

1. (2 points) Using propositional resolution, show the following propositional sentence is unsatisfiable.

$$(p \mid q \mid \neg r) \ \& \ ((\neg r \mid q \mid p) \rightarrow ((r \mid q) \ \& \ \neg q \ \& \ \neg p))$$

To do this, convert this sentence to clausal form and derive the empty clause using resolution.

Conversion to clausal form

I

$$(p \mid q \mid \neg r) \ \& \ ((\neg r \mid q \mid p) \rightarrow ((r \mid q) \ \& \ \neg q \ \& \ \neg p))$$

N

$$(p \mid q \mid \neg r) \ \& \ (\neg(\neg r \mid q \mid p) \mid ((r \mid q) \ \& \ \neg q \ \& \ \neg p))$$

D

$$(p \mid q \mid \neg r) \ \& \ ((r \ \& \ \neg q \ \& \ \neg p) \mid ((r \mid q) \ \& \ \neg q \ \& \ \neg p))$$

$$\begin{aligned} &(p \mid q \mid \neg r) \ \& \\ &((r \ \& \ \neg q \ \& \ \neg p) \mid (r \mid q)) \ \& \\ &((r \ \& \ \neg q \ \& \ \neg p) \mid \neg q) \ \& \\ &((r \ \& \ \neg q \ \& \ \neg p) \mid \neg p) \end{aligned}$$

$$\begin{aligned} &(p \mid q \mid \neg r) \ \& \\ &((r \mid r \mid q) \ \& \ (\neg q \mid r \mid q) \ \& \ (\neg p \mid r \mid q)) \ \& \ ((r \mid \neg q) \ \& \ (\neg q \mid \neg q) \ \& \ (\neg p \mid \neg q)) \ \& \ ((r \mid \neg p) \ \& \ (\neg q \mid \neg p) \\ &\ \& \ (\neg p \mid \neg p)) \end{aligned}$$

O

$\{p, q, \neg r\}$
 $\{r, q\}$
 $\{\neg q, r, q\}$
 $\{\neg p, r, q\}$
 $\{r, \neg q\}$
 $\{\neg q\}$
 $\{\neg p, \neg q\}$
 $\{r, \neg p\}$
 $\{\neg q, \neg p\}$
 $\{\neg p\}$

1. $\{p, q, \neg r\}$	Premise
2. $\{r, q\}$	Premise
3. $\{\neg p\}$	Premise
4. $\{\neg q\}$	Premise
5. $\{p, q\}$	1,2
6. $\{q\}$	3,5
7. $\{\}$	4,6

I have shown only those steps that quickly produce the empty clause.

2. (8 points) Every horse can outrun every dog. Some greyhounds can outrun every rabbit. Show that every horse can outrun every rabbit.

$$\forall x. \forall y. (\text{Horse}(x) \wedge \text{Dog}(y) \Rightarrow \text{Faster}(x, y))$$

$$\exists y. (\text{Greyhound}(y) \wedge \forall z. (\text{Rabbit}(z) \Rightarrow \text{Faster}(y, z)))$$

$$\forall y. (\text{Greyhound}(y) \Rightarrow \text{Dog}(y)) \quad (\text{background knowledge})$$

$$\forall x. \forall y. \forall z. (\text{Faster}(x, y) \wedge \text{Faster}(y, z) \Rightarrow \text{Faster}(x, z)) \quad (\text{background knowledge})$$

$$\neg \forall x. \forall y. (\text{Horse}(x) \wedge \text{Rabbit}(y) \Rightarrow \text{Faster}(x, y)) \quad \text{negated conclusion}$$

In clausal form they become:

$$\{\neg \text{Horse}(x), \neg \text{Dog}(y), \text{Faster}(x, y)\}$$

$$\{\text{Greyhound}(\text{Greg})\}$$

$$\{\neg \text{Rabbit}(z), \text{Faster}(\text{Greg}, z)\}$$

$$\{\neg \text{Greyhound}(y), \text{Dog}(y)\}$$

$$\{\neg \text{Faster}(x, y), \neg \text{Faster}(y, z), \text{Faster}(x, z)\}$$

Let's transform the goal into clausal form:

$$\neg \forall x. \forall y. (\text{Horse}(x) \wedge \text{Rabbit}(y) \Rightarrow \text{Faster}(x, y))$$

I $\neg \forall x. \forall y. (\neg(\text{Horse}(x) \wedge \text{Rabbit}(y)) \vee \text{Faster}(x, y))$

N $\neg \forall x. \forall y. (\neg \text{Horse}(x) \vee \neg \text{Rabbit}(y) \vee \text{Faster}(x, y))$

$$\exists x. \exists y. (\text{Horse}(x) \wedge \text{Rabbit}(y) \wedge \neg \text{Faster}(x, y))$$

S $\exists x. \exists y. (\text{Horse}(x) \wedge \text{Rabbit}(y) \wedge \neg \text{Faster}(x, y))$

