6/2/17 CSC 349A - Assignment 3 Devraj

1) $\frac{\sin x}{1+\cos x}$ $\quad \frac{\tilde{x} f'(\tilde{x})}{f(\tilde{x})}$

a) $\tilde{x} = 1.0005$ radians

$f(x) = \frac{\sin x}{1+\cos x}$

$\Rightarrow f'(x) = \frac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2}$

$= \frac{(\cos x + \cos^2 x + 1) - \cos^2 x}{(1+\cos x)(1+\cos x)}$

$\Rightarrow f'(x) = \frac{1}{1+\cos x}$

$\frac{\tilde{x} f'(\tilde{x})}{f(\tilde{x})}$

$\Rightarrow \frac{\tilde{x}(1+\cos\tilde{x})^{-1}}{\sin\tilde{x}(1+\cos\tilde{x})^{-1}}$

$= \frac{\tilde{x}}{\sin\tilde{x}}$

$= \frac{1.0005}{\sin(1.0005)}$

≈ 1.1886

$\therefore$ Ans $\Rightarrow \frac{\tilde{x} f(\tilde{x})}{f(\tilde{x})}$ is well conditioned [close to 1].

b) $\tilde{x} = 1.0005\pi$ radians.

From previously done calculations, we know that $\frac{\tilde{x} f'(\tilde{x})}{f(\tilde{x})}$ comes down to $\frac{\tilde{x}}{\sin\tilde{x}} = \frac{1.0005\pi}{\sin(1.0005\pi)} \approx -2001.000$

$\therefore$ Ans $\Rightarrow \frac{\tilde{x} f(\tilde{x})}{f(\tilde{x})}$ is ill conditioned [huge value]

2. a) $f(x) = \frac{(\sin x - e^x) + 1}{x^2}$ $\quad x \neq 0 \quad b = 10,\ k = 4$

$fl(f(0.123))$

$fl(\sin(0.123)) = 0.1226$

$fl(e^{0.123}) = 1.130$ or 0.1130×10^1

$fl((\sin(0.123) - e^{0.123}) + 1) = -0.0074$ or -0.7400×10^{-2}

$fl(x^2) = 0.01512$ or 0.1512×10^{-1}

$fl(f(0.123)) = (-0.7400 \times 10^{-2})/(0.1512 \times 10^{-1}) = -0.4894$

$\therefore$ Ans $\Rightarrow -0.4894$

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b) $e^{x} \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$ $\qquad a = 0,\ n = 4$

$g(x) = \sin x \qquad g(0) = 0$

$g'(x) = \cos x \qquad g'(0) = 1$

$g''(x) = -\sin x \qquad g''(0) = 0$

$g'''(x) = -\cos x \qquad g'''(0) = -1$

$g^{(4)}(x) = \sin x \qquad g^{(4)}(0) = 0$

$$\sin x \approx g(a) + g'(a)(x-a) + \frac{g''(a)(x-a)^2}{2!} + \frac{g'''(a)(x-a)^3}{3!} + \frac{g^{(4)}(a)(x-a)^4}{4!}$$

$$\approx g(0) + g'(0)x + \frac{g''(0)x^2}{2} + \frac{g'''(0)x^3}{6} + \frac{g^{(4)}(0)x^4}{24}$$

$$\approx x - \frac{x^3}{6} = \frac{6x - x^3}{6} = \frac{x(6-x^2)}{6}$$

$$f(x) = \frac{(\sin x - e^{x}) + 1}{x^2}$$

$$= \frac{\left\{\frac{x(6-x^2)}{6} - \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}\right) + 1\right\}}{x^2}$$

$$= \frac{\left(\cancel{x} - \frac{x^3}{6} - \cancel{1} - \cancel{x} - \frac{x^2}{2} - \frac{x^3}{6} - \frac{x^4}{24} + \cancel{1}\right)}{x^2}$$

$$= \frac{-x^2}{2x^2} - \frac{x^3}{3x^2} - \frac{x^4}{24x^2} = \frac{-x^2(12 + 8x + x^2)}{24x^2}$$

$\therefore$ **Ans** $\Rightarrow \frac{-(x^2 + 8x + 12)}{24}$

c) $y(x) = \frac{-(x^2 + 8x + 12)}{24}$ $\qquad x = 0.123$, $\frac{\varepsilon}{x}$ has to be small [close to 0]

