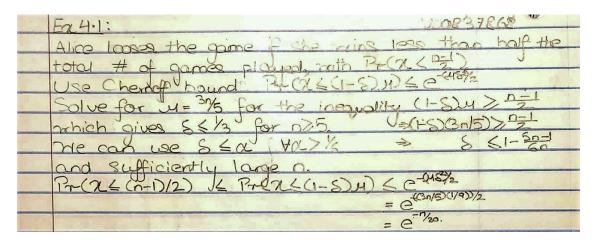
CSC 423 - A3

Devroop Banerjee V00837868

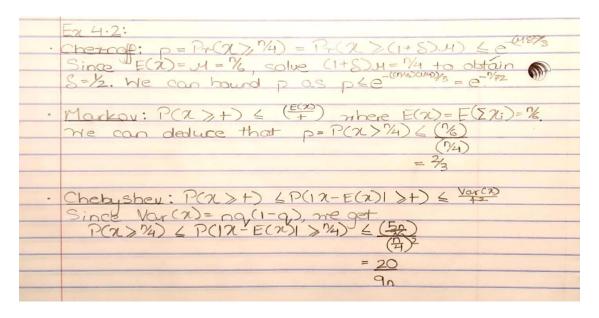
1. **Exercise 4.1**

Alice and Bob play checkers often. Alice is a better player, so the probability that she wins any given game is 0.6, independent of all other games. They decide to play a tournament of n games. Bound the probability that Alice loses the tournament using a Chernoff bound.



2. Exercise 4.2

We have a standard six-sided die. Let X be the number of times that a 6 occurs over n throws of the die. Let p be the probability of the event $X \ge n/4$. Compare the best upper bounds on p that you can obtain using Markov's inequality, Chebyshev's inequality, and Chernoff bounds.



3. Exercise 4.10

A casino is testing a new class of simple slot machines. Each game, the player puts in \$1, and the slot machine is supposed to return either \$3 to the player with probability 4/25, \$100 with probability 1/200, or nothing with all remaining probability. Each game is supposed to be independent of other games.

The casino has been surprised to find in testing that the machines have lost \$10,000 over the first million games. Derive a Chernoff bound for the probability of this event. You may want to use a calculator or program to help you choose appropriate values as you derive your bound.

423	Ex 4.10:
	Let my 2 denote net loss to the online over 1st
	million garnes. 1 = 11+ Notes The
	Let 71; for i= 1,2,, 10° he the maines net loss in the
A ST	ith coars
Pecc	ni= 32 mp. 1/25 (et m. p. 1/25)
-13	99 mp. 200 => et = 3 e m. p. 200
	$\chi_{i} = \frac{5}{2} \frac{\pi \cdot \rho}{100} = \frac{1}{25}$ $(2^{4} \times 10^{4})^{25}$ $(2^{4} \times $
	-(+2) = 1/25 C + 200 C + 200 C
	F(e') = [-(e') - (+7.4)
	$= \underbrace{E(e^{+\chi_{i}}) \cdot E(e^{+\chi_{i}}) \cdot \cdot E(e^{+\chi_{i}})}_{= \underbrace{(25e^{2t} + \frac{167}{200}e^{+})^{106}}$ (% S are independent)
	= (25 + 200 + 200)
5-1	Pr(x>104) - Pr(0+2>0104+)
mini	$P_{\tau}(\chi) O^{4} \rangle = P_{\tau}(e^{+\chi}) e^{10^{4}+}$ $\leq E(e^{+\chi}) \text{(by Markov's)}$ $e^{10^{4}+}$
- 100	EIOTH (S)
	$= \left(\frac{4}{25}e^{2t} + \frac{1}{200}e^{99t} + \frac{167}{200}e^{+1}\right)^{106} - 10^{41}$
CIRS	This bound holds for any + 70, so we can
	choose the best value fort lets pick a smilling to
	like +=0.0006, which gives us a bound of 0.0000, - suggesting that the rasing has faulty machines.
	suggesting that the mains has faulty machines.
-	

4. Exercise 4.12

Consider a collection $X1, \ldots, Xn$ of n independent geometrically distributed random variables with mean 2. Let X = ni = 1 Xi and $\delta > 0$.

(a) Derive a bound on $\Pr(X \ge (1 + \delta)(2n))$ by applying the Chernoff bound to a sequence of $(1 + \delta)(2n)$ fair coin tosses.

Ex 4.12:

Oli: Sequence of coin flips till 1st head $\Sigma \lambda i$: λi :

(b) Directly derive a Chernoff bound on $Pr(X \ge (1 + \delta)(2n))$ using the moment generating function for geometric random variables.

