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1. a) $f(x_0) = f(\pi/4) = \cos(\pi^2/4)$

$\rightarrow c_0 + (\pi/4)c_1 + (\pi/4)^2 c_2 + (\pi/4)^3 c_3 = \cos(\pi^2/4)$

$f(x_1) = f(\pi/2) = \cos(\pi^2/2)$

$\rightarrow c_0 + (\pi/2)c_1 + (\pi/2)^2 c_2 + (\pi/3)^3 c_3 = \cos(\pi^2/2)$

$f(x_2) = f(2\pi/3) = \cos(2\pi^2/3)$

$\rightarrow c_0 + (2\pi/3)c_1 + (2\pi/3)^2 c_2 + (2\pi/3)^3 c_3 = \cos(2\pi/3)$

$f(x_3) = f(5\pi/6) = \cos(5\pi^2/6)$

$\rightarrow c_0 + (5\pi/6)c_1 + (5\pi/6)^2 c_2 + (5\pi/6)^3 c_3 = \cos\left(\frac{5\pi^2}{6}\right)$

$$\Rightarrow \begin{bmatrix} 1 & \pi/4 & (\pi/4)^2 & (\pi/4)^3 \\ 1 & \pi/2 & (\pi/2)^2 & (\pi/2)^3 \\ 1 & 2\pi/3 & (2\pi/3)^2 & (2\pi/3)^3 \\ 1 & 5\pi/6 & (5\pi/6)^2 & (5\pi/6)^3 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} \cos(\pi^2/4) \\ \cos(\pi^2/2) \\ \cos(2\pi^2/3) \\ \cos(5\pi^2/6) \end{bmatrix}$$

Ans 1 (b)

```
>> A=[1 pi/4 (pi/4)^2 (pi/4)^3
1 pi/2 (pi/2)^2 (pi/2)^3
1 2*pi/3 (2*pi/3)^2 (2*pi/3)^3
1 5*pi/6 (5*pi/6)^2 (5*pi/6)^3]

A =

    1.0000    0.7854    0.6169    0.4845
    1.0000    1.5708    2.4674    3.8758
    1.0000    2.0944    4.3865    9.1870
    1.0000    2.6180    6.8539   17.9434

>> b=[cos(pi^2/4) cos(pi^2/2) cos(2*pi^2/3) cos(5*pi^2/6)]'

b =

   -0.7812
    0.2206
    0.9564
   -0.3623

>> c=A\b

c =

    3.7616
  -11.9028
    9.4389
   -2.0986
```

1c)
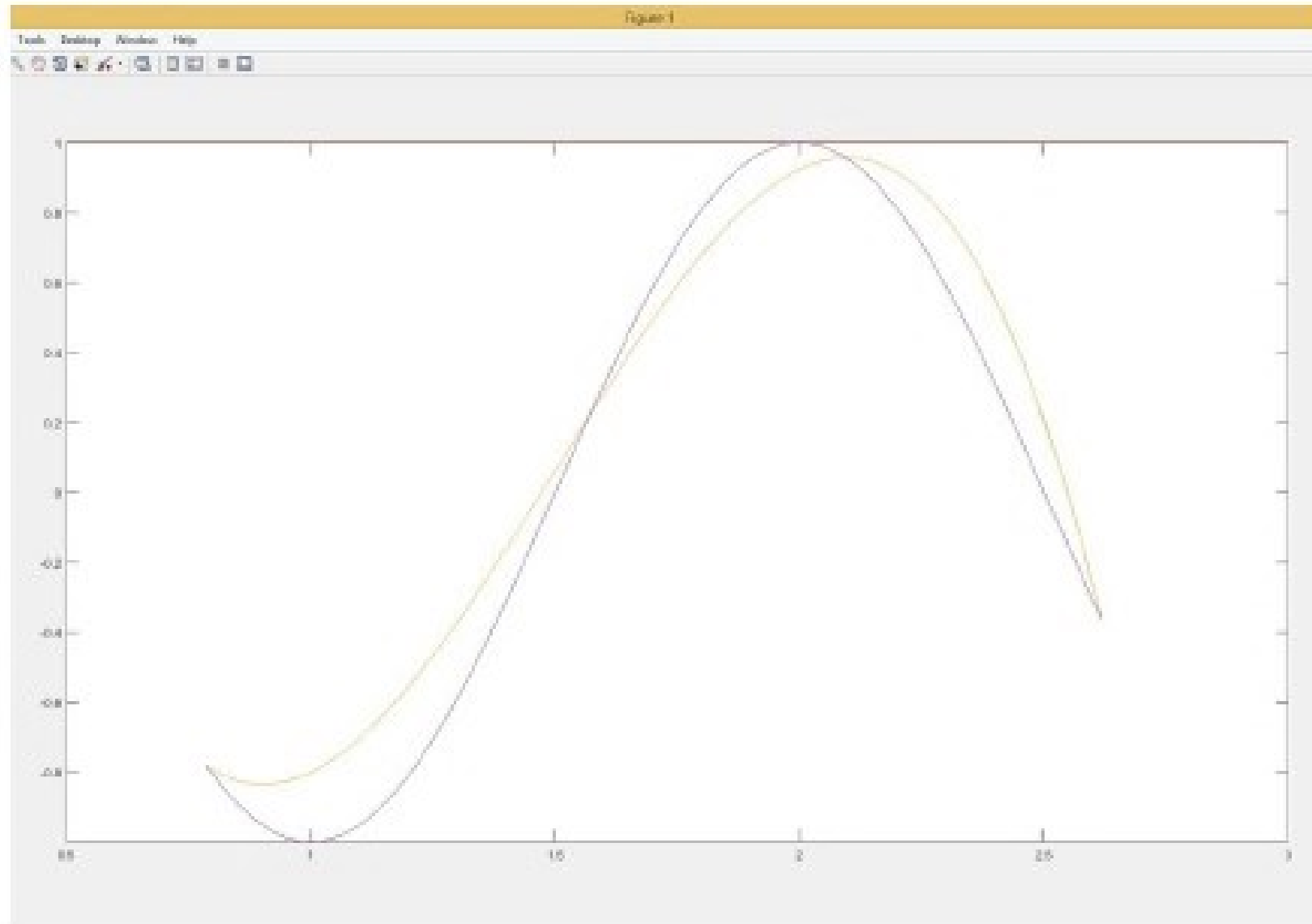
```
>> x=[pi/4; pi/2; 2*pi/3; 5*pi/6]

x =

    0.7854
    1.5708
    2.0944
    2.6180

>> b=cos(pi*x);
>> p=polyfit(x,b,3);
```

```
>> x1=linspace(pi/4,5*pi/8);
>> b1=polyval(p,x1);
>> hold on
>> plot(x1,b1)
>> plot(x1,cos(pi*x1))
>> xlim([0,3])
>> xlim([0.5,3])
```