E $\text{Horse}(\text{Jack}) \wedge \text{Rabbit}(\text{Smith}) \wedge \neg \text{Faster}(\text{Jack}, \text{Smith})$

A $\text{Horse}(\text{Jack}) \wedge \text{Rabbit}(\text{Smith}) \wedge \neg \text{Faster}(\text{Jack}, \text{Smith})$

D $\text{Horse}(\text{Jack}) \wedge \text{Rabbit}(\text{Smith}) \wedge \neg \text{Faster}(\text{Jack}, \text{Smith})$

O $\{\text{Horse}(\text{Jack})\}$

$$\{\text{Rabbit}(\text{Smith})\}$$

$$\{\neg \text{Faster}(\text{Jack}, \text{Smith})\}$$

Let's try to infer the {} using resolution:

- | | | |
|-----|---|-------|
| 1. | $\{\neg\text{Horse}(x), \neg\text{Dog}(y), \text{Faster}(x, y)\}$ | |
| 2. | $\{\text{Greyhound}(\text{Greg})\}$ | |
| 3. | $\{\neg\text{Rabbit}(z), \text{Faster}(\text{Greg}, z)\}$ | |
| 4. | $\{\neg\text{Greyhound}(y), \text{Dog}(y)\}$ | |
| 5. | $\{\neg\text{Faster}(x, y), \neg\text{Faster}(y, z), \text{Faster}(x, z)\}$ | |
| 6. | $\{\text{Horse}(\text{Jack})\}$ | |
| 7. | $\{\text{Rabbit}(\text{Smith})\}$ | |
| 8. | $\{\neg\text{Faster}(\text{Jack}, \text{Smith})\}$ | |
| 9. | $\{\neg\text{Dog}(y), \text{Faster}(\text{Jack}, y)\}$ | 1,6 |
| 10. | $\{\text{Faster}(\text{Jack}, y), \neg\text{Greyhound}(y)\}$ | 4,9 |
| 11. | $\{\text{Faster}(\text{Jack}, \text{Greg})\}$ | 2,10 |
| 12. | $\{\text{Faster}(\text{Greg}, \text{Smith})\}$ | 3,7 |
| 13. | $\{\neg\text{Faster}(\text{Greg}, z), \text{Faster}(x, z)\}$ | 5,11 |
| 14. | $\{\text{Faster}(x, \text{Smith})\}$ | 12,13 |
| 15. | $\{\}$ | 8,14 |

3. (2 points)

All hummingbirds are richly colored.

No large birds live on honey.

Birds that do not live on honey are dull in color.

Conclusion: All hummingbirds are small.

$\text{all } x (\text{hummingbird}(x) \rightarrow \text{richly_colored}(x)).$

$\neg(\text{exists } x (\text{bird}(x) \ \& \ \text{large}(x) \ \& \ \text{lives_on_honey}(x))).$

$\text{all } x (\text{bird}(x) \ \& \ \neg\text{lives_on_honey}(x) \rightarrow \neg\text{richly_colored}(x)).$

%background knowledge

$\text{all } x (\text{hummingbird}(x) \rightarrow \text{bird}(x)).$

Theorem

$\text{all } x (\text{hummingbird}(x) \rightarrow \neg\text{large}(x)).$

4. (2 points)

My gardener is well worth listening to on military subjects;

No one can remember the battle of Waterloo, unless he is very old;

Nobody is really worth listening to on military subjects, unless he can remember the battle of Waterloo.

Conclusion My gardener is very old.

all x ($\text{gardener}(\text{Me}, x) \rightarrow \text{worthListening}(x)$).

all x ($\text{remember}(x, \text{Waterloo}) \rightarrow \text{old}(x)$).

$\neg(\exists x (\text{worthListening}(x) \ \& \ \neg\text{remember}(x, \text{Waterloo})))$.

Theorem

all x ($\text{gardener}(\text{Me}, x) \rightarrow \text{old}(x)$).

Wumpus world

- Each of the three reachable squares, [1,3], [2,2], and [3,1], might contain a pit.
- Find

$$P(p_{13}|\text{evidence})$$

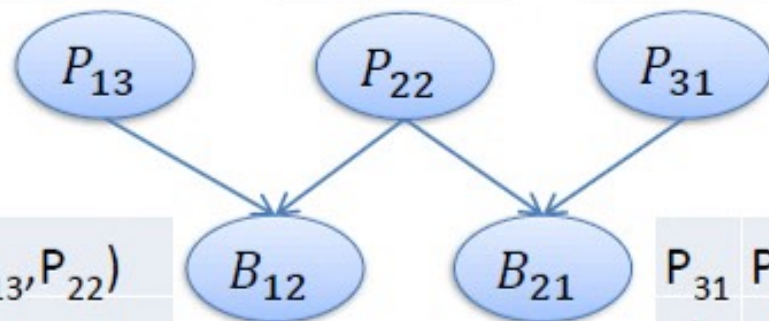
$$P(p_{22}|\text{evidence})$$

$$P(p_{31}|\text{evidence})$$

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 B OK	2,2	3,2	4,2
1,1 OK	2,1 B OK	3,1	4,1

