MIDDLE EAST TECHNICAL UNIVERSITY
COMPUTER ENGINEERING DEPARTMENT

CENG 280
Formal Languages and Abstract Machines

Homework 1

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Question 1

(a) R the regular expression that generates L_0 is given in Equation 1.

$$R = (\Sigma^* aa \Sigma^* bb \Sigma^*) \cup (\Sigma^* bb \Sigma^* aa \Sigma^*) \tag{1}$$

, or equivalently it can be written by substituting $\Sigma^* = (a \cup b)^*$.

(b) The NFA recognizing L_0 is given as $M = (Q, \Sigma, \delta, q, F)$, where

- $Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}\}$
- $\Sigma = \{a, b\}$
- δ, where tabulation of δ and transitions are shown in Table 1.
- $q = q_0$
- $F = \{q_5, q_{10}\}$

q	σ	$\delta(q, \sigma)$
q_0	e	q_1
q_0	e	q_6
q_1	a	q_2
q_1	b	q_1
q_2	a	q_3
q_2	b	q_1
q_3	a	q_3
q_3	b	q_4
q_4	a	q_3
q_4	b	q_5
q_5	a, b	q_5
q_6	a	q_6
q_6	b	q_7
q_7	a	q_6
q_7	b	q_8
q_8	a	q_9
q_8	b	q_8
q_9	a	q_{10}
q_9	b	q_8
q_{10}	a, b	q_{10}

Table 1: Table of δ transitions. Some symbols are written in the same row separated by comma for convenience.

and the machine is given in Figure 1. The transition

(c) The DFA M', $M' = (Q', \Sigma, \delta', q', F')$, constructed from M recognizing language L_0, where

- $Q = \{q'_0, q'_1, q'_2, q'_3, q'_4, q'_5, q'_6, q'_7, q'_8, q'_9, q'_{10}\}$

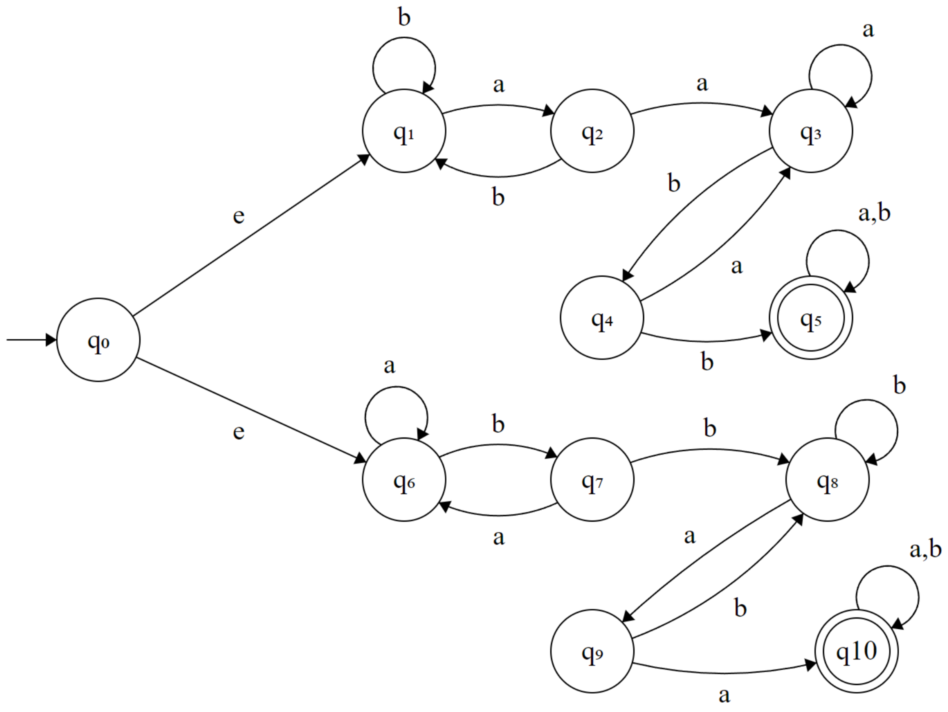

Figure 1: NFA M recognizing L_0.

