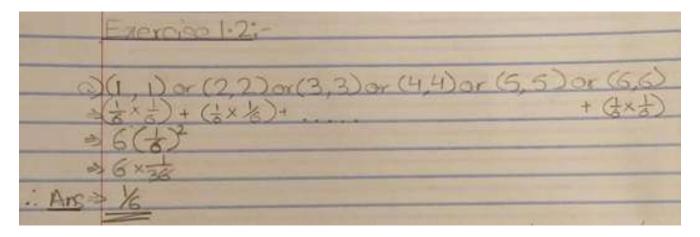
CSC 423 - A1

Devroop Banerjee V00837868

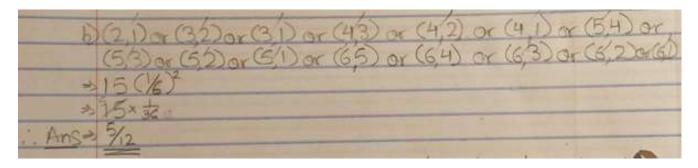
1. Exercise 1.2

We roll two standard six-sided dice. Find the probability of the following events, assuming that the outcomes of the rolls are independent.

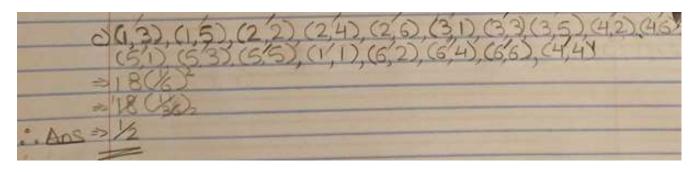
(a) The two dice show the same number.



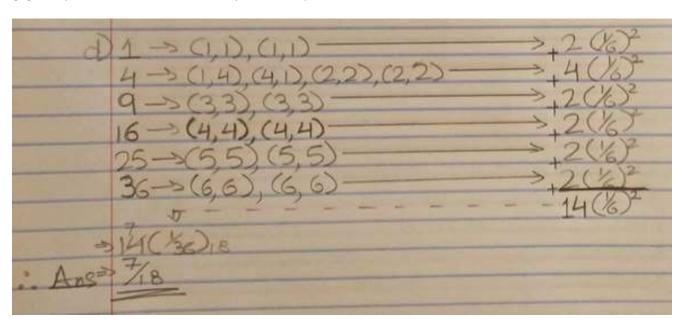
(b) The number that appears on the first die is larger than the number on the second.



(c) The sum of the dice is even.



(d) The product of the dice is a perfect square.



2. Exercise 1.8

I choose a number uniformly at random from the range [1, 1,000,000]. Using the inclusion–exclusion principle, determine the probability that the number chosen is divisible by one or more of 4, 6, and 9.

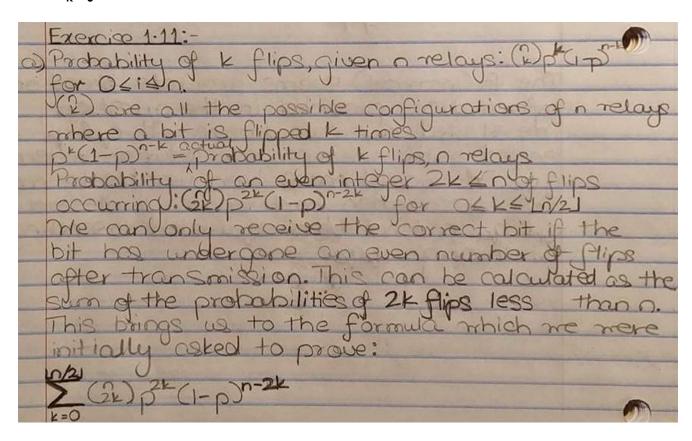
Exercise 1.8:-
Let 2 be a uniform number in the range I toman
lot tip the event more a sound
Satal divisors Y:-
Set of divisors Y:- Pr[[0,E] _ Lipoppoo/LCMGD] (LCM= Lordent
1000000 common multiple
Pr(EnUE, UEq) - Pr(En) + Pr(En) + Pr(Eq) + Pr(En) + Pr(En
+ Pr (En Ea (Ea) = (25,0000 + 166,666 + 111,111) - (83333+27,177+55,655)
100000
127777
= 527,777-166,665+27,777
1000000
= 3GI,TT2
vocato
:. Ans = Pr(E, UE, UE, 139 or 0361112
1,25,000

3. Exercise 1.11

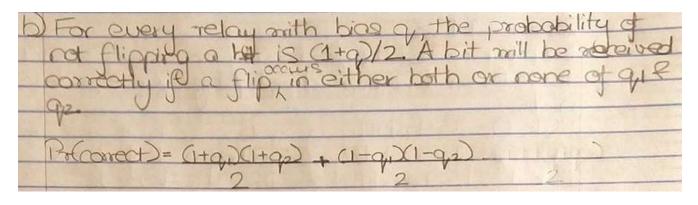
I am trying to send you a single bit, either a 0 or a 1. When I transmit the bit, it goes through a series of n relays before it arrives to you. Each relay flips the bit independently with probability p.