Bayesian Net for the Wumpus World

P_{13}	$P(P_{13})$	P_{22}	$P(P_{22})$	P_{31}	$P(P_{31})$
t	0.01	t	0.01	t	0.01



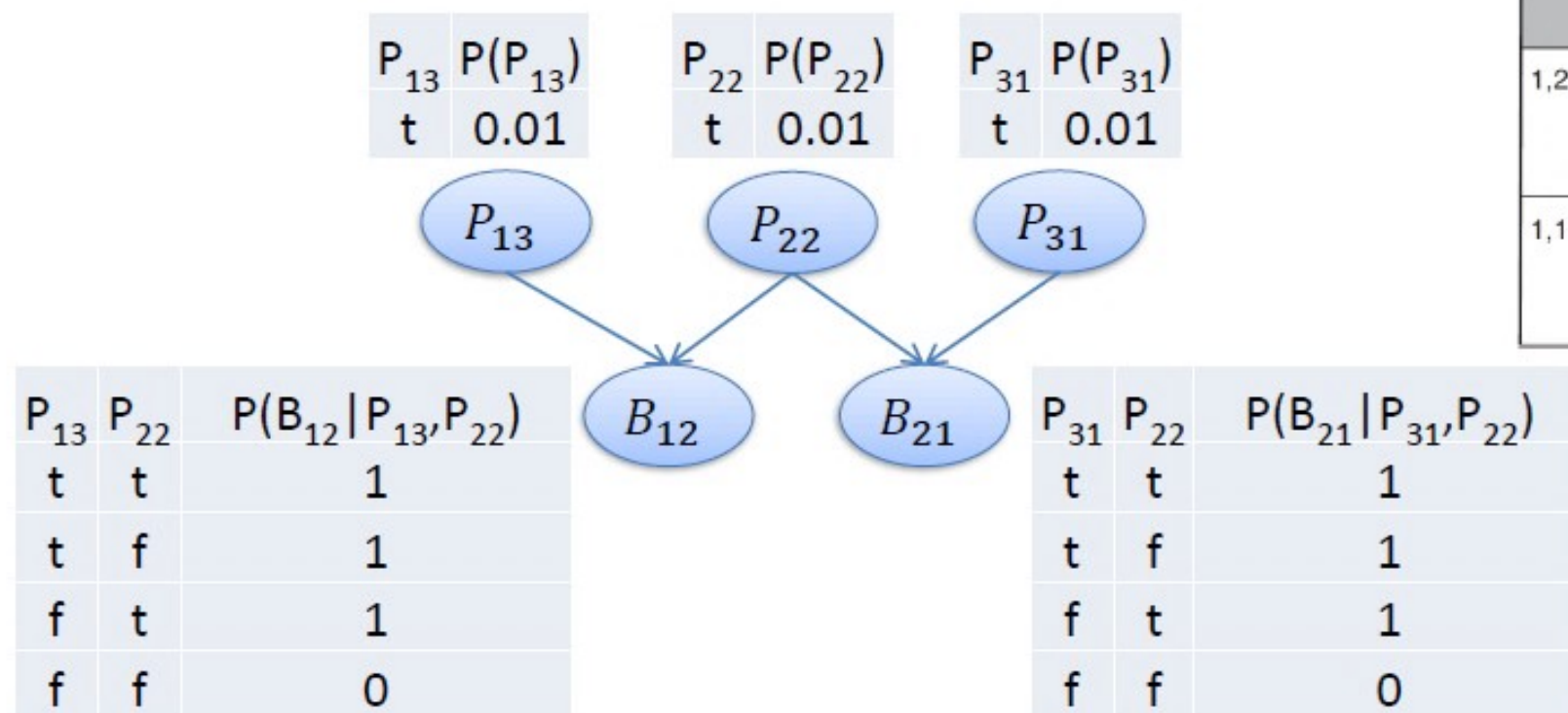
P_{13}	P_{22}	$P(B_{12} P_{13}, P_{22})$
t	t	1
t	f	1
f	t	1
f	f	0

