4/11/16 **CSC 226 - Assignment 3**

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1. Let's assume that G has 2 minimum spanning trees:- M1 & M2.
If M1 is an MST, then adding another edge e to it should form a cycle within G. This new tree G′ would now be 1 edge e away from becoming an MST. To make an MST, the most weighted edge in the cycle needs to be removed. Since all the edges have different weights, the heaviest edge must also be unique. The edge "e" is the heaviest edge, then we can't get multiple MSTs M1 & M2. If edge e isn't the heaviest edge then M1 wasn't a MST to begin with. Either way, this proves that G has exactly one MST.

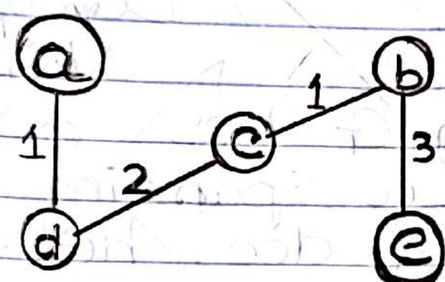
2. By the definition of a minimum spanning tree, an edge "e" which is connecting 2 vertices(ie:- making sure the graph isn't disjoint) could/must be in the graph irrespective of its weight.

To prove this, we can observe 2 cases:-

i) Where the graph G would become disjoint without said repeated edge. Ofcourse the disjoint sets might contain their own MSTs, but it wouldn't be G's MST.

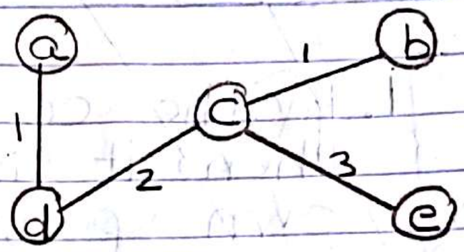
ii) Where a heavier edge e′ remains but repeated edge e is removed. If there's a way of accessing every vertex using a repeated edge, which is cheaper, then the current tree containing distint edge e′ is not a MST by definition.

3.



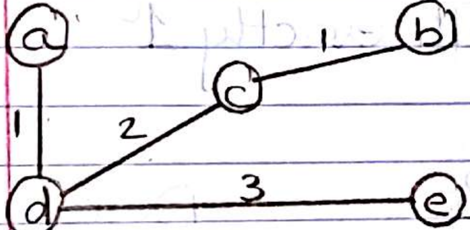
$\{\{a,d\},\{b,c\},\{c,d\},\{e,b\}\}$

$W = 7$; 1,1,2,3.



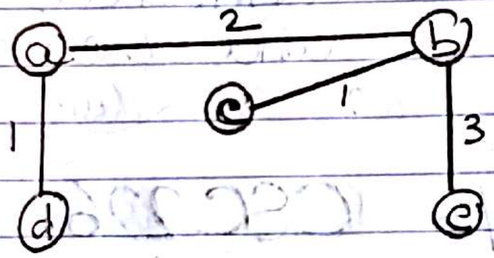
$\{\{a,d\},\{b,c\},\{c,d\},\{c,e\}\}$

$W = 7$; 1,1,2,3.



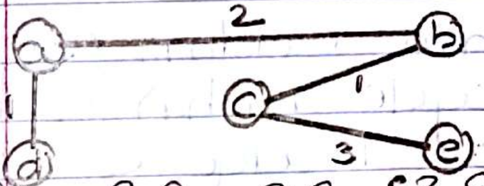
$\{\{a,d\},\{b,c\},\{c,d\},\{d,e\}\}$

$W = 7$; 1,1,2,3.



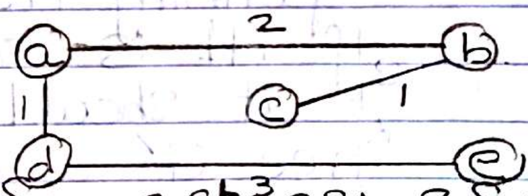
$\{\{a,d\},\{b,a\},\{b,c\},\{b,e\}\}$

$W = 7$; 1,1,2,3.



$\{\{a,d\},\{b,a\},\{b,c\},\{c,e\}\}$

$W = 7$; 1,1,2,3.



