Middle East Technical University

Computer Engineering Department

CENG 280
Formal Languages and Abstract Machines

Homework 1

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Question 1

(a) CFG $G_1 = (V_1, \Sigma, R_1, S)$ for L_1, where

- $V_1 = \{ S \} \cup \Sigma$
- $\Sigma = \{ a, b \}$
- $R_1 = \{ S \to aSbSb \mid bSaSb \mid bSbSa \mid e \}$

(b) CFG $G_2 = (V_2, \Sigma, R_2, T)$ for L_2, where

- $V_2 = \{ T \} \cup \Sigma$
- $R_2 = \{ T \to aaTb \mid aTb \mid e \}$

(c) $M_1 = (\{ q_0, q_1 \}, \Sigma, V_1, \Delta, q_0, \{ q_1 \})$ is a PDA accepting L_1, and transitions Δ are given below and the sketch of the PDA is shown in Figure 1.

$$\begin{aligned}\Delta = \{ & ((q_0, e, e), (q_1, S)), \\ & ((q_1, e, S), (q_1, aSbSb)), ((q_1, e, S), (q_1, bSaSb)), \\ & ((q_1, e, S), (q_1, bSbSa)), ((q_1, e, S), (q_1, e)), \\ & ((q_1, a, a), (q_1, e)), ((q_1, b, b), (q_1, e)) \}\end{aligned}$$

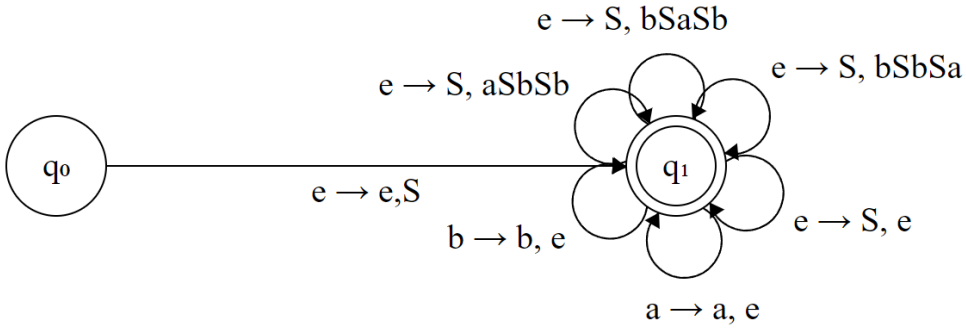

Figure 1: PDA M_1 recognizing L_1.

(d) CFG $G_3 = (V_3, \Sigma, R_3, Q)$ for L_3, where

- $V_3 = V_1 \cup V_2 \cup \{ Q \}$
- $R_2 = R_1 \cup R_2 \cup \{ Q \to S \mid T \}$

Question 2

(a) Let $L_{G_1} = L(G_1)$ and $s = 00111$ where $s \in L_{G_1}$. The grammar G_1 is ambiguous as there exists at least two parse trees for s to be generated. The two parsing routes to generate s is given in Table 1.

Rule	**Application**	**Result**
Start → S	Start	S
S → AS	S	AS
A → A1	AS	A1S
A → 0A1	A1S	0A11S
A → 01	0A11S	00111S
S → e	00111S	00111

Rule	**Application**	**Result**
Start → S	Start	S
S → AS	S	AS
A → 0A1	AS	0A1S
A → A1	0A1S	0A11S
A → 01	0A11S	00111S
S → e	00111S	00111

Table 1: Two different generation path for s.

(b) Let $G_2 = (V, \Sigma, R', S)$ where $R' = \{ S \to SA1 \mid e, A \to 0A1 \mid e \}$, and $L(G_2) = L(G_1)$. The language generated by both grammars are the same; however, G_2 is an unambiguous CFG whereas G_1 is ambiguous.

(c) The leftmost derivation of s is given in Table 2 and the parse tree is given in Figure 2.

Rule	Application	Result
Start → S	Start	S
S → SA1	S	SA1
S → e	SA1	A1
A → 0A1	A1	0A11
A → 0A1	0A11	00A111
A → e	00A111	00111

Table 2: The leftmost derivation for s.

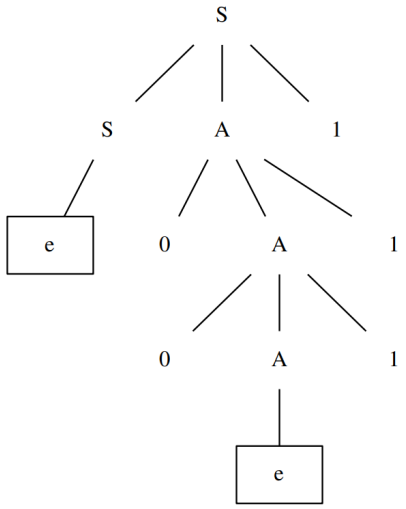

Figure 2: Parse tree for derivation of s by G_2.