P_{31}	P_{22}	$P(B_{21} P_{31}, P_{22})$
t	t	1
t	f	1
f	t	1
f	f	0

1,3	2,3	3,3	4,3
1,2 B OK	2,2	3,2	4,2
1,1 OK	2,1 B OK	3,1	4,1

$$\begin{aligned}
 P(p_{13} | b_{12}, b_{21}) &= \alpha \sum_{p_{22}} \sum_{p_{31}} P(b_{12} | p_{13}, p_{22}) \cdot P(b_{21} | p_{31}, p_{22}) \cdot P(p_{13}) \cdot P(p_{22}) \cdot P(p_{31}) = \\
 &\alpha [P(b_{12} | p_{13}, p_{22}) \cdot P(b_{21} | p_{31}, p_{22}) \cdot P(p_{13}) \cdot P(p_{22}) \cdot P(p_{31}) + \\
 &\quad P(b_{12} | p_{13}, \neg p_{22}) \cdot P(b_{21} | p_{31}, \neg p_{22}) \cdot P(p_{13}) \cdot P(\neg p_{22}) \cdot P(p_{31}) + \\
 &\quad P(b_{12} | p_{13}, p_{22}) \cdot P(b_{21} | \neg p_{31}, p_{22}) \cdot P(p_{13}) \cdot P(p_{22}) \cdot P(\neg p_{31}) + \\
 &\quad P(b_{12} | p_{13}, \neg p_{22}) \cdot P(b_{21} | \neg p_{31}, \neg p_{22}) \cdot P(p_{13}) \cdot P(\neg p_{22}) \cdot P(\neg p_{31})] \\
 &= \alpha (1 \cdot 1 \cdot 0.01 \cdot 0.01 \cdot 0.01 + 1 \cdot 1 \cdot 0.01 \cdot 0.99 \cdot 0.01 + 1 \cdot 1 \cdot 0.01 \cdot 0.01 \cdot 0.99 + 0) = \alpha \cdot 0.000199
 \end{aligned}$$

Bayesian Net for the Wumpus World



1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 B OK	2,2	3,2	4,2
1,1 OK	2,1 B OK	3,1	4,1

$$P(\neg p_{13} | b_{12}, b_{21}) = \alpha \sum_{p_{22}} \sum_{p_{31}} P(b_{12} | \neg p_{13}, p_{22}) \cdot P(b_{21} | p_{31}, p_{22}) \cdot P(\neg p_{13}) \cdot P(p_{22}) \cdot P(p_{31}) =$$

$$\alpha [P(b_{12} | \neg p_{13}, p_{22}) \cdot P(b_{21} | p_{31}, p_{22}) \cdot P(\neg p_{13}) \cdot P(p_{22}) \cdot P(p_{31}) +$$

$$P(b_{12} | \neg p_{13}, \neg p_{22}) \cdot P(b_{21} | p_{31}, \neg p_{22}) \cdot P(\neg p_{13}) \cdot P(\neg p_{22}) \cdot P(p_{31}) +$$

$$P(b_{12} | \neg p_{13}, p_{22}) \cdot P(b_{21} | \neg p_{31}, p_{22}) \cdot P(\neg p_{13}) \cdot P(p_{22}) \cdot P(\neg p_{31}) +$$

$$P(b_{12} | \neg p_{13}, \neg p_{22}) \cdot P(b_{21} | \neg p_{31}, \neg p_{22}) \cdot P(\neg p_{13}) \cdot P(\neg p_{22}) \cdot P(\neg p_{31})]$$