$\{\{a,d\},\{b,c\},\{b,a\},\{d,e\}\}$

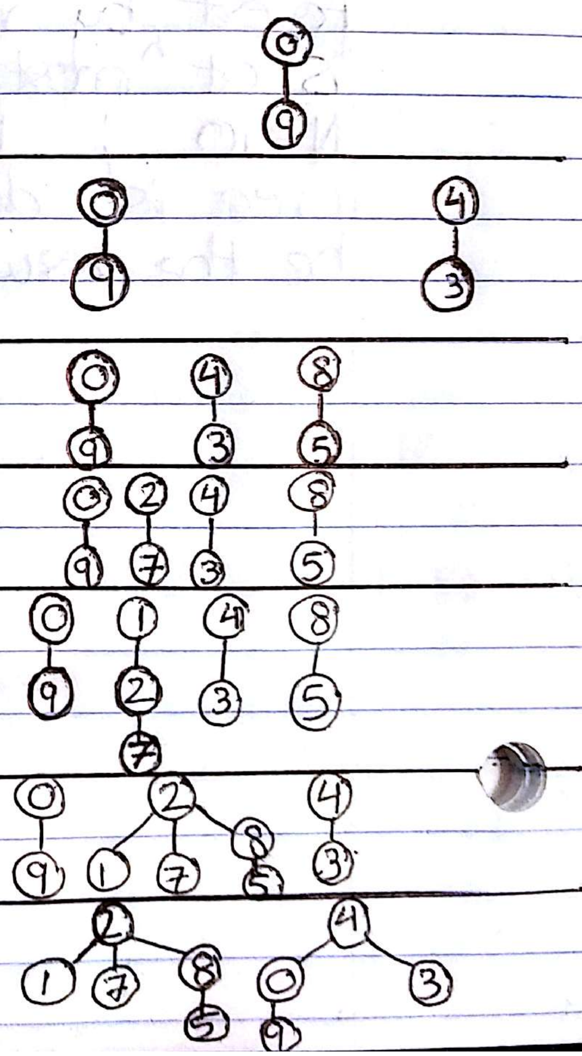
$W = 7$; 1,1,2,3.

The profile is basically the weights of the edges listed in non decreasing order and there are always n-1 edges listed where n = # of vertices. This makes sense because 2 vertices are connected by 1 edge and since MSTs don't have cycles, there would always be 1 less edge compared to the # of vertices.

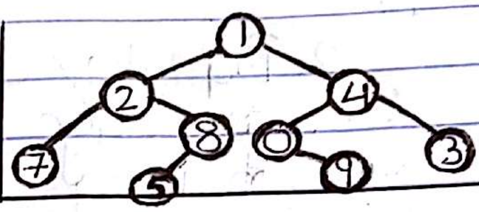
id[]

4.

p	q	0	1	2	3	4	5	6	7	8	9
9	0	0	1	2	3	4	5	6	7	8	9
		0	1	2	3	4	5	6	7	8	0
3	4	0	1	2	3	4	5	6	7	8	0
		0	1	2	4	4	5	6	7	8	0
5	8	0	1	2	4	4	5	6	7	8	0
		0	1	2	4	4	8	6	7	8	0
7	2	0	1	2	4	4	8	6	7	8	0
		0	1	2	4	4	8	6	2	8	0
2	1	0	1	2	4	4	8	6	2	8	0
		0	1	1	4	4	8	6	2	8	0
5	7	0	1	1	4	4	8	6	2	8	0
		0	1	1	4	4	2	6	2	8	0
0	3	0	1	1	4	4	2	6	2	8	0
		4	1	1	4	4	2	6	2	8	0



4	2	4 1 1 4 4 2 6 2 8 0
		4 1 1 4 1 2 6 2 8 0



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i	0	1	2	3	4	5	6	7	8	9
id[i]	1	1	3	1	5	6	1	3	4	5

Tree of size k ; k=10

$\Rightarrow 1 + \lg i = \lg(i+i) \leq \lg(i+j) = \lg k$

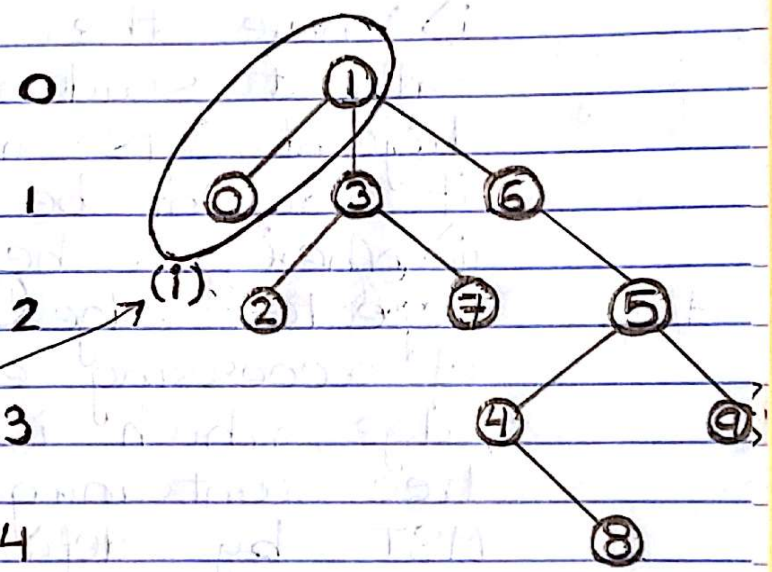
$\Rightarrow 1 + \lg i = \lg 10$

$\Rightarrow \cancel{1} + \lg i = \cancel{1}$

$\Rightarrow \lg i = 0$

$\Rightarrow e^{\lg i} = e^0$

$\Rightarrow i = 1$ $\left\{ \begin{matrix} 0 & (1) \\ 1 & (0) \end{matrix} \right\}$



This makes sense,
which means the depth of any node in a forest built by weighted quick union for N sites is at most lg N.
N = 10 ; height ⇒ lg N ⇒ lg 10 ⇒ 1. The height of this tree is definitely not 1. Hence no, this can not be the result of running weighted quick union.

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