$\hat{x} = (0.123 + \varepsilon)$

$$\Rightarrow y(\hat{x}) = \frac{-\{(0.123 + \varepsilon)^2 + 8(0.123 + \varepsilon) + 12\}}{24}$$

$$\Rightarrow y(\hat{x}) = \frac{-(0.01512 + 0.246\varepsilon^2 + \varepsilon^2 + 0.984 + 8\varepsilon + 12)}{24}$$

$$\Rightarrow y(\hat{x}) = \frac{-(\varepsilon^2 + 8.246\varepsilon + 12.9991)}{24}$$

$$\Rightarrow \cancel{-}0.4894 = \frac{\cancel{-}(\varepsilon^2 + 8.246\varepsilon + 12.9991)}{24}$$

$$\Rightarrow \varepsilon^2 + 8.246\varepsilon + 12.9991 = 11.7456$$

$$\Rightarrow \varepsilon^2 + 8.246\varepsilon + 1.2535 = 0$$

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$$\Rightarrow \mathcal{E} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \mathcal{E} = \frac{-8.246 \pm \sqrt{8.246^2 - (4 \times 1 \times 1.2535)}}{2 \times 1}$$

$$\Rightarrow \mathcal{E} = \frac{-8.246 \pm 7.9361}{2} = \underbrace{-0.1549}_{\text{Use this coz smaller}} \text{ or } -8.0910$$

$$\Rightarrow \frac{|\mathcal{E}|}{|x|} = \frac{|-0.1549|}{|0.1231|} = 1.2593$$

Since $1.2593 > 1$, the computation is unstable.

Bisect.m | criticalDepth.m

```matlab
function root = Bisect(xl, xu, eps, imax, f, enablePlot)
i = 1;
fl = feval(f,xl);
fprintf ( ' iteration approximation \n')
while(i <= imax)
    xr = (xl  + xu)/2;
    fprintf ( ' %6.0f %18.8f \n', i, xr )
    fr = feval(f,xr);
    if (((i == 1) || (i == 2) || (i == 4) || (i == 6)) && (enablePlot == 1))
        x = [xl:0.01:xu];
        fx = feval(f, x);
        z = [xl,xr,xu];
        fz = feval(f, z);
        figure;
        plot(x, fx, z, fz, '*r');
    end
    if ((fr == 0) || ((xu - xl)/abs(xu + xl)) < eps)
        root = xr;
        %exit;
    end
    i = i + 1;
    if (fl * fr) < 0
        xu = xr;
    else
        xl = xr;
        fl = fr;
    end
end
fprintf ( ' failed to converge in %g iterations\n', imax )
```

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3.b) Trapezoid; $Q = 20 m^3/s$; $0 = 1 - \frac{Q^2}{g A_c^3} B$; $B = 3 + y$

$g = 9.81 m/s^2$; A_c in m^2; B in m ; $A_c = 3y + y^2/2$

$\Rightarrow 1 - \frac{Q^2 (3+y)}{g(3y + y^2/2)^3} = 0$

$\Rightarrow Q^2 (3+y) = 1(g(3y + y^2/2)^3)$

$\Rightarrow 1200 + 400y = 9.81(27y^3 + 13.5y^4 + 2.25y^5 + 0.125y^6)$

$(3y + \frac{y^2}{2})(3y + \frac{y^2}{2})(3y + \frac{y^2}{2})$

$\Rightarrow (9y^2 + 3y^3 + \frac{y^4}{4})(3y + \frac{y^2}{2})$

$\Rightarrow 27y^3 + \frac{9y^4}{2} + 9y^4 + \frac{3y^5}{2} + \frac{3y^5}{4} + \frac{y^6}{8}$

$\Rightarrow 27y^3 + 13.5y^4 + 2.25y^5 + 0.125y^6$

$\therefore$ **Ans** $\Rightarrow 400y - 264.87y^3 - 132.435y^4 - 22.0725y^5 - 1.22625y^6 + 1200 = 0$

Bisect.m   criticalDepth.m

```
function y = criticalDepth(y)
    fy = 400*y - 264.87*(y.^3) - 132.435*(y.^4) - 22.0725*(y.^5) - 1.22625*(y.^6) + 1200;
end
```

```
>> Bisect(0.5, 2.5, 200, 11, @criticalDepth, 1)
 iteration approximation
      1          1.50000000
      2          2.00000000
      3          2.25000000
      4          2.37500000
      5          2.43750000
      6          2.46875000
      7          2.48437500
      8          2.49218750
      9          2.49609375
     10          2.49804688
     11          2.49902344
 failed to converge in 11 iterations

ans =

    2.4990
```