2. $S(x)$ $\begin{cases} S_0(x) = a_0 + b_0(x-0) + d_0(x-0)^3 & 0 \le x \le 1 \\ S_1(x) = a_1 + b_1(x-1) + c_1(x-1)^2 + d_1(x-1)^3 & 1 \le x \le 3 \end{cases}$

b) $f(x_0 = 0) = 1$; $S_0(x_0) = f(x_0) \Rightarrow (a_0 = 1)$

$f(x_1 = 1) = 2$; $S_1(x_1) = f(x_1) \Rightarrow (a_1 = 2)$

$f(x_2 = 3) = -20$; $S_1(x_2) = f(x_2) \Rightarrow a_1 + 2b_1 + 4c_1 + 8d_1 = -20$

$\Rightarrow 2b_1 + 4c_1 + 8d_1 = -22$ ④

c) $S_1(x_1) = S_0(x_1) = f(x_1) \Rightarrow a_0 + b_0 + d_0 = 2 \Rightarrow b_0 + d_0 = 1$ ①

$S_1'(x) = b_1 + 2c_1(x-1) + 3d_1(x-1)^2$

$S_1''(x) = 2c_1 + 6d_1(x-1)$

$S_0'(x) = b_0 + 3d_0 x^2$

$S_0''(x) = 6d_0 x$

d) $S_1'(x_1) = S_0'(x_1) \Rightarrow b_1 = b_0 + 3d_0$

$\Rightarrow b_0 + 3d_0 - b_1 = 0$ ②

e) $S_1''(x_1) = S_0''(x_1) \Rightarrow 2c_1 = 6d_0$

$$\Rightarrow 6d_0 - 2c_1 = 0 \quad \text{③}$$

f) $S_0''(x_0) = 0 \quad ; \quad S_1(x_n) = 0$

$$\Rightarrow 2c_1 + 12d_1 = 0 \quad \text{⑤}$$

From ① : $b_0 = 1 - d_0$ ⑥

* Substituting ⑥ into ② * $d_0 = (b_1 - 1)/2$ ⑦

*   "   ⑦   "   ③ * $b_1 = \frac{2}{3}c_1 + 1$ ⑧

*   "   ⑧   "   ④ * $c_1 = -\frac{9}{2} - \frac{3}{2}d_1$ ⑨

*   "   ⑨   "   ⑤ * $d_1 = 1$ ⑩

*   "   ⑩   "   ⑨ * $c_1 = -6$ ⑪

*   "   ⑪   "   ⑧ * $b_1 = -3$ ⑫

*   "   ⑫   "   ⑦ * $d_0 = -2$ ⑬

*   "   ⑬   "   ⑥ * $b_0 = 3$

$\therefore$ Ans $\Rightarrow$ $a_0 = 1$ $b_0 = 3$ $c_1 = -6$ $d_0 = -2$

$a_1 = 2$ $b_1 = -3$ $d_1 = 1$

<u>Answer 3)</u>

```
>> x = [1:2:5;6:7:8;10:13:17]

x =

      1
      2
      5
      6
      7
      8
     10
     13
     17

>> y = [1:3;3.7;3.9;4.2;5.7;6.6;7.1;6.7;4.5;-0.67]

y =

   1.0000
   3.0000
   3.7000
   3.9000
   4.2000
   5.7000
   6.6000
   7.1000
   6.7000
   4.5000
  -0.6700

>> pp = spline(x,y)

pp =

      form: 'pp'
    breaks: [1 2 5 6 7 8 10 13 17]
     coefs: [8x4 double]
    pieces: 8
     order: 4
       dim: 1
```

```
>> [i,j]=unmkpp(pp)

i =

    1    2    5    6    7    8    10    13    17

j =

    0.0468   -0.3488    1.0000    3.0000
    0.0288   -0.2064    0.4468    3.7000
    0.3419    0.0326   -0.0745    3.9000
   -0.5745    1.0582    1.0163    4.2000
    0.1563   -0.6854    1.4091    5.7000
    0.0239   -0.1965    0.5472    6.6000
   -0.0026   -0.0529    0.0485    7.1000
    0.0057   -0.0759   -0.3381    6.7000

>> for a = 1:8,
X(a,:) = linspace(i(a), i(a+1),50);
end
for a = 1:8,
Y(a,:) = j(a,1)*(X(a,:)-i(a)).^3+j(a,2)*(X(a,:)-i(a)).^2+j(a,3)*(X(a,:)-i(a))+j(a,4);
end
for a=1:4,
plot(X(2*a-1,:),Y(2*a-1,:), '.', X(2*a,:), Y(2*a,:),':');
hold on;
End
```