 $be^{\frac{1}{2}} = \frac{1}{2}e^{\frac{1}{4}} + \frac{1}{4}e^{2\frac{1}{4}} + \frac{1}{4$

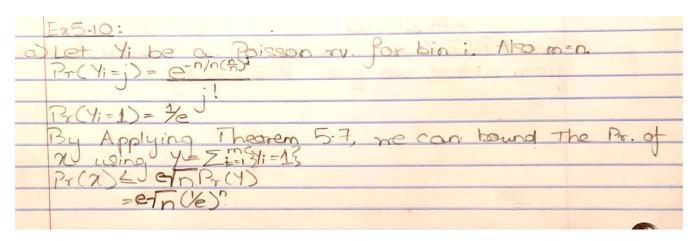
(c) Which bound is better?

c) when S is small, st e is abse to 0, the bound in (b) is tighter than the one in (a).

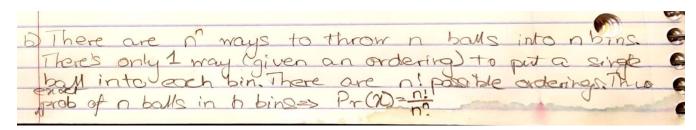
5. Exercise 5.10

Consider the probability that every bin receives exactly one ball when n balls are thrown randomly into n bins.

(a) Give an upper bound on this probability using the Poisson approximation.



(b) Determine the exact probability of this event.



(c) Show that these two probabilities differ by a multiplicative factor that equals the probability that a Poisson random variable with parameter n takes on the value n. Explain why this is implied by Theorem 5.6

6. Exercise 5.14

We prove that if Z is a Poisson random variable of mean μ , where $\mu \ge 1$ is an integer, then $\Pr(Z \ge \mu) \ge 1/2$.

(a) Show that $Pr(Z = \mu + h) \ge Pr(Z = \mu - h - 1)$ for $0 \le h \le \mu - 1$.

(b) Using part (a), argue that $Pr(Z \ge \mu) \ge 1/2$.

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b) Using part (a) me can argue that P(Z/u) > \frac{1}{2}.

P(Z/u) = \sum_{n=u}^{\infty} P(Z=h).

= \sum_{n=0}^{\infty} P(Z=u+h)

\Rightarrow \sum_{n=0}^{\omega-1} P(Z=u+h)

\Rightarrow \sum_{n=0}^{\omega-1} P(Z=u+h) (by part (a))

= P(Z/u)

Since P(Z/u) + P(Z/u) = 1 + P(Z/u) > P(Z/u)

from above, me get P(Z/u) > \frac{1}{2}.
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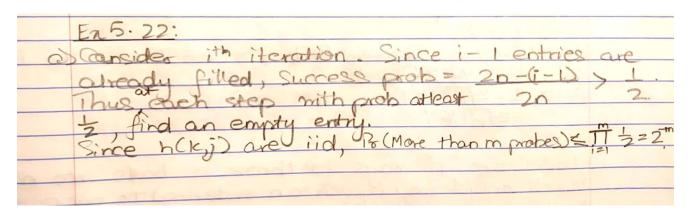
7. Exercise 5.22

In hashing with open addressing, the hash table is implemented as an array and there are no linked lists or chaining. Each entry in the array either contains one hashed item or is empty. The hash function defines, for each key k, a probe sequence h(k, 0), h(k, 1), . . . of table locations. To insert the key k, we first examine the sequence of table locations in the order defined by the key's probe sequence until we find an empty location; then we insert the item at that position. When searching for an item in the hash table, we examine the sequence of table locations in the order defined by the key's probe sequence until either the item is found or we have found an empty location in the sequence.

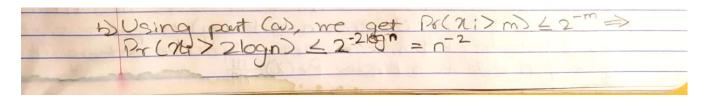
If an empty location is found, this means the item is not present in the table.

An open-address hash table with 2n entries is used to store n items. Assume that the table location h(k, j) is uniform over the 2n possible table locations and that all h(k, j) are independent.

(a) Show that, under these conditions, the probability of an insertion requiring more than k probes is at most 2-k.



(b) Show that, for i = 1, 2, ..., n, the probability that the *i*th insertion requires more than $2 \log n$ probes is at most 1/n2.



Let the random variable Xi denote the number of probes required by the ith insertion. You have shown in part (b) that $Pr(Xi > 2 \log n) \le 1/n2$. Let the random variable $X = \max 1 \le i \le n$ Xi denote the maximum number of probes required by any of the n insertions.

- (c) Show that $Pr(X > 2 \log n) \le 1/n$.
- (d) Show that the expected length of the longest probe sequence is $\mathbf{E}[X] = O(\log n)$.