(a) Argue that the probability you receive the correct bit is

$$\sum_{k=0}^{(n/2)} {n \choose 2k} * (p^{2k}) * (1 - p)^{n-2k}$$



(b) We consider an alternative way to calculate this probability. Let us say the relay has bias q if the probability it flips the bit is (1 - q)/2. The bias q is therefore a real number in the range [-1, 1]. Prove that sending a bit through two relays with bias q1 and q2 is equivalent to sending a bit through a single relay with bias q1q2.



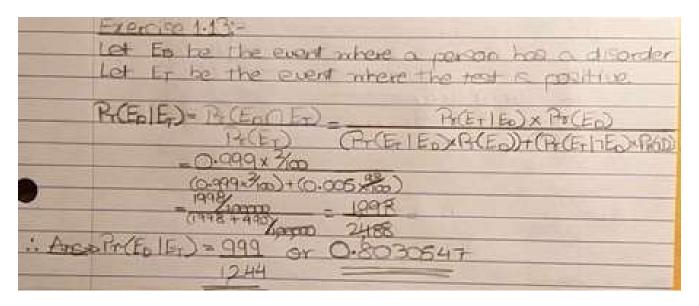
(c) Prove that the probability you receive the correct bit when it passes through n relays as described before (a) is

$$\frac{1 + (1 - 2p)^n}{2}$$

	the bit ariving ingreatly till the oth relay
	The one control of the control of th
	the bit arriving incorrectly till the ath relay
	. The probability of receiving the bit correctly:
	1+(1-20) 1-(1-20)
	2 2 2
>	1+(1-205(1-0),1-(1-2050
3	(1-2p)(1-p)-p)+1
B. Barrier	2'
3	(1-2)(1-2)+1
	2
· Ans >	$(1-2p)^{n+1}+1$
-	2 1

4. Exercise 1.13

A medical company touts its new test for a certain genetic disorder. The false negative rate is small: if you have the disorder, the probability that the test returns a positive result is 0.999. The false positive rate is also small: if you do not have the disorder, the probability that the test returns a positive result is only 0.005. Assume that 2% of the population has the disorder. If a person chosen uniformly from the population is tested and the result comes back positive, what is the probability that the person has the disorder?



5. Exercise 1.15

Suppose that we roll ten standard six-sided dice. What is the probability that their sum will be divisible by 6, assuming that the rolls are independent? (Hint: Use the principle of deferred decisions, and consider the situation after rolling all but one of the dice.)

6. Exercise 1.18

We have a function $F: \{0, ..., n-1\} \rightarrow \{0, ..., m-1\}$. We know that, for $0 \le x, y \le n-1$, $F((x + y) \mod n) = (F(x) + F(y)) \mod m$. The only way we have for evaluating F is to use a lookup table that stores the values of F. Unfortunately, an Evil Adversary has changed the value of 1/5 of the table entries when we were not looking.

Describe a simple randomized algorithm that, given an input z, outputs a value that equals F(z) with probability at least 1/2. Your algorithm should work for every value of z, regardless of what values the Adversary changed. Your algorithm should use as few lookups and as little computation as possible.

Suppose I allow you to repeat your initial algorithm three times. What should you do in this case, and what is the probability that your enhanced algorithm returns the correct answer?

Exercise 1.18:-
Chare & (uniformly) from (0,, n-1)
Let 4= Z-2 mod a.
F(2) = F(2) + F(y) mod m . 7 ? ? O Uniform w between
(= 7-2) 02 n-1 1
2 Decembert
Pr(F(x) is incorrect) = Pr(F(x) is incorrect) = 1/5
is an house'
Processon = Profa is incorrect UFCW is incorrect) 4 3/5
ie: - Po (correct) = 1 - Po (empr)
= 1-2/5
: Area Proceed = 35 (>12)
10 11 1 11 in an in the in
If the algorithm is repeated thrice then: Brick the made, if a result is repeated, or
Drick the mode, if a result is reported, or
@Pick the first result, if no repetition.
Then Priceron needs attenst 2 mins to produce an
immed result. The probability of the apprithm
running once on some input i, is Pi Pizza Cfrom
our collowation above? Building on this, the protob
-ility of the algorithm giving right are at least 2 out of
the 3 times it's run!
(2)Pr2(1-Pr) + (3)Pr3
(3) D.2(1 B) + (3) D.3/ (3) (3/2)(2/6) + (3) (2/3)
(3)P;2(1-P) + (3)P;34(3)(3/5)2(2/5) + (3)(3/5)3 = (3)P;34(5)(3/5)(3/5)(3/5)(3/5)(3/5)(3/5)(3/5)(3
542-124/
281 0 0 0
≥ 68/25 or 0.648
The probability of getting the correct result
upon running the algorithm thrice is 8/25 or 0.648