- δ, where tabulation of δ and transitions are shown in Table 1.
- $q = q'_0$
- $F = \{q'_5, q'_{10}\}$

The transition table for M' is shown in Table 2. For illustration purposes and convenience state names of M are used. Note that the transition table can be simplified further, but we will stick with this also for sketching the machine as it is more descriptive.

DFA

Alias	**State**	**a**	**b**
q'_0	$\{q_0, q_1, q_6\}$	$\{q_2, q_6\}$	$\{q_1, q_7\}$
q'_1	$\{q_2, q_6\}$	$\{q_3, q_6\}$	$\{q_1, q_7\}$
q'_2	$\{q_1, q_7\}$	$\{q_2, q_6\}$	$\{q_1, q_8\}$
q'_3	$\{q_3, q_6\}$	$\{q_3, q_6\}$	$\{q_4, q_7\}$
q'_4	$\{q_1, q_8\}$	$\{q_2, q_9\}$	$\{q_1, q_8\}$
q'_5	$\{q_4, q_7\}$	$\{q_3, q_6\}$	$\{q_5, q_8\}$
q'_6	$\{q_2, q_9\}$	$\{q_3, q_{10}\}$	$\{q_1, q_8\}$
q'_7	$\{q_5, q_8\}$	$\{q_5, q_9\}$	$\{q_5, q_8\}$
q'_8	$\{q_3, q_{10}\}$	$\{q_3, q_{10}\}$	$\{q_4, q_{10}\}$
q'_9	$\{q_5, q_9\}$	$\{q_5, q_{10}\}$	$\{q_5, q_{10}\}$
q'_{10}	$\{q_4, q_{10}\}$	$\{q_3, q_{10}\}$	$\{q_5, q_{10}\}$

Table 2: The transitions for M, specifically δ'. We added Alias & State columns in the table for illustration purposes which makes the conversion steps more descriptive.

(d) The yields in one step for DFA M' is given in Equation 2.

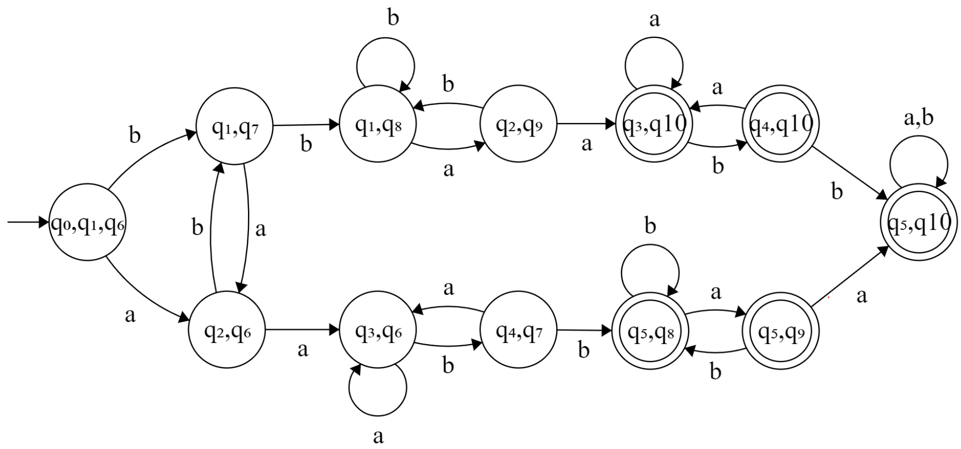

Figure 2: DFA M' recognizing L_0. Notice that the final states F' can be simply reduced to a single state as once the machine gets in either (q_3, q_{10}) or (q_5, q_8), then there is no transition to any of non-accepting states and the machine accepts the input anyway regardless of the remaining part.

$$\begin{aligned}(q'_0, bbabb) &\vdash_{M'} (q'_2, babb)\\ &\vdash_{M'} (q'_4, abb)\\ &\vdash_{M'} (q'_6, bb)\\ &\vdash_{M'} (q'_4, b)\\ &\vdash_{M'} (q'_4, e)\end{aligned} \tag{2}$$

Therefore, $(q'_0, bbabb) \vdash *_{M'}(q'_4, e)$ and $q_4 \notin F'$ and thus ω' is not accepted by M'.