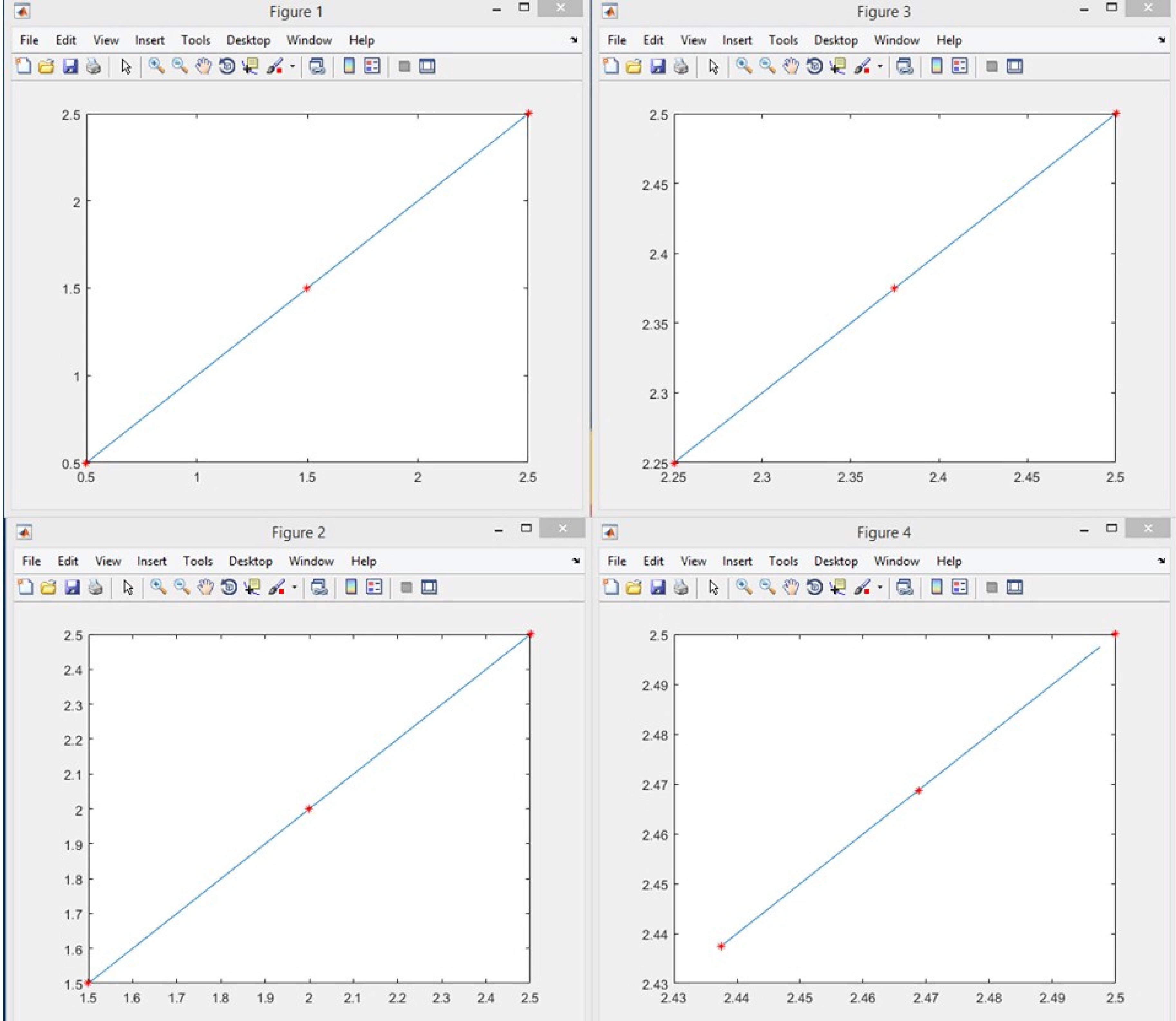

Figure 1
File Edit View Insert Tools Desktop Window Help
2.5
2
1.5
1
0.5
0.5 1 1.5 2 2.5
Figure 3
File Edit View Insert Tools Desktop Window Help
2.5
2.45
2.4
2.35
2.3
2.25
2.25 2.3 2.35 2.4 2.45 2.5
Figure 2
File Edit View Insert Tools Desktop Window Help
2.5
2.4
2.3
2.2
2.1
2
1.9
1.8
1.7
1.6
1.5
1.5 1.6 1.7 1.8 1.9 2 2.1 2.2 2.3 2.4 2.5
Figure 4
File Edit View Insert Tools Desktop Window Help
2.5
2.49
2.48
2.47
2.46
2.45
2.44
2.43
2.43 2.44 2.45 2.46 2.47 2.48 2.49 2.5