25/1/17 CSC 349A - Assignment #2 Devrup Banerjee

V00837868

1. a) $b=10$, $k=4$, chopping

$P(x) = 1.234x^2 + 76.54x - 1.216 = 0$

(a) (b) (c)

i) $\dfrac{-2c}{b-\sqrt{b^2-4ac}}$

• $-2c = -2 \times -1.216$
$= \underline{2.432}$

• $b^2 = 76.54^2$
$= fl(5858.3716)$
$= \underline{5858}$

• $4ac = 4 \times 1.234 \times (-1.216)$
$= fl(4 \times -fl(1.500544))$
$= \underline{-6.000}$

• $\sqrt{b^2-4ac} = (5858+6)^{1/2}$
$= fl(76.57675888)$
$= \underline{76.57}$

$\therefore \dfrac{-2c}{b-\sqrt{b^2-4ac}} = \dfrac{2.432}{76.54-76.57} = -fl(81.066667)$

$\therefore$ Ans $\Rightarrow -\underline{81.06}$ or $-\underline{0.8106 \times 10^2}$

ii) $\dfrac{-b-\sqrt{b^2-4ac}}{2a}$

• $2a = 2 \times 1.234$
$= \underline{2.468}$

$\therefore \dfrac{-b-\sqrt{b^2-4ac}}{2a} = \dfrac{-76.54-76.57}{2.468} = \dfrac{-fl(153.11)}{2.468} = -fl(62.03403$

$\therefore$ Ans $\Rightarrow -\underline{62.03}$ or $-\underline{0.6203 \times 10^2}$

b) Relative error $= \dfrac{|\text{Real value} - \text{approx value}|}{|\text{Real value}|}$

i) $-2c/(b-\sqrt{b^2-4ac}) \Rightarrow \dfrac{|-62.0418+81.06|}{|-62.0418|}$

$\Rightarrow \dfrac{19.0182}{62.0418} = \underline{0.3065}$ or $\underline{30.65\%}$ error

ii) $(-b-\sqrt{b^2-4ac})/2a \Rightarrow \dfrac{|-62.0418+62.03|}{|-62.0418|}$

$\Rightarrow \dfrac{0.0118}{62.0418} = \underline{0.0002}$ or $\underline{0.019\%}$ error

CSC 349A – Assignment #2 – Denmp Banerjee - V00837868

c)

Polynomial	i) is more accurate	ii) is more accurate
$0.01x^2 - 125x + 0.05$		X
$-0.3x^2 + 125x + 0.025$		X

2. a) $f(x) = -x - \sqrt{x^2 - c}$; $x < 0$ $|x| > |c|$

$= \frac{-x - \sqrt{x^2 - c}}{1} \times \frac{(-x + \sqrt{x^2 - c})}{(-x + \sqrt{x^2 - c})}$

$= \frac{+\cancel{x^2} - \cancel{x^2} + c}{-x + \sqrt{x^2 - c}}$

$\therefore$ Ans $\Rightarrow f(x) = \frac{c}{-x + \sqrt{x^2 - c}}$

b) $g(x) = \sin x / (1 + \cos x)$; x close to π

$= \frac{\sin x}{1 + \cos x}$ $[\sin 2\theta = 2\sin\theta\cos\theta]$ $[\cos 2\theta = 2\cos^2 x - 1]$

$= \frac{2\sin(x/2)\cos(x/2)}{\cancel{1} + (2\cos^2(x/2) - \cancel{1})} = \frac{\cancel{2}\sin(x/2)\cancel{\cos(x/2)}}{\cancel{2}(\cancel{\cos(x/2)}\cos(x/2))}$

$= \frac{\sin(x/2)}{\cos(x/2)} = \tan(x/2)$

$\therefore$ Ans $\Rightarrow g(x) = \tan(x/2)$

c) $h(x) = x/(x+1) - 1$; $|x| =$ large

$= \frac{x}{x+1} - \frac{x+1}{x+1}$

$= \frac{\cancel{x} - (\cancel{x} + 1)}{x+1}$

$\therefore$ Ans $\Rightarrow h(x) = \frac{-1}{x+1}$

CSC 349A – Assignment #2 – Devroop Banerjee – V00837868

3. a) $f(x) = \sqrt{x+1}$; $a=3$; $n=2$; factors of $(x-3)$

$\Rightarrow f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!}$

$f(x) = \sqrt{x+1}$

$\Rightarrow f'(x) = \frac{1}{2}(x+1)^{-1/2} = \frac{1}{2\sqrt{x+1}}$

$\Rightarrow f''(x) = \frac{1}{2}\left(\frac{-1}{2}\right)(x+1)^{-3/2} = \frac{-1}{4\sqrt{(x+1)^3}}$

$\Rightarrow f'''(x) = \frac{-3}{2}\left(\frac{-1}{4}\right)(x+1)^{-5/2} = \frac{3}{8\sqrt{(x+1)^5}}$

$\rightarrow f(3) = \sqrt{4} = 2$

$\Rightarrow f'(3) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$\Rightarrow f''(3) = \frac{-1}{4\sqrt{4^3}} = -1/32$

$\Rightarrow f'''(\varepsilon) = \frac{3}{8\sqrt{(\varepsilon+1)^5}}$

$\Rightarrow f(x) = f(3) + f'(3)(x-3) + \frac{f''(3)(x-3)^2}{2!} + \underbrace{\frac{f'''(3)(x-3)^3}{3!}}_{\text{Remainder}}$

$\therefore$ Ans $\Rightarrow f(x) = 2 + \frac{(x-3)}{4} - \frac{(x-3)^2}{64} + \frac{3}{8\sqrt{(\varepsilon+1)^5}}$

b) $f(3.08) = \sqrt{4.08}$

$\Rightarrow f(3.08) = 2 + \frac{(3.08-3)}{4} - \frac{(3.08-3)^2}{64}$

$\Rightarrow f(3.08) = 2 + 1/50 - 1/10{,}000$

$\Rightarrow f(3.08) = \frac{20{,}000 + 200 - 1}{10{,}000}$

$\therefore$ Ans $\Rightarrow f(3.08) = \frac{20{,}199}{10{,}000}$

c) $f(3.08) = \sqrt{4.08} \approx 2.01990099$; 9 sd

$|E_t| = |2.01990099 - 2.01990000|$

$\therefore$ Ans = 0.00000099 or 9.9×10^{-7} or 0.000099%

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d) Upper bound for $\sqrt{4.08}$; Exact ans ; $x = 3.08$; $|E| < U.B$

$$P(x) = 2 + \frac{(x-3)}{4} - \frac{(x-3)^2}{16}$$

$$\Rightarrow P_2(3.08) = 2.0199$$

$$R_2 = \frac{(\varepsilon+1)^{-5/2}(x-3)^3}{16}$$

$$R_2(3.08) = \frac{(\varepsilon+1)^{-5/2}(3.08-3)^3}{16}$$

$[3, 3.08]$

$$= \frac{(\varepsilon+1)^{-5/2}}{1} \times \frac{1}{16} \times \frac{8}{15625}$$

$$= \frac{1}{31250(\varepsilon+1)^{5/2}}$$

$$|R_2| = \left|\frac{1}{31250(\varepsilon+1)^{5/2}}\right| \leq \frac{1}{31250(4)^{5/2}}$$

$$= \frac{1}{10,00,000} \text{ truncation}$$

$\therefore$ Ans $\Rightarrow$ Max truncation error $= \frac{1}{10,00,000}$