For the NFA we need to show all possible scenarios since the string is rejected if and only if all possible tracks end up in a non-accepting state. We know that this string will not be accepted by the NFA M as well since we have already shown that M' rejected it.

There are three possible traces for string ω' because we have only 2 ϵ-transitions from q_0 and no other. For these three possible options which are starting at state either q_0, q_1 or q_6 are shown in Equation 3, 4 and 5 respectively.

$$(q_0, bbabb) \vdash_M (q_0, e) \tag{3}$$

$$\begin{aligned}(q_1, bbabb) &\vdash_M (q_1, babb)\\ &\vdash_M (q_1, abb)\\ &\vdash_M (q_2, bb)\\ &\vdash_M (q_1, b)\\ &\vdash_M (q_1, e)\end{aligned} \tag{4}$$

$$\begin{aligned}(q_6, bbabb) &\vdash_M (q_7, babb)\\ &\vdash_M (q_8, abb)\\ &\vdash_M (q_9, bb)\\ &\vdash_M (q_8, b)\\ &\vdash_M (q_8, e)\end{aligned} \tag{5}$$

And we can see that $(q_0, bbabb) \vdash *_M(\{q_0, q_1, q_6\}, e)$ and $q_0, q_1, q_6 \notin F$, and thus ω' is not accepted by M.

Question 2

(a) The proof is given below.

Proof. Assume that L_1 is regular. Since L_1 is regular, it must satisfy the Pumping Lemma.

Let's say we have a string $s \in L_1$,

$$s = a^k b^j \textit{ where } k > j \textit{ and } k, j \in \mathbb{N}$$

Now, if we set $p = k + 1$ as a pumping length, and $s = xyz \textit{ where } |xy| \leq p$. With that we have x, y, z as below where the vertical bars separates x, y and z in order.

$$s = \underbrace{aaaa...aaa}_{k} | \underbrace{b}_{1} | \underbrace{bbbb...bbbb}_{j-1}$$

Now, when we pump y c times, the pumped string becomes

$$s' = a^{k'} b^{j'} \textit{ where } k' \leq j' \textit{ and } k', j' \in \mathbb{N}$$

for some $c \in \mathbb{N}$ and by Pumping lemma $s'' \in L_1$, where $s'' = xy^i z \mid i \geq 0$. However, we have a contradiction, s cannot be pumped (not staying under L_1), and thus L_1 is not regular.

We have $L_2 = \overline{L_1}$ and we showed that L_1 is not regular. Since regular languages are closed under complement operation, we can say that L_2 is not regular since L_1 is not regular. Therefore, L_2 is not regular.

□

(b) The proof is given below.

Proof. Since the regular languages are closed under union operation, it is sufficient to show that either of L_4, L_5 or L_6 are not regular (if possible).

Assume that L_4 is regular. Let's say we have a string $s \in L_4$,

$$s = a^k b^k \textit{ where } k \in \mathbb{N}^+$$

Now, if we set $p = k + 1$ as a pumping length, and $s = xyz \textit{ where } |xy| \leq p$. With that we have x, y, z as below where the vertical bars separates x, y and z in order.

$$s = \underbrace{aaaa...aaa}_{k} | \underbrace{b}_{1} | \underbrace{bbbb...bbbb}_{k-1}$$

Now, when we pump y c times, the pumped string becomes

$$s' = a^{k'} b^{j'} \textit{ where } k' \leq j' \textit{ and } k', j' \in \mathbb{N}^+$$

for some $c \in \mathbb{N}$ and by Pumping lemma $s'' \in L_4$, where $s'' = xy^i z \mid i \geq 0$. However, we have a contradiction, s cannot be pumped (not staying under L_4), and thus L_4 is not regular.

Since we showed that L_4 is not regular, and by the closure property of regular languages, we can also say that $L_4 \cup L_5 \cup L_6$ is not regular. Therefore $L_4 \cup L_5 \cup L_6$ is not regular.

□