COMPUTER SCIENCE 349A, SPRING 2017
ASSIGNMENT #2 - 20 MARKS

DUE MONDAY JANUARY 30, 2017 (11:30 p.m. PST)

This is a really large class and the logistics of grading assignments are challenging. Me and the markers require your help in making this process go smoothly. Please ensure that your assignments conform to the following requirements - any violation will result in getting a zero for the particular assignment.

- All assignments should be submitted electronically through the ConneX course website and shoud be **SINGLE PDF FILES**. No other formats will be accepted.. Handwritten answers are ok but they will need to be scanned and merged into a single pdf file together with any code examples and associated plots.
- The assignment number, student name and student number should be clearly visible on the top of every page of your assignment submission.
- **PLEASE DO NOT COPY THE ASSIGNMENT DESCRIPTION IN YOUR SUBMISSION**
- The asnwers to the questions should be in the same order as in the assignment specification.
- Some of the questions of the assignments are recycled from previous years but typically with small changes in either the description or the numbers. Any submission that contains numbers from previous years in any questions will be immediately graded with zero.
- Any assignment related email questions should have a subject line of the form CSC349A Assignment X, where X is the number of the corresponding assignment.
- The total number of points for this assignment is 20.

Question #1 - 6 marks.
The polynomial $P(x) = x^2 - 83.12x + 3.123$ has two roots, at approximately 0.0375892 and 83.0824. The roots of a quadratic polynomial $ax^2 + bx + c$ can be computed by

$$(i)\quad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{or equivalently} \quad (ii)\quad \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}$$

Using floating-point arithmetic, one of these formulas is often much more accurate than the other. For example, if $(-b + \sqrt{b^2 - 4ac})/(2a)$ is used to compute one of the roots of $P(x) = x^2 - 83.12x + 3.123 = 0$ with base $b = 10$, precision $k = 4$, idealized chopping arithmetic, the results are as follows:

$$\begin{aligned}
fl(b^2) &= fl(()6908.9344) = 6908 \text{ or } 0.6908 \times 10^4. \\
fl(4a) &= 4 \text{ or } 0.4000 \times 10^1. \\
fl(4ac) &= fl(4 \times 3.123) = fl(12.492) = 12.49 \\
fl(b^2 - 4ac) &= fl(()6908 - 12.49) = fl(()6895.51) = 6895 \\
fl(\sqrt{b^2 - 4ac}) &= fl(\sqrt{6895}) = fl(83.036136\cdots) = 83.03 \\
fl(-b + \sqrt{b^2 - 4ac}) &= fl(83.12 + 83.03) = fl(166.15) = 166.1 \\
fl(2a) &= 2 \\
fl\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right) &= fl\left(\frac{166.1}{2}\right) = 83.05 \text{ or } 0.8305 \times 10^2
\end{aligned}$$

which is very accurate. The relative error is about 0.00039 or 0.039%.

On the other hand, it can be shown (similar to the above) that

$$fl\left(\frac{-2c}{b + \sqrt{b^2 - 4ac}}\right) = 69.40 \text{ or } 0.6940 \times 10^2$$

which (using the exact value of $83.0824\cdots$) has a large relative error of 0.165 or 16.5%.

(a) Use base $b = 10$, precision $k = 4$, idealized chopping arithmetic and each of the mathematically equivalent formulas

$$\frac{-2c}{b - \sqrt{b^2 - 4ac}} \text{ and } \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

to compute an approximation to one root of $P(x) = 1.234x^2 + 76.54x - 1.216 = 0$. As above, specify each step of the computation. Note that many of the computations for the two formulas are identical, and need only be done once. Use your calculator to do this, not MATLAB.

(b) Compute the relative errors of each of the approximations in (a) using the fact that the exact value of the root is $-62.0418\cdots$. Give at least 2 significant digits.

(c) One of the two zeros of a quadratic polynomial $ax^2 + bx + c$ can be computed using either the formula

$$(i)\frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ or } (ii)\frac{-2c}{b + \sqrt{b^2 - 4ac}}$$

For each of the specified polynomials in the table below, place an X in the appropriate box to indicate which of these formulas is more accurate in precision $k = 4$ floating-point arithmetic. Put exactly one X in each row of the table. (No justification for your answers is required. It is NOT necessary to do any floating-point computation to answer this question.)

polynomial	(i) is more accurate	(ii) is more accurate
$0.01x^2 - 125x + 0.05$		
$-0.3x^2 + 125x + 0.025$		

Question #2 - 6 Marks.

For each of the functions below, the floating-point evaluation of the given function is very inaccurate for the specified range of values of the parameters. In each case, find another expression that is mathematically identical to the given function and that is more accurate using floating-point computation than is the given form of the function (for the specified values of the parameters).

Note. Do not use any Taylor polynomial approximations in this question.

(a) $f(x) = -x - \sqrt{x^2 - c}$, when x is negative and $|x|$ is much larger than $|c|$.

(b) $g(x) = \dfrac{\sin x}{1 + \cos x}$, when x is close (but not equal) to π radians.

(c) $h(x) = \dfrac{x}{x+1} - 1$, when $|x|$ is very large.

Question #3 - 8 Marks

(a) Determine the second order ($n = 2$) Taylor series expansion for $f(x) = \sqrt{x+1}$ expanded about $a = 3$ including the remainder term. Leave your answer in terms of factors $(x - 3)$ (that is, do not simplify). Show all your work.

(b) Use the polynomial approximation in (a) (without the remainder term) to approximate $f(3.08) = \sqrt{4.08}$. Use either hand computation, your calculator or MATLAB. Give an exact answer.

(c) To 9 correct significant digits, the exact value of $f(3.08) = \sqrt{4.08}$ is 2.01990099. Use this value to compute the absolute error of your computed approximation in (b).

(d) Determine a good upper bound for the truncation error of the Taylor polynomial approximation for $\sqrt{4.08}$ in (b) by bounding the remainder term in (a). Give an exact answer. Note: Here $x = 3.08$ only. The actual absolute error in (c) should be less than this upper bound.