Bayesian Net for the Wumpus World

$$P(p_{13}|b_{12}, b_{21}) = \alpha * .000199$$

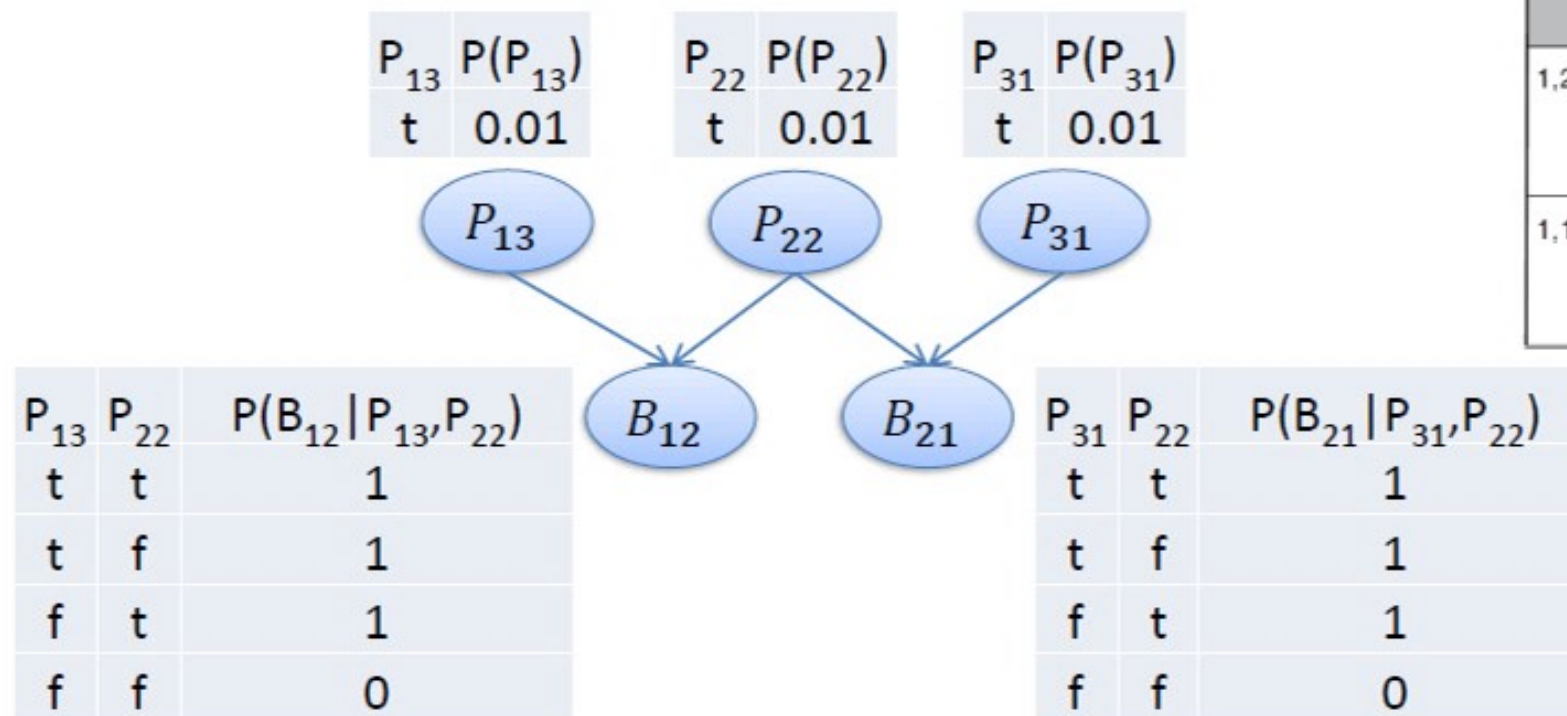
$$P(\neg p_{13}|b_{12}, b_{21}) = \alpha * .0099$$

