25/03/17 CSC 349A Assignment 6

Devaap Banerjee
V00837868

1\. a) $f(x)$ at $x = 0, h \,\&\, 4h;\ h > 0$
$(x_0)\ (x_1)\ (x_2)$

$$P_2(x) = \left(\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\right)f(x_0) + \left(\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}\right)f(x_1) + \left(\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\right)f(x_2)$$

$$= \frac{(x-h)(x-4h)}{(0-h)(0-4h)}f(0) + \frac{(x-0)(x-4h)}{(h-0)(h-4h)}f(h) + \frac{(x-0)(x-h)}{(4h-0)(4h-h)}f(4h)$$

$\therefore$ Ans $\Rightarrow P_2(x) = f(0)\dfrac{(x^2-5hx+4h^2)}{4h^2} - f(h)\dfrac{(x^2-4hx)}{3h^2} + f(4h)\dfrac{(x^2-hx)}{12h^2}$

b) $I = \int_0^{4h} f(x)\,dx \approx \int_0^{4h} P_2(x)\,dx$

$\Rightarrow \dfrac{f(0)}{4h^2}\displaystyle\int_0^{4h}(x^2-5hx+4h^2)\,dx - \dfrac{f(h)}{3h^2}\int_0^{4h}(x^2-4hx)\,dx + \dfrac{f(4h)}{12h^2}\int_0^{4h}(x^2-hx)\,dx$

$\Rightarrow \dfrac{f(0)}{4h^2}\left[\dfrac{x^3}{3} - \dfrac{5hx^2}{2} + 4h^2x\right]_0^{4h} - \dfrac{f(h)}{3h^2}\left[\dfrac{x^3}{3} - \dfrac{4hx^2}{2}\right]_0^{4h} + \dfrac{f(4h)}{12h^2}\left[\dfrac{x^3}{3} - \dfrac{hx^2}{2}\right]_0^{4h}$

$\Rightarrow \dfrac{f(0)}{4h^2}\left(\dfrac{64h^3}{3} - \dfrac{80h^3}{2} + \dfrac{16h^3}{1}\right) - \dfrac{f(h)}{3h^2}\left(\dfrac{64h^3}{3} - \dfrac{64h^3}{2}\right) + \dfrac{f(4h)}{12h^2}\left(\dfrac{64h^3}{3} - \dfrac{16h^3}{2}\right)$

$\Rightarrow \dfrac{f(0)}{4h^2}\left(\dfrac{-8h^3}{3}\right) + \dfrac{f(h)}{3h^2}\left(\dfrac{32h^3}{3}\right) + \dfrac{f(4h)}{12h^2}\left(\dfrac{40h^3}{3}\right)$

$\therefore$ Ans $\Rightarrow \dfrac{-2h}{3}f(0) + \dfrac{32h}{9}f(h) + \dfrac{10}{9}f(4h)$
$(a_0)\quad (a_1)\quad (a_2)$

c) $\dfrac{-2h}{3}f(0) + \dfrac{32h}{9}f(h) + \dfrac{10}{9}f(4h)$

$\Rightarrow -\dfrac{2(0.1)}{3}(1) + \dfrac{32(0.1)}{9}(1.11091) + \dfrac{10(0.1)}{9}(1.63778)$

$\Rightarrow -0.066667 + 0.39499 + 0.181976$

$\therefore$ Ans $\Rightarrow 0.510299$

2 a) Considering Simpson's rule

$f(x)$	$\int_a^b f(x)\,dx$	$h/3\,[f(a)+4f(\frac{a+b}{2})+f(b)]$
1	2 $(b-a)$	$5/9[(-\sqrt{3/5})+5/9(\sqrt{3/5})] = 2$
x	0 $(\frac{b^2-a^2}{2})$	$5/9[(-\sqrt{3/5})^2+5/9(\sqrt{3/5})^2] = 0$
x^2	$2/3$ $(\frac{b^3-a^3}{3})$	$5/9[(-\sqrt{3/5})^3+5/9(\sqrt{3/5})^3] = 2/3$
x^3	0 ⋮	⋮ $=0$
x^4	$2/5$ ⋮	⋮ $=2/5$
$\rightarrow x^5$	0 ⋮	⋮ $=0$
x^6	$2/7 \leftarrow$ (c)	(d) $\rightarrow$ $= 270/1125$

Since the values (c) & (d) are different, the degree of precision is 5 [from x^{5}]

b) $\int_{-1}^{1} e^{-x}\sqrt{x+2}\,dx \approx 5/9 f(-\sqrt{3/5}) + 8/9 f(0) + 5/9 f(\sqrt{3/5})$

$f(-\sqrt{3/5}) = e^{\sqrt{3/5}}\sqrt{-\sqrt{3/5}+2} = 2.401832$

$f(0) = e^0\sqrt{2} = 1.414213$

$f(\sqrt{3/5}) = e^{-\sqrt{3/5}}\sqrt{\sqrt{3/5}+2} = 0.7677094$

$2.401832 + 1.414213 + 0.7677094 = \underline{\underline{3.017935}} \Rightarrow$ Ans

3. $f(x_0+h) \approx f(x_0) + hf'(x_0) + \frac{h^2}{2}f''(x_0) + \frac{h^3}{6}f'''(x_0) + O(h^4)$

$f(x_0+2h) \approx f(x_0) + 2hf'(x_0) + 2h^2f''(x_0) + \frac{4h^3}{3}f'''(x_0) + O(h^4)$

$-3f(x_0) + 4f(x_0+h) - f(x_0+2h)$

$\Rightarrow -3f(x_0) + 4f(x_0) + 4hf'(x_0) + \cancel{2h^2 f''(x_0)} + \frac{2h^3}{3}f'''(x_0) + O(h^4))$

$- f(x_0) - 2hf'(x_0) - \cancel{2h^2f''(x_0)} - \frac{4h^3}{3}f'''(x_0) + O(h^4)$

$\Rightarrow 2hf'(x_0) - \frac{2h^3}{3}f'''(x_0) + O(h^4)$

$\Rightarrow 2hf'(x_0) = -3f(x_0) + 4f(x_0+h) - f(x_0+2h) - \frac{2h^3}{3}f'''(x_0)$

$- O(h^4)$

$$\Rightarrow f'(x_0) = \frac{3f(x_0)}{2h} + \frac{2f(x_0+h)}{h} - \frac{f(x_0+2h)}{2h} + \frac{h^2 f'''(x_0)}{3} - O(h^3)$$

* After differentiation $O(h^3) \rightarrow O(h^2)$ *

$\therefore$ Order of error $\Rightarrow \underline{\underline{O(h^2)}}$