cancel{T_3(2)}=T_3(1)+2^1$$
$$\cancel{T_3(4)}=\cancel{T_3(2)}+2^2$$
$$\cancel{T_3(8)}=\cancel{T_3(4)}+2^3$$
$$\cancel{T_3(16)}=\cancel{T_3(8)}+2^4$$
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$$. \quad . \quad . \quad .$$
$$T_3(n)=\cancel{T_3\left(\frac{n}{2}\right)}+2^n$$
$$T_3(n)=T_3(1)+\sum_{i=1}^{n} 2^i$$
$$T_3(n)=1+2^n+\frac{(2^n-1)}{(2-1)}$$
$$T_3(n)=2^{(n+1)}$$
$$Cevap \rightarrow T_3(n)\in \theta(2^n)$$

$$CSE\,321$$
$$HOMEWORK-2$$
$$QUESTION\,6(PART-B)$$

$$Part-1 \rightarrow T_1(n)=6T_1(n-1)-9T_1(n-2)\quad T_1(0)=1\ T_1(1)=6$$
$$T_1(n)-6T_1(n-1)+9T_1(n-2)=0$$
$$k^n-6k^{(n-1)}+9k^{(n-2)}=0(\text{Her iki tarafı } k^{(n-2)}\text{'ye bölersek})$$
$$k^2-6k+9=0 \rightarrow (k-3)^2=0 \rightarrow k=3(\text{Çift katlı kök})$$
$$T_1(n)=a(3)^n+bn(3)^n$$
$$T_1(0)=a3^0+b03^0 \rightarrow a=1$$
$$T_1(1)=1(3)^1+b(3)^1 \rightarrow 3+3b=6 \rightarrow b=1$$
$$T_1(n)=3^n(n+1)$$
$$(I)\,T_1(2)=6T_1(1)-9T_1(0) \rightarrow T_1(2)=27$$
$$(II)\,T_1(2)=3^2(2+1) \rightarrow T_1(2)=27$$

I ve II deki denklemler birbirlerine denk oldukları için;

$$\text{Cevap} \rightarrow T_1(n)=3^n(n+1)$$

$$Part-2 \rightarrow T_2(n)=5T_2(n-1)-6T_2(n-2)+7^n$$
$$a_n=a_n^h+a_n^p$$
$$for\,a_n^h \rightarrow T_2(n)-5T_2(n-1)+6T_2(n-2)=0$$
$$k^2-5k+6=0 \rightarrow (k-3)(k-2)=0 \rightarrow k=3\ k=2$$
$$a_n^h=a(2)^n+b(3)^n$$
$$for\,a_n^p \rightarrow T_2(n)-5T_2(n-1)+6T_2(n-2)=7^n \quad GUESS \rightarrow X7^n$$
$$X7^{(n+2)}-5X7^{(n+1)}+6X7^n=7^n\ (\text{Her iki tarafı } 7^n\text{'e bölersek})$$
$$49X-35X+6X=1 \rightarrow 20X=1 \rightarrow X=\frac{1}{20} \qquad X7^n \rightarrow \frac{7^n}{20} \rightarrow a_n^p$$

$$\text{Cevap} \rightarrow T_2(n)=a(2)^n+b(3)^n+\frac{7^n}{20}$$