$$\alpha = \frac{1}{.000199 + .0099}$$

$$P(p_{13}|b_{12}, b_{21}) = \frac{.000199}{.000199 + .0099} = 1.97\%$$

$$P(\neg p_{13}|b_{12}, b_{21}) = \frac{.0099}{.000199 + .0099} = 98.03\%$$

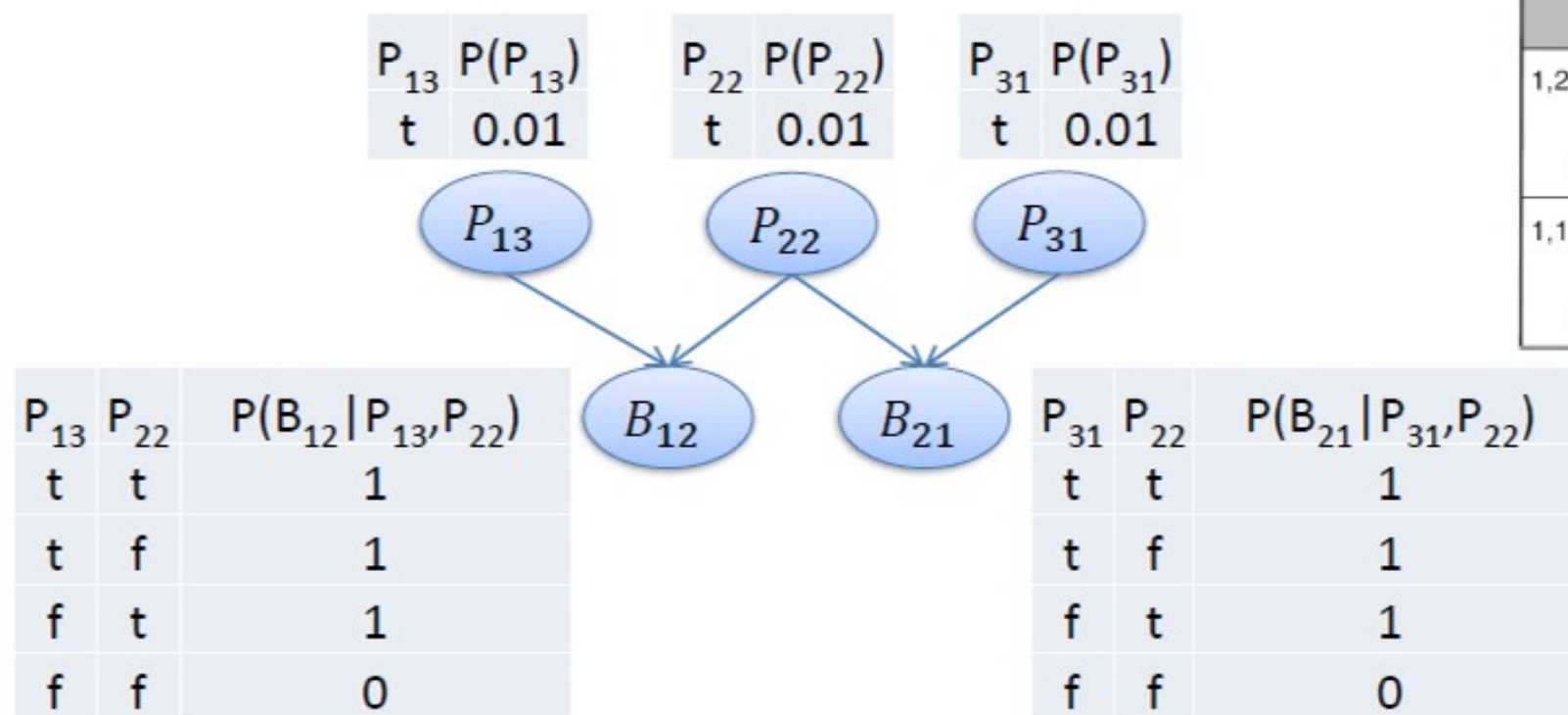
Bayesian Net for the Wumpus World



1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 B OK	2,2	3,2	4,2
1,1 OK	2,1 B OK	3,1	4,1

$$\begin{aligned}
 P(p_{22} | b_{12}, b_{21}) &= \alpha \sum_{p_{13}} \sum_{p_{31}} P(b_{12} | p_{13}, p_{22}) \cdot P(b_{21} | p_{31}, p_{22}) \cdot P(p_{13}) \cdot P(p_{22}) \cdot P(p_{31}) = \\
 &\alpha [P(b_{12} | p_{13}, p_{22}) \cdot P(b_{21} | p_{31}, p_{22}) \cdot P(p_{13}) \cdot P(p_{22}) \cdot P(p_{31}) + \\
 &\quad P(b_{12} | \neg p_{13}, p_{22}) \cdot P(b_{21} | p_{31}, p_{22}) \cdot P(\neg p_{13}) \cdot P(p_{22}) \cdot P(p_{31}) + \\
 &\quad P(b_{12} | p_{13}, p_{22}) \cdot P(b_{21} | \neg p_{31}, p_{22}) \cdot P(p_{13}) \cdot P(p_{22}) \cdot P(\neg p_{31}) + \\
 &\quad P(b_{12} | \neg p_{13}, p_{22}) \cdot P(b_{21} | \neg p_{31}, p_{22}) \cdot P(\neg p_{13}) \cdot P(p_{22}) \cdot P(\neg p_{31})] \\
 &= \alpha (1 \cdot 1 \cdot 0.01 \cdot 0.01 \cdot 0.01 + 1 \cdot 1 \cdot 0.99 \cdot 0.01 \cdot 0.01 + 1 \cdot 1 \cdot 0.01 \cdot 0.01 \cdot 0.99 + 1 \cdot 1 \cdot 0.99 \cdot 0.01 \cdot 0.99) = \alpha \cdot 0.01
 \end{aligned}$$

Bayesian Net for the Wumpus World



1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 B OK	2,2	3,2	4,2
1,1 OK	2,1 B OK	3,1	4,1

$$\begin{aligned}
 P(\neg p_{22} | b_{12}, b_{21}) &= \alpha \sum_{p_{13}} \sum_{p_{31}} P(b_{12} | p_{13}, p_{22}) \cdot P(b_{21} | p_{31}, p_{22}) \cdot P(p_{13}) \cdot P(p_{22}) \cdot P(p_{31}) = \\
 &\alpha [P(b_{12} | p_{13}, \neg p_{22}) \cdot P(b_{21} | p_{31}, \neg p_{22}) \cdot P(p_{13}) \cdot P(\neg p_{22}) \cdot P(p_{31}) + \\
 &\quad P(b_{12} | \neg p_{13}, \neg p_{22}) \cdot P(b_{21} | p_{31}, \neg p_{22}) \cdot P(\neg p_{13}) \cdot P(\neg p_{22}) \cdot P(p_{31}) + \\
 &\quad P(b_{12} | p_{13}, \neg p_{22}) \cdot P(b_{21} | \neg p_{31}, \neg p_{22}) \cdot P(p_{13}) \cdot P(\neg p_{22}) \cdot P(\neg p_{31}) + \\
 &\quad P(b_{12} | \neg p_{13}, \neg p_{22}) \cdot P(b_{21} | \neg p_{31}, \neg p_{22}) \cdot P(\neg p_{13}) \cdot P(\neg p_{22}) \cdot P(\neg p_{31})] \\
 &= \alpha (1 \cdot 1 \cdot 0.01 \cdot 0.99 \cdot 0.01 + 0 + 0 + 0) = \alpha \cdot 0.000099
 \end{aligned}$$

