

15/01/16

CSC 225 - Assignment 1

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1.	<u>A</u>	<u>B</u>	<u>O</u>	<u>Ω</u>
	$(\log n)^3$	n	Yes	No
	$2n^2 + 4n$	$4n^2$	No	No
	$n!$	2^n	No	Yes
	n^5	n^4	No	Yes
	100	10000	No	No
	n^2	$(1.5)^n$	Yes	No

$$2. \quad 2^{\log n} = O(\log \log n) = O(\sqrt{\log n}) = O(n \log n) = O(\log n)^2$$

$$\rightarrow O(4^{\log n}) = O(3n^{0.5}) = O(n^3) = O(4^n) = O(2^{2n})$$

3. Pg 208-

1.4.6

$$a) N + \frac{N}{2} + \frac{N}{4} + \frac{N}{8} + \dots + 1$$

$$\left[a \left(\frac{r^n - 1}{r - 1} \right) \right]$$

$$\Rightarrow 1 \left(\frac{2^{\log_2 N + 1} - 1}{2 - 1} \right)$$

$$\Rightarrow ((2 \cdot N) - 1)$$

$$\therefore \text{Ans} \Rightarrow \underline{\underline{O(N)}}$$

$$b) 1 + 2 + 4 + 8 + \dots + N$$

$$\left[a \left(\frac{r^n - 1}{r - 1} \right) \right]$$

$$\Rightarrow 1 \left(\frac{2^{\log_2 N + 1} - 1}{2 - 1} \right)$$

$$\Rightarrow ((2 \cdot N) - 1)$$

$$\therefore \text{Ans} \Rightarrow \underline{\underline{O(N)}}$$

~~$$c) 1(N) + 2(N) + 4(N) + 8(N) + \dots + N(N)$$~~

$$c) 1(N) + 2(N) + 4(N) + 8(N) + \dots + N(N)$$

$$\Rightarrow N(1 + 2 + 4 + 8 + \dots + N)$$

$$\left[a \left(\frac{r^n - 1}{r - 1} \right) \right]$$

$$\Rightarrow N \left(1 \left(\frac{2^{\log_2 N + 1} - 1}{2 - 1} \right) \right)$$

$$\Rightarrow N((2 \cdot N) - 1)$$

$$\Rightarrow 2 \cdot N^2 - N$$

$$\therefore \text{Ans} \Rightarrow \underline{\underline{O(N^2)}}$$

$$4. \sum_{i=1}^n 2i-1 = n^2 \quad \forall n \geq 1$$

When $n=1$ $\sum_{i=1}^1 2i-1 = 1 \quad \& \quad 1^2 = 1$

For $n+1$

$$\sum_{i=1}^{n+1} 2(i+1)-1$$

$$\Rightarrow 2(n+1) + \sum_{i=1}^n 2(i+1)-1$$

$$\Rightarrow 2n+2 + n^2$$

$$\Rightarrow n^2 + 2n + 1$$

$$\Rightarrow (n+1)^2$$

$$(n^2+1)^2$$

$$\Rightarrow (n+1)(n+1)$$

$$\Rightarrow n^2 + 2n + 1$$

\therefore Proved $\sum_{i=1}^n 2i-1 = n^2$ by induction.

$$5. \sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1} \quad \forall n \geq 1$$

When $n=1$, $\sum_{i=1}^1 \frac{1}{i(i+1)} = \frac{1}{2}$ & $\frac{1}{1+1} = \frac{1}{2}$

For $n+1$

$$\sum_{i=1}^{n+1} \frac{1}{i(i+1)} = \frac{1}{(n+1)(n+2)} + \sum_{i=1}^n \frac{1}{i(i+1)}$$

$$\Rightarrow \frac{1}{(n+1)(n+2)} + \frac{n}{n+1}$$

$$\frac{n}{n+1} \rightarrow \frac{(n+1)}{(n+2)}$$

$$\Rightarrow \frac{1 + (n+2)n}{(n+1)(n+2)} = \frac{1 + n^2 + 2n}{(n+1)(n+2)} = \frac{(n+1)}{(n+2)} \therefore \text{Proved by induction.}$$

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6. Make integer sum;
   Make integer missingValue;
   Make integer formulaCantUse;
   For (integer i = 0; till i is lesser than 0A.length; increment i by 1) {
       Add value of A at index i to sum;
   }

   For (integer j = 1; till j is lesser than / equal to A.length; increment i by 1) {
       formulaCantUse = formulaCantUse + j;
   }

   missingValue = formulaCantUse - sum;
   return missingValue;
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