Bayesian Net for the Wumpus World

$$P(p_{22}|b_{12}, b_{21}) = \alpha * .01$$

$$P(\neg p_{22}|b_{12}, b_{21}) = \alpha * .000099$$

$$\alpha = \frac{1}{.01 + .000099}$$

$$P(p_{22}|b_{12}, b_{21}) = \frac{.01}{.01 + .000099} = 99\%$$

$$P(\neg p_{22}|b_{12}, b_{21}) = \frac{.000099}{.01 + .000099} = 1\%$$

Conclusion

- Going to $[2,2]$ is almost certain death.
- So, a probabilistic agent will never choose to go to $[2,2]$.
- On the other hand, to a logical agent, squares $[1,3]$, $[2,2]$, $[3,1]$ look the same. So, the logical agent would choose either one with equal chance ($1/3$). By doing that, the agent will die with a chance of about $1/3$.

6 . (9 pts) [Adapted from a CMU machine learning assignment]

As part of a comprehensive study of the role of CMU 10-601 (Machine Learning) on people's happiness CMU has been collecting data from graduating students. In an optional survey that all students are required to complete, the following questions were asked:

- Do you party frequently [Party: Yes/No]?
- Are you wicked smart [Smart: Yes/No]?
- Are you creative [Creative: Yes/No]? (Please only answer Yes or No)
- Did you do well on all your homework assignments? [HW: Yes/No]
- Do you use a Mac? [Mac: Yes/No]
- Did your 10-601 project succeed? [Project: Yes/No]
- Did you succeed in your most important class (which is 10-601)? [Success: Yes/No]
- Are you currently Happy? [Happy: Yes/No]

You can obtain the comma-separated survey results from

<http://www.cs.cmu.edu/~ggordon/10601/hws/hw2/students.csv.zip>

Each row in students.csv corresponds to the responses of a separate student.

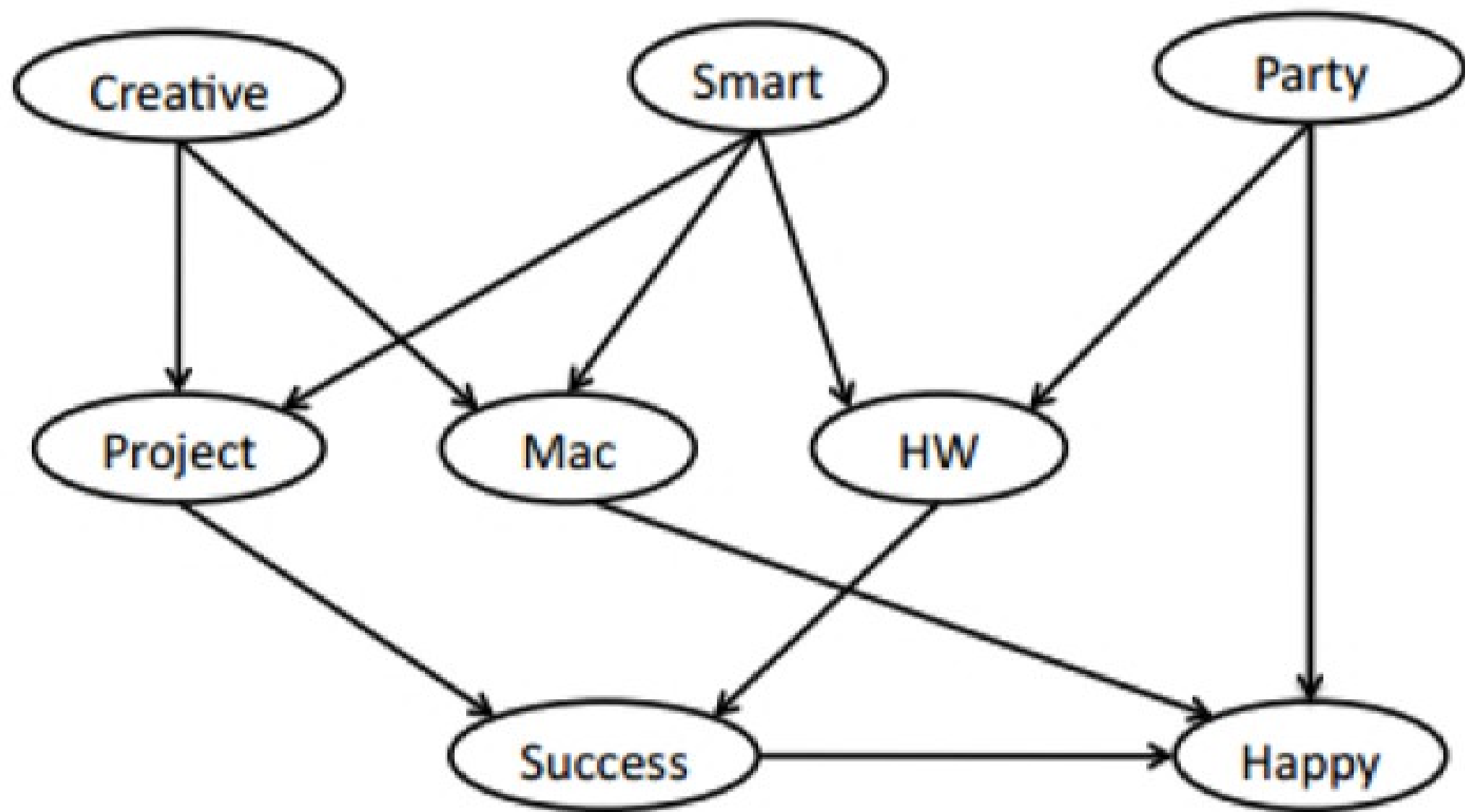
The columns in students.csv correspond to each question (random variable) in the order Party, Smart, Creative, HW, Mac, Project, Success, and Happy.

The entries are either zero, corresponding to No response, or one, corresponding to a Yes response.

After consulting a behavioral psychologist they obtained the following complete set of conditional relationships:

- HW depends only on Party and Smart
- Mac depends only on Smart and Creative
- Project depends only on Smart and Creative
- Success depends only on HW and Project
- Happy depends only on Party, Mac, and Success

1. (1 pt) Draw the Bayesian network.



2. (2 pt) Estimate the probabilities of the conditional probability tables using the data provided (you can use Excel pivot tables for counting).

$$\mathbf{P}(\text{creative} = T) = 0.69932$$

$$\mathbf{P}(\text{smart} = T) = 0.70472$$

$$\mathbf{P}(\text{party} = T) = 0.60216$$

creative	smart	$\mathbf{P}(\text{project} = T \mid \text{creative}, \text{smart})$
T	T	0.90484
T	F	0.40307
F	T	0.79326
F	F	0.10731

creative	smart	$\mathbf{P}(\text{mac} = T \mid \text{creative}, \text{smart})$
T	T	0.68564
T	F	0.89635
F	T	0.41347
F	F	0.12329

smart	party	$\mathbf{P}(\text{hw} = T \mid \text{smart}, \text{party})$
T	T	0.80252
T	F	0.89790
F	T	0.09447
F	F	0.30556

project	hw	$\mathbf{P}(\text{success} = T \mid \text{project}, \text{hw})$
T	T	0.89633
T	F	0.20737
F	T	0.30714
F	F	0.05066

success	mac	party	$\mathbf{P}(\text{happy} = T \mid \text{success}, \text{mac}, \text{party})$
T	T	T	0.95842
T	T	F	0.35837
T	F	T	0.72082
T	F	F	0.30769
F	T	T	0.49234
F	T	F	0.20619
F	F	T	0.42043
F	F	F	0.09646

3. (2 pts) What is the probability of being happy given that you party often, are wicked smart, but not very creative?
Show details of computation.

$$P(\text{happy} = T \mid \text{party} = T, \text{smart} = T, \text{creative} = F) = 0.6922$$

$$\begin{aligned} &P(h \mid p, s, c) \\ &= \alpha * \sum_{hw, m, pr, su} (P(c)P(s)P(p)P(pr|c,s)P(m|c,s)P(hw|s,p)P(su|pr,hw)P(h|p,m,su)) \\ &= \alpha * () \\ &P(-h \mid p, s, c) \\ &= \alpha * \sum_{hw, m, pr, su} (P(c)P(s)P(p)P(pr|c,s)P(m|c,s)P(hw|s,p)P(su|pr,hw)P(-h|p,m,su)) \\ &= \alpha * () \end{aligned}$$

$$\alpha = 1 / (P(h \mid p, s, c) + P(-h \mid p, s, c))$$

4. (2 pts) What is the probability of being happy given that you are wicked smart and very creative?
No details required. Use the AIspace tool.

$$P(\text{happy} = T \mid \text{smart} = T, \text{creative} = T) = 0.58132$$

5. (0.5 pts) What is the probability of being happy given you do not party, and do well on all your homework and class project?
No details required. Use the AIspace tool.

$$P(\text{happy} = T \mid \text{party} = F, \text{hw} = T, \text{project} = T) = 0.32108$$

6. (0.5 pts) What is the probability of being happy given you own a mac?
No details required. Use the AIspace tool.

$$P(\text{happy} = T \mid \text{mac} = T) = 0.56269$$

7. (0.5 pts) What is the probability that you party often given you are wicked smart?
No details required. Use the AIspace tool.

$$P(\text{party} = T \mid \text{smart} = T) = 0.60216$$

8. (0.5 pts) What is the probability that you party often given you are wicked smart and happy? No details required. Use the AIspace tool.

$$P(\text{party} = T \mid \text{smart} = T, \text{happy} = T